Far-from-equilibrium QFT from a symmetry perspective

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with Brants, Heller and Yin, 2511.11555

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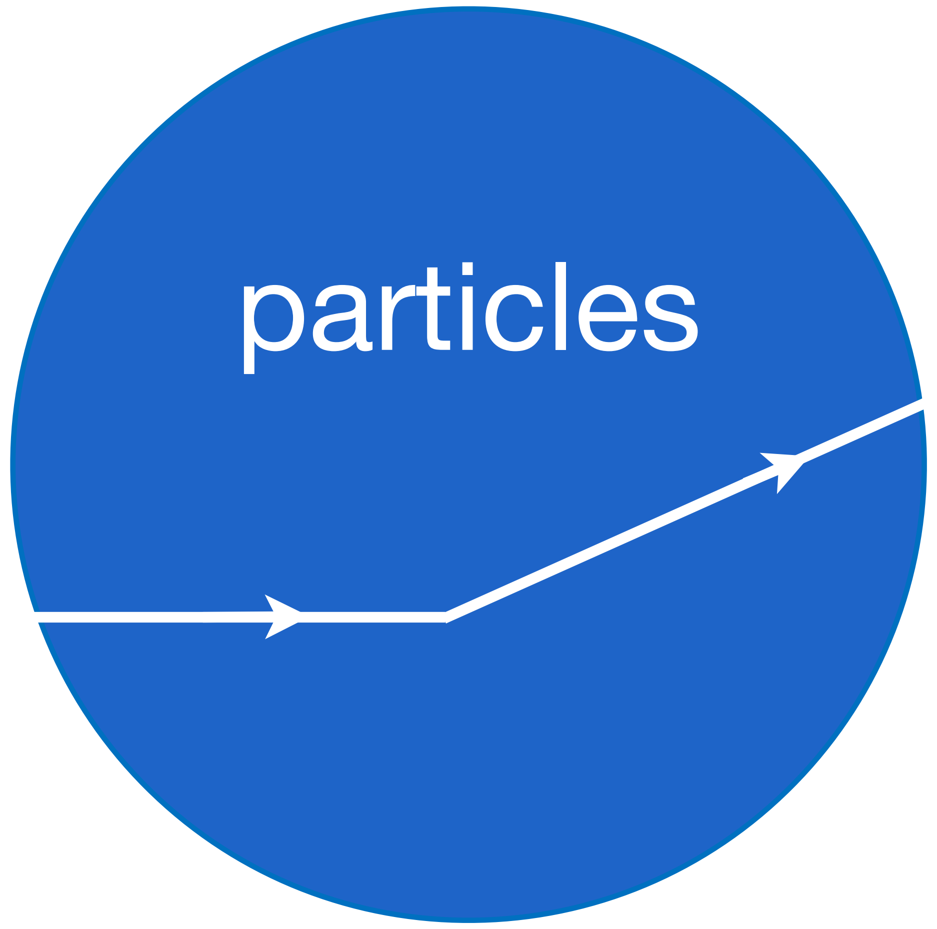
QFT: particles, waves and fields

Relativistic scalar particles



Dispersion relation for free massive particles

$$E^2 = \mathbf{p}^2 + m^2$$

Relativistic scalar particles

Wave equation for real scalar fields

$$(\partial_t^2 - \nabla^2 + m^2)\,\phi = 0$$

Solution: dispersive waves

$$\phi \sim \int_q e^{-i\omega_q t + i\,\mathbf{q}\cdot\mathbf{x}} + c.c.$$

where $\omega_q = \sqrt{q^2 + m^2} \xrightarrow{q\to 0} m + \dfrac{q^2}{2m}$ (gapped)

Relativistic scalar particles

Action for real scalar fields

$$I[\phi] = \frac{1}{2}\int_x (\partial_\mu \phi)^2 - m^2\phi^2$$

- Invariant under P, T, Z(2):

$$I[\phi; -x] = I[\phi; x], \;\; I[-\phi] = I[\phi]$$

- Not invariant under shift

$I[\phi + \lambda] \neq I[\phi]$ unless $m = 0$ (gapless)

A lesson we learned and a conjecture we made

A lesson we learned

The shift $\phi \to \phi + \lambda$ controls whether a mass term (a gap) is allowed.

A conjecture we made

Shift-like symmetry
may play an essential role in describing
the far-from-equilibrium (non-hydro) behavior.

Towards U(1) charge

Z(2) action

$$I[\Phi] = \frac{1}{2}\int_x (\partial_\mu \Phi)^2 - m^2\Phi^2 + \ldots$$

Complexified to $\Phi \in \mathbb{C}$

$$I[\Phi] = I[e^{i\theta}\Phi]$$

Gauged by local U(1)

$$D_\mu \Phi \equiv (\partial_\mu - igA_\mu)\Phi$$

Fluctuation around VEV

$$\Phi = \langle \Phi \rangle + \rho e^{i\phi}$$

Stuckelberg

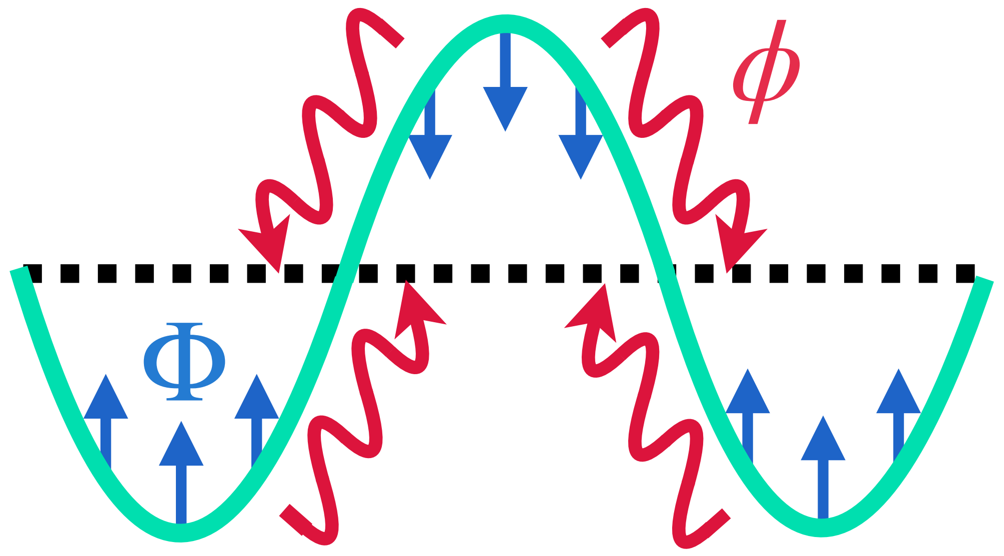
U(1) action

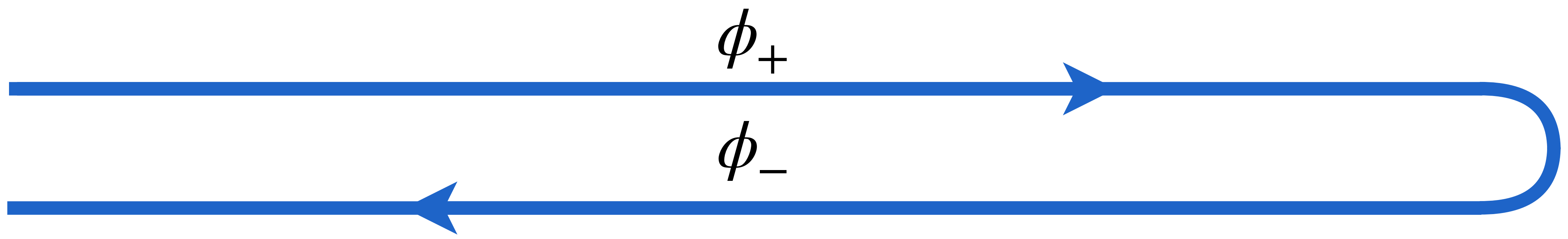
$$I[\phi, A_\mu]?$$

“Integrating-out”: SK formalism

Schwinger-Keldysh method: construct $I[\phi, A_\mu]$ from inherited UV properties

UV: $I_0[\phi, \Phi]$ → **Integrate out Φ** → **IR**: $I[\phi]$





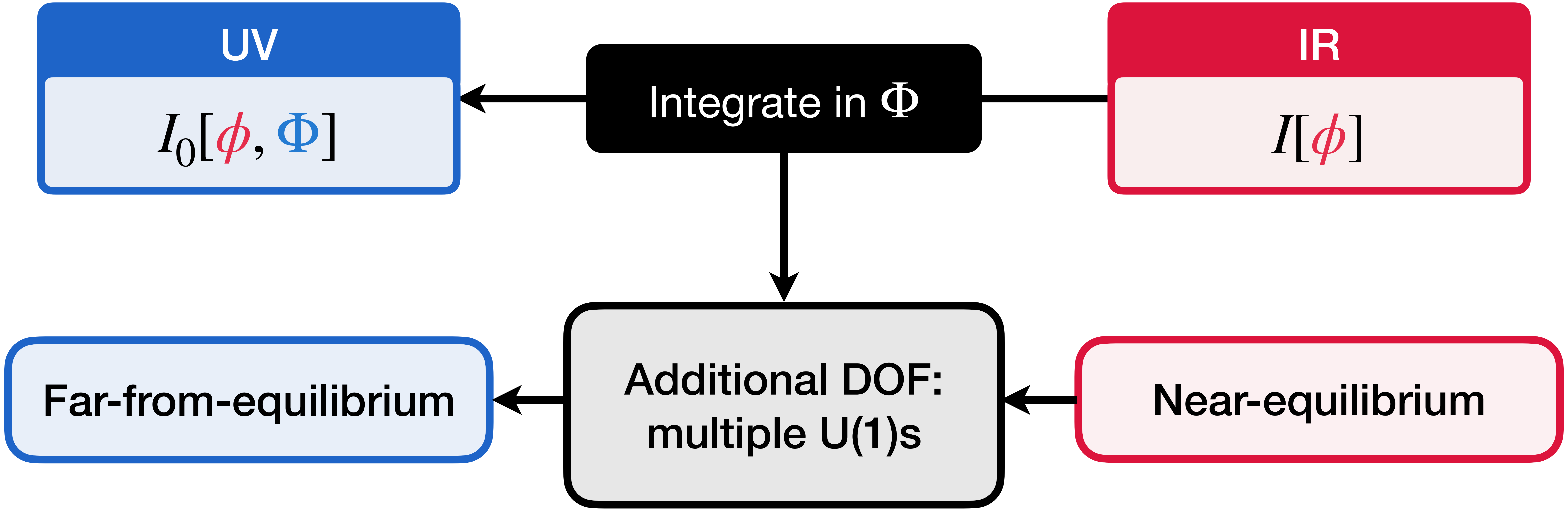
$$\phi^r = \frac{1}{2}(\phi_+ + \phi_-) \qquad \phi^a = \phi_+ - \phi_-$$

Schwinger, 1961; Keldysh, 1965
Martin et al, 1973
Glorioso et al, 1805.09331

Unitarity

$$I[\phi^a = 0] = 0 \quad \Longrightarrow \quad I = \underbrace{E_i \phi_i^a}_{\text{linear: EOM}} + \underbrace{iQ^{ij}\phi_i^a \phi_j^a}_{\text{quadratic: noise}} + \dots$$

“Integrating-in”: multiple U(1)s

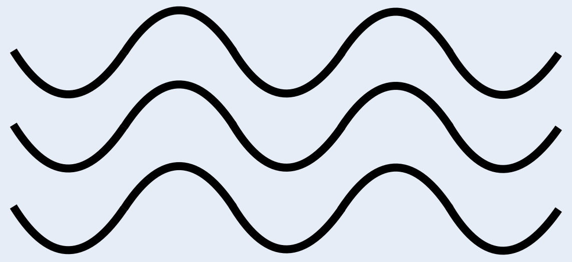


What is the underlying principle to organize and distinguish far-from-equilibrium theories?

Strongly and weakly coupled regimes

Strongly coupled regime

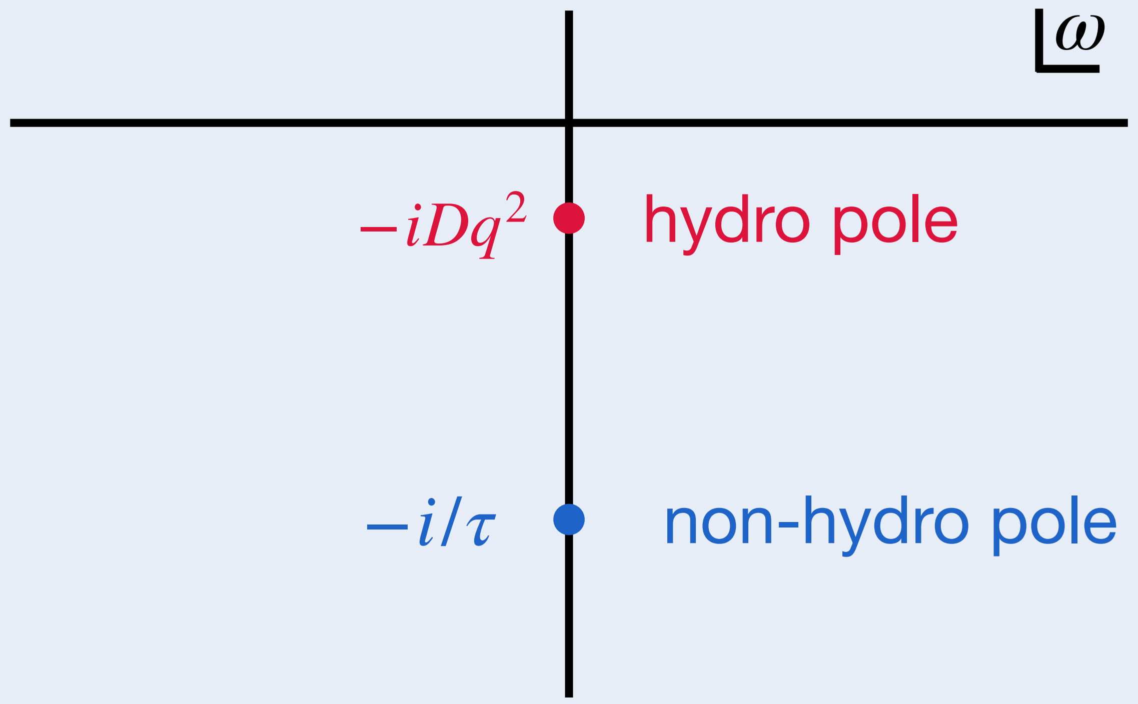
Collective excitations



Hydro-like (MC)

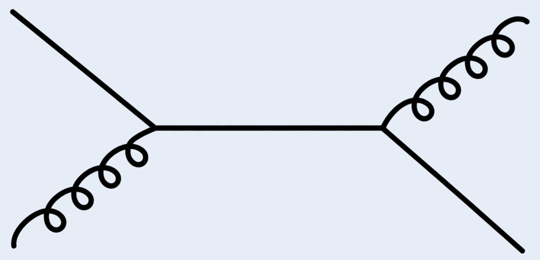
$$(1 + \tau\partial_t)\partial_t n = D\partial^2 n$$
$$\tau \sim \ell_{\text{mic}} \sim 1/T$$

Mode structure: poles



Weakly coupled regime

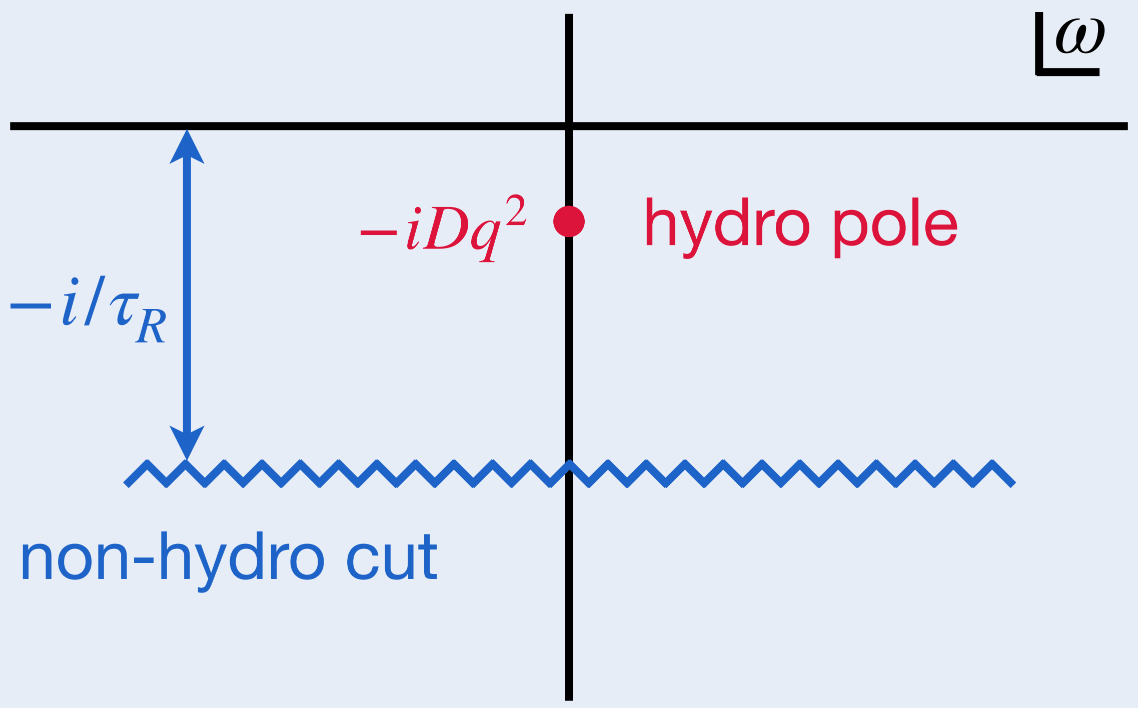
Quasiparticles



Boltzmann (RTA kinetic)

$$(\partial_t + v \cdot \partial) f(x, p) \approx -(f - f_{\text{eq}})/\tau_R$$
$$\tau_R \sim \ell_{\text{mfp}} \sim 1/(\alpha_s^2 T)$$

Mode structure: cuts



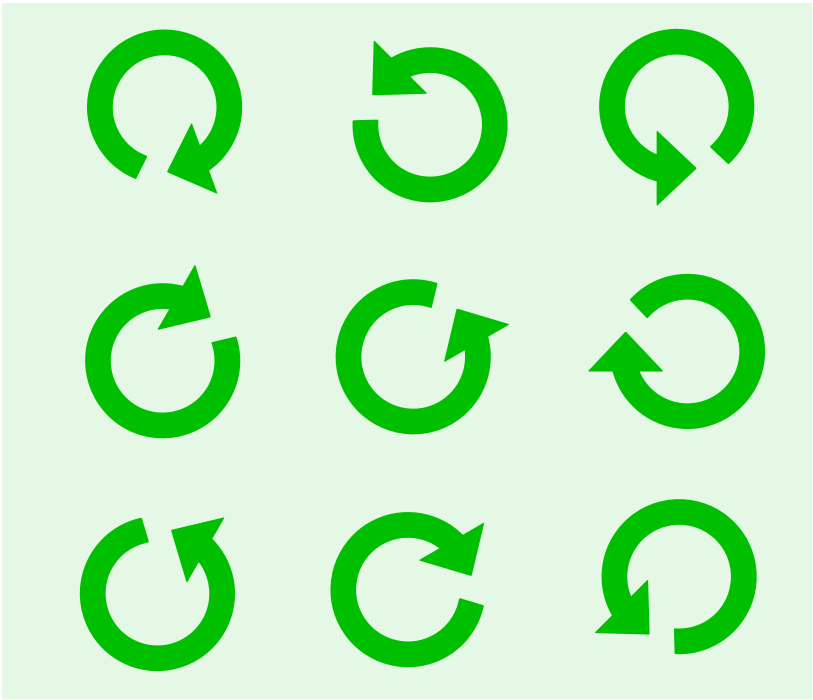
Explicit and emergent symmetries

The action is required to be invariant under

- *Explicit* **U(1) gauge transformation**

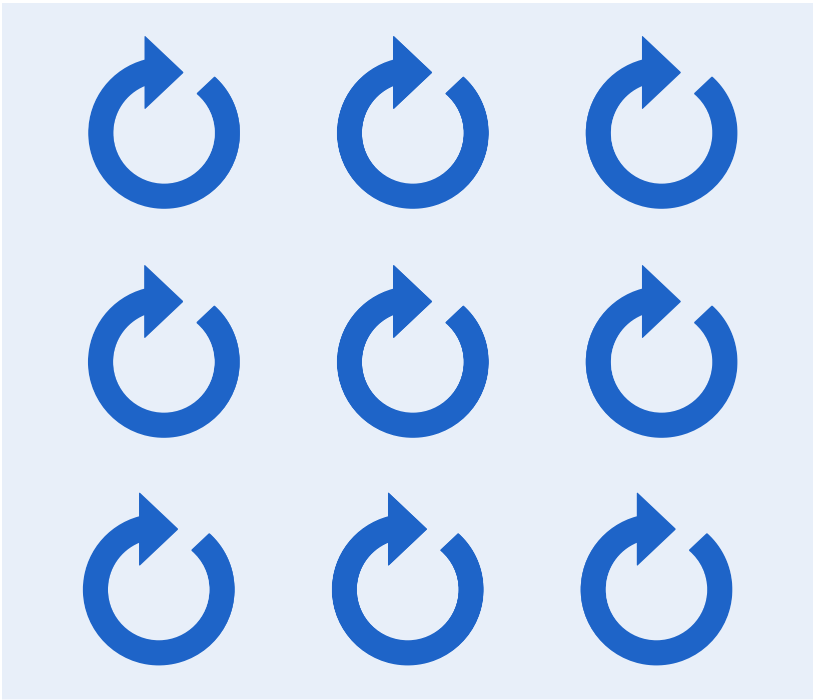
$$\phi \to \phi + \theta(x), \quad A_\mu \to A_\mu - \partial_\mu \theta(x), \qquad \Longrightarrow \qquad \mathbb{A}_\mu = A_\mu + \partial_\mu \phi \to \mathbb{A}_\mu$$

- *Emergent* **shift**



Unbroken (disordered) phase: local/independent shift

$$\phi_r \to \phi_r + \lambda(\mathbf{x}), \quad \mathbb{A}_\mu \to \mathbb{A}_\mu + \partial_\mu \lambda(\mathbf{x})$$



Broken (ordered) phase: global/common shift

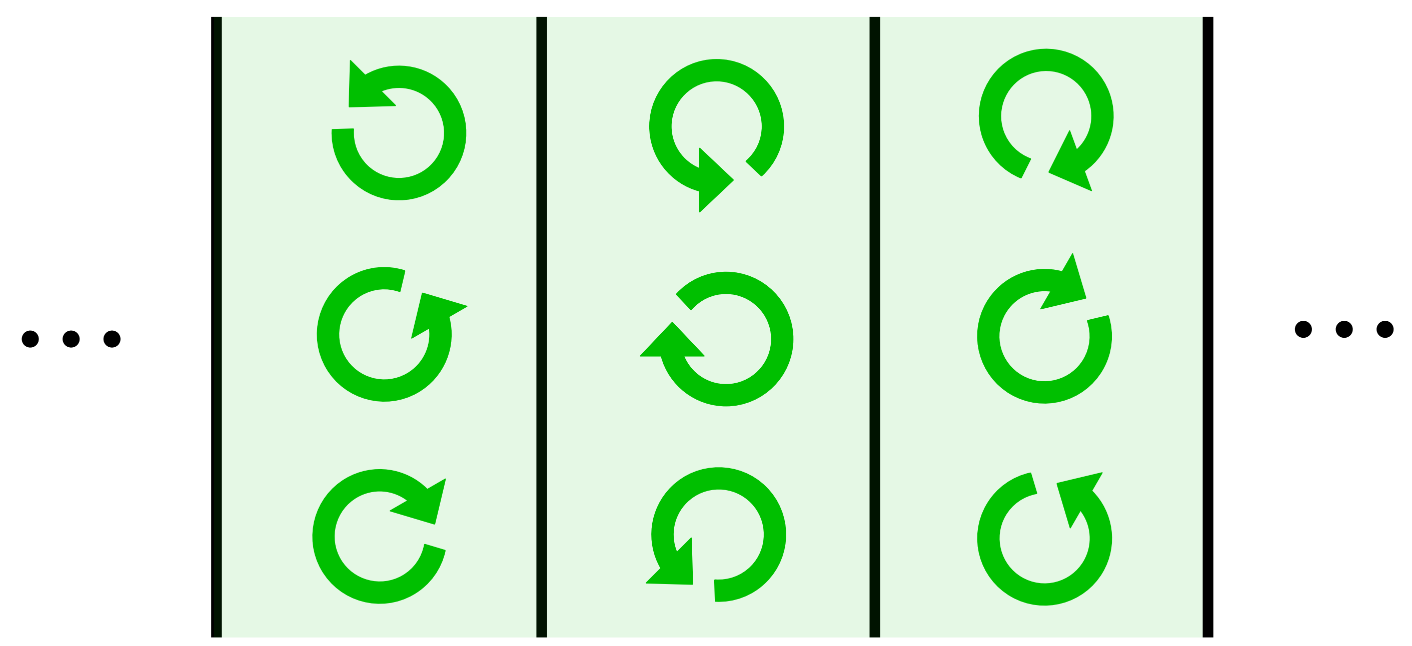
$$\phi_r \to \phi_r + \lambda, \quad \mathbb{A}_\mu \to \mathbb{A}_\mu$$

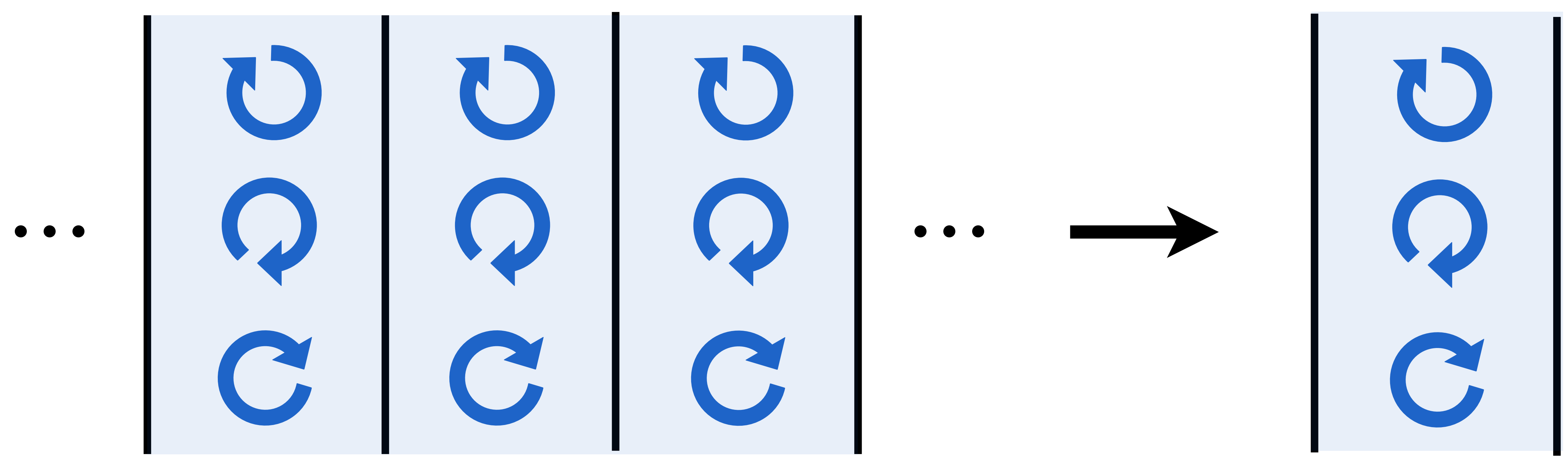
Scenarios for multiple U(1)s

Generalization to a collection of systems (layers) with multiple U(1)s.

I All U(1)s are **unbroken**

II All U(1)s are **broken** into a diagonal U(1)

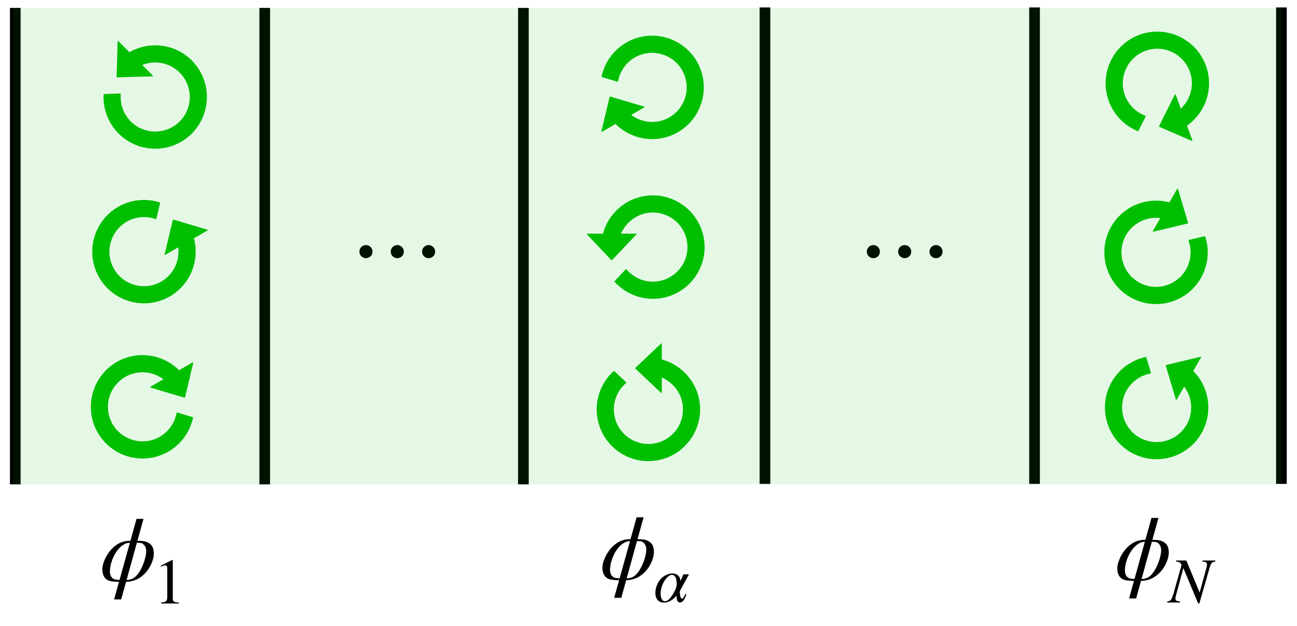




III **Hybridization** of the two

Scenario I: all U(1)s are unbroken

Delacretaz et al, 2203.05004
York et al, 0811.0729
Mo et al, 2512.23960



Layer-dependent shift

$$I_{\mathrm{U}}[\{\phi^r_\alpha, \phi^a_\alpha\}] = I_{\mathrm{U}}[\{\phi^r_\alpha + \lambda_\alpha(\mathbf{x}), \phi^a_\alpha\}]$$

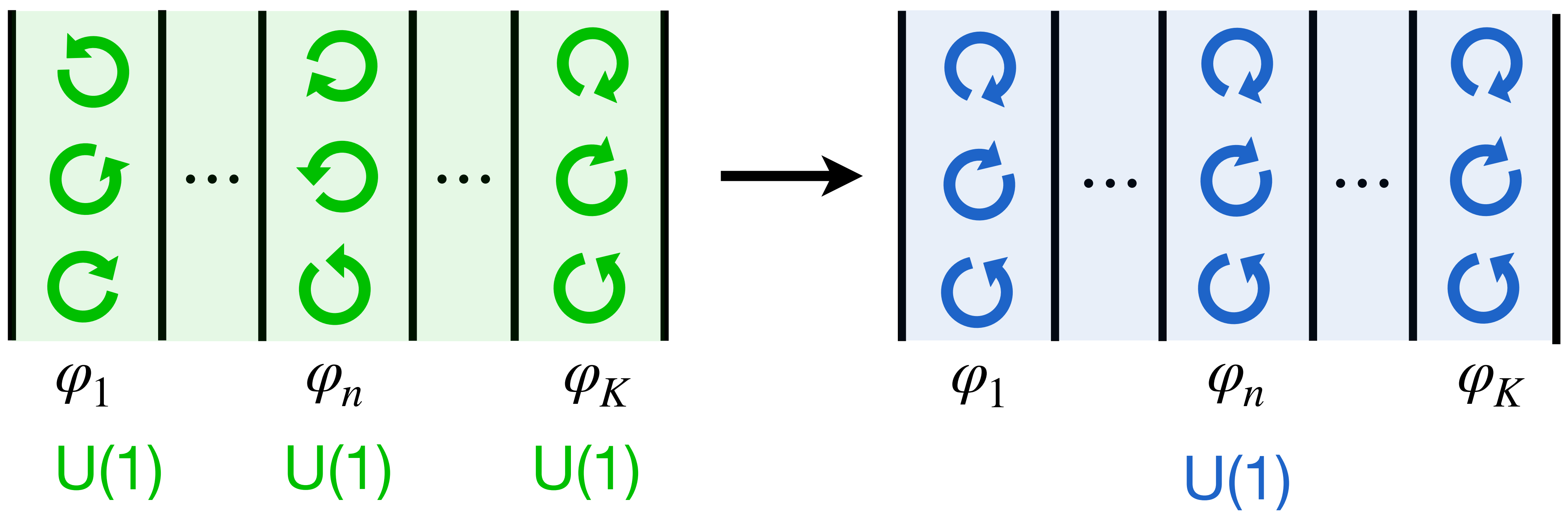
Action for unbroken scenario

$$I_{\mathrm{U}}[\{\phi_\alpha\}] = \sum_{\alpha=1}^{N} \int_x \chi_0 \left\{ (\partial_t \phi^r)\partial_t \phi^a + \frac{1}{2}\left[(\mathbf{v}\cdot\partial)\phi^r \partial_t \phi^a + (r \leftrightarrow a)\right] \right\}_\alpha$$

Kinetic-like theory

Boltzmann equation for $\delta f = \chi_0(\partial_t \phi^r)$

Scenario II: all U(1)s are broken into a diagonal U(1)



Layer-independent (diagonal) shift

$$I_{\rm B}[\{\mathbb{A}^r_n, \mathbb{A}^a_n\}] = I_{\rm B}\{\mathbb{A}^r_n + \partial\lambda(x), \mathbb{A}^a_n\}]$$

Action for broken scenario

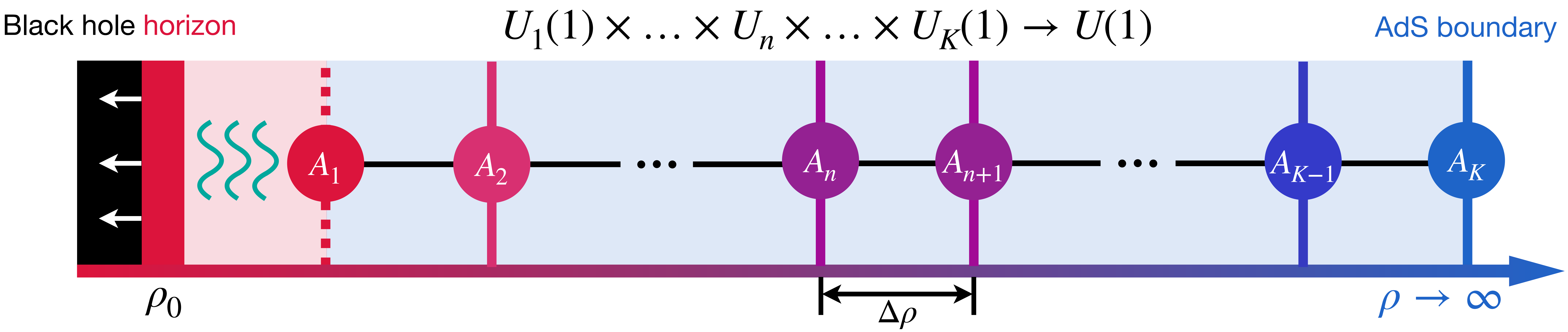
$$I_{\rm B}[\{\mathbb{A}_n\}] = \sum_{n=1}^{K}\int_x \Big(f^2\,(\mathbb{A}_{n+1} - \mathbb{A}_n)^r_t\,(\mathbb{A}_{n+1} - \mathbb{A}_n)^a_t - g^2(\mathbb{A}_{n+1} - \mathbb{A}_n)^r_i(\mathbb{A}_{n+1} - \mathbb{A}_n)^a_i$$
$$+\epsilon\mathbf{E}^r\cdot\mathbf{E}^a - \kappa\mathbf{B}^r\cdot\mathbf{B}^a + \sigma\mathbb{A}^a\cdot\mathbf{E}^r\Big)_n$$

Holographic-like theory

Maxwell equations for hidden photon $\mathbb{A}_\mu$

Scenario II: all U(1)s are broken into a diagonal U(1)

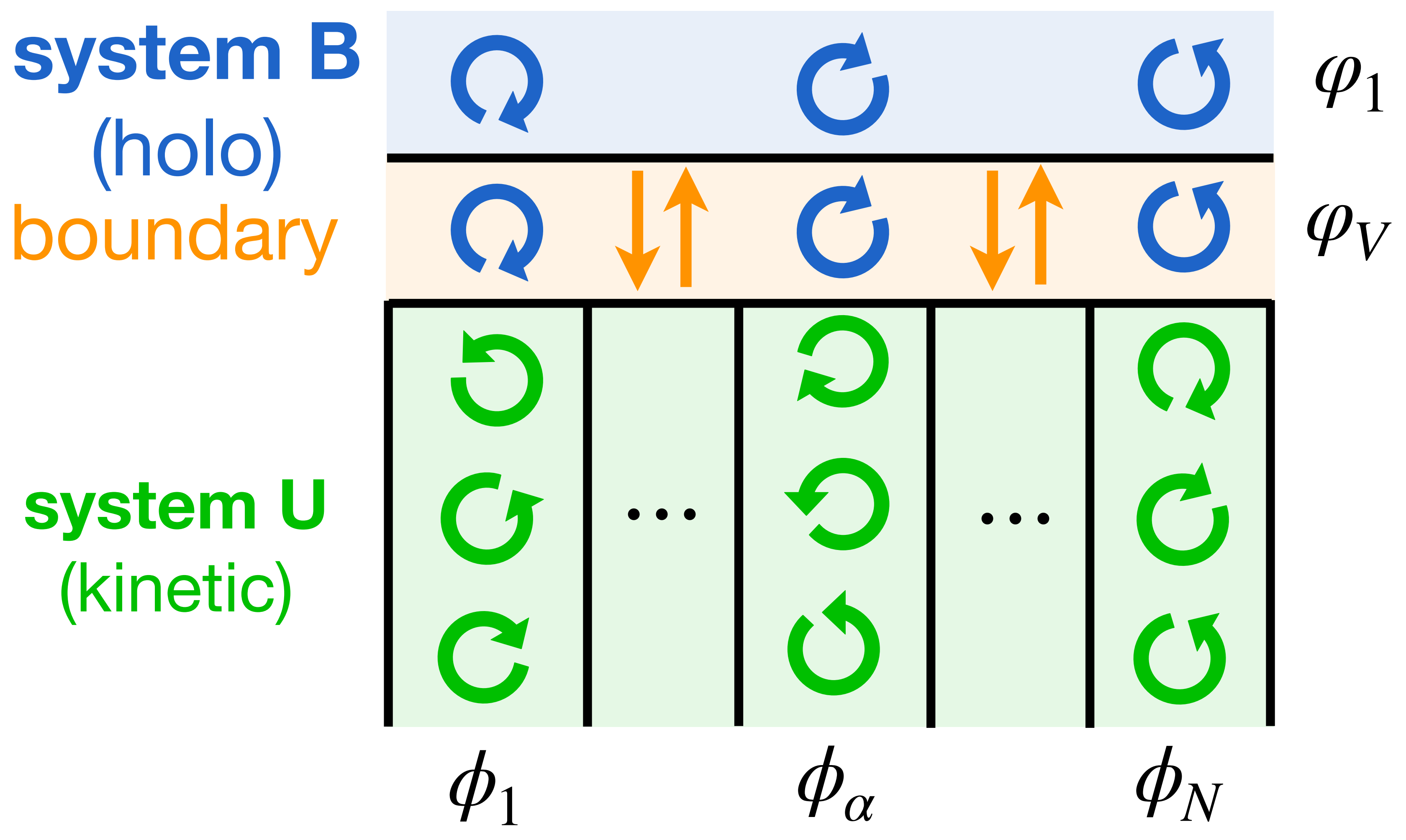
When layer number $K \to \infty$ (continuous limit), this scenario reproduces holography via dimensional deconstruction. Arkani-Hamed et al, 0104005; Nickel and Son, 1009.3094

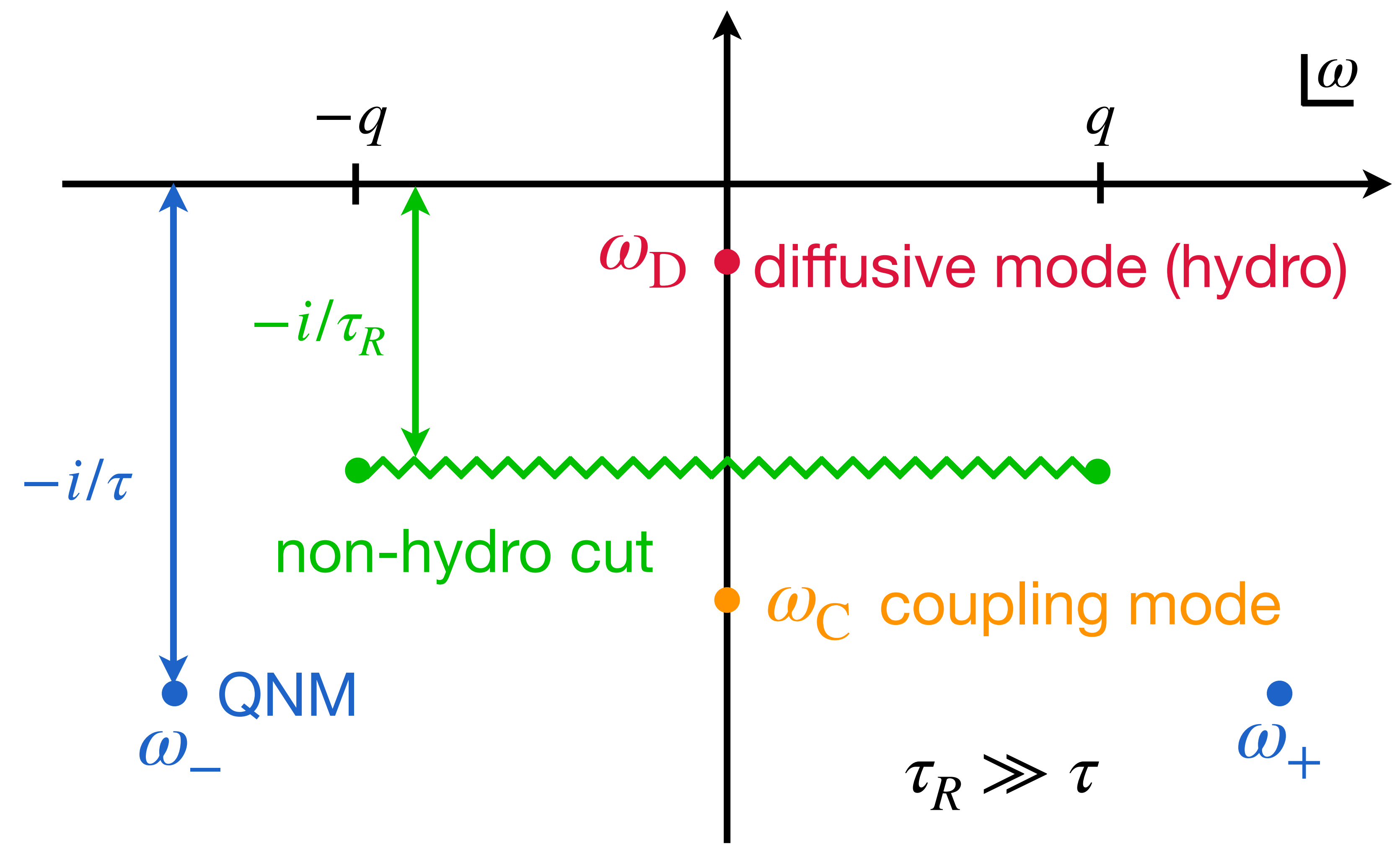


Action for 5D holography

$$I_{\mathrm{B}}[\{\mathbb{A}_n\}] \xrightarrow{K\to\infty} -\frac{1}{4}\int_{x,\rho} \sqrt{-g}\, g^{MM'} g^{NN'} F^r_{MN} F^a_{M'N'}$$

Scenario III: Hybridization of the two





Action for hybrid scenario

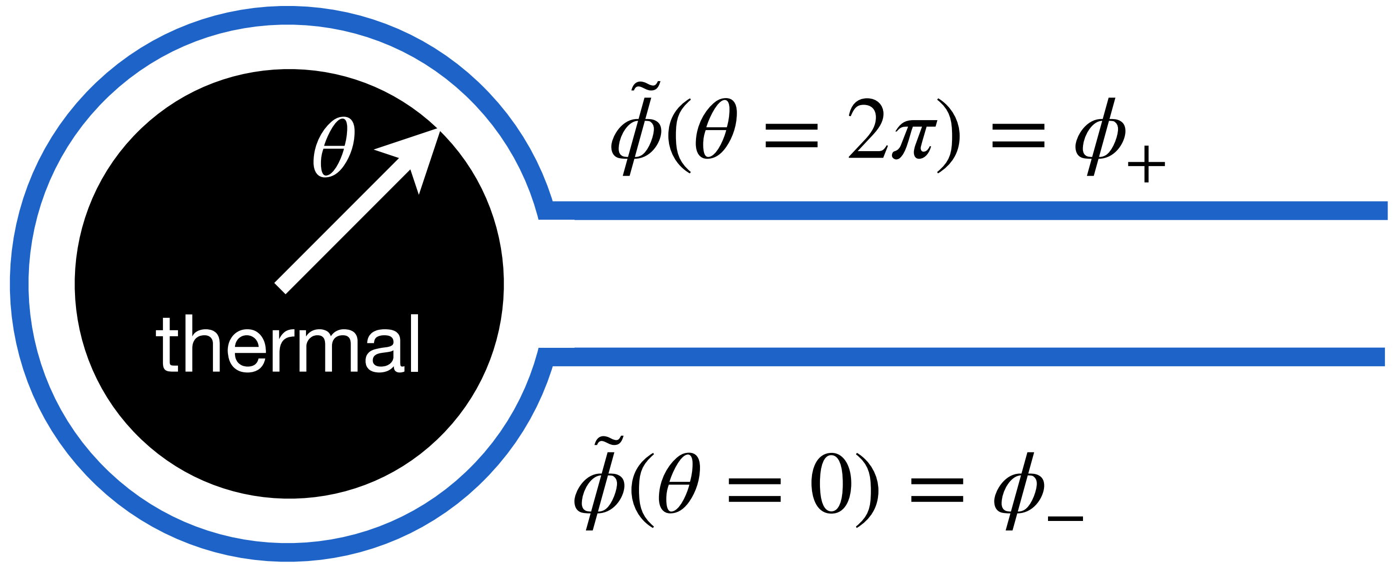
$$I_{\rm H} = I_{\rm U}[\{\phi^r_\alpha, \phi^a_\alpha\}] + I_{\rm B}[\{\mathbb{A}^r_n, \mathbb{A}^a_n\}] + \Delta I[\phi_\alpha, \varphi_V]$$

$$\Delta I = -\frac{\chi_{\rm U}}{\tau_{\rm R}} \sum_\alpha \int_x (\partial_t \phi^r_\alpha - \partial_t \varphi^r_V)(\phi^a_\alpha - \varphi^a_V)$$

- Mutual equilibration of two open systems with chemical potential $\mu_U \sim \partial_t \phi_\alpha$, $\mu_B = \partial_t \varphi_V$.
- Transition between strongly and weakly coupled regimes.

Fluctuations and shift symmetry

"Integrated in" action **invariant** under **shift**



Shift on auxiliary field

$$\tilde{I}[\tilde{\phi}(\theta, x)] = \tilde{I}[\tilde{\phi}(\theta, x) + \lambda(x)]$$

Local action for $\tilde{\phi}$

$$\tilde{I}[\tilde{\phi}] = \int_x \int_0^{2\pi} d\theta \, \frac{\sigma}{2} \left[i(\partial_\theta \tilde{\phi})^2 - \frac{1}{2\pi T} (\partial_\theta \tilde{\phi}) \, \partial_t \tilde{\phi} \right]$$

integrate out $\tilde{\phi}$

Nonlocal action for ϕ

$$I_{\text{diss}} + I_{\text{fluct}} = \int_{\mathbf{x},\omega} \phi^a(-\omega) \left[E' \phi^r(\omega) + iQ\phi^a(\omega) \right]$$

KMS/FDR

$$Q = \coth\left(\frac{\omega}{2T}\right) \mathrm{Im} E'$$

Recap

Far-from-equilibrium QFT $\leftarrow$ fundamental+emergent (shift) symmetries

Outlook

- Energy-momentum
- Other symmetry groups
- AI

Thank You