Bubble wall dynamics from nonequilibrium quantum field theory

Bridging two regimes

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Bubble dynamics: A balance of pressure

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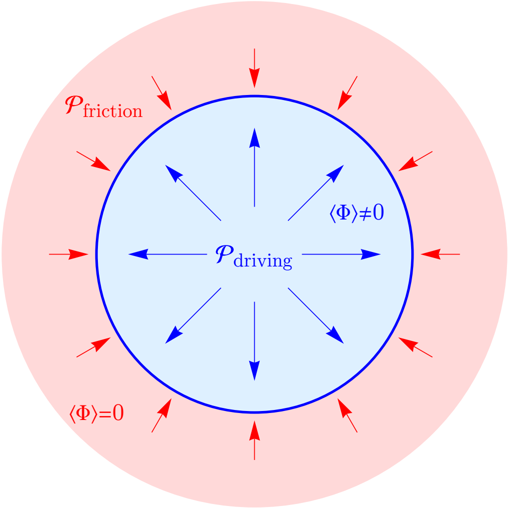
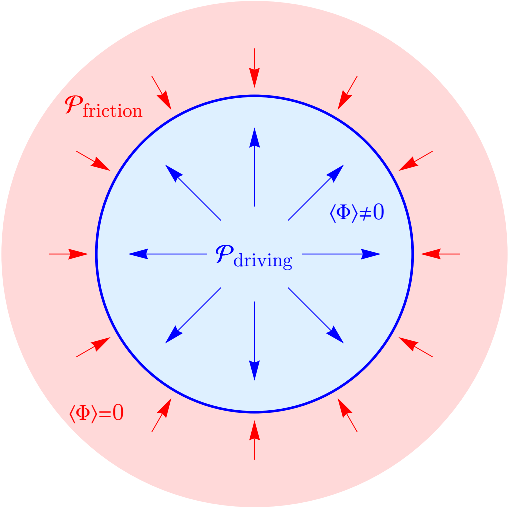


Bubble dynamics: A balance of pressure



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$$\mathcal{P}_{\text{friction}} = \mathcal{P}_{\text{driving}}$$

the system reaches a *steady state*

$\Longrightarrow$terminal wall velocity

$$\equiv v_w$$

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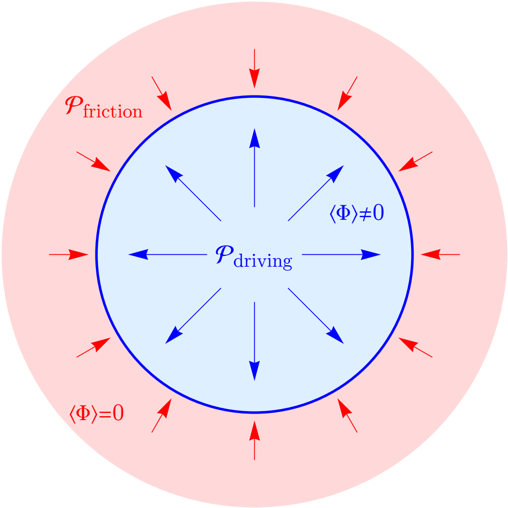

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Goal: does a stable steady state exist? If so, what is v_w?

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

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Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} \delta f_i(\mathbf{p}, x) = 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

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[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

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The two are **not equivalent**: they can give **different** predictions, e.g. when **particle production** is relevant.

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- captures **non-equilibrium** phenomena (like the kinetic picture),
- and includes **quantum processes**, such as **particle production** (like the kick picture),
- from **first principles**, in a single **closed, self-consistent** scheme.

The framework

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closed and **self-consistent**: no double counting

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$\downarrow$

bubble EoM

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$\rightsquigarrow$ Boltzmann eqs. / kinetic theory

The bubble wall equation of motion

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The Kadanoff-Baym equations are reduced to the known Boltzmann equations for the plasma distribution functions. As for the bubble EoM

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

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- Packing it into a more familiar form

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recall kinetic approach: $\Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) = 0$

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NEW TERM!

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The vertex pressure in Wigner space

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Focus on the new term

$$\mathcal{P}_{\text{vertex}} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \;\equiv\; -\,\mathrm{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

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It is a **trace**. In **Wigner space** it stays a trace,

$$\mathrm{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad\Longrightarrow\quad \mathcal{P}_{\mathrm{vertex}} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\frac{1}{2}\partial_u - ik\Big]\widetilde{\pi}^R(k,u)$$

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enters only at **NLO** in gradients

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$\mathrm{Im}\,\widetilde{\pi}^R$: even integrand $\rightsquigarrow$ **LO**

genuine **dissipation** (fluctuation–dissipation)

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Particle production

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- Introduce a heavy scalar field χ in the Lagrangian

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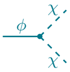

$$\Big|^2$$

optical theorem $=$ cutting rule

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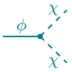

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$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

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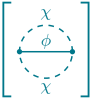

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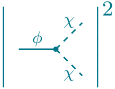

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density of incoming particles

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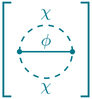

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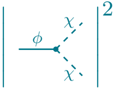


$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

density of incoming particles

$$\times\, f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad \left|\tilde{\varphi}(\Delta p_\perp)\right|^2$$

momentum exchange

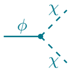
Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\tilde{\varphi}(q_\perp)|^2\ \mathrm{Im}\,\tilde{\pi}^{\mathrm{R}}(q_\perp)\Big\} \sim \mathrm{Im}\Big[\ \Big] = \Big|\ \Big|^2$$



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density of incoming particles $\quad \times\, f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2 \longleftarrow$ Fourier tf. of the wall

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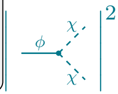
recall: $\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\mathbf{p}} \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p})\, f_i(\mathbf{p})\, \Delta p_{\perp\,,i\to X}$

$\mathcal{P}_{\text{vertex}} \approx$ 

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Summary and outlook

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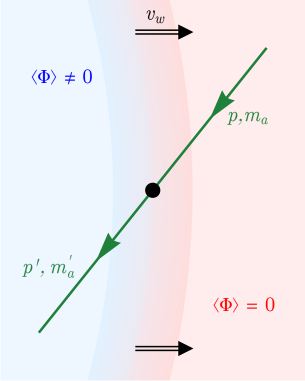
THANK YOU

arXiv:2504.13725

+ upcoming work

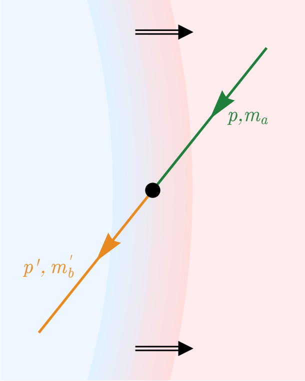
BACK-UP SLIDES

(some) Sources of friction in the kick picture

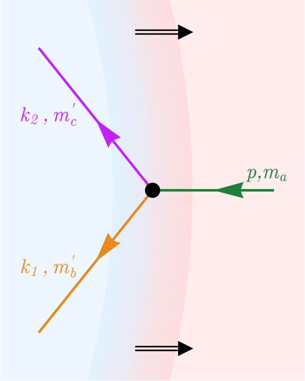


Mass gain

Captured by kinetic pic



Mixing



Particle production

$\sim \log \gamma_w$ for scalars

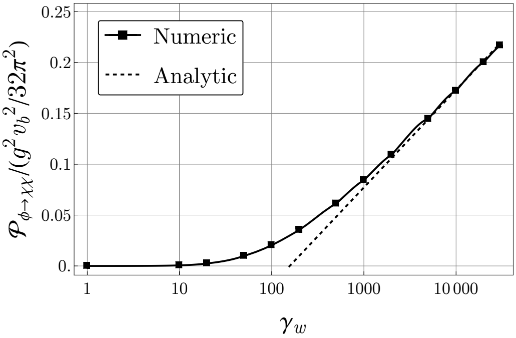
$\sim \gamma_w$ for gauge bosons

The ultrarelativistic regime

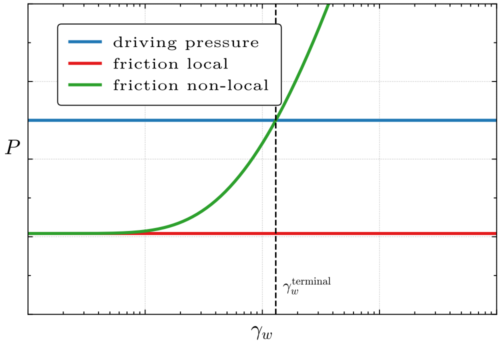
Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

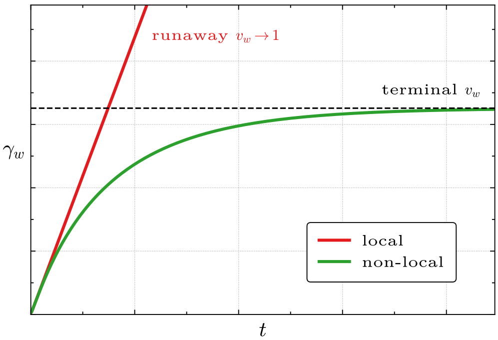
$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2}\log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

which approaches the result from the kick picture.
Similarly, we show in our work that particle mixing and transition radiation are also captured within this framework.



Dramatic consequences!





Evolution of a *rigid* tanh-wall with $L = 1/T = 1$, $g = 3$, $v = 2$, $\Delta V = 0.005$ and $\Delta m_\phi^2 = 0.05$.

In a region of parameter space, neglecting the non-local self-energy (i.e. particle production) can mischaracterise whether a wall accelerates indefinitely or reaches a terminal velocity.

Entropy production in local thermal equilibrium

[Ai, Laurent, Van de Vis '23]: the dissipative friction (total minus the hydrodynamic obstruction) is proportional to the **entropy production** Δs.

in LTE: $\delta f = 0 \;\Rightarrow\; \Delta s = 0 \;\Rightarrow\;$ friction vanishes $\;\Rightarrow\;$ **upper bound on** v_w

But this rests on the *incomplete* EoM. With the new term

$$\Delta s \;\propto\; \underbrace{\mathcal{P}_{\text{dissipative}}}_{\propto\,\delta f\,=\,0 \text{ in LTE}} \;+\; \underbrace{\mathcal{P}_{\text{vertex}}}_{\neq 0 \text{ in LTE!}}$$

- entropy production **does not vanish** in LTE
- the LTE upper bound on v_w must be **revisited**
- dissipative effects are captured already in LTE, at LO in the gradients

Identifying sources of friction

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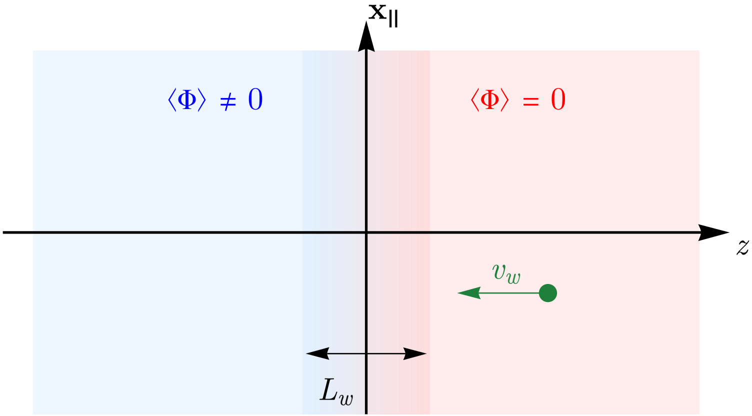
In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**



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$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\begin{aligned}
\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2}\left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\right)^2} + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \, \Bigg[& \frac{\partial U}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z} && \equiv \mathcal{P}_{\text{LTE}} \\
& - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) && \equiv \mathcal{P}_{\text{dissipative}} \\
& - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \Bigg] && \equiv \mathcal{P}_{\text{vertex}}
\end{aligned}$$

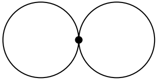
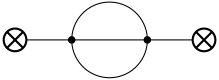
The full dynamical equations

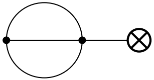
The equations of motion are now readily obtained

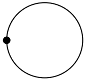
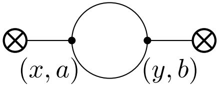
$$\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^{+}(x)}\bigg|_{\varphi^{+}=\varphi^{-}=\varphi} = \frac{\delta S}{\delta\varphi(x)} - \frac{1}{2}\frac{\mathrm{d}m_{\varphi}^{2}}{\mathrm{d}\varphi(x)}\Delta^{T}(x,x) + \frac{\delta\Gamma_{2}}{\delta\varphi^{+}(x)}\bigg|_{\varphi^{+}=\varphi} = 0$$

$$\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta^{ab}(x,y)} = 0 \quad \Rightarrow \quad \Delta^{ab,-1}(x,y) - G_{\varphi}^{ab,-1}(x,y) + 2i\frac{\delta\Gamma_{2}}{\delta\Delta^{ab}(x,y)} = 0$$

For a scalar theory with quartic self-interaction, we have

$$\mathcal{L}_{\mathrm{int}} = -\frac{\lambda}{4!}\phi^{4} \longrightarrow i\Gamma_{2} =$$  $+$  $+\dots$

$$i\frac{\delta\Gamma_{2}}{\delta\varphi(x)} \supset \quad x$$ 

$$2i\frac{\delta\Gamma_{2}}{\delta\Delta^{ab}(x,y)} \supset \quad (x,a)$$  $+$ 

The self-energy

At leading order in the gradient expansion

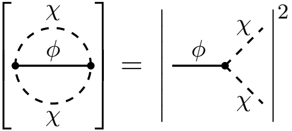
$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \left|\tilde{\varphi}(q^z)\right|^2 \tilde{\pi}^R(-q^z)$$

$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \left|\tilde{\varphi}(q^z)\right|^2 \mathrm{Im}\tilde{\pi}^R(q^z)$$

Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$



$\implies$ pair production!

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$$k \sim \gamma_w T \;\equiv\; \text{typical momentum of a particle } \textbf{in the wall frame}$$

$$\Longrightarrow \gamma_w T L_w \gg 1$$

The gradient expansion is valid if the wall is either **fast** or **thick**. For the numerical and analytical results, we assumed the plasma outside the bubble to be **in equilibrium**, which is once again only valid if the wall is very fast.

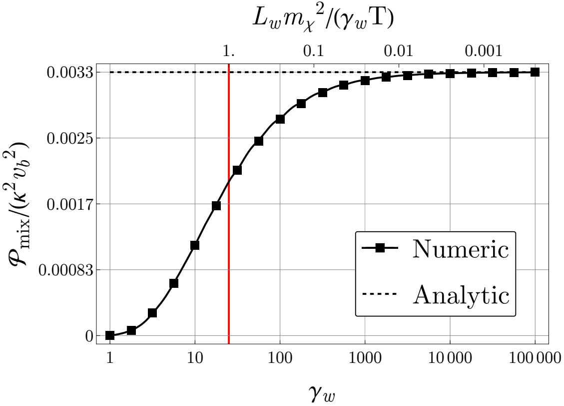
Mixing

Assume two mixing scalar species χ and s interacting through the background

$$\mathcal{L}_{\text{int}} \supset -\kappa\varphi\chi s\,, \qquad \text{and} \quad m_\chi \gg m_s$$

Particles χ are absent in the plasma but are generated via mixing as s-particles go through the wall. In the ultrarelativistic limit

$$\mathcal{P}_{s\to\chi}^{\gamma_w\to\infty} = \frac{2\kappa^2 v_b^2}{m_\chi^2}\frac{T^2}{24}$$



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