QFT Thermodynamics from Entanglement Entropy

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Based on [2603.07635, 2607.06286]

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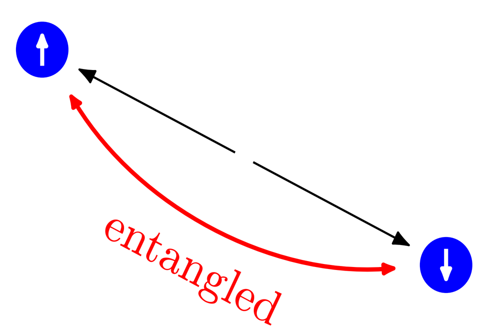
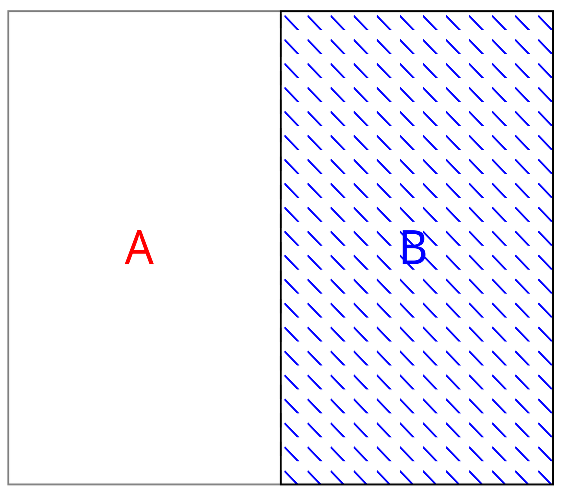
Entanglement

- A fundamental propery of quantum systems
- Entanglement measures give precise way to quantify quantum correlations
- For QFTs applications in
    - Critical Phenomena
    - RG flow
    - Confinement
    - Thermodynamics



Entanglement Entropy



- An entanglement measure for a 2-part system
- Obtained with the reduce density matrix

$$\rho_A = \mathrm{tr}_B \rho$$

- Entanglement entropy is the entropy of ρ_A

$$S_{EE}(A) = -\mathrm{tr}\left(\rho_A \log \rho_A\right)$$

- In QFTs, Replica trick [Calabrese, Cardy; hep-th/0405152]

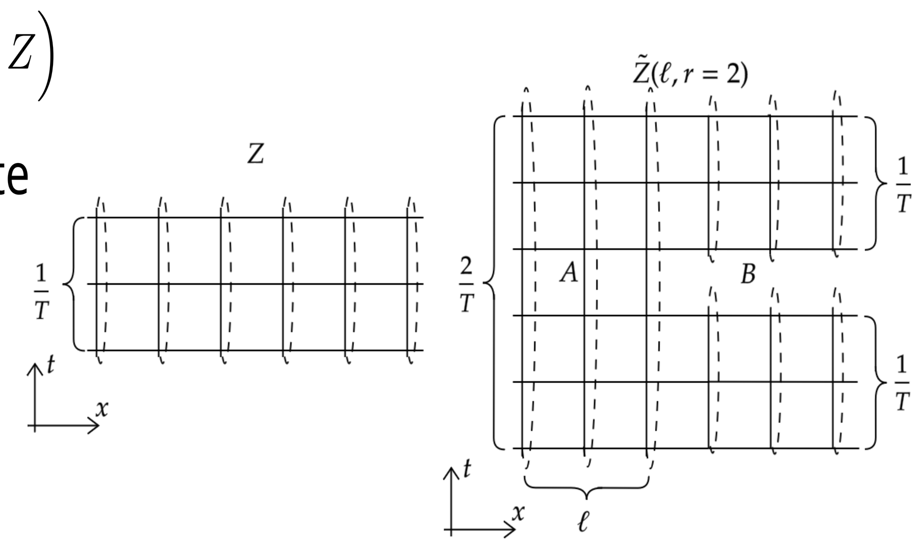
$\mathrm{tr}(\rho_A^r) = \tilde{Z}(\ell, r)/Z^r$, different BCs in *t* for A and B in $\tilde{Z}(\ell, r)$

- Entanglement entropy can be defined with Z and path integrals

$$S_{EE} = -\lim_{r\to 1}\left(\partial_r \log \tilde{Z}(\ell, r) - \log Z\right)$$

- EE UV-divergent, ℓ derivative finite

$$\partial_\ell S_{EE} = -\lim_{r\to 1} \frac{\partial^2 \log \tilde{Z}(\ell, r)}{\partial \ell \partial r}$$



EE and Thermodynamics

- At finite T EE gets contribution from the thermal entropy of A
- Can this contribution be isolated?

- Consider $\partial_\ell S_{EE}$ and express ∂_ℓ as

$$\frac{\log \tilde{Z}(\ell + \delta\ell, \beta, V, \mu, r) - \log \tilde{Z}(\ell, \beta, V, \mu, r)}{\delta \ell}$$

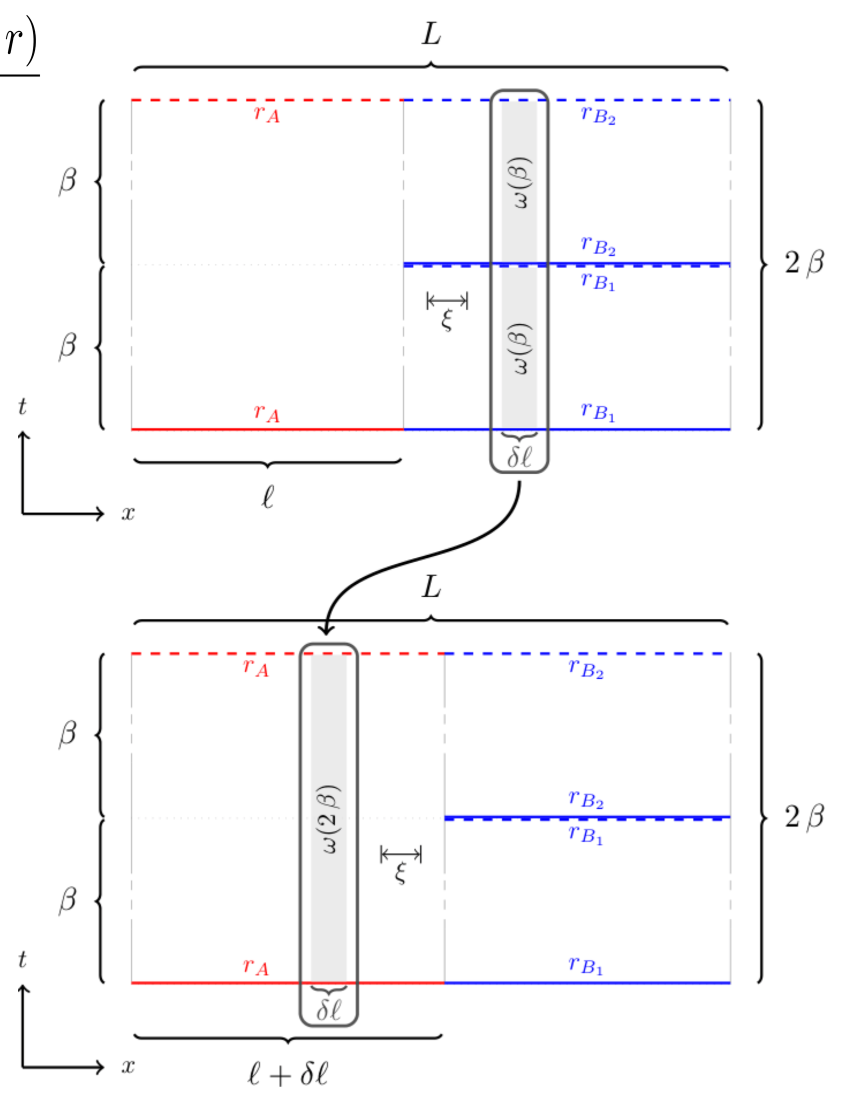
- At $\xi \ll \ell \ll L$ only contr. from $\delta\ell$ remains in the numerator
- Multiply numerator by $V_\perp$ to obtain densities $\omega = -\frac{1}{V}\log Z$

$$-\frac{1}{V_\perp}\frac{\partial \log \tilde{Z}(\ell, r)}{\partial \ell} = \omega(r\,\beta, \mu) - r\,\omega(\beta, \mu)$$

- Then the EE derivative gives

$$\boxed{\frac{1}{V_\perp}\frac{\partial S_{EE}}{\partial \ell} = \beta\,\frac{\partial \omega}{\partial \beta} - \omega = s}$$

- So, at small correlation lengths, and large ℓ, the ℓ derivative of entanglement entropy approaches the thermal entorpy

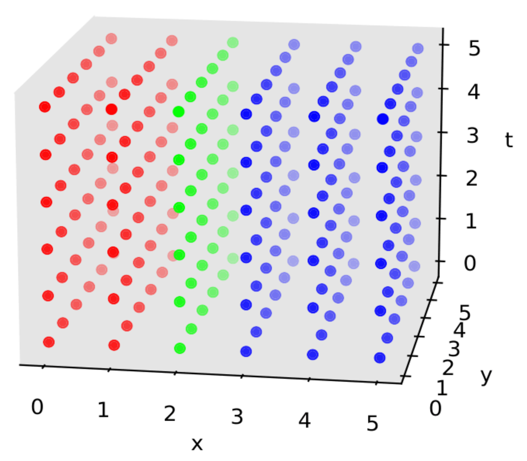


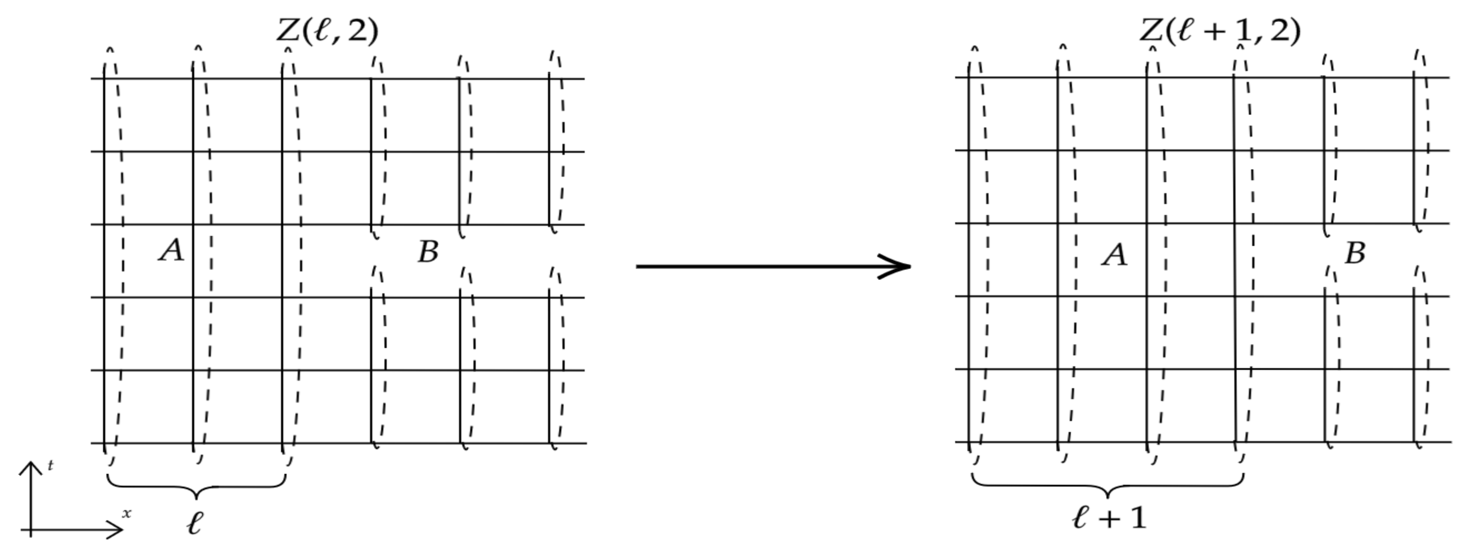
EE on the lattice

- $\partial_\ell \partial_r$ in $\partial_\ell S_{EE}$ need to be discretized

$$\left.\frac{\partial S_{EE}(\ell)}{\partial \ell'}\right|_{\ell'=\ell+1/2} \approx -\log \tilde{Z}(\ell+1, 2) + \log \tilde{Z}(\ell, 2)$$

-Measurable by deforming the boundary site by site and interpolating between ℓ and $\ell + 1$
[Jokela et al.; 2304.08949]
- $\partial_\ell S_{EE}$ evaluated from histograms of boundary configurations





Results

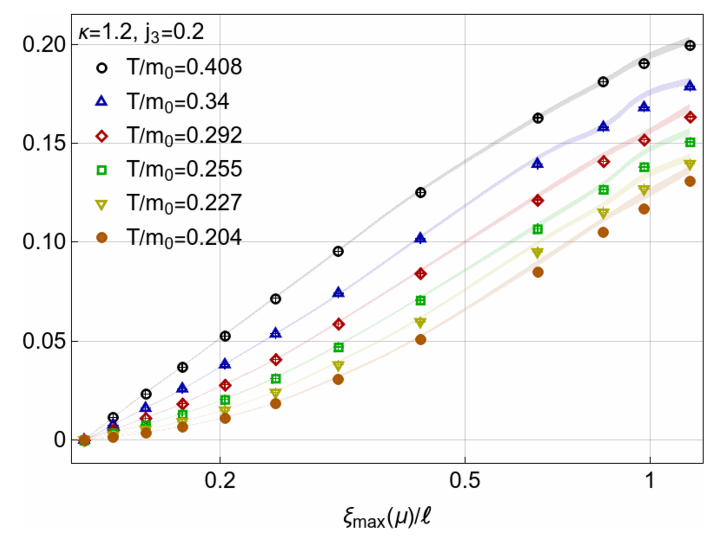
- *O(4)* scalar field theory in *d=3* at finite T and μ
- Wish to test connection between $\partial_\ell S_{EE}$ and s
- Set $r = 2$ and take ∂_μ in the relation between $\partial_\ell \log \tilde{Z}$ and ω

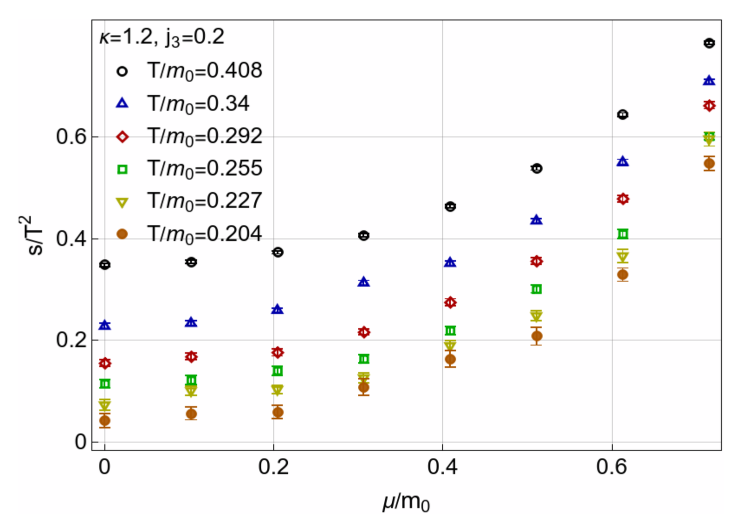
$$-\frac{1}{V_\perp}\frac{\partial^2 \log \tilde{Z}(\ell, 2)}{\partial \mu\, \partial \ell} \overset{\xi \ll \ell}{=} -2\,N_t(n(2\,N_t, \mu) - n(N_t, \mu))$$

- Can be thought of as placing $\partial_\ell S_{EE}$ in the Maxwell relation of s and n

- Both sides computable on the lattice
- Match well up to $\xi(\mu)/\ell \leq 0.5$ corresponding to μ_c
    - Holds longer for high T

- Can use $\partial_\ell S_{EE}$ to estimate s in the applicable regime





Outlook

- The connection between $\partial_\ell S_{EE}$ and s at $\xi \ll \ell$ should hold for general QFTS
- Repeat our approach for pure Yang-Mills and QCD
    - Comparison of results possible
- Could other thermodynamic quantities be derived from S?
- Phase transitions could be studied and critical behavior could be extracted with entanglement entropy