Attractodynamics

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arXiv:2607.16393

August 18, 2026

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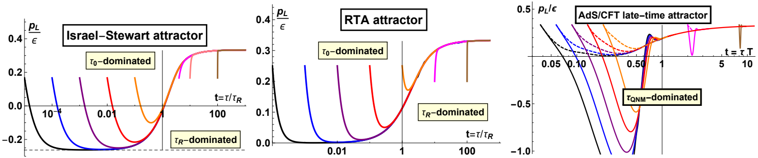
Attractors vs Hydrodynamics



Kurkela, van der Schee, Wiedemann, Wu, arXiv:1907.08101

- Experiments show signs of collective behavior in small systems.
- We know the far-from-equilibrium QGP has attractor behavior. In what ways are the long-wavelength excitations near attractors similar to hydrodynamics?
- Plan:
  - Derive a theory describing these excitations by doing an analog to the 14-moment approximation.

    c.f. Berges, Denicol, Heller, Preis, arXiv:2504.18754

  - Apply to a simple known far-from-equilibrium attractor.

See Michal Heller's talk!

Review: 14-moment approximation

We start with a Boltzmann equation

$$p^\mu \partial_\mu f = C[f]$$

with equilibrium distribution

$$f_{\mathrm{eq}} = f_{\mathrm{eq}}\left(\frac{\mu - u_\mu p^\mu}{T}\right)$$

$$\begin{array}{lccccccc} \text{DOF:} & 1 & + & 1 & + & 3 & & = 5 \\ & (\mu) & & (T) & & (u^\mu) & & \end{array}$$

and expand around equilibrium by writing

$$f = f_{\mathrm{eq}} + \delta f\ .$$

Review: 14-moment approximation

In the 14-moment approximation, we then derive Israel-Stewart hydrodynamics by taking moments of the Boltzmann equation to find equations of motion for

$$N^\mu = \langle p^\mu \rangle_f \,, \qquad T^{\mu\nu} = \langle p^\mu p^\nu \rangle_f \,.$$

$$\text{DOF:} \quad 4 \quad + \quad 10 \quad = 14$$

The f_{eq} piece gives the ideal hydro quantities $\mathcal{E}, \mathcal{N}, u^\mu$ (5 DOF), while the viscous corrections $\Pi, \pi^{\mu\nu}, V^\mu$ (9 DOF) are related to the relaxation of δf,

$$V^\mu = \langle p^{\langle\mu\rangle} \rangle_{\delta f} \quad \pi^{\mu\nu} = \langle p^{\langle\mu} p^{\nu\rangle} \rangle_{\delta f} \quad \Pi = -\frac{1}{3} \langle \Delta^{\alpha\beta} p_\alpha p_\beta \rangle_{\delta f}$$

Israel and Stewart, Annals Phys. 118 (1979) 341–372.

Review: 14-moment approximation

We choose an ansatz for δf,

$$\delta f = f_{\mathrm{eq}}(\mu, T, u^\mu) \quad \cdot \quad \left(\alpha_\mu \frac{p^\mu}{E} \quad + \quad \beta_{\mu\nu} \frac{p^\mu p^\nu}{E}\right).$$

$$\text{DOF:} \qquad 4 \qquad + \qquad 10 \qquad\qquad = 14$$

Some of these overlap with the equilibrium degrees of freedom μ, T, u^μ, so make these well-defined by imposing constraints

$$\langle E \rangle_{\delta f} = \langle E p^\mu \rangle_{\delta f} = 0\,,$$

$$\text{DOF:} \quad 1 \quad + \quad 4 \qquad = 5$$

so δf has the correct 9 DOF.

Attractodynamics in the 30-moment approximation

We again start with a Boltzmann equation

$$p^\mu \partial_\mu f = C[f]$$

and expand around a generalized attractor surface

$$f_{\mathcal{A}} = f_{\mathcal{A}}\left(\alpha^{\mathcal{A}} + \beta^{\mathcal{A}}_\mu p^\mu + \frac{1}{2} p^\mu \mathcal{U}^{\mathcal{A}}_{\mu\nu} p^\nu + \dots\right)$$

$$\text{DOF:} \quad 1 \quad + \quad 4 \quad + \quad 9 \quad = 14$$

and write

$$f = f_{\mathcal{A}} + \delta f \, .$$

These $\mathcal{U}^{\mathcal{A}}_{\mu\nu}$ represent non-hydrodynamic degrees of freedom that parametrize the attractor.

Du Plessis, Scheihing-Hitschfeld, **RS**, arXiv:2607.16393

Attractodynamics in the 30-moment approximation

The extra degrees of freedom suggest we should calculate additional dynamical moments

$$N^{\mu} = \langle p^{\mu} \rangle_f, \quad T^{\mu\nu} = \langle p^{\mu} p^{\nu} \rangle_f, \quad I^{\mu\nu\lambda} = \langle p^{\mu} p^{\nu} p^{\lambda} \rangle_f .$$

$$\text{DOF:} \quad 4 \quad + \quad 10 \quad + \quad 16 \quad = 30$$

These will break down into 14 "ideal" on-attractor degrees of freedom and 16 "viscous" off-attractor degrees of freedom.

Du Plessis, Scheihing-Hitschfeld, **RS**, arXiv:2607.16393

Attractodynamics in the 30-moment approximation

We also therefore need extra terms in our ansatz

$$\delta f = f_{\mathcal{A}}(\alpha^{\mathcal{A}}, \beta^{\mathcal{A}}, \mathcal{U}^{\mathcal{A}}) \quad \cdot \left(\alpha_\mu \frac{p^\mu}{E} \quad + \quad \beta_{\mu\nu} \frac{p^\mu p^\nu}{E} \quad + \quad w_{\mu\nu\lambda} \frac{p^\mu p^\nu p^\lambda}{E} \right).$$

$$\text{DOF:} \quad 4 \quad + \quad 10 \quad + \quad 16 \quad = 30$$

and new matching conditions

$$\langle E \rangle_{\delta f} = \langle E p^\mu \rangle_{\delta f} = \langle E p^\mu \mathcal{U}^{\mathcal{A}}_{\mu\nu} p^\nu \rangle_{\delta f} = 0\,,$$

$$\text{DOF:} \quad 1 \quad + \quad 4 \quad + \quad 9 \quad = 14$$

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Connection to Adiabatic Hydrodynamization (AH)

Consider Bjorken expansion in an infinite homogeneous medium with diffusive elastic small-angle scattering,

$$\frac{\partial f}{\partial \tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} = C[f] \text{ where } C[f] = q[f]\nabla_p^2 f\,.$$

Now following AH, apply a time-dependent coordinate rescaling,

$$f(p_\perp, p_z, \tau) = A(\tau) w\left(\frac{p_\perp}{B(\tau)}, \frac{p_z}{C(\tau)}, \tau\right).$$

and rewrite as an effective Hamiltonian evolution,

$$Hw \equiv \underset{\uparrow}{-}\partial_y w, \quad \text{where } y \equiv \log\left(\frac{\tau}{\tau_0}\right)$$

Note: -1 instead of i

Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

Connection to Adiabatic Hydrodynamization (AH)

This effective Hamiltonian is adiabatic and exactly solvable with ground state

$$\ln f_{\mathcal{A}} = \left(\alpha^{\mathcal{A}} + \beta_{\mu}^{\mathcal{A}} p^{\mu} + \frac{1}{2} p^{\mu} \mathcal{U}_{\mu\nu}^{\mathcal{A}} p^{\nu} \right)$$

where

$$\alpha^{\mathcal{A}} = \ln A \,, \quad \mathcal{U}_{xx}^{\mathcal{A}} = \mathcal{U}_{yy}^{\mathcal{A}} = -\frac{1}{B^2} \,, \quad \mathcal{U}_{zz}^{\mathcal{A}} = -\frac{1}{C^2}$$

and all others zero. Anisotropic and far from thermal!

Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

The on-attractor degrees of freedom $A(\tau)$, $B(\tau)$, and $C(\tau)$ can be used to write the attractodynamics equivalent of ideal hydrodynamics.

Simplifications in 0+1D

We have 3 on-attractor degrees of freedom $(\alpha^{\mathcal{A}}, \mathcal{U}_{xx}^{\mathcal{A}}, \mathcal{U}_{zz}^{\mathcal{A}})$, and 2 off-attractor degrees of freedom Π and π, where $\pi^{\mu\nu} = (0, -\pi/2, -\pi/2, \pi)$. The ansatz for δf simplifies to

$$\delta f = f_{\mathcal{A}}(\alpha^{\mathcal{A}}, \mathcal{U}_{xx}^{\mathcal{A}}, \mathcal{U}_{zz}^{\mathcal{A}}) \cdot \left(\alpha + \beta_0 p + w_{00} p^2 + w_{33} p_z^2 + v_{33} \frac{p_z^2}{p} \right) .$$

We need to calculate five moments,

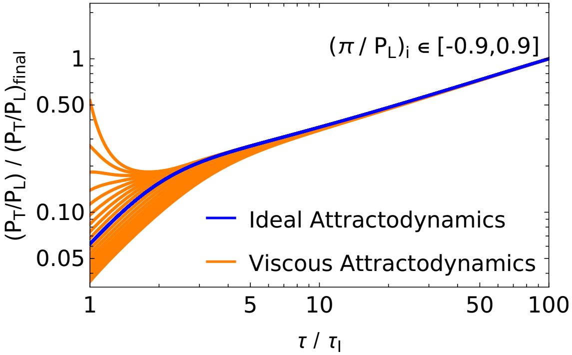
$$\mathcal{N} = \langle E \rangle_f \ , \ \ \mathcal{E} = \langle E^2 \rangle_f \ , \ \ \mathcal{P}_z = \langle p_z^2 \rangle_f \ ,$$
$$\mathcal{I}_0 = \langle E^3 \rangle_f \ , \ \ \text{and} \ \ \mathcal{I}_z = \langle E p_z^2 \rangle_f \ .$$

and we need three matching conditions

$$\langle E \rangle_{\delta f} = \langle E p_z^2 \rangle_{\delta f} = \langle E^3 \rangle_{\delta f} = 0 \, .$$

Attractodynamics: proof of principle

$$\partial_y \pi = \int \frac{d^3p}{(2\pi)^3} p \left(\left(\frac{p_z}{p} \right)^2 - \frac{1}{3} \right) \partial_y \delta f \,, \quad \partial_y \Pi = \frac{1}{3} \int \frac{d^3p}{(2\pi)^3} p \, \partial_y \delta f$$



As expected, excitations relax towards the attractor surface.

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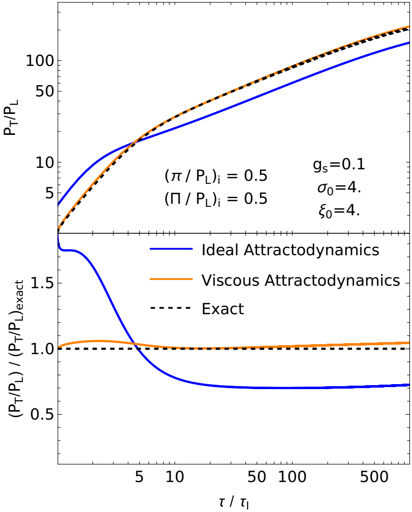
Attractodynamics: proof of principle



Choosing one initial condition, we can check against a numerical solution to the Boltzmann equation.

Including "viscous" corrections gives a marked improvement over "ideal" attractodynamics!

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Outlook

- We have derived a macroscopic effective theory of dynamics near an attractor, directly analogous to hydrodynamics.
- In a highly simplified example, we have demonstrated that attractodynamics behaves as expected!
- The next step is developing attractodynamics into a full 3+1D macroscopic theory.
- This will allow us to calculate observables like ellipticity, potentially giving insight into collectivity in small systems!