Preheating
after strongly supercooled first-order phase transitions

Mateusz Kulejewski

in collaboration with

Bogumiła Świeżewska and **Jorinde van de Vis**

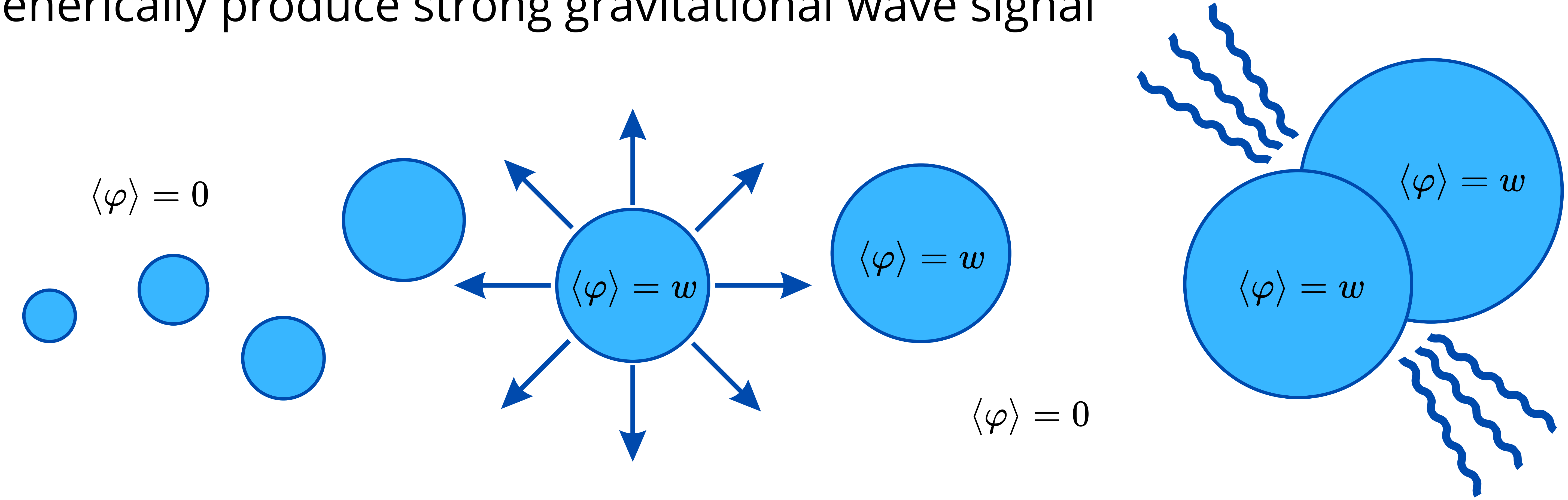
Strong and Electro-Weak Matter
University of Helsinki
19.8.2026

*

Supercooled phase transitions

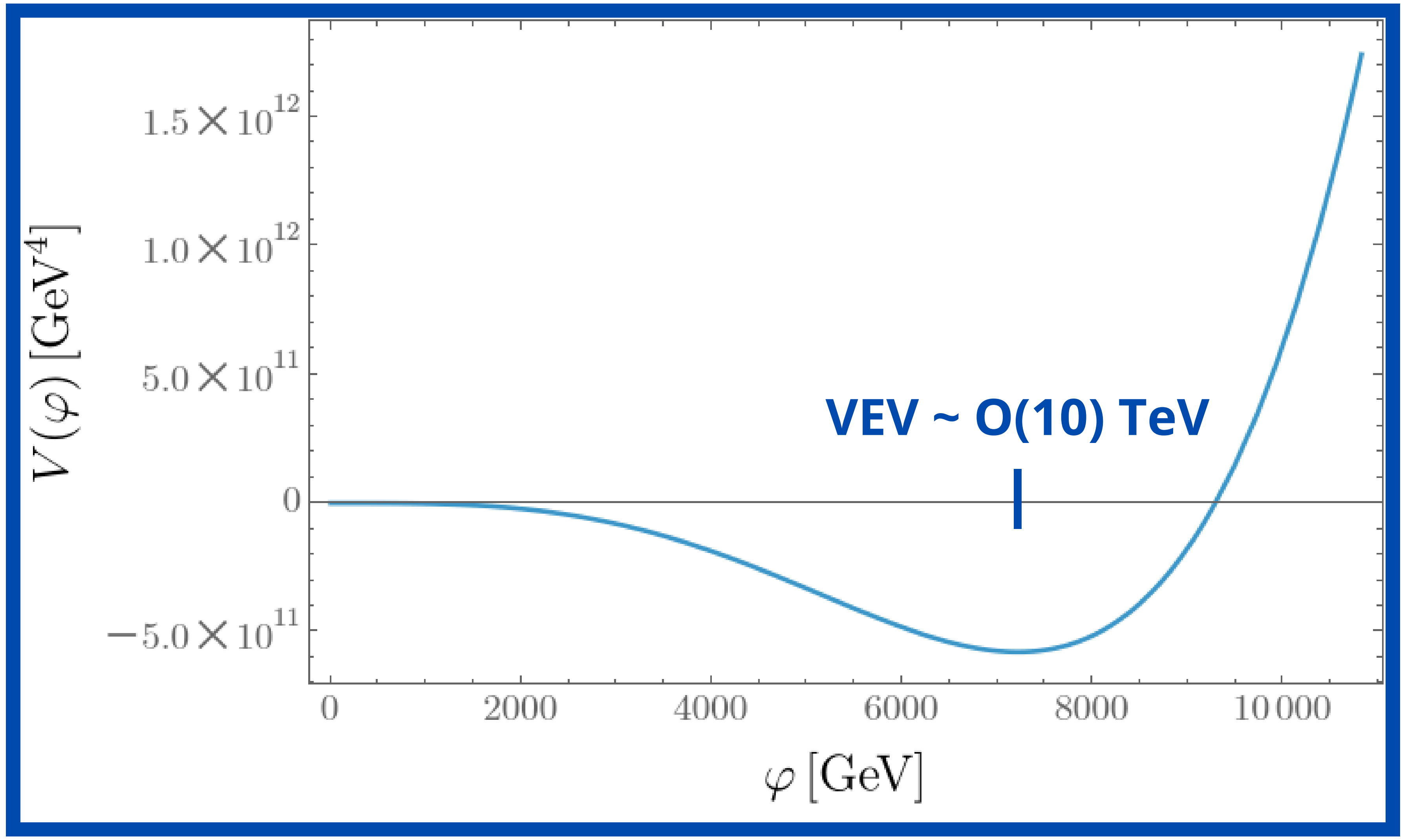
Classically scale invariant models

- motivated by the EW hierarchy problem
- provide a radiative origin of EW symmetry breaking
- include dark matter candidates
- generically produce strong gravitational wave signal



Phase transition dynamics

The effective potential

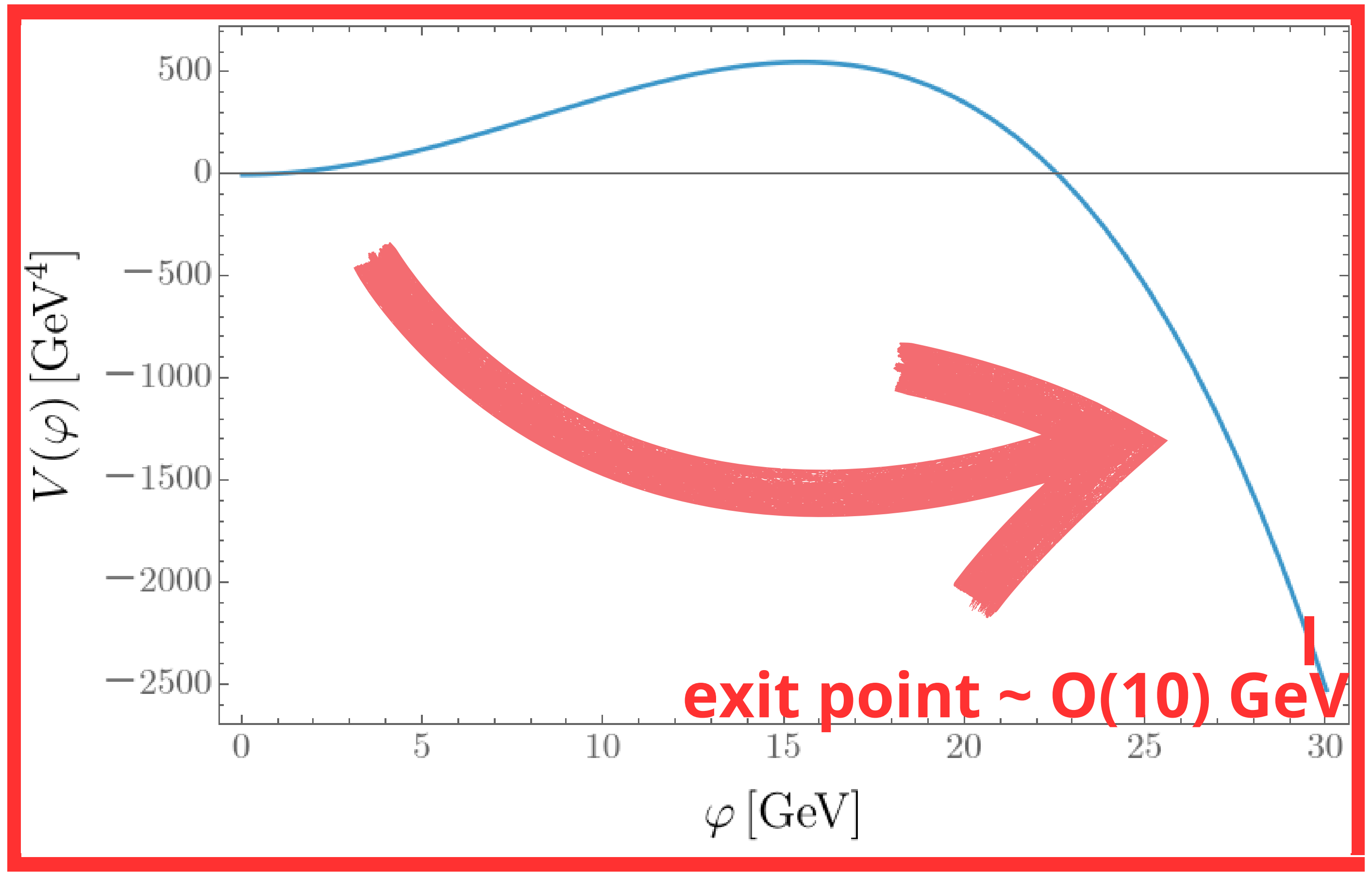


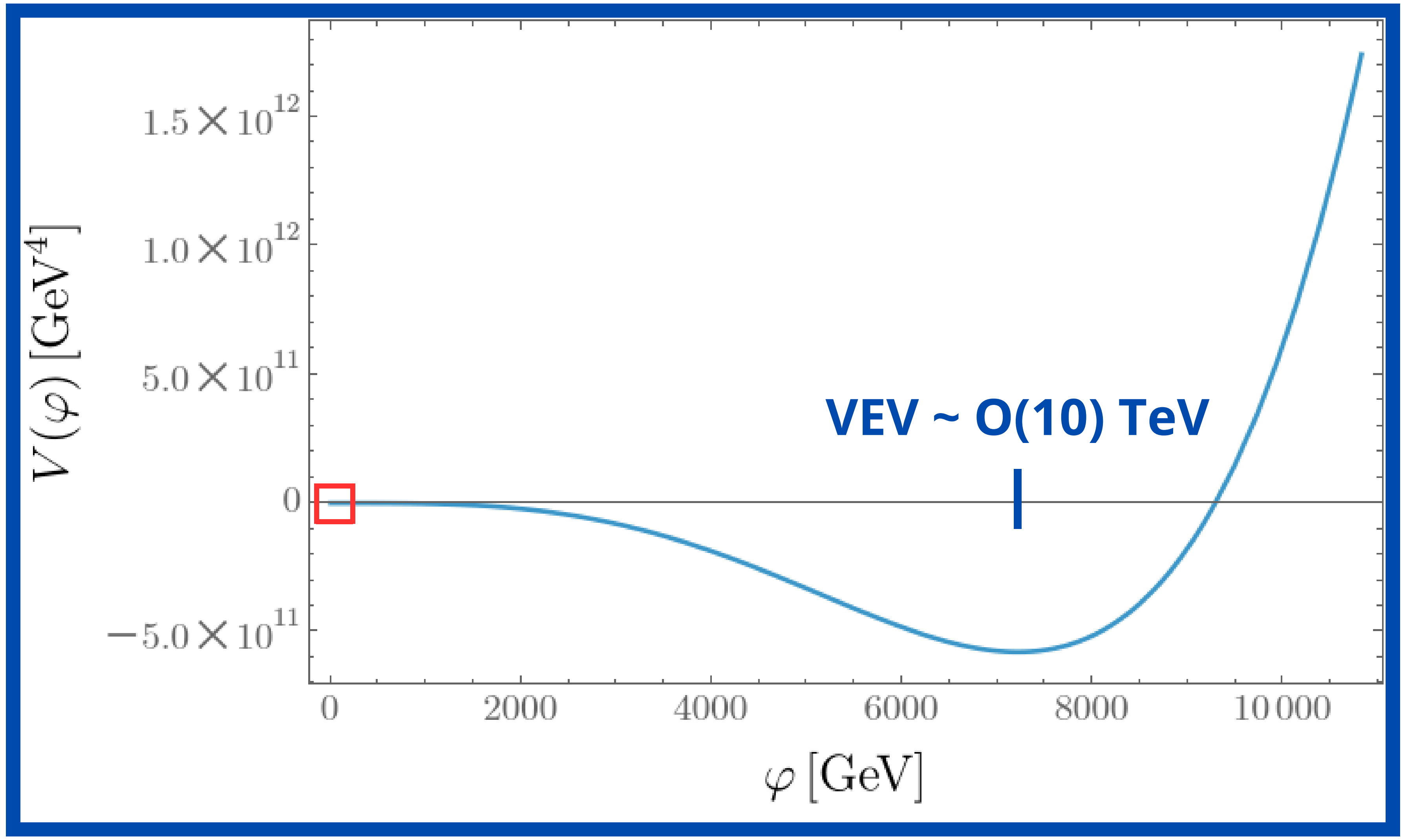
$$V(\varphi) = \frac{\lambda_{\mathrm{CW}}}{4}\varphi^4\left(\log\frac{\varphi^2}{w^2} - \frac{1}{2}\right) + V_T(\varphi)$$

M. Kierkla, A. Karam, B. Świeżewska, JHEP 03 (2023) 007

Phase transition dynamics

The effective potential



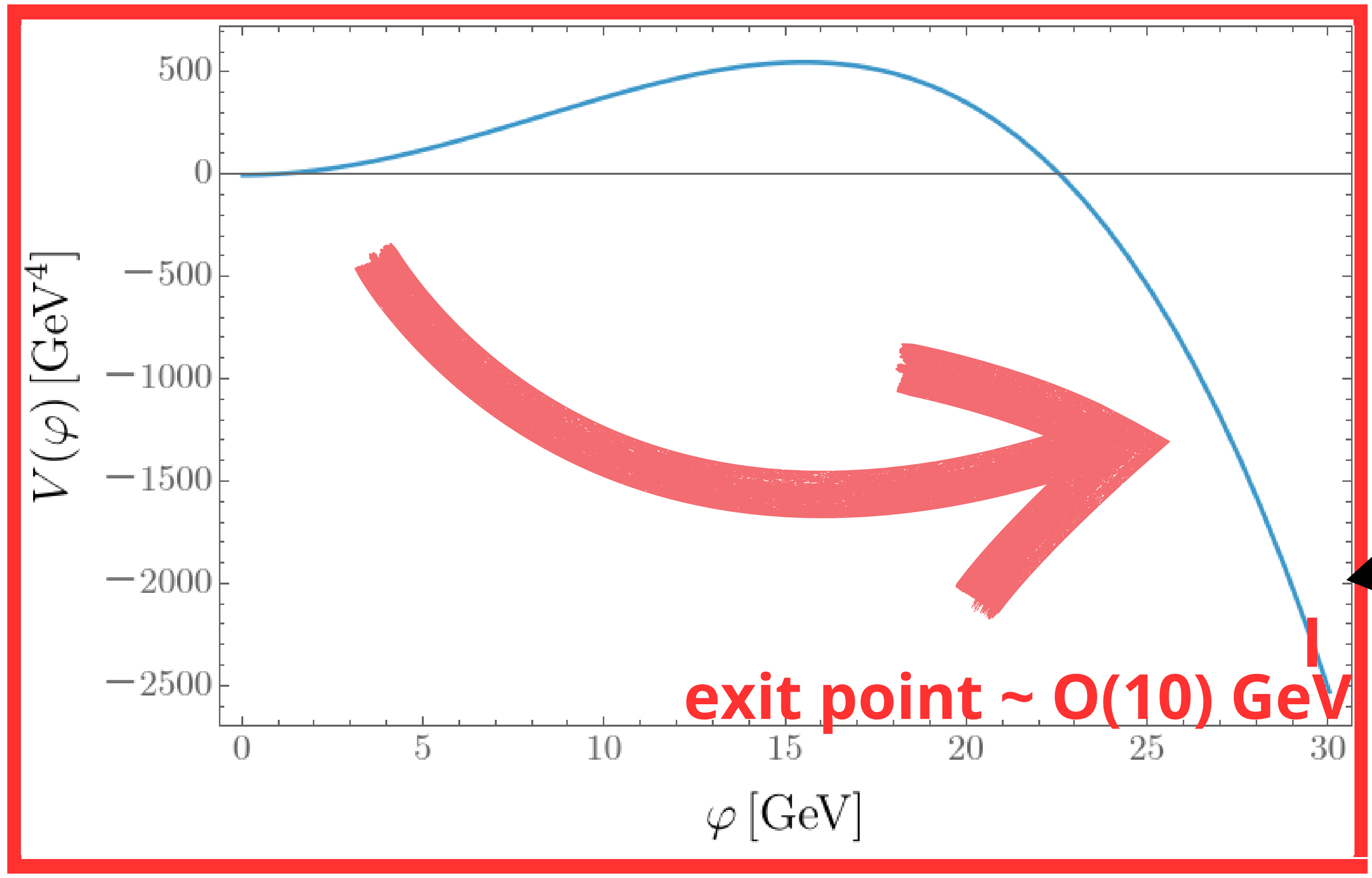


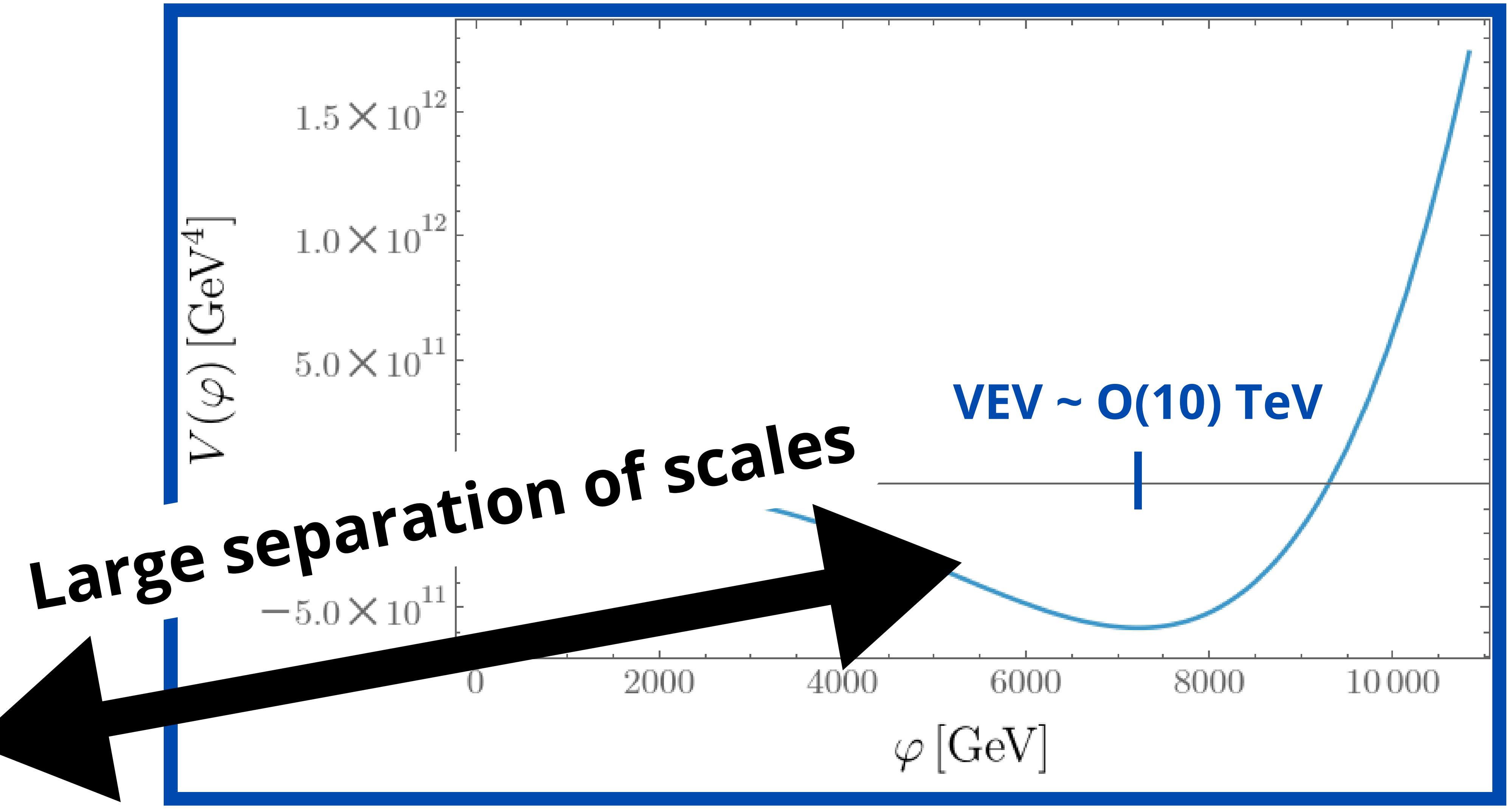
$$V(\varphi) = \frac{\lambda_{\mathrm{CW}}}{4}\varphi^4\left(\log\frac{\varphi^2}{w^2} - \frac{1}{2}\right) + V_T(\varphi)$$

M. Kierkla, A. Karam, B. Świeżewska, JHEP 03 (2023) 007

Phase transition dynamics

The effective potential



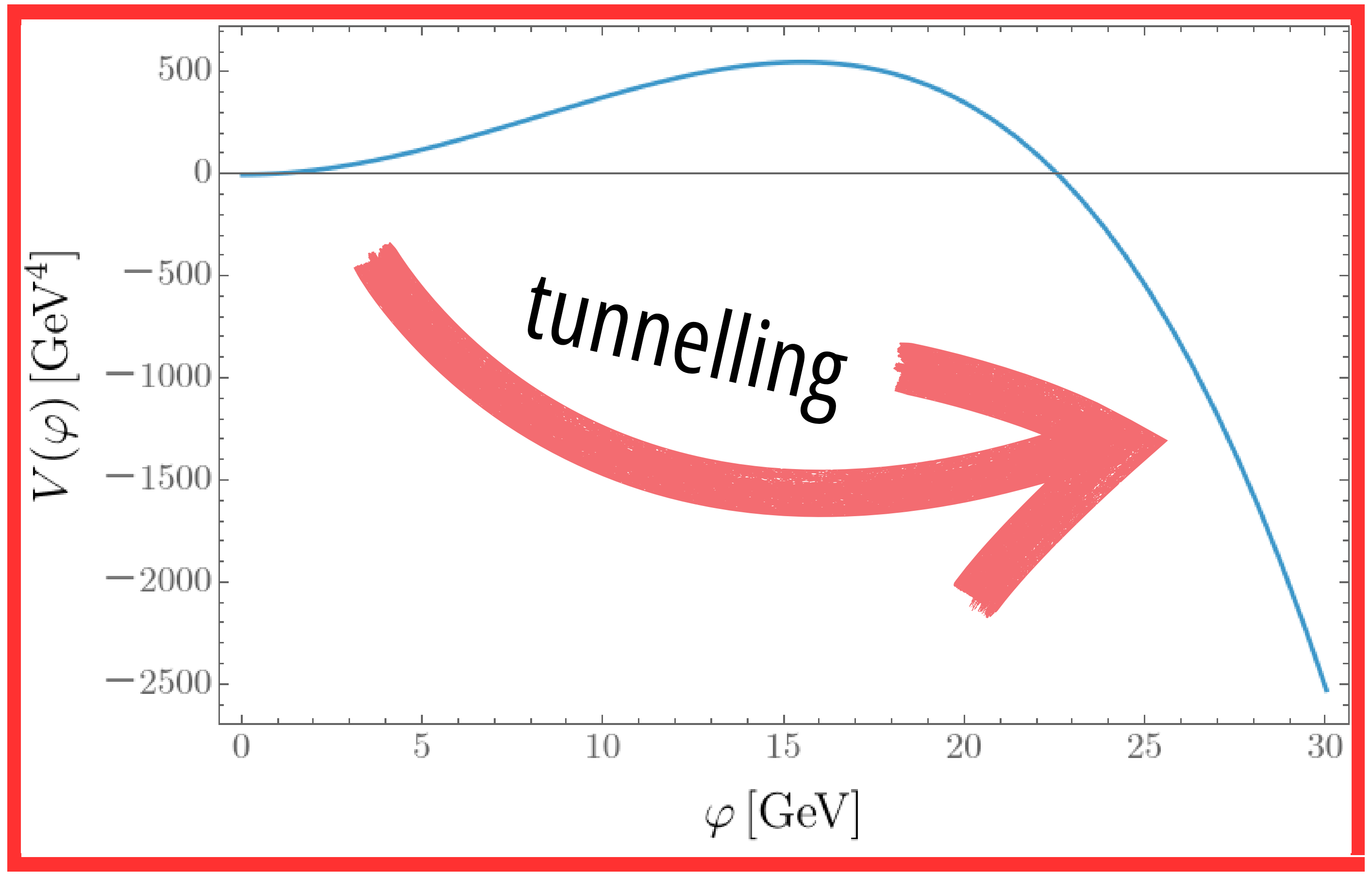


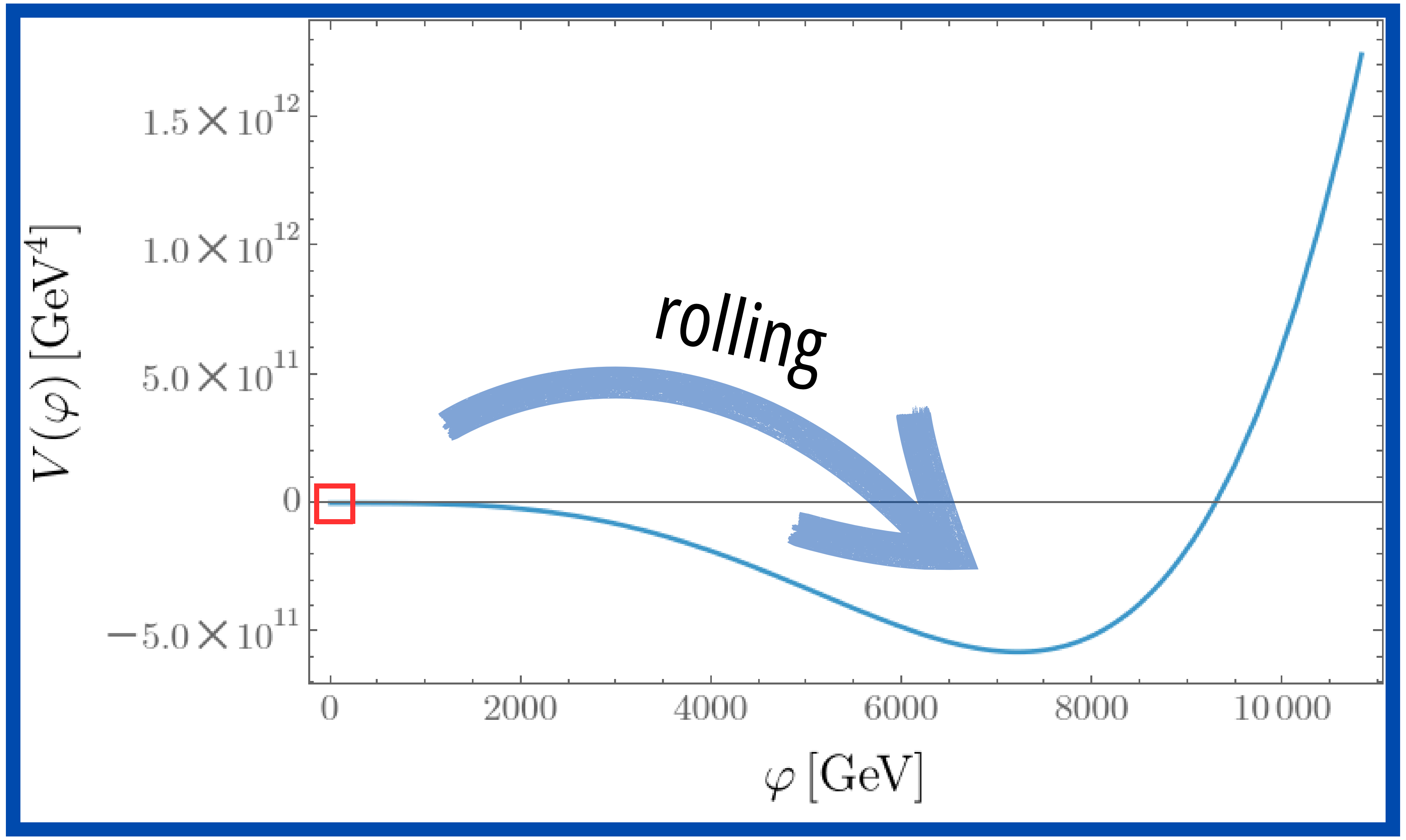
$$V(\varphi) = \frac{\lambda_{\rm CW}}{4}\varphi^4\left(\log\frac{\varphi^2}{w^2} - \frac{1}{2}\right) + V_T(\varphi)$$

M. Kierkla, A. Karam, B. Świeżewska, JHEP 03 (2023) 007

Phase transition dynamics

The effective potential





$$V(\varphi) = \frac{\lambda_{\mathrm{CW}}}{4}\varphi^4\left(\log\frac{\varphi^2}{w^2} - \frac{1}{2}\right) + V_T(\varphi)$$

M. Kierkla, A. Karam, B. Świeżewska, JHEP 03 (2023) 007

Background field dynamics

3+1-dimensional vacuum transition: nucleation

O(4)-symmetric bounce

$$\frac{\partial^2}{\partial \xi^2}\varphi(\xi) + \frac{3}{\xi}\frac{\partial}{\partial \xi}\varphi(\xi) - V'(\varphi) = 0$$

$$\xi^2 = \tau^2 + r^2$$

Background field dynamics

3+1-dimensional vacuum transition: nucleation and evolution

O(4)-symmetric bounce

$$\frac{\partial^2}{\partial \xi^2}\varphi(\xi) + \frac{3}{\xi}\frac{\partial}{\partial \xi}\varphi(\xi) - V'(\varphi) = 0$$

$$\xi^2 = \tau^2 + r^2$$

analytic continuation

SO(1,3)-symmetric EoM

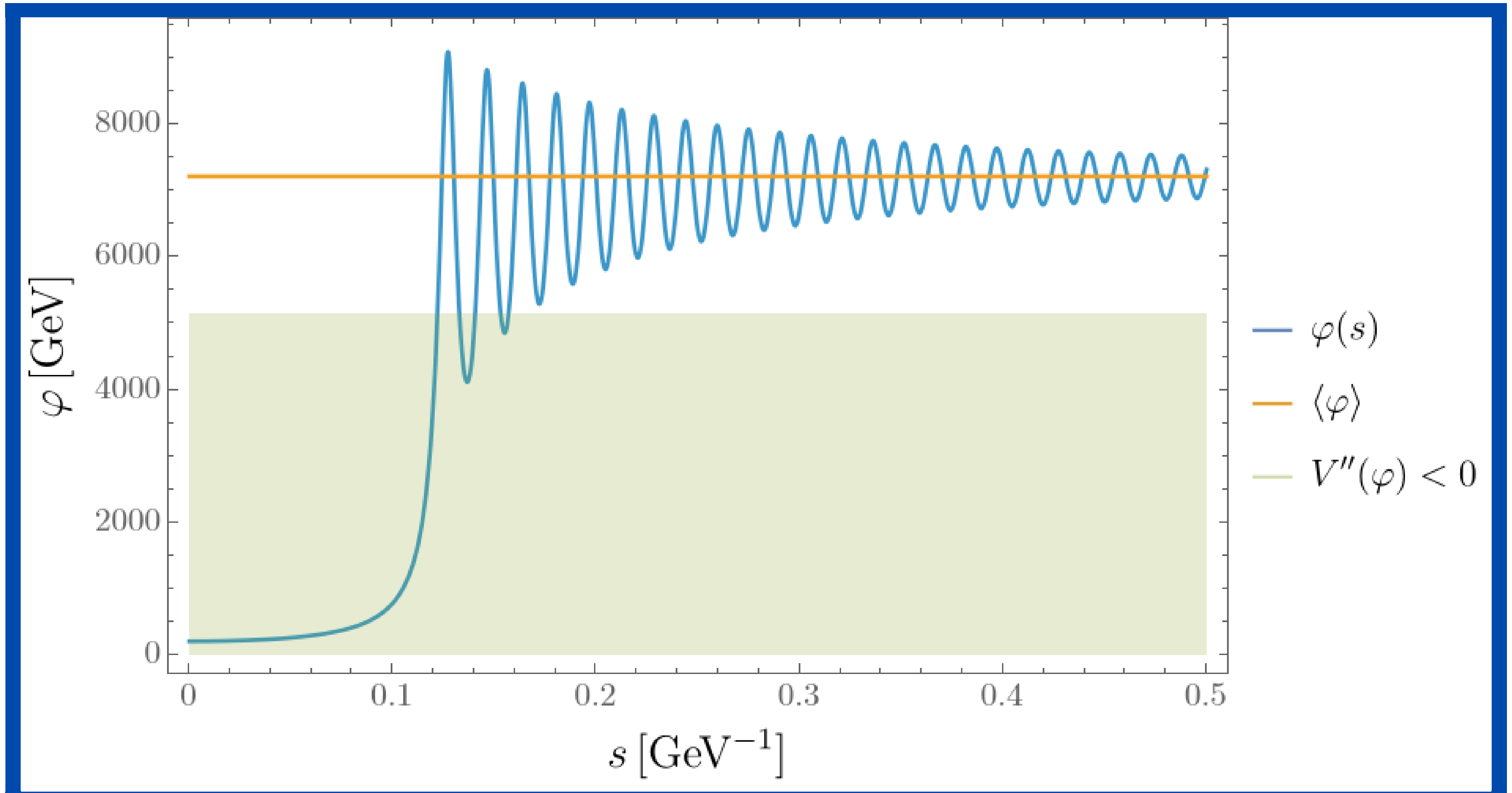
$$\frac{\partial^2}{\partial s^2}\varphi(s) + \frac{3}{s}\frac{\partial}{\partial s}\varphi(s) + V'(\varphi) = 0$$

$$s^2 = t^2 - r^2$$

Background field dynamics

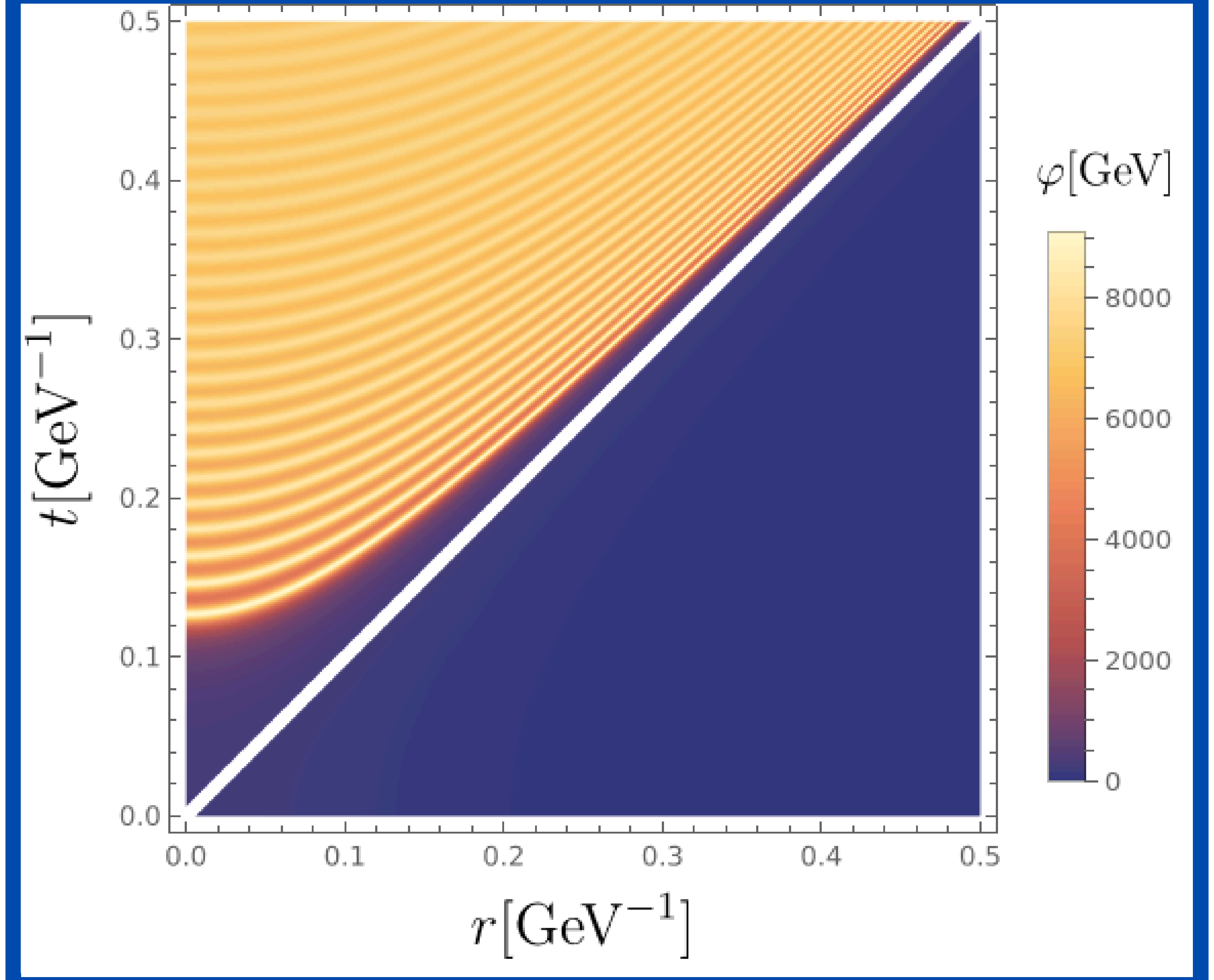
3+1-dimensional vacuum transition

$$\frac{\partial^2}{\partial s^2}\varphi(s) + \frac{3}{s}\frac{\partial}{\partial s}\varphi(s) + V'(\varphi) = 0$$



Background field dynamics

3+1-dimensional vacuum transition

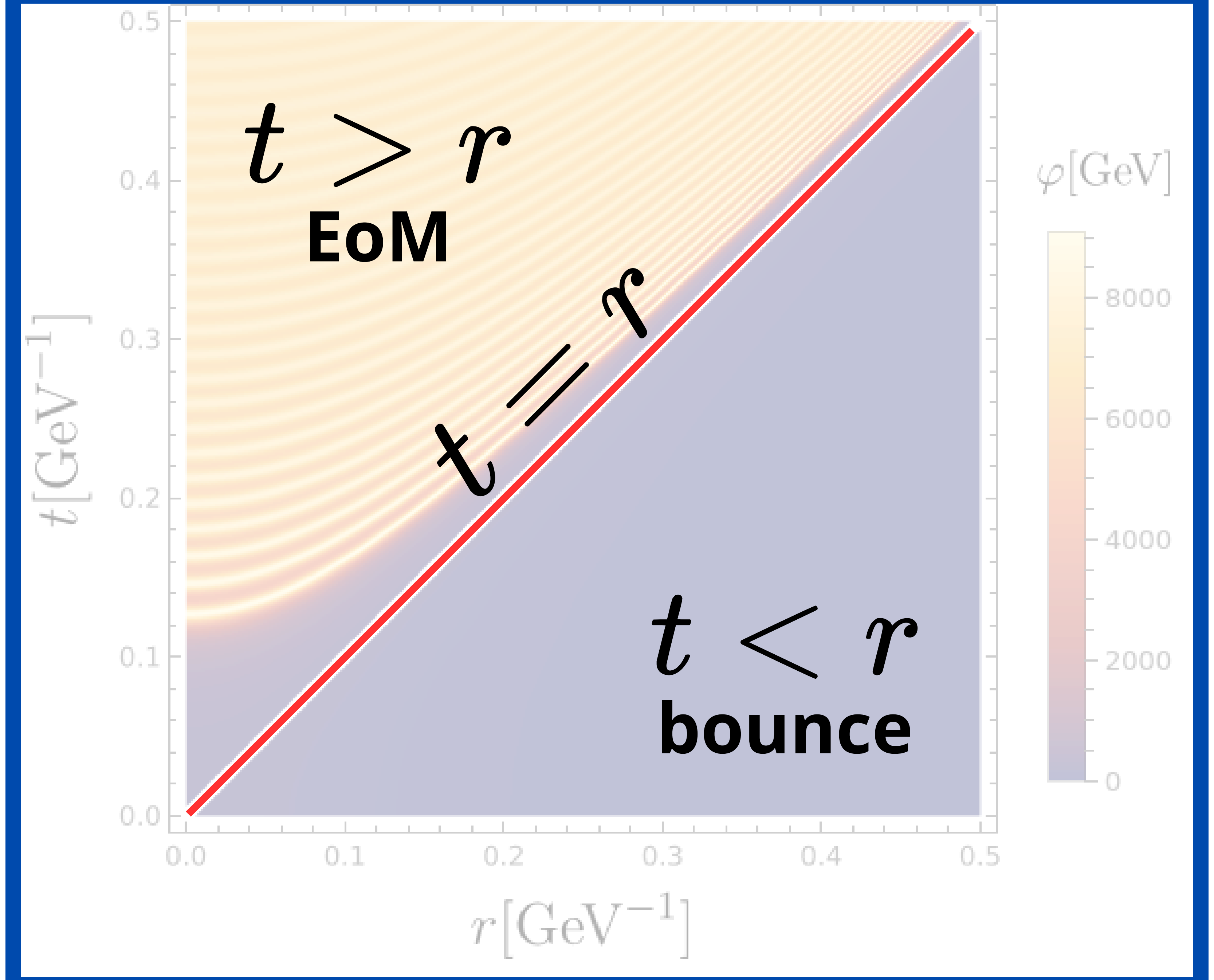


$$\frac{\partial^2}{\partial s^2}\varphi(s) + \frac{3}{s}\frac{\partial}{\partial s}\varphi(s) + V'(\varphi) = 0$$

$$s^2 = t^2 - r^2$$

Background field dynamics

3+1-dimensional vacuum transition



$$\frac{\partial^2}{\partial s^2}\varphi(s) + \frac{3}{s}\frac{\partial}{\partial s}\varphi(s) + V'(\varphi) = 0$$

$$s^2 = t^2 - r^2$$

Fluctuations dynamics

3+1-dimensional vacuum transition

$$\hat{\varphi}(s,\chi,\Omega) = \varphi(s) + \delta\hat{\varphi}(s,\chi,\Omega)$$

$$\delta\hat{\varphi}(s,\chi,\Omega) = \int_0^\infty \mathrm{d}p \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[\delta\varphi_p(s) Y_{plm}(\chi,\Omega)\,\hat{b}_{plm} + \mathrm{h.\ c.}\right]$$

K. Yamamoto, T. Tanaka, M. Sasaki, Phys.Rev.D 51 (1995), 2968-2978

Fluctuations dynamics

3+1-dimensional vacuum transition

$$\hat{\varphi}(s, \chi, \Omega) = \varphi(s) + \delta\hat{\varphi}(s, \chi, \Omega)$$

$$\delta\hat{\varphi}(s, \chi, \Omega) = \int_0^\infty \mathrm{d}p \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[\delta\varphi_p(s) Y_{plm}(\chi, \Omega)\, \hat{b}_{plm} + \mathrm{h.\ c.}\right]$$

K. Yamamoto, T. Tanaka, M. Sasaki, Phys.Rev.D 51 (1995), 2968-2978

$$\frac{\partial^2}{\partial s^2}\delta\varphi_p(s) + \frac{3}{s}\frac{\partial}{\partial s}\delta\varphi_p(s) + \left(\frac{p^2+1}{s^2} + V''(\varphi)\right)\delta\varphi_p(s) = 0$$

Fluctuations dynamics

3+1-dimensional vacuum transition

$$\hat{\varphi}(s,\chi,\Omega) = \varphi(s) + \delta\hat{\varphi}(s,\chi,\Omega)$$

$$\delta\hat{\varphi}(s,\chi,\Omega) = \int_0^\infty \mathrm{d}p \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[\delta\varphi_p(s) Y_{plm}(\chi,\Omega)\, \hat{b}_{plm} + \mathrm{h.\,c.}\right]$$

K. Yamamoto, T. Tanaka, M. Sasaki, Phys.Rev.D 51 (1995), 2968-2978

$$\frac{\partial^2}{\partial s^2}\delta\varphi_p(s) + \frac{3}{s}\frac{\partial}{\partial s}\delta\varphi_p(s) + \left(\frac{p^2+1}{s^2} + V''(\varphi)\right)\delta\varphi_p(s) = 0$$

background-dependent mass:
- tachyonic instability
- parametric resonance

Fluctuations dynamics

3+1-dimensional vacuum transition

$$\hat{\varphi}(s,\chi,\Omega) = \varphi(s) + \delta\hat{\varphi}(s,\chi,\Omega)$$

$$\delta\hat{\varphi}(s,\chi,\Omega) = \int_0^\infty \mathrm{d}p \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[\delta\varphi_p(s) Y_{plm}(\chi,\Omega)\, \hat{b}_{plm} + \mathrm{h.\ c.}\right]$$

K. Yamamoto, T. Tanaka, M. Sasaki, Phys.Rev.D 51 (1995), 2968-2978

$$\frac{\partial^2}{\partial s^2}\delta\varphi_p(s) + \frac{3}{s}\frac{\partial}{\partial s}\delta\varphi_p(s) + \left(\frac{p^2+1}{s^2} + V''(\varphi)\right)\delta\varphi_p(s) = 0$$

$$\frac{\partial^2}{\partial s^2}\varphi + \frac{3}{s}\frac{\partial}{\partial s}\varphi + V'(\varphi) + \frac{1}{2}V'''(\varphi)\left\langle \delta\varphi^2 \right\rangle = 0$$

N. Herring, S. Cao, D. Boyanovsky, Phys.Rev.D 109 (2024) 10, 105021

Fluctuations dynamics

3+1-dimensional vacuum transition

$$\hat{\varphi}(s,\chi,\Omega) = \varphi(s) + \delta\hat{\varphi}(s,\chi,\Omega)$$

$$\delta\hat{\varphi}(s,\chi,\Omega) = \int_0^\infty \mathrm{d}p \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[\delta\varphi_p(s) Y_{plm}(\chi,\Omega)\, \hat{b}_{plm} + \mathrm{h.\ c.}\right]$$

K. Yamamoto, T. Tanaka, M. Sasaki, Phys.Rev.D 51 (1995), 2968-2978

$$\frac{\partial^2}{\partial s^2}\delta\varphi_p(s) + \frac{3}{s}\frac{\partial}{\partial s}\delta\varphi_p(s) + \left(\frac{p^2+1}{s^2} + V''(\varphi)\right)\delta\varphi_p(s) = 0$$

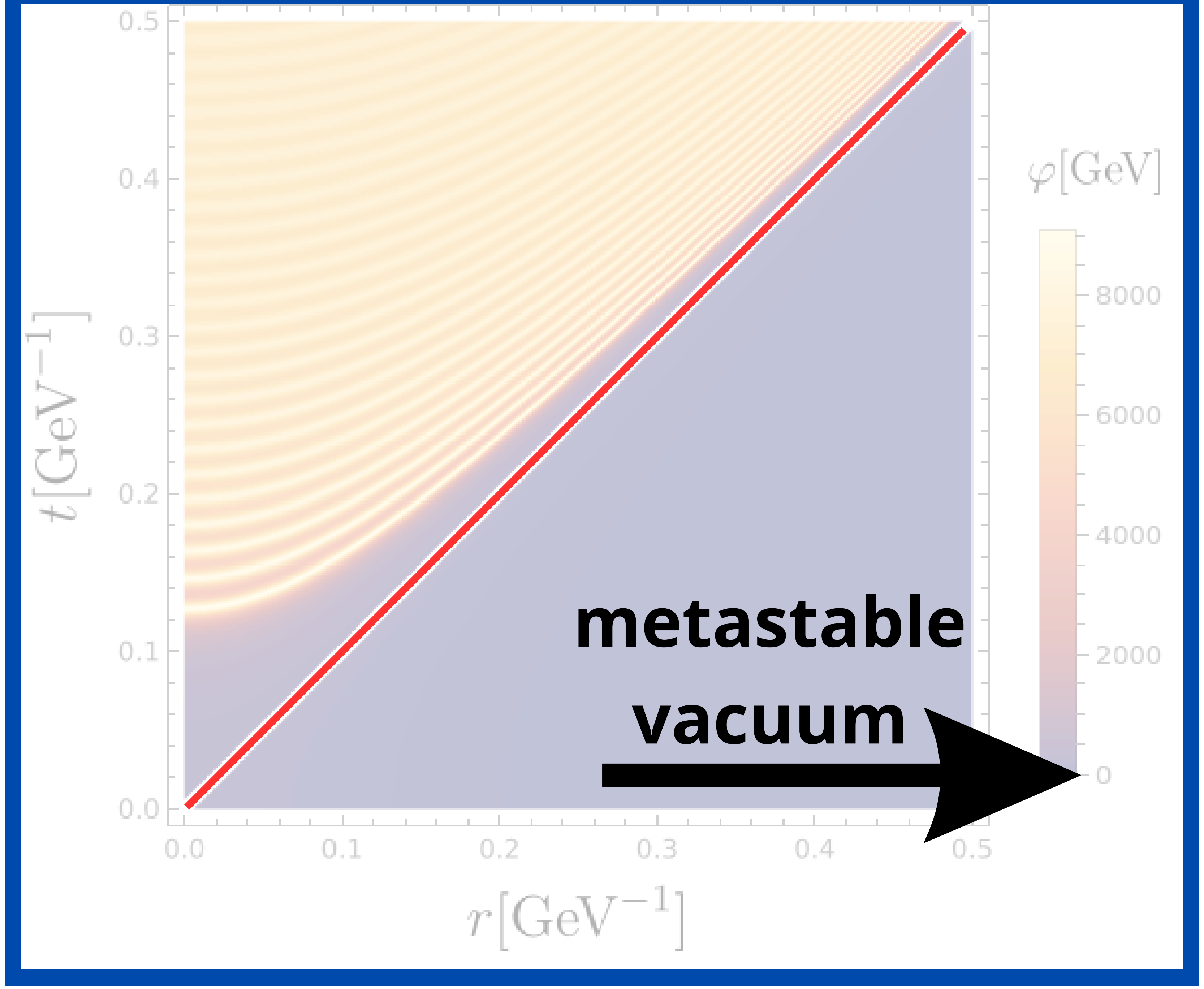
$$\frac{\partial^2}{\partial s^2}\varphi + \frac{3}{s}\frac{\partial}{\partial s}\varphi + V'(\varphi) + \frac{1}{2}V'''(\varphi)\left\langle\delta\varphi^2\right\rangle = 0$$

backreaction!

N. Herring, S. Cao, D. Boyanovsky, Phys.Rev.D 109 (2024) 10, 105021

Initial conditions for the fluctuations

3+1-dimensional vacuum transition



$$\frac{\partial^2}{\partial s^2}\delta\varphi_p(s) + \frac{3}{s}\frac{\partial}{\partial s}\delta\varphi_p(s) + \left(\frac{p^2+1}{s^2} + V''(\varphi)\right)\delta\varphi_p(s) = 0$$

K. Yamamoto, T. Tanaka, M. Sasaki, Phys.Rev.D 51 (1995), 2968-2978

Energy inside the bubble

Preliminary results

$$\langle \delta\varphi^2 \rangle \to \langle \delta\varphi^2 \rangle_{\text{ren}}$$

EoM

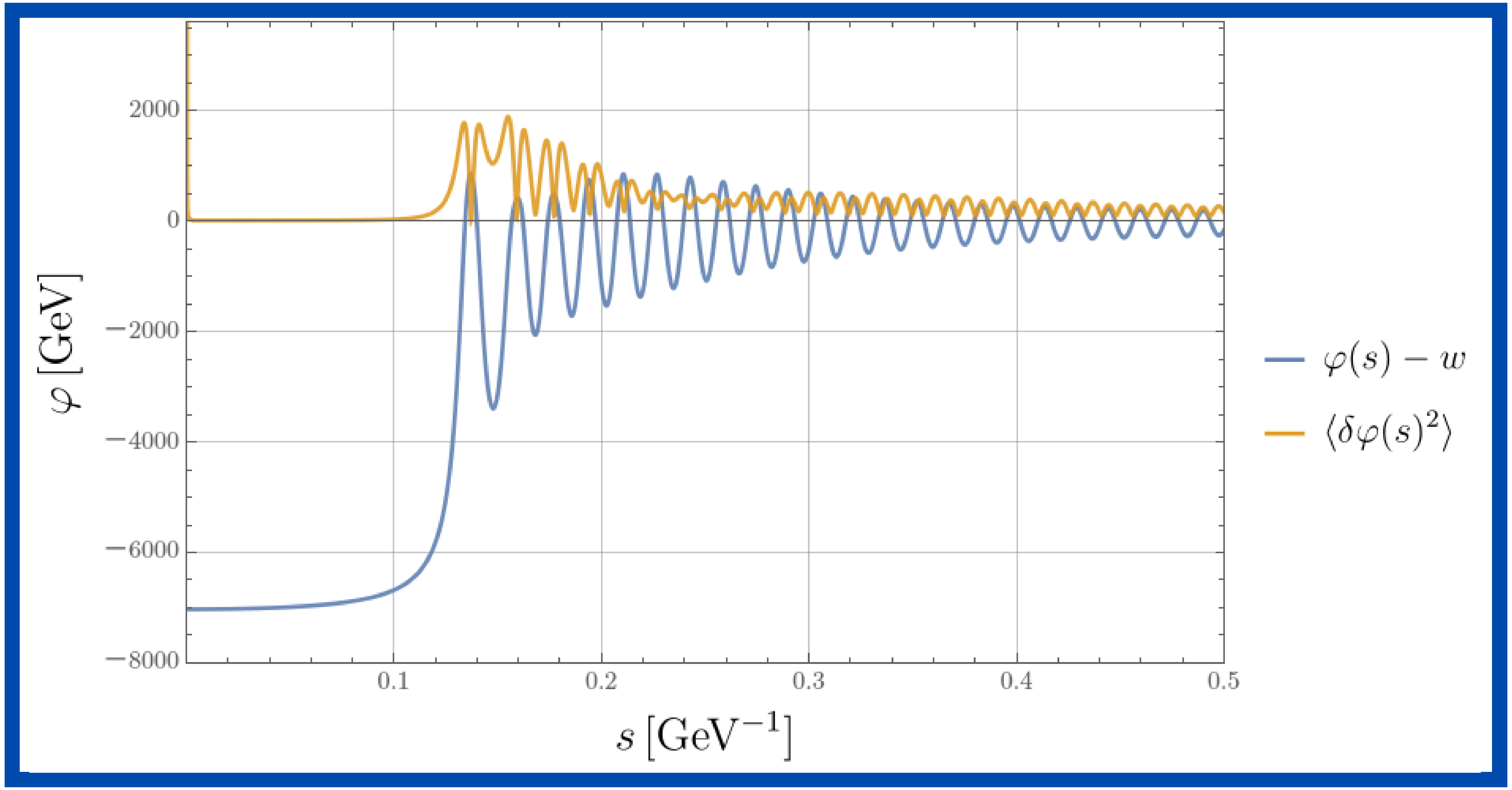
N. Herring, S. Cao, D. Boyanovsky, Phys.Rev.D 109 (2024) 10, 105021

Energy inside the bubble

Preliminary results

$$\langle \delta\varphi^2 \rangle \to \langle \delta\varphi^2 \rangle_{\text{ren}}$$

EoM



Energy inside the bubble

Preliminary results

$$\langle \delta\varphi^2 \rangle \to \langle \delta\varphi^2 \rangle_{\text{ren}}$$

EoM

$$\rho = \langle 0 | \hat{\mathcal{H}}_{\text{ren}} | 0 \rangle$$

N. Herring, S. Cao, D. Boyanovsky, Phys.Rev.D 109 (2024) 10, 105021

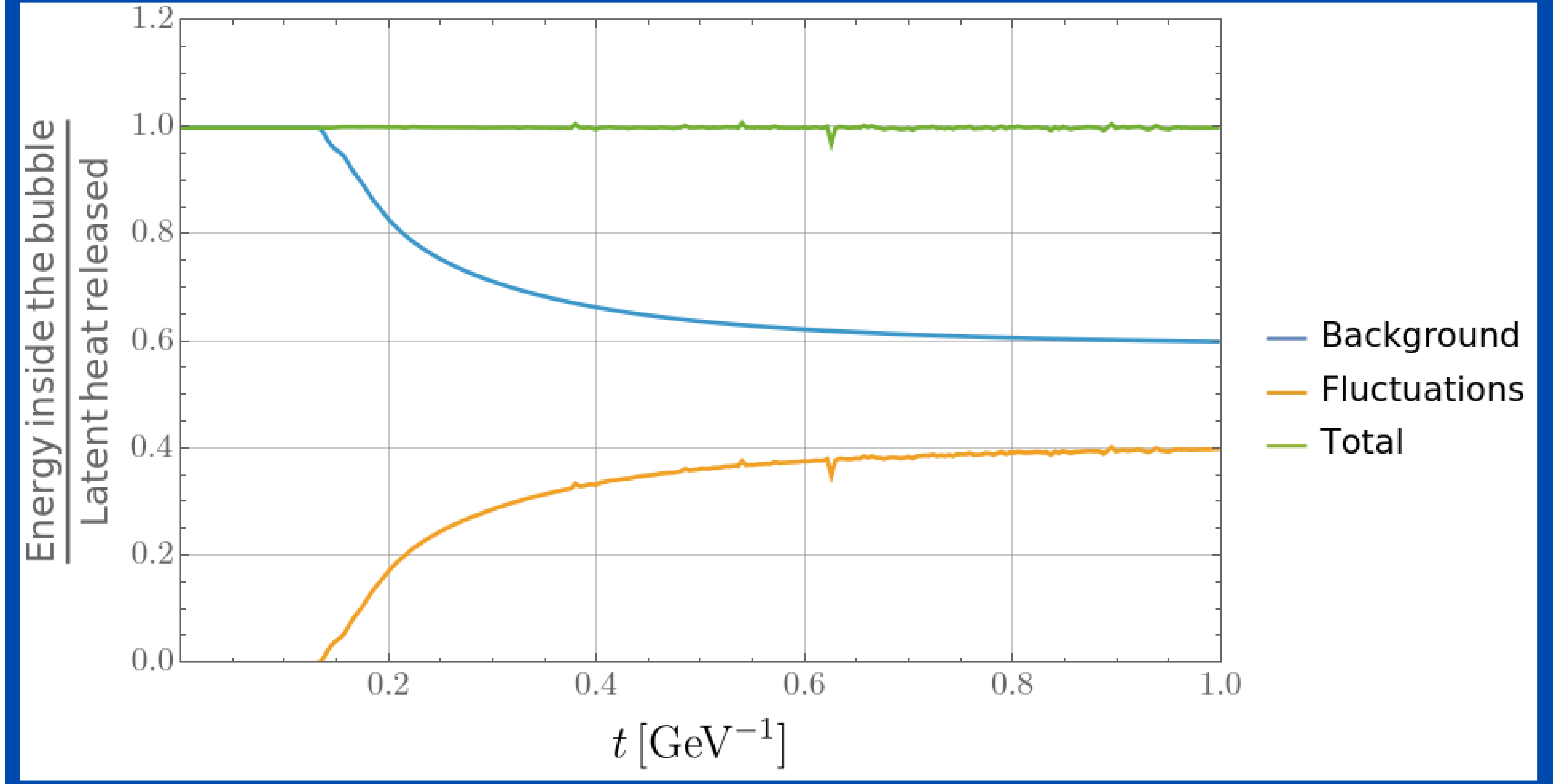
Energy inside the bubble

Preliminary results

$$\langle \delta\varphi^2 \rangle \to \langle \delta\varphi^2 \rangle_{\mathrm{ren}}$$

EoM

$$\rho = \langle 0 | \hat{\mathcal{H}}_{\mathrm{ren}} | 0 \rangle$$



Consequences

- energy budget of the bubble
- gravitational-wave signal
- bubble-wall velocity
- equation of state

Consequences

- energy budget of the bubble
- gravitational-wave signal
- bubble-wall velocity
- equation of state

A strongly supercooled transition releases its large vacuum energy, but particle production can absorb a sizeable fraction of it

Thank you for your attention!

NATIONAL SCIENCE CENTRE
POLAND

Backup slides

Symmetry & background dynamics

In real time with t>r:

$$\frac{\partial^2}{\partial s^2}\phi(s) + \frac{3}{s}\frac{\partial}{\partial s}\phi(s) + V'(\phi) = 0$$

$$s^2 = t^2 - r^2$$

In real time with t<r:

$$\frac{\partial^2}{\partial \zeta^2}\phi(\zeta) + \frac{3}{\zeta}\frac{\partial}{\partial \zeta}\phi(\zeta) - V'(\phi) = 0$$

$$\zeta^2 = r^2 - t^2$$

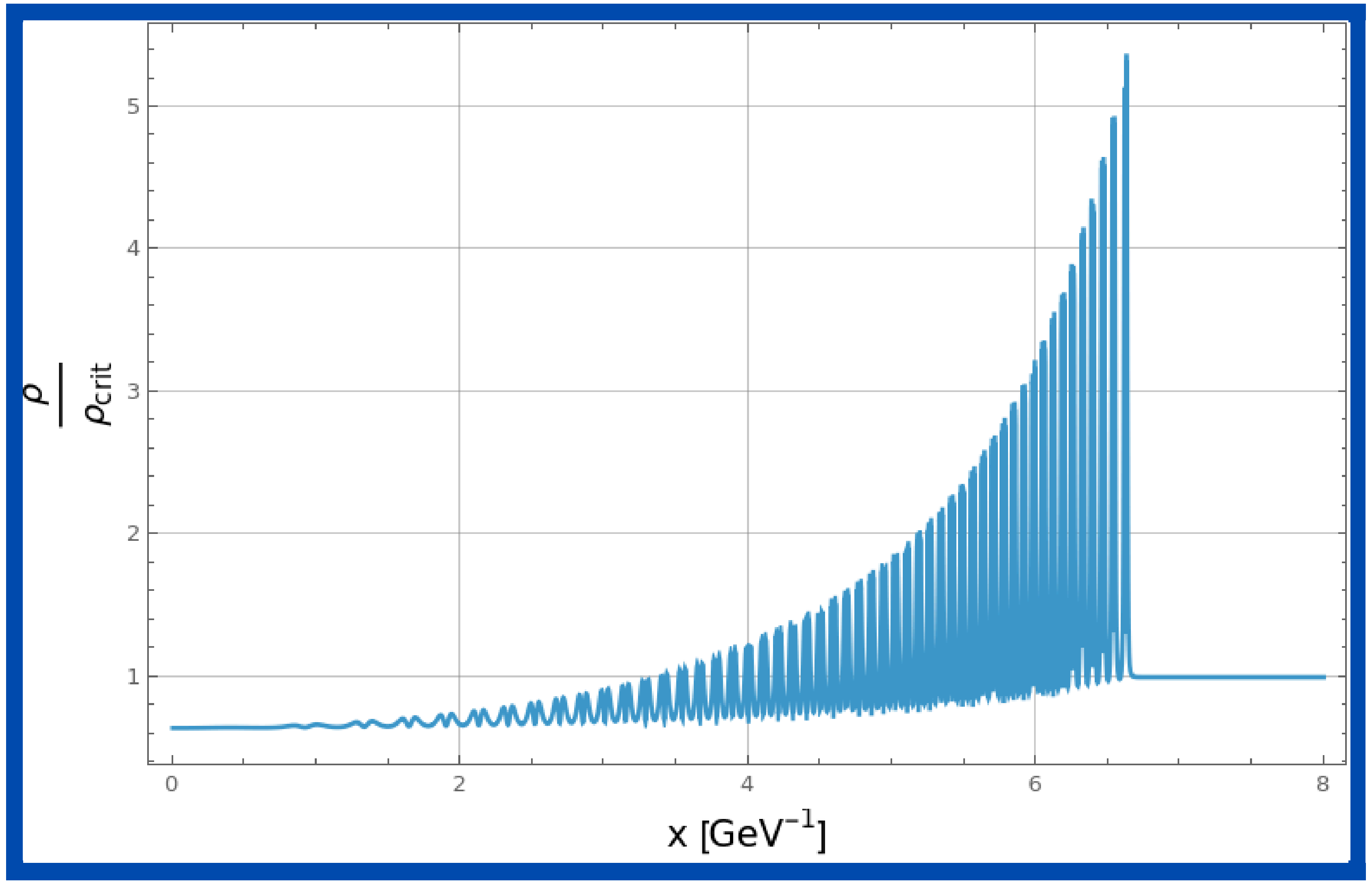
In imaginary time:

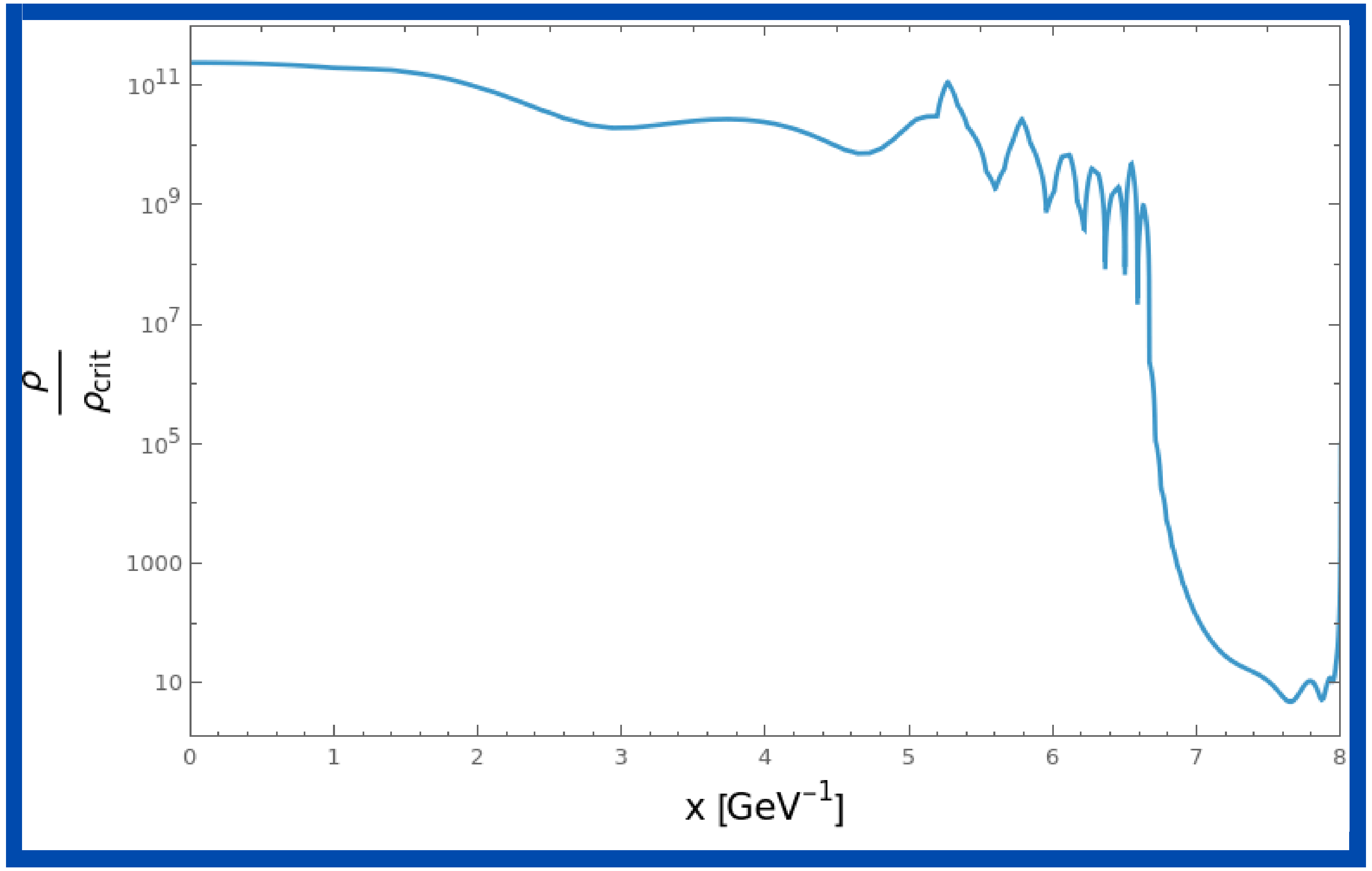
$$\frac{\partial^2}{\partial \xi^2}\phi(\xi) + \frac{3}{\xi}\frac{\partial}{\partial \xi}\phi(\xi) - V'(\phi) = 0$$

$$\xi^2 = \tau^2 + r^2 \equiv r^2 - (i\tau)^2$$

Fluctuations and background energy

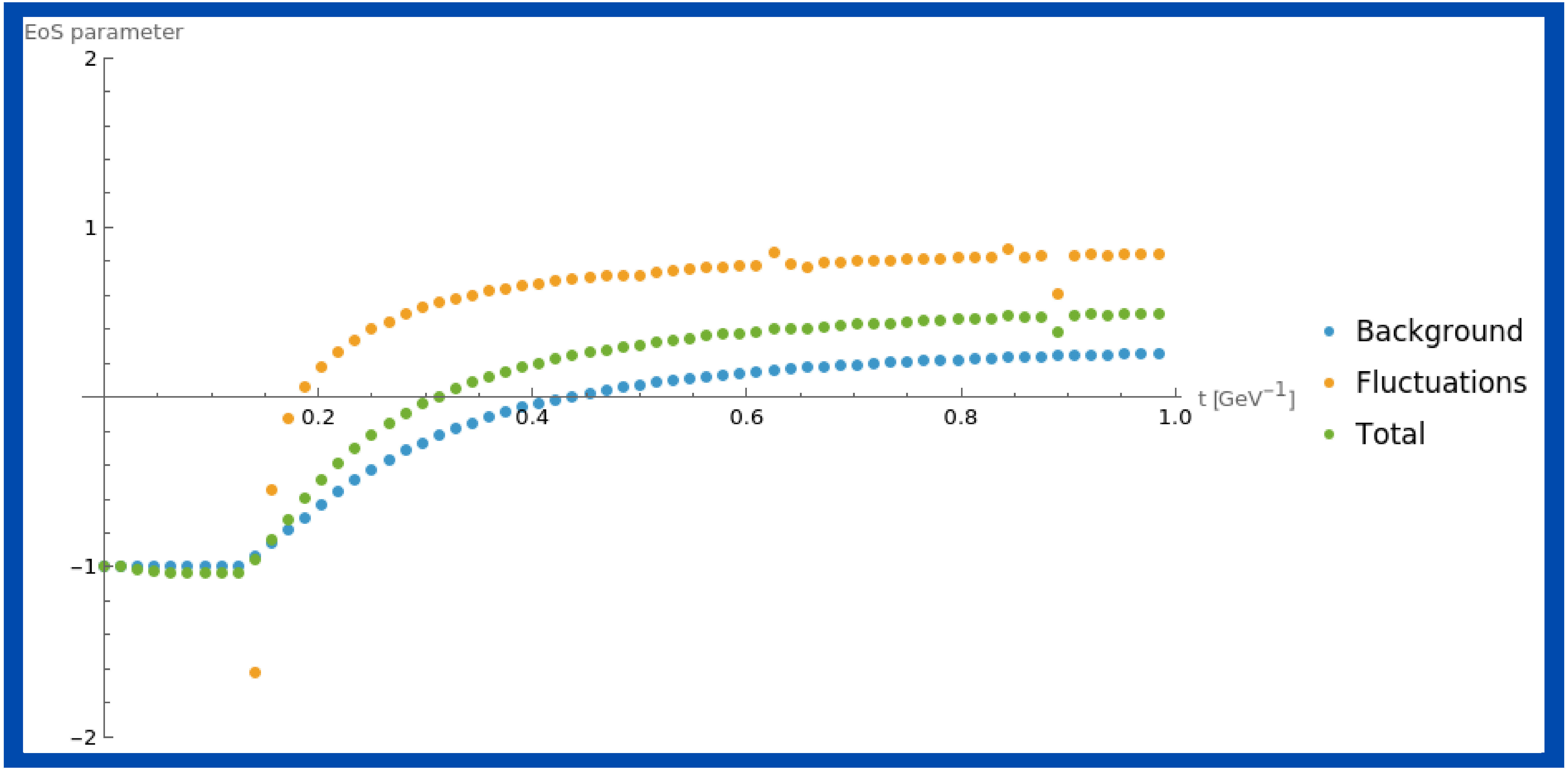
Without backreaction





Background vs Fluctuations

Equation of state



Particle spectrum