Gravitational waves from phase transitions in dark sectors

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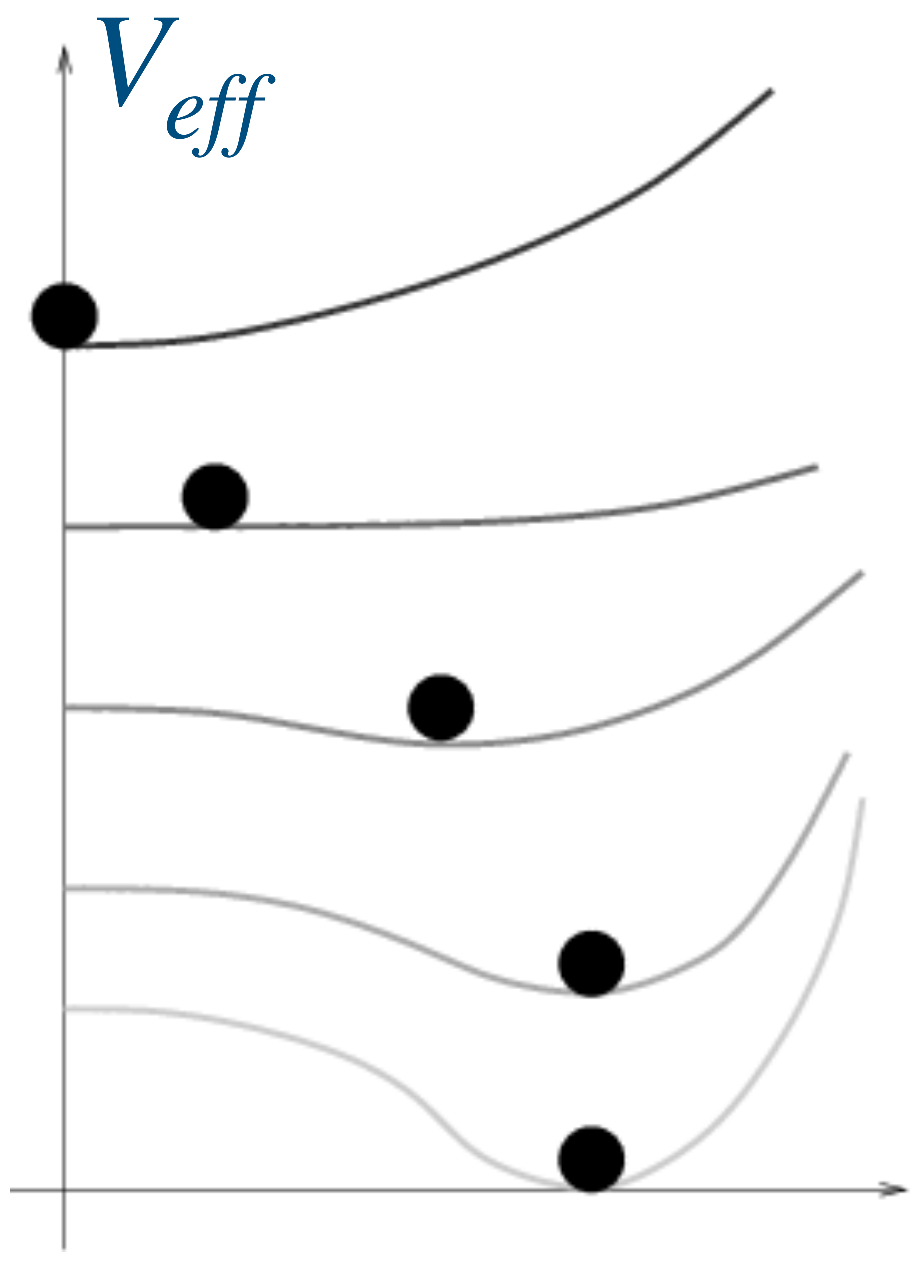
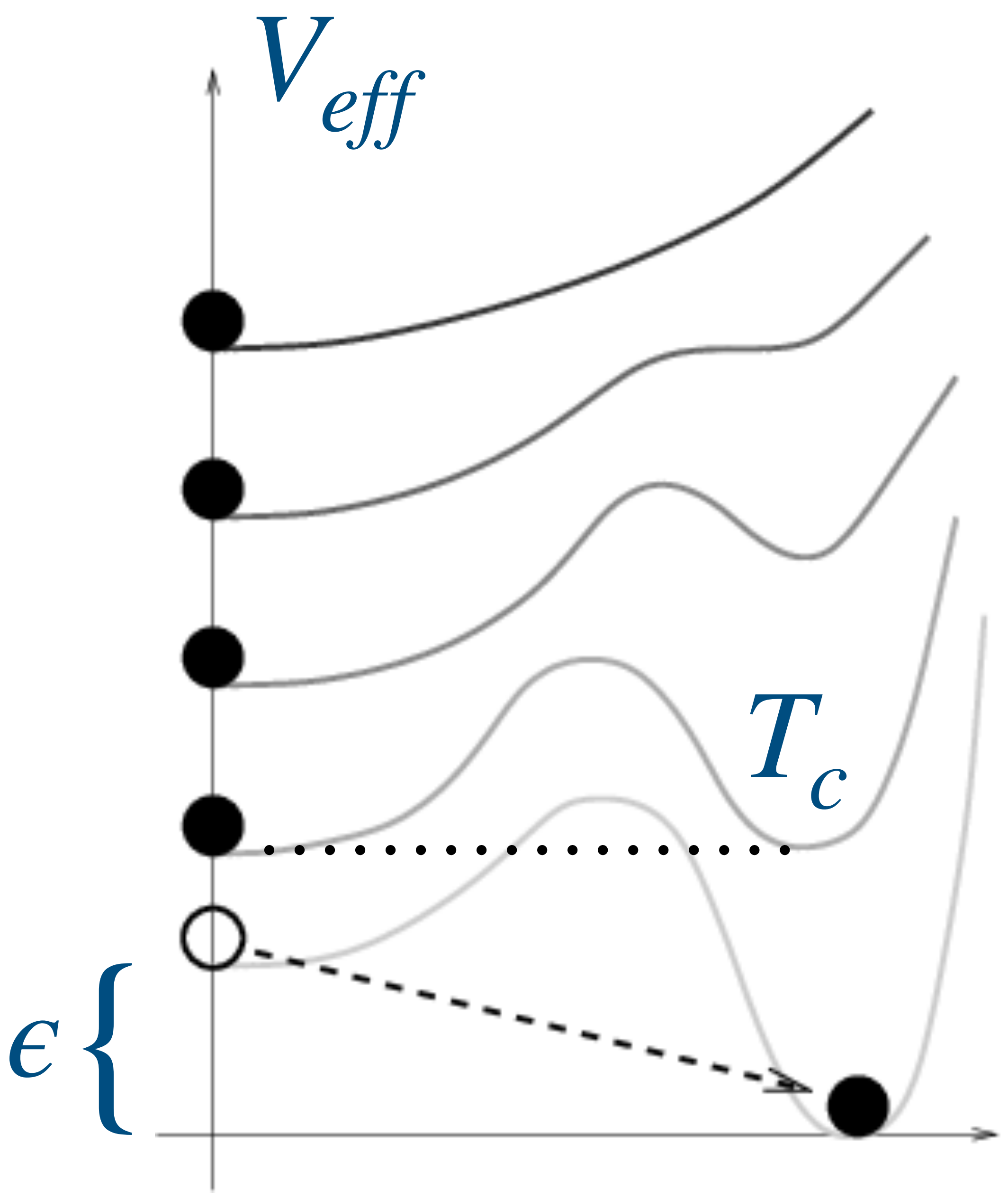
- First order phase transition within the dark sector
- GW from fully coupled dark sector **vs** secluded dark sector
- Hydrodynamic model for the intermediate regime
- Implications for the GW spectrum using the Sound Shell Model

First order phase transition within the dark sector



Smooth crossover

$T \downarrow$



1st order PT

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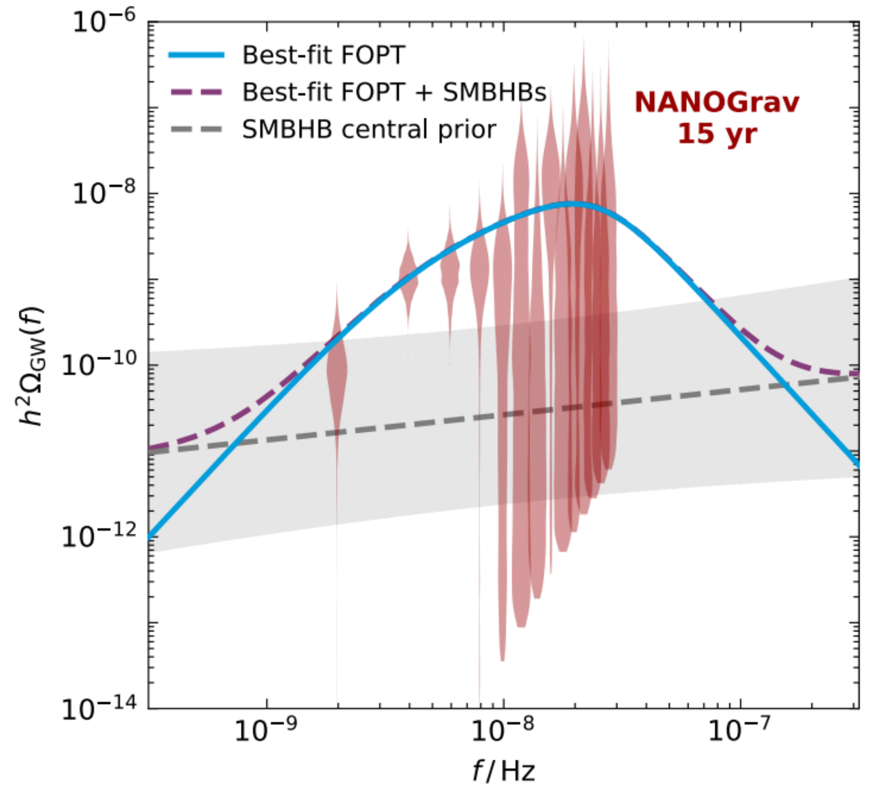


[2602.09092]

- No GW
- Realised in SM (EW PT, QCD confinement PT)

- Produces GW!
- Requires some additional fields coupled to SM Higgs or new dark sector with scalars…

Image credit: [1504.03587]

First order phase transition within the dark sector

We assume PT to happen but do not discuss any concrete details!

Sources of GWs in the 1st order PT:

- bubble-collisions wall,
- **sound waves in plasma - main source for the moderately strong phase transitions** [1910.13125],
- turbulence.

Sound waves propagate long after phase transition completion:

$$\tau_{\rm sw} \sim R_*/\sqrt{\Omega_K} \sim \mathcal{O}(10)\, R_* \text{ with } R_* \sim 1/\beta$$

(R_* - mean bubble separation, β - inverse PT duration)

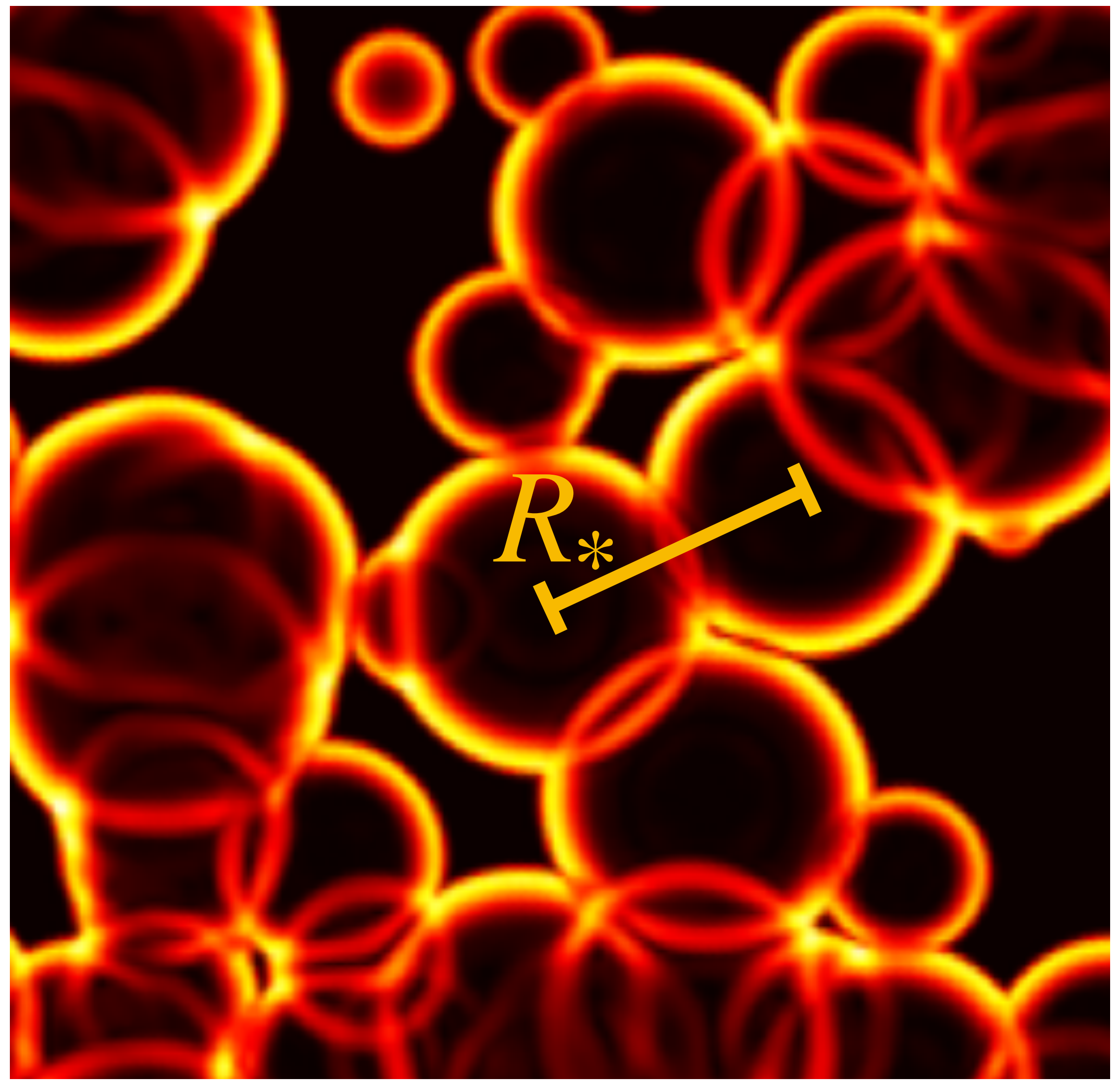
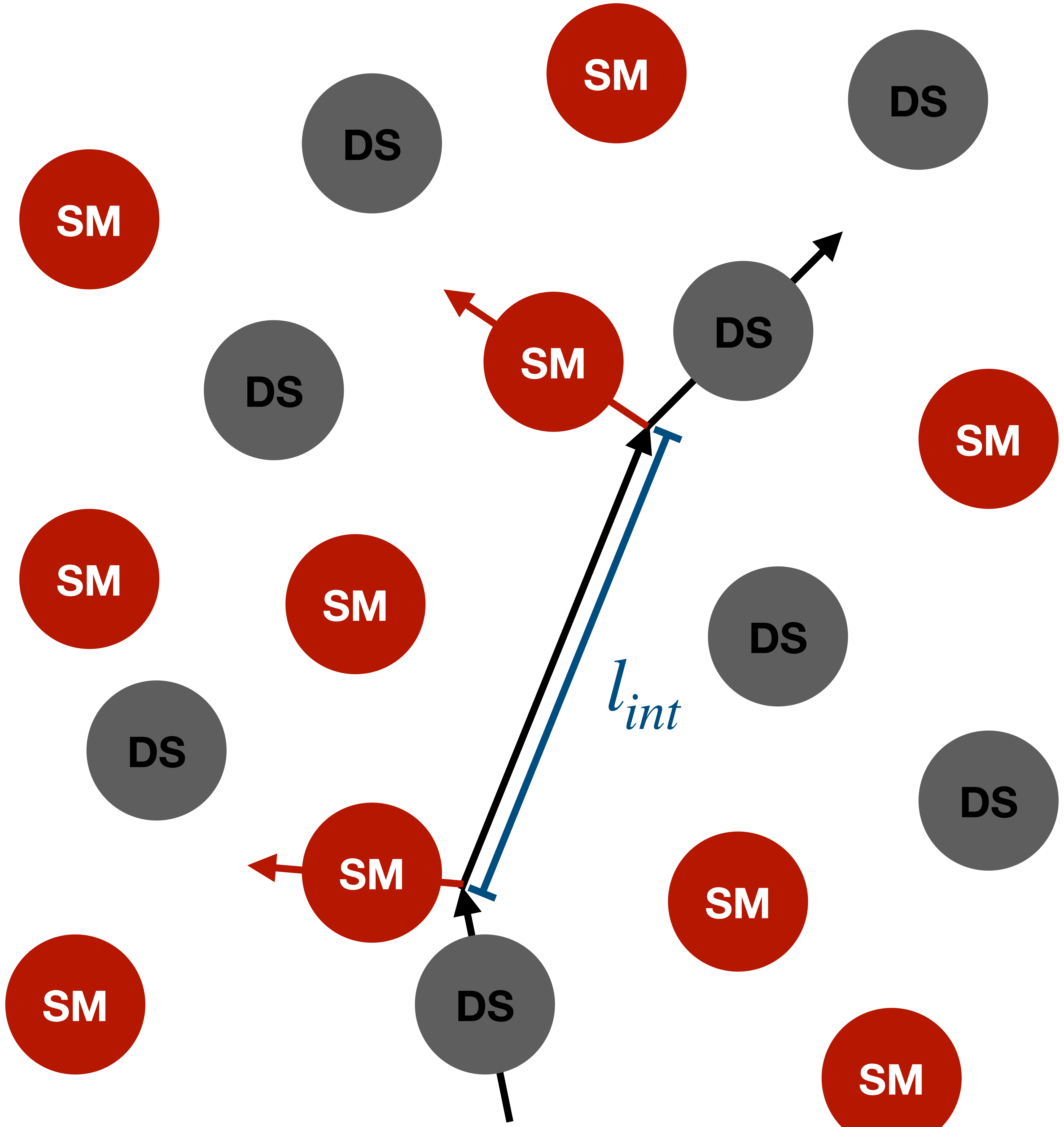


Image credit: [2209.04369]

Interaction between SM and dark sector

We assume some interaction rate between two relativistic fluids.



- Interaction length scale l_{int}:

$$\bar{l}_{\mathrm{int}} \sim \tau_{\mathrm{int}} \sim \frac{1}{\Gamma_{\mathrm{int}}}$$

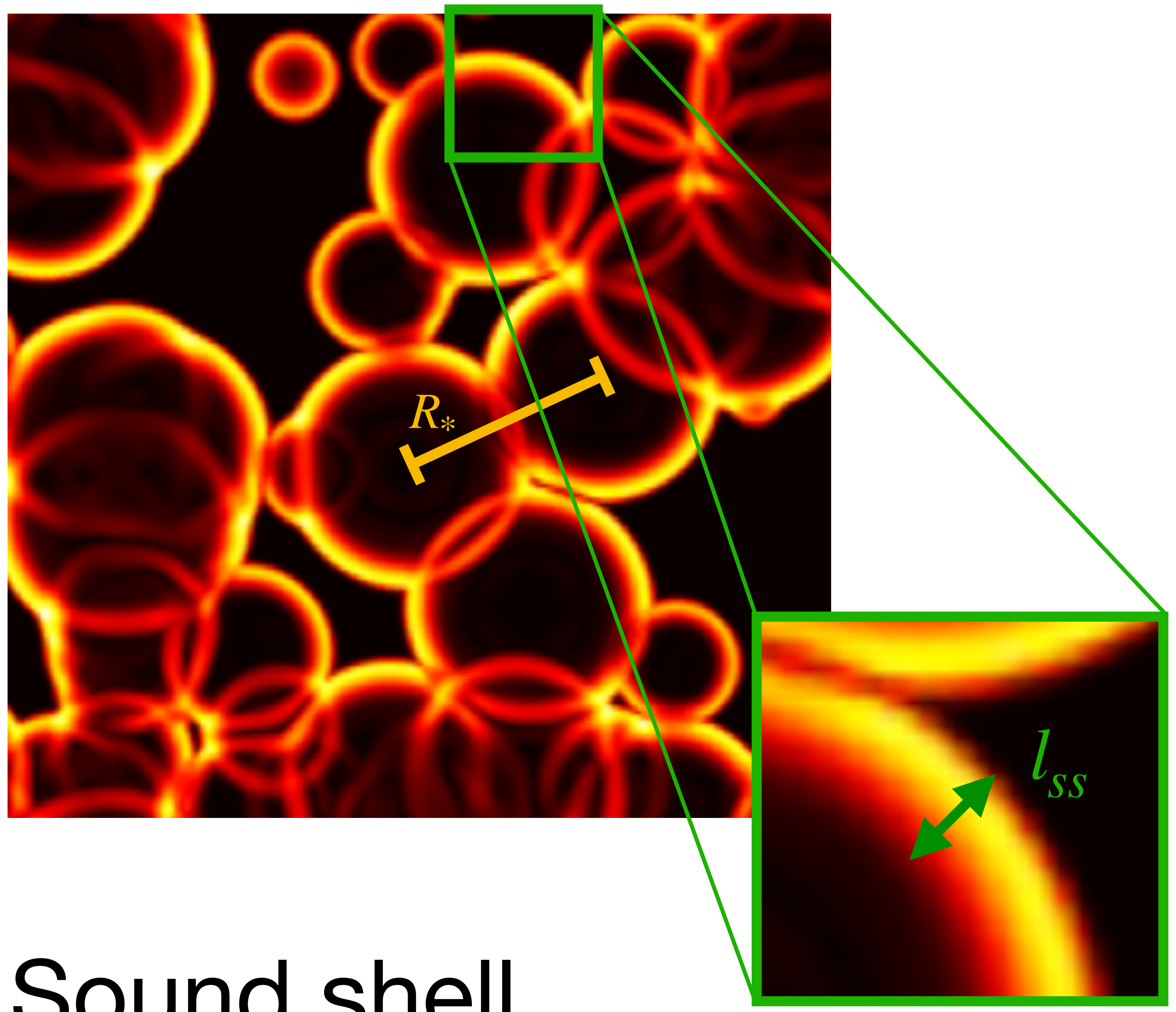
- Interaction rate Γ_{int} for 2→2 scattering:

$$\Gamma_{\mathrm{int}} \sim \sigma_{2\to 2}\, n$$

Cross section Number density

Fully coupled vs secluded dark sector

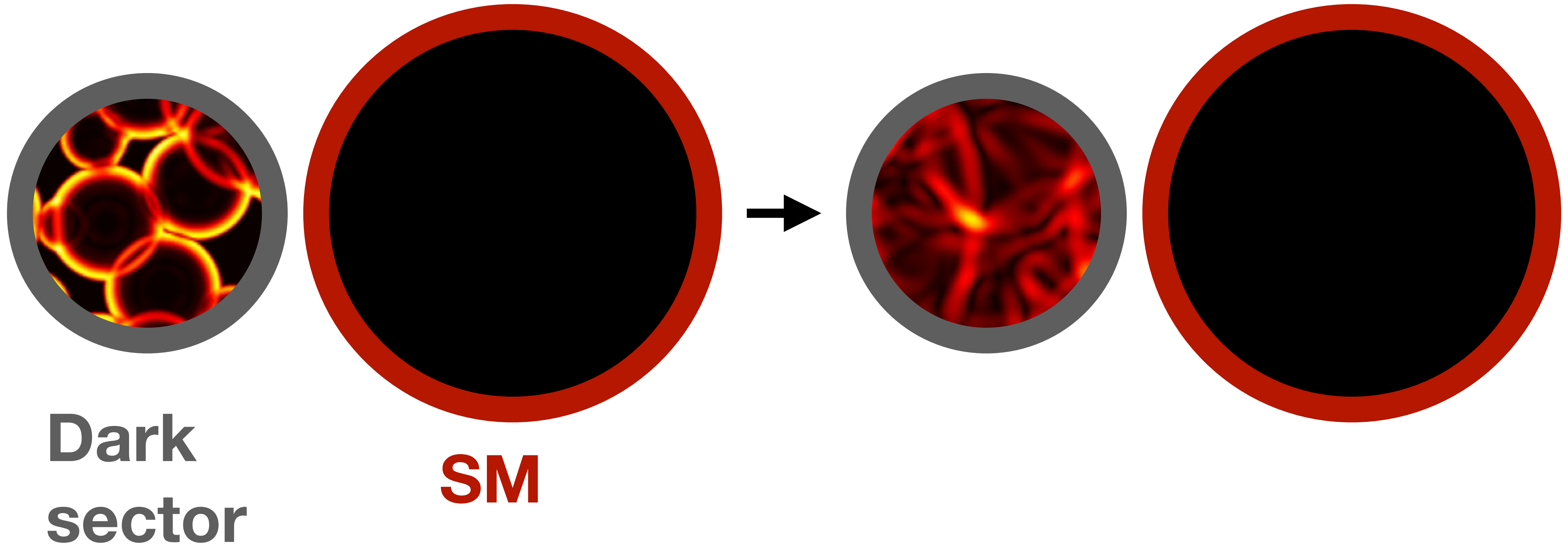
Image credit: [2209.04369]



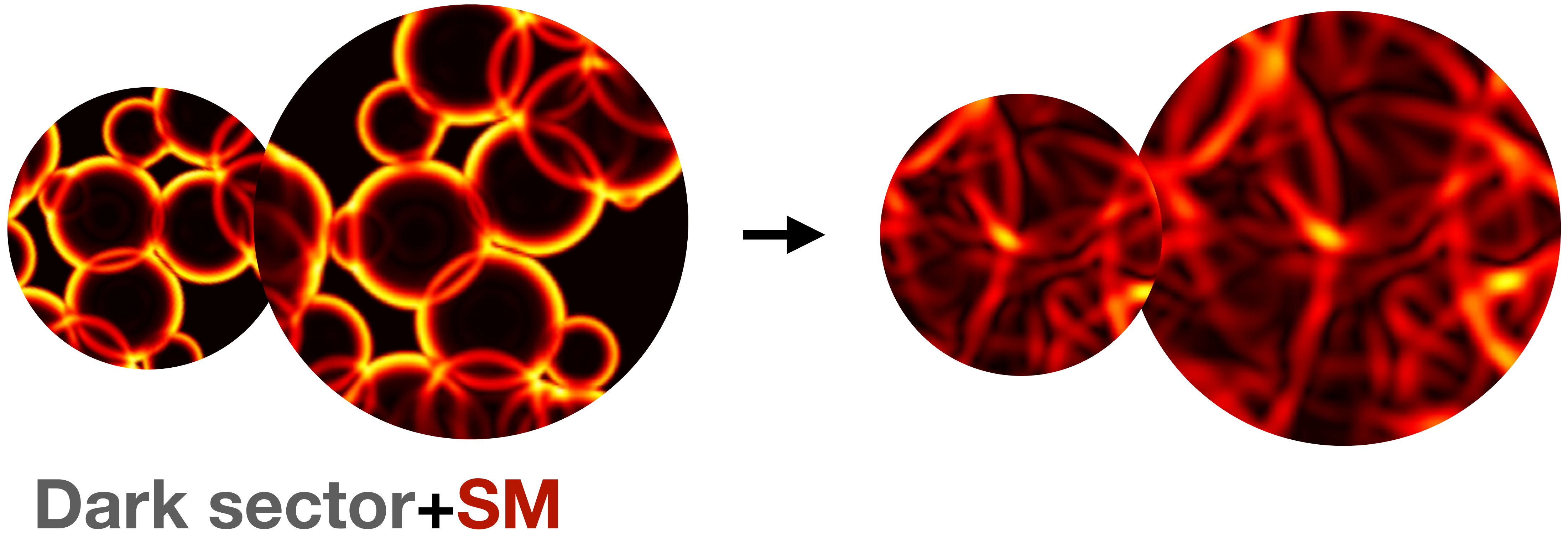
Sound shell thickness:

$$l_{ss} = R_* \frac{|v_w - c_s|}{v_w}$$

- If $l_{\text{int}} \gg l_{sw} \sim \tau_{sw}$



- If $l_{\text{int}} \ll l_{ss}$



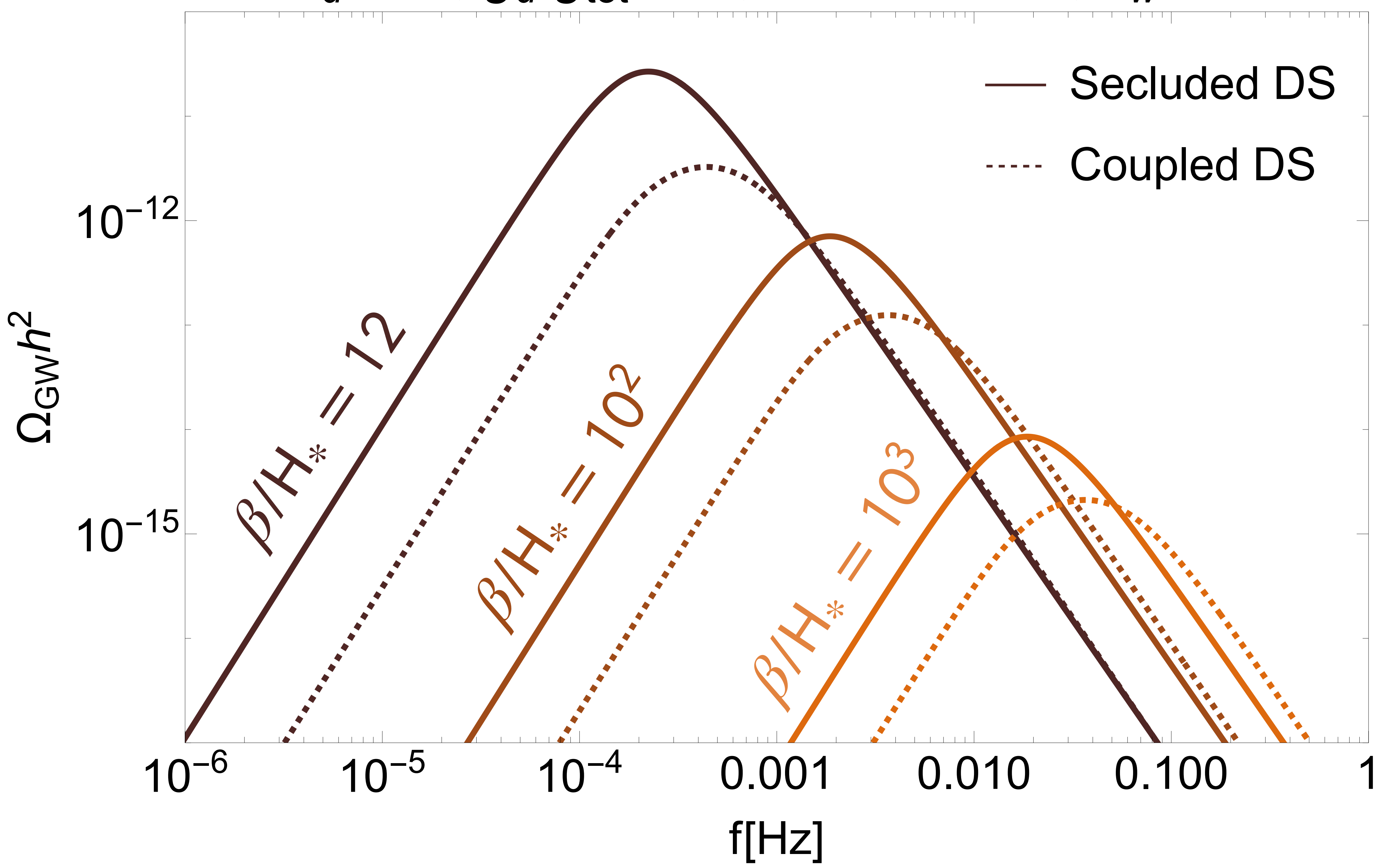
Fully coupled vs secluded dark sector

Strength parameter

Effective number of relativistic d.o.f.

Percolation temperature

α_d=0.5, $g_d/g_{\rm tot}$=0.1, T_*=500 GeV, v_w=0.6



GW spectra based on [2409.03651], plotted using **CosmoGW** package

GW signal suppressed for fully coupled dark sector compared to the secluded dark sector for the same latent heat released in the phase transition!

Intermediate regime: weakly coupled dark sector

Bubble growth assumed to be not affected, while the sound-wave propagation is.

$$R_* \ll l_{\rm int} < l_{\rm sw}$$

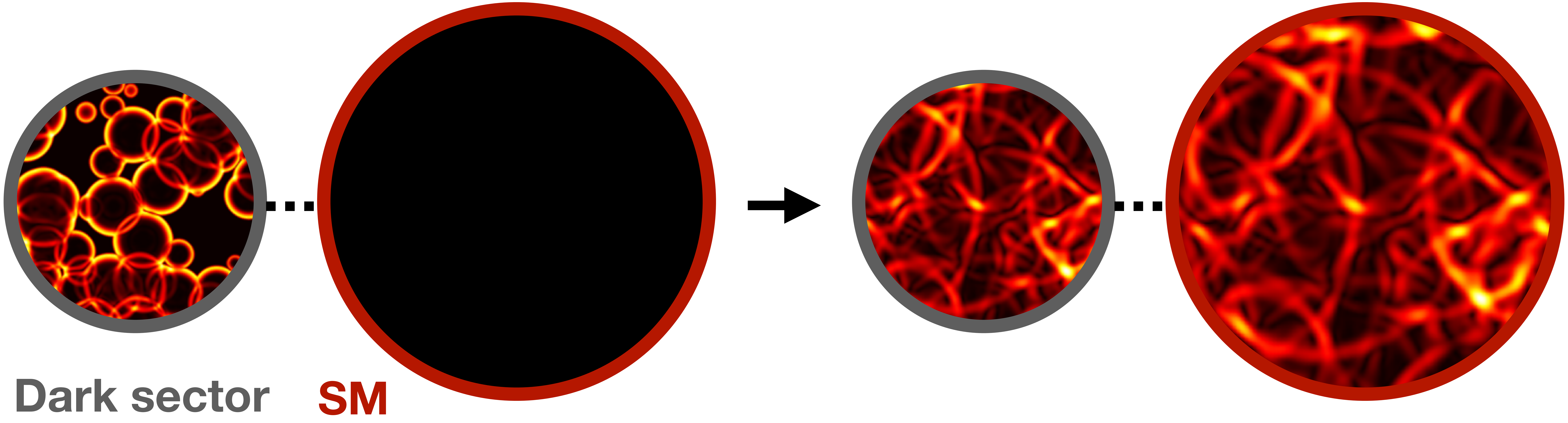
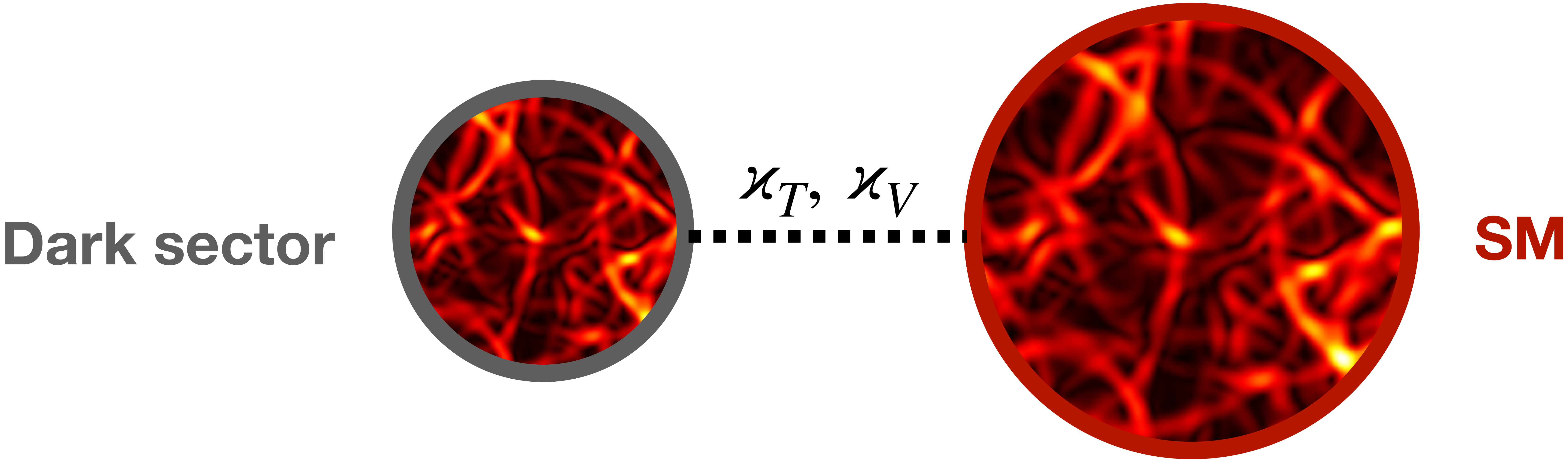


Image credit: [2209.04369]

Hydrodynamic model

We consider sound-waves propagation stage to 1st order in near-equilibrium expansion.



$$\begin{cases} \partial_\mu T^{\mu\nu}_d = Q^\nu, \\ \partial_\mu T^{\mu\nu}_v = -\,Q^\nu, \end{cases} \quad \text{with} \quad Q^\nu = \varkappa_T \left(\frac{T_v - T_d}{\overline{T}} \right) \bar{u}^\nu + \varkappa_V \left(u_v - u_d \right)^\nu$$

Model-dependent phenomenological parameters,
(can, in principle, be derived from the collision integral)
$\partial_\mu S^\mu \geq 0 \Rightarrow \varkappa_{V,T} > 0$

Image credit: [2209.04369]

Hydrodynamic model

Four linearised equations for small perturbations.

For $\lambda_\alpha \equiv \delta e_\alpha / \bar{w}_\alpha$ and $v^i_\alpha \ll 1$, we get

$$\begin{cases} \lambda'_\alpha + \partial_i v^i_\alpha = \dfrac{(-1)^{\delta_{\alpha d}+1}}{3\bar{w}_\alpha} \varkappa_T \left(\bar{T}\right) \left(\lambda_v - \lambda_d\right), & \alpha = d, v \\ (v^j_\alpha)' + c_s^2 \partial_j \lambda_\alpha = \dfrac{(-1)^{\delta_{\alpha d}+1}}{\bar{w}_\alpha} \varkappa_V \left(\bar{T}\right) \left(v^j_v - v^j_d\right), & \alpha = d, v, \end{cases}$$

Solve in Fourier space $\Rightarrow$ 4 eigenmodes:

$$\Omega_{+,\pm} = \pm\, i\omega, \quad \Omega_{-,\pm} = -\,\Gamma_{\text{int}} \pm \omega \sqrt{-1 + \frac{\delta\Gamma^2}{\omega^2}} = \overbrace{-\,\Gamma_{\text{int}} \pm i\omega}^{\text{LO ✅}} + \mathcal{O}\left(\frac{\delta\Gamma^2}{\omega^2}\right), \quad \omega = c_s k$$

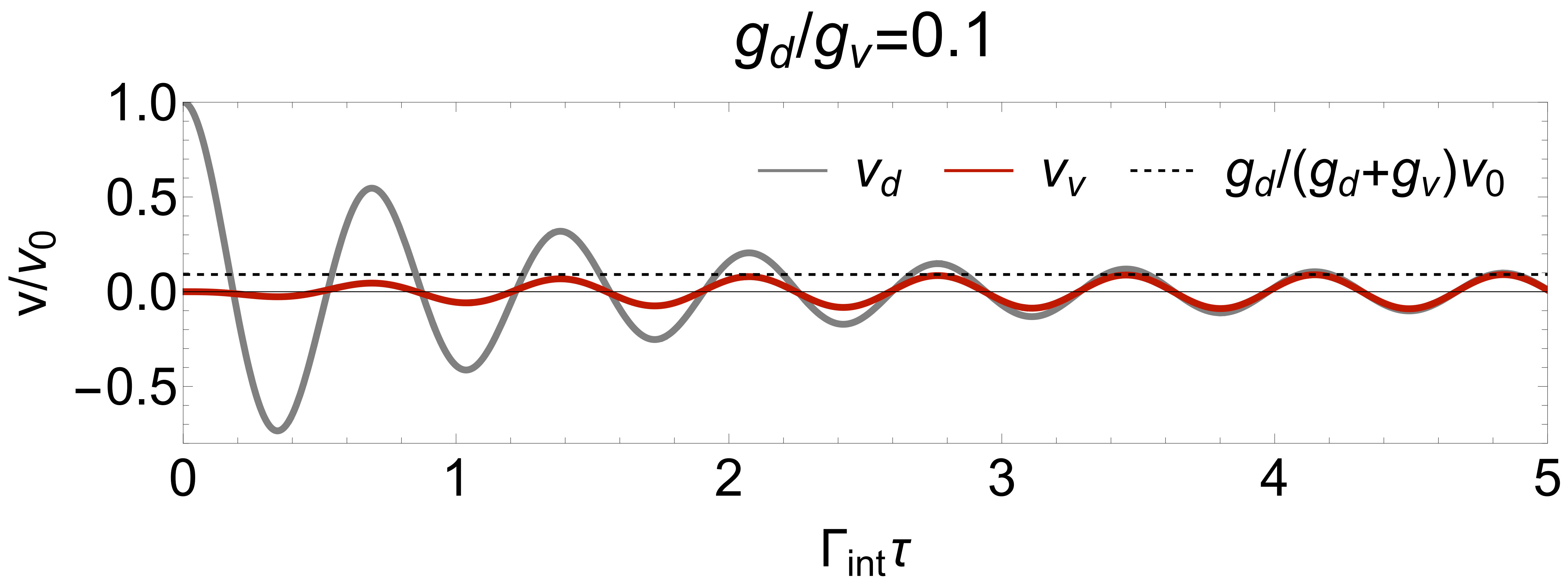
$$\Gamma_{\text{int}} \equiv \left(\frac{\varkappa_T}{3} + \varkappa_V\right) \frac{\bar{w}_d + \bar{w}_v}{2\bar{w}_d \bar{w}_v}, \qquad \delta\Gamma \equiv \left(\frac{\varkappa_T}{3} - \varkappa_V\right) \frac{\bar{w}_d + \bar{w}_v}{2\bar{w}_d \bar{w}_v}$$

Hydrodynamic model

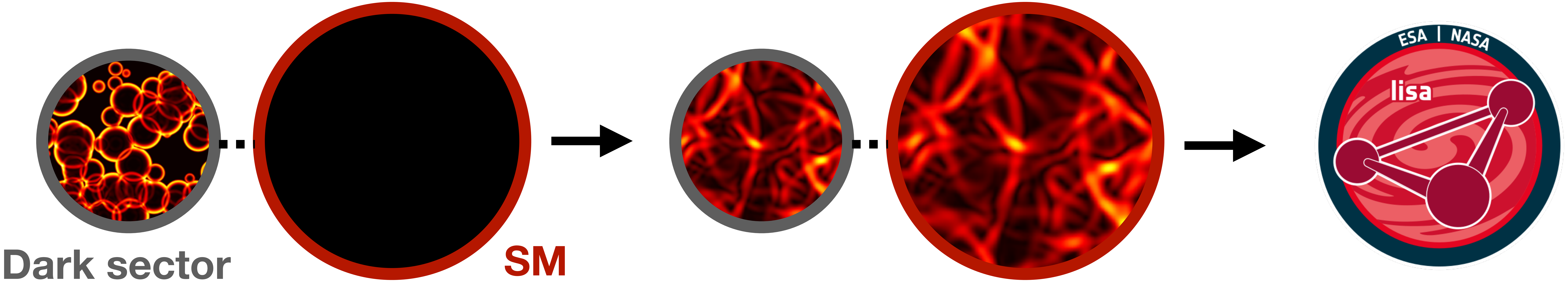
Simplified isotropic case

If the initial conditions are set to $v_d = v_0$ and $v_v = 0$ at $\tau = 0$, the velocity fields in the two fluids tend to equal value $v_{\text{fin}} = \dfrac{\bar{w}_d}{\bar{w}_d + \bar{w}_v} v_0$.

Effective numbers of relativistic degrees of freedom in dark/visible fluids, $g_d/g_v = \bar{w}_d/\bar{w}_v$



GW spectrum within the sound shell model



- Initial conditions for the velocity fields and energy density fluctuations:
  - Zero for the visible fluid
  - Given by a set of randomly distributed bubbles with self-similar velocity profiles in the dark fluid

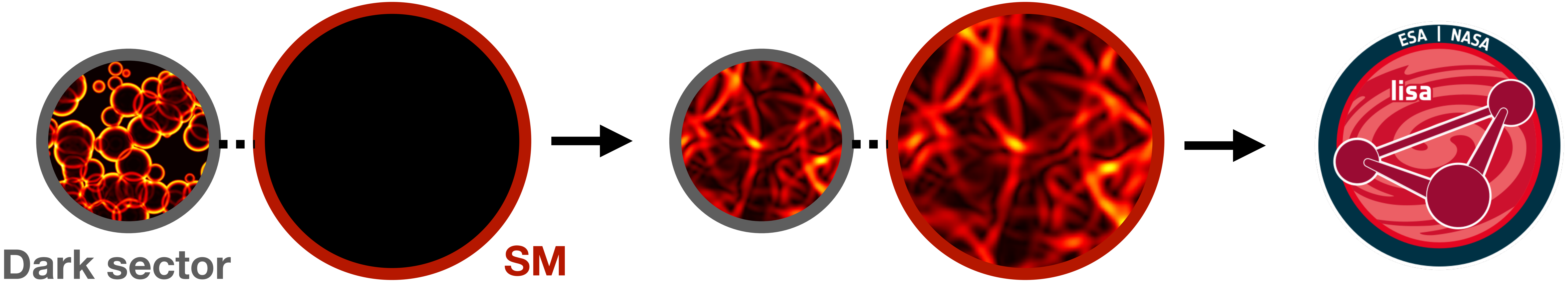
- Anisotropic velocity fields obtained as the solutions of the coupled wave equations

$$\delta T_{kl}(\tau, \mathbf{k}) = \sum_{\alpha=v,d} a^2 \bar{w}_\alpha v_{\alpha,k} v_{\alpha,l}$$

- Analytic results for the GW spectrum obtained by generalisation of the calculation within the sound shell model [1909.10040, 2308.12943]

Image credit: [2209.04369]

GW spectrum within the sound shell model



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$$\Omega_{\rm GW}(k) \approx \frac{3k}{2}\mathcal{T}_{\rm GW}\int_{\tau_*}^{\tau_f}\frac{{\rm d}\tau_1}{\tau_1}\int_{\tau_*}^{\tau_f}\frac{{\rm d}\tau_2}{\tau_2}\cos[k(\tau_1-\tau_2)]\,E_\Pi^{\rm sec}(\tau_1,\tau_2,k)\,F(\tau_1)\,F(\tau_2)$$

Transfer function accounting for the expansion of the Universe since GW production

$$\tau_f - \tau_* = \tau_{\rm sw}$$

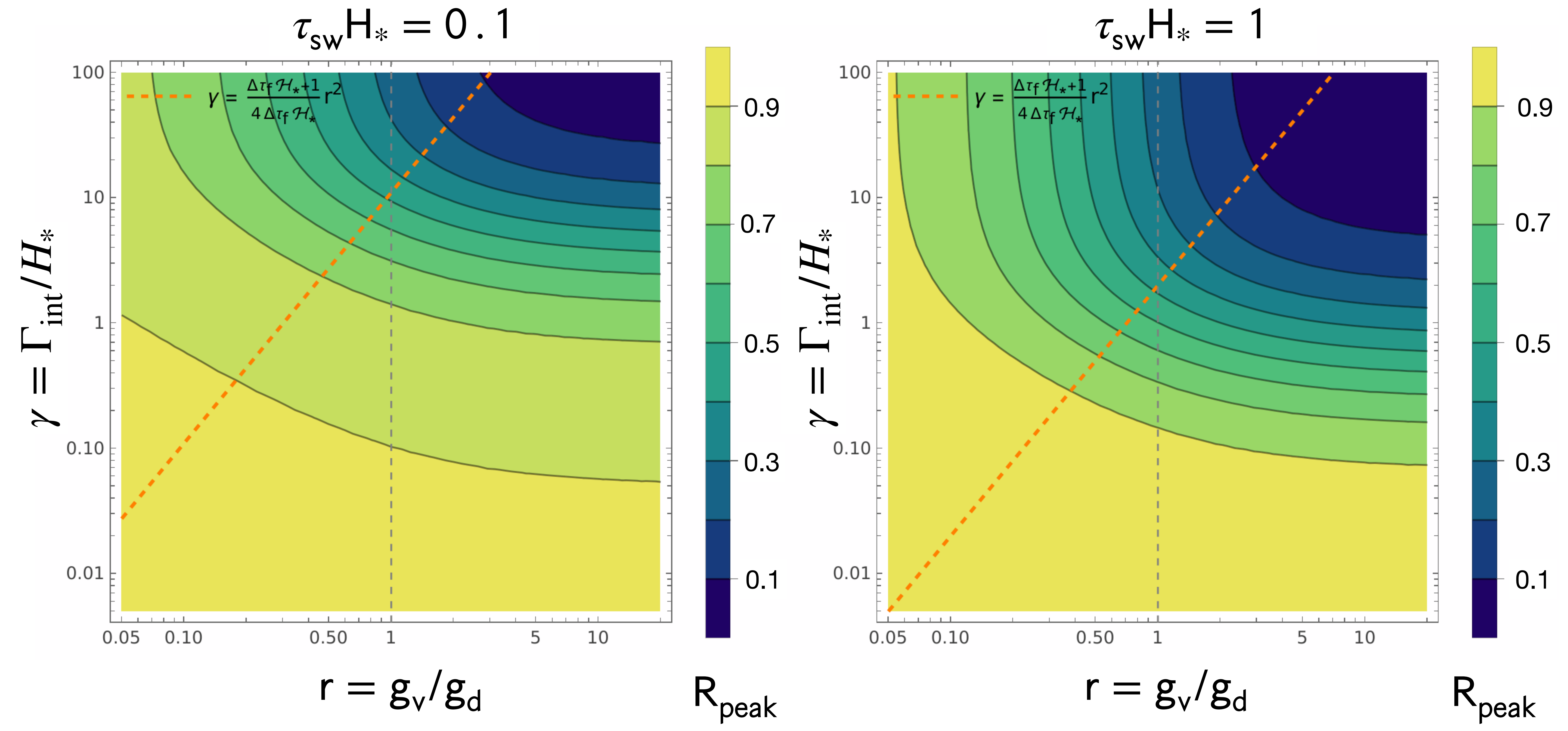
Unequal time correlator of the anisotropic stresses in the case of the secluded dark sector.

$$F(\tau) = \frac{\bar{w}_d + \bar{w}_v e^{-2\Gamma_{\rm int}(\tau-\tau_*)}}{\bar{w}_d + \bar{w}_v}$$

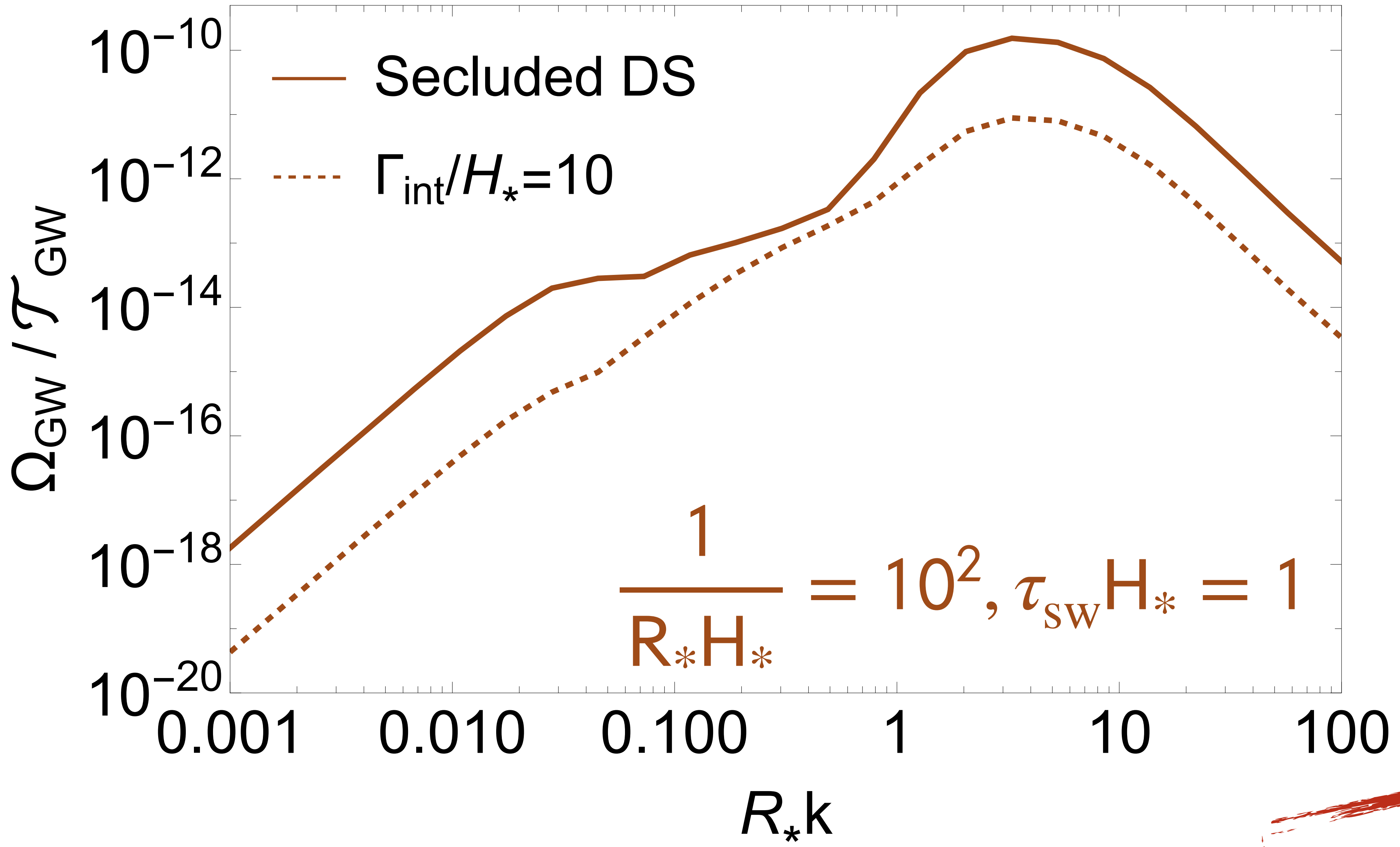
Image credit: [2209.04369]

Suppression of the peak amplitude

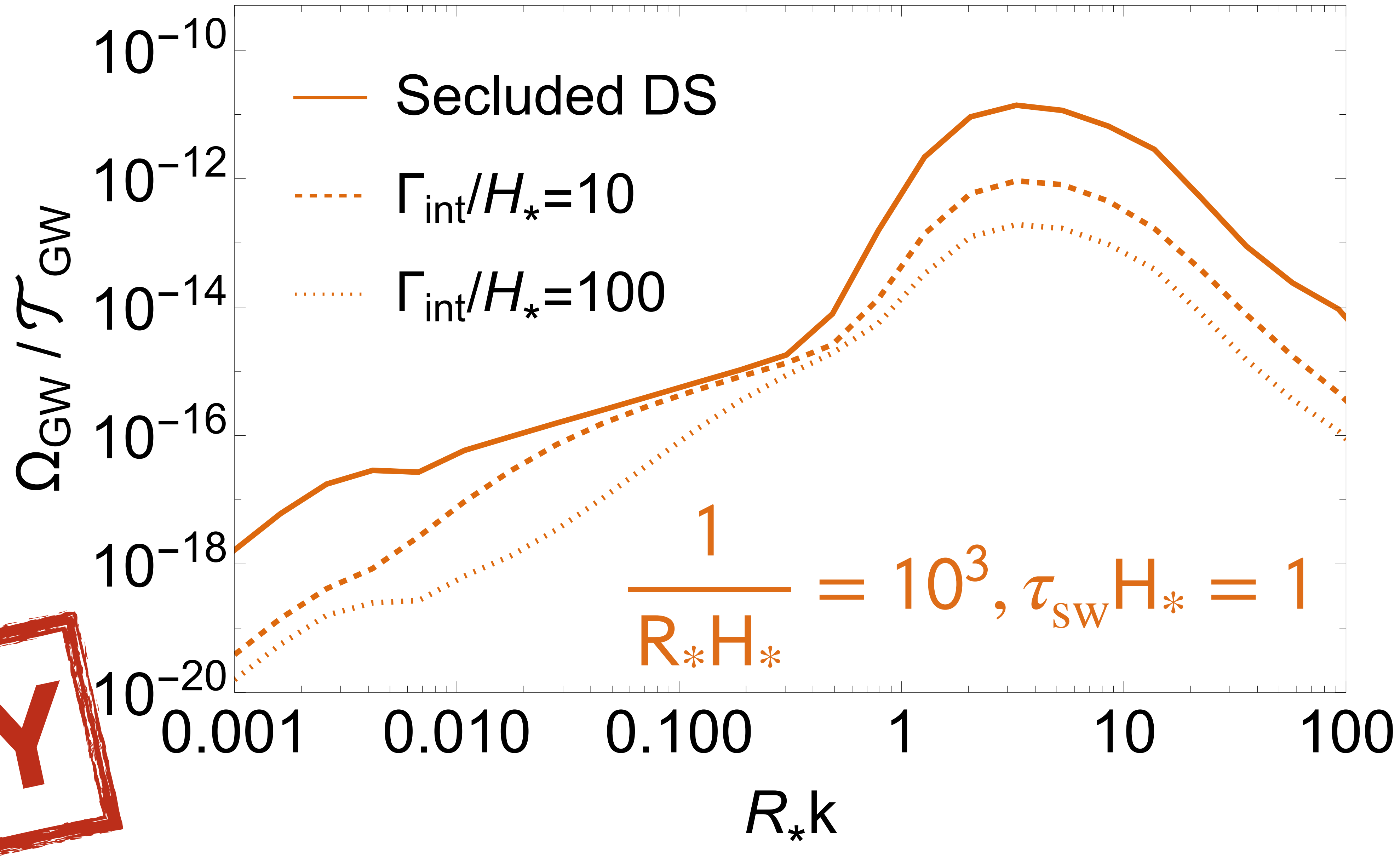
⇒ peak amplitude suppressed by a suppression factor $R_{\rm peak} \equiv \dfrac{\int_{\tau_*}^{\tau_f} \frac{d\tau}{\tau^2} F(\tau)^2}{\int_{\tau_*}^{\tau_f} \frac{d\tau}{\tau^2}}$:



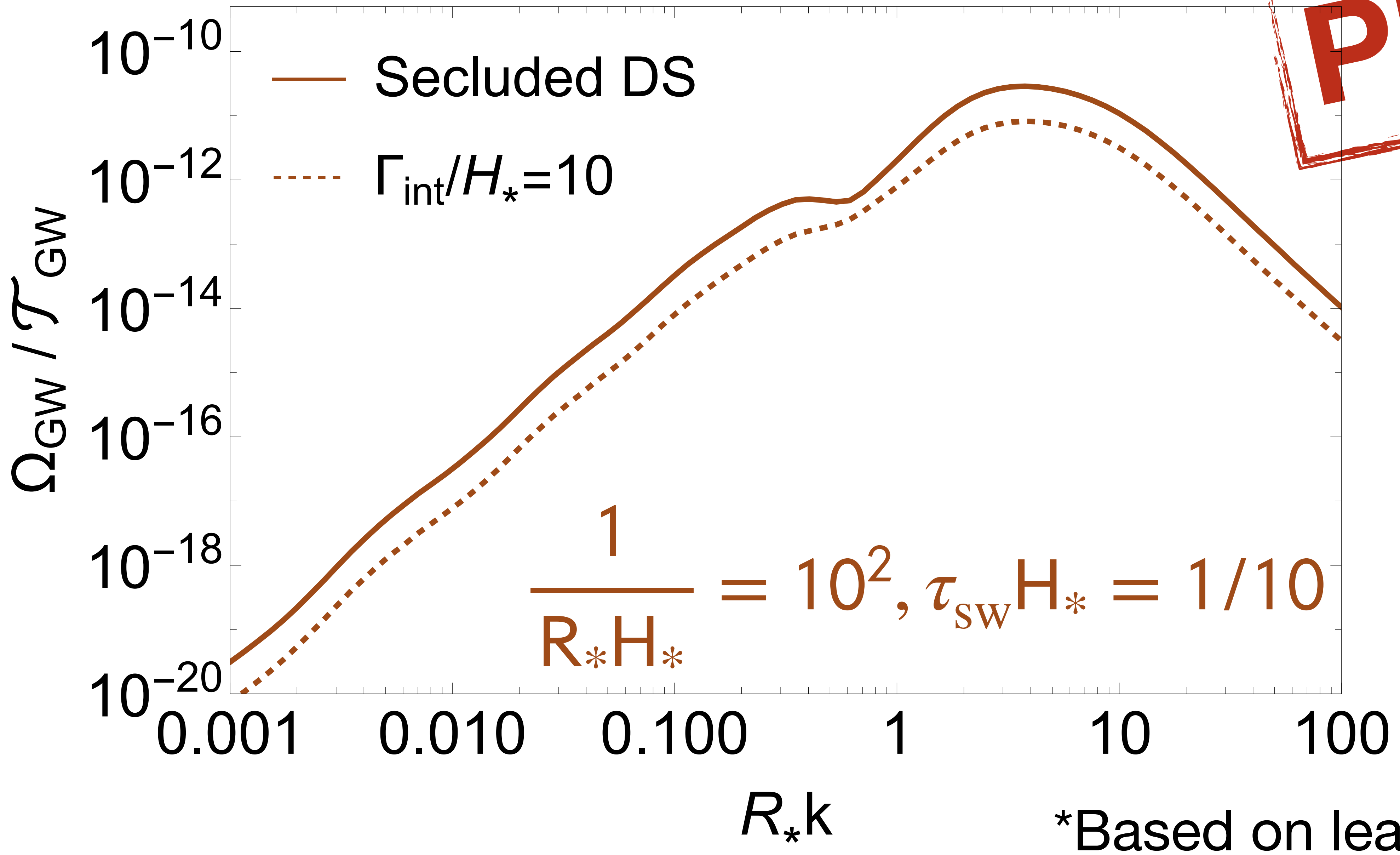
Full GW spectrum*

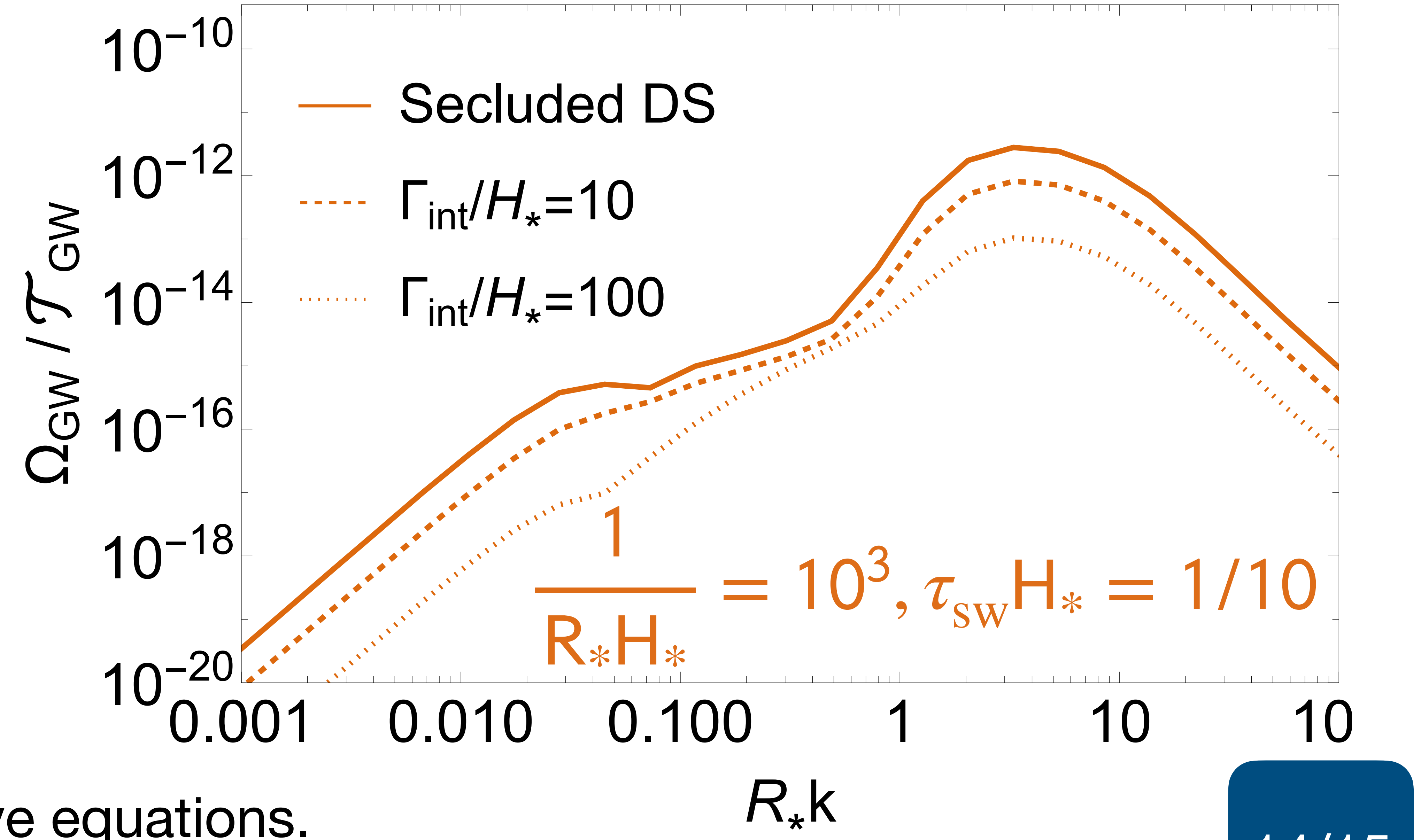


$\alpha_d=0.1$, $g_d/g_v=0.1$, $v_w=0.9$



PRELIMINARY





*Based on leading-order solution of the wave equations.

Conclusions

- GW signal from a dark sector phase transition is suppressed in presence of coupling to SM!

  (compared to secluded dark sector that shares the same temperature with SM)

- Further directions:
  - Inclusion of the NLO correction (work in progress!)
  - What if interactions are stronger and also bubble dynamics is affected?
  - Calculation of $\Gamma_{\rm int}$ in specific models.
  - All calculations assume sound waves propagating in a relativistic plasma.
    What if this is not true?
  - Is $\tau_{\rm sw} \sim \tau_{\rm nl}$ affected by the coupling between the fluids?

Thank you for your attention!

Back up

Validity of the leading order solution?

$$\begin{cases} \lambda'_\alpha + \partial_i v^i_\alpha = \dfrac{(-1)^{\delta_{\alpha d}+1}}{3\bar{w}_\alpha} \varkappa_T\left(\bar{T}\right)\left(\lambda_v - \lambda_d\right), & \alpha = d, v \\ (v^j_\alpha)' + c_s^2 \partial_j \lambda_\alpha = \dfrac{(-1)^{\delta_{\alpha d}+1}}{\bar{w}_\alpha} \varkappa_V\left(\bar{T}\right)\left(v^j_v - v^j_d\right), & \alpha = d, v, \end{cases}$$

Only the combination $\Gamma_{\rm int} \equiv \left(\dfrac{\varkappa_T}{3} + \varkappa_V\right)\dfrac{\bar{w}_d + \bar{w}_v}{2\bar{w}_d\bar{w}_v}$, appears in the **leading order solution**!

Assuming $\delta\Gamma \equiv \left(\dfrac{\varkappa_T}{3} - \varkappa_V\right)\dfrac{\bar{w}_d + \bar{w}_v}{2\bar{w}_d\bar{w}_v} \sim \mathcal{O}(\Gamma_{\rm int}) \ll c_s k$

- Requirement $\Gamma_{\rm int}\, R_* \ll c_s\, k\, R_*$ satisfied around the peak of the GW spectrum ($k\, R_* \sim \mathcal{O}(1)$) since $\Gamma_{\rm int} R_* \ll 1$ is assumed anyway
- But at small k, LO solution cannot be trusted!

GW from secluded or fully coupled dark sectors

- Let us assume a first order phase transition in a dark sector releasing vacuum energy ϵ. At the time of phase transition, total energy density of the Universe consists of dark and visible sector energy densities $\rho_{\rm tot} = \rho_d + \rho_v$. We define following phase transition parameters (assuming bag equation of state and denoting the dark/visible sector temperatures and effective numbers of relativistic degrees of freedom by $T_{d/v}$ and $g_{d/v}$):

$$\alpha_d = \frac{\epsilon}{\rho_d - \epsilon}\,, \qquad \alpha_{\rm tot} = \frac{\epsilon}{\rho_{\rm tot} - \epsilon} = \alpha_d\,\frac{g_d T_d^4}{g_d T_d^4 + g_v T_v^4}$$

- When calculating the gravitational wave (GW) spectrum sourced by the sound waves, the key quantity is the efficiency factor $\kappa_{\rm sw}$ determining the fraction of vacuum energy transferred to fluid motion:

Secluded dark sector:
$\kappa_{\rm sw}$ evaluated at α_d

Fully coupled dark sector:
$\kappa_{\rm sw}$ evaluated at $\alpha_{\rm tot} < \alpha_d \Rightarrow$ suppressed!