Soft fluctuations and higher-dimensional operators affect bubble wall dynamics

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Introduction

The bubble wall velocity v_w controls the observable signatures of first-order phase transitions, shaping both the predicted gravitational wave spectrum and the baryon asymmetry produced in electroweak baryogenesis. It follows from the dynamics of the scalar field in the plasma; integrating out the hard modes generates a Langevin equation for mean the soft field:

$$-\Box\phi + V'_{\text{eff}}(\phi) - \underbrace{\eta\, u^\mu \partial_\mu \phi}_{\text{friction}} + \zeta(x) = 0, \tag{1}$$

where ζ represents noise. While simulating eq. (1), we estimate the soft fluctuations perturbatively and compare their effect on v_w with that of hard-scale higher-dimensional operators in the context of high-temperature EFT [8].

Theoretical Setup

Assuming a planar wall moving at constant velocity, the profile $\phi(z)$ across the wall obeys the following equation:

$$-\partial_z^2\phi(z) + V'_{\text{eff}}(\phi) - \underbrace{\boldsymbol{\eta\gamma_w v_w}}_{\omega_0}\,\partial_z\phi(z) = 0.$$

(N.B: in our computations, we always set η to 1.)
Soft fluctuations are included perturbatively via

$$\phi_0(z) \to \phi_0(z) + g\,\phi_1(z),$$
$$\omega_0 \to \omega_0 + g\,\omega_1.$$

The resulting NLO equation for ϕ_1 reads:

$$\left[-\partial_z^2 - \omega_0\partial_z + W(z)\right]\phi_1(z) + \Omega(\phi_0(z), z) - \omega_1\partial_z\phi_0(z) = 0. \tag{2}$$

where $\Omega(\phi_0(z), z) = \frac{1}{2}V'''(\phi)\,G_H(z,z)$, $W(z) = V''(\phi(z)) + \frac{\omega_0^2}{4}$, and $G_H(z,z)$ is the scalar propagator in the wall background.

Results

At finite temperature, the hard modes naturally generate higher-dimensional operators. We first consider the sextic effective potential:

$$V_{\text{eff}}(\phi) = \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 + \frac{\kappa}{6}\phi^6, \quad \kappa = c_6. \tag{3}$$

▶ The sextic potential solution is asymmetric compared to the `tanh` ansatz; the wall narrows as the transition strengthens.

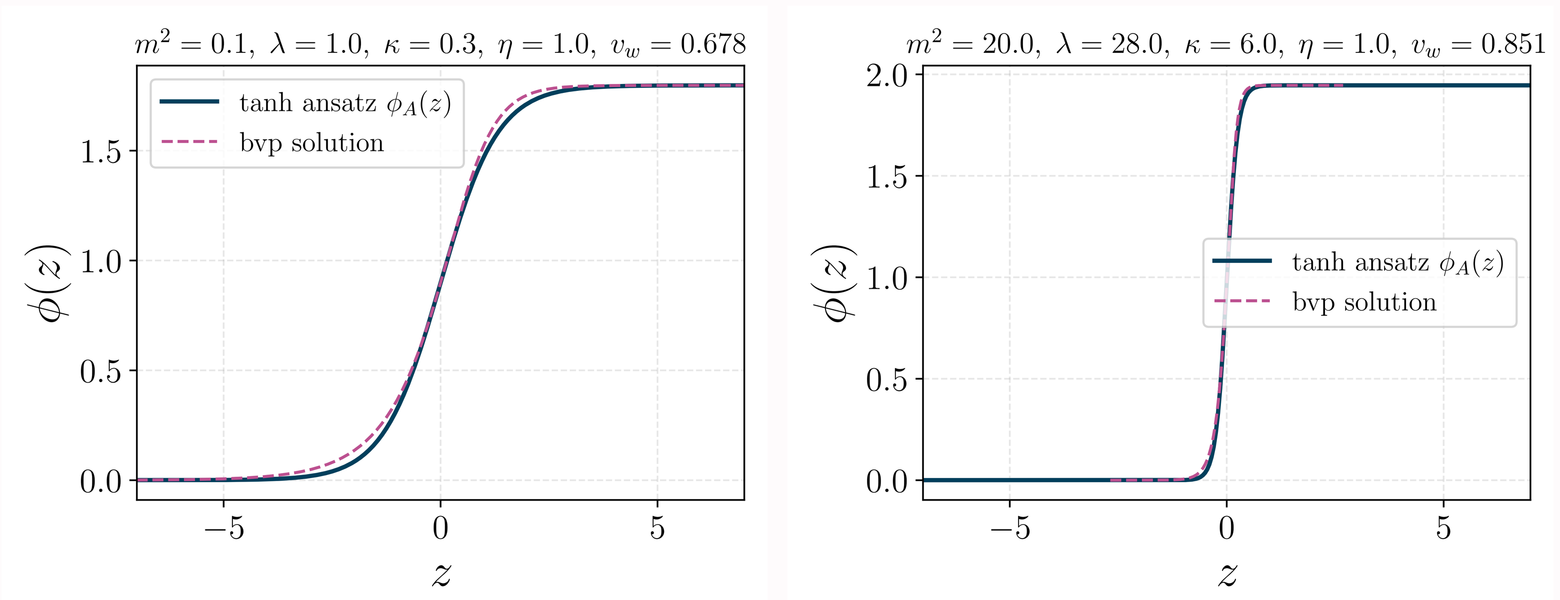

Figure 1: *Wall profiles for the sextic potential eq. (3); parameters in the panel titles. $\phi_A(z) = \frac{\phi_{TV}}{2}[1 + \tanh(z/L)]$; bvp: Dirichlet solution of the wall equation.*

▶ The soft effects are localised inside the wall; far from the wall the solution matches the WKB approximation [1].

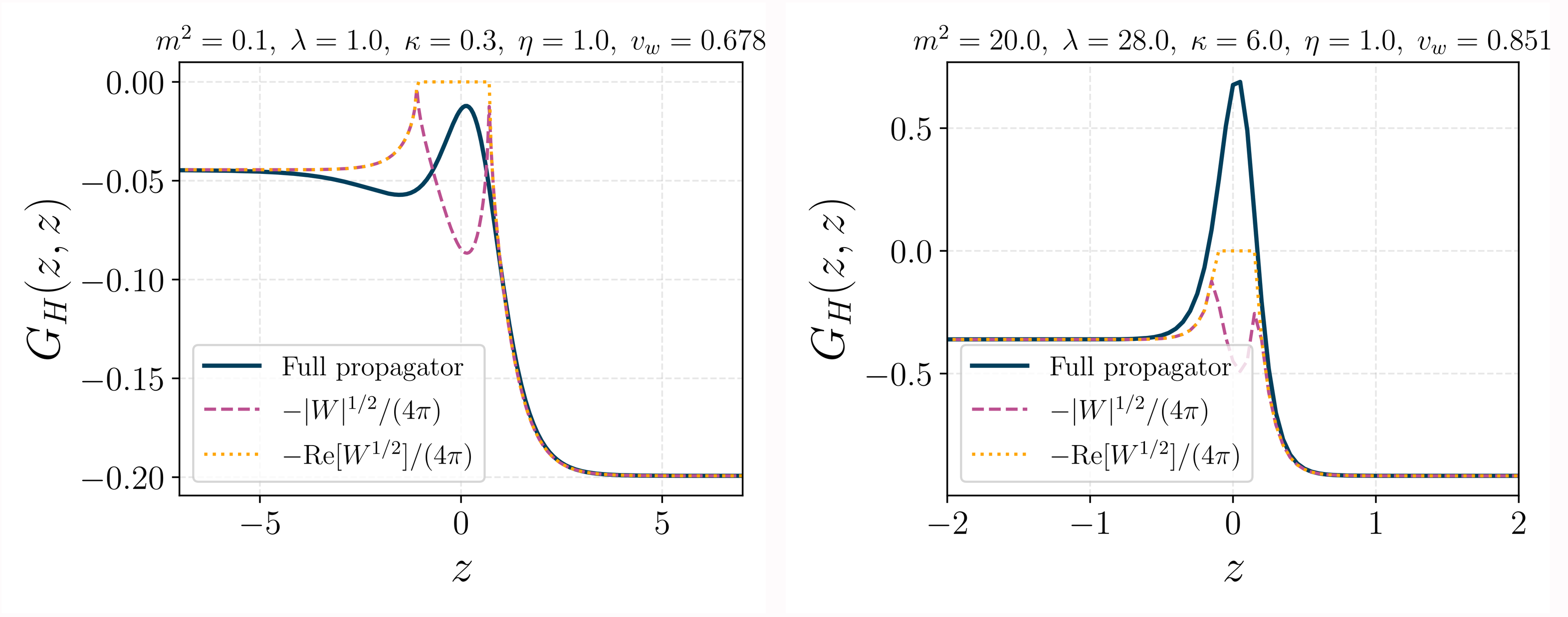

Figure 2: *Corresponding propagator $G_H(z,z)$ across the wall with the same benchmarks.*

Solving for (ϕ_0, ϕ_1) yields the LO and NLO friction terms ω_0 and ω_1, hence the corrected wall velocity, which we scan over the coupling c_6.

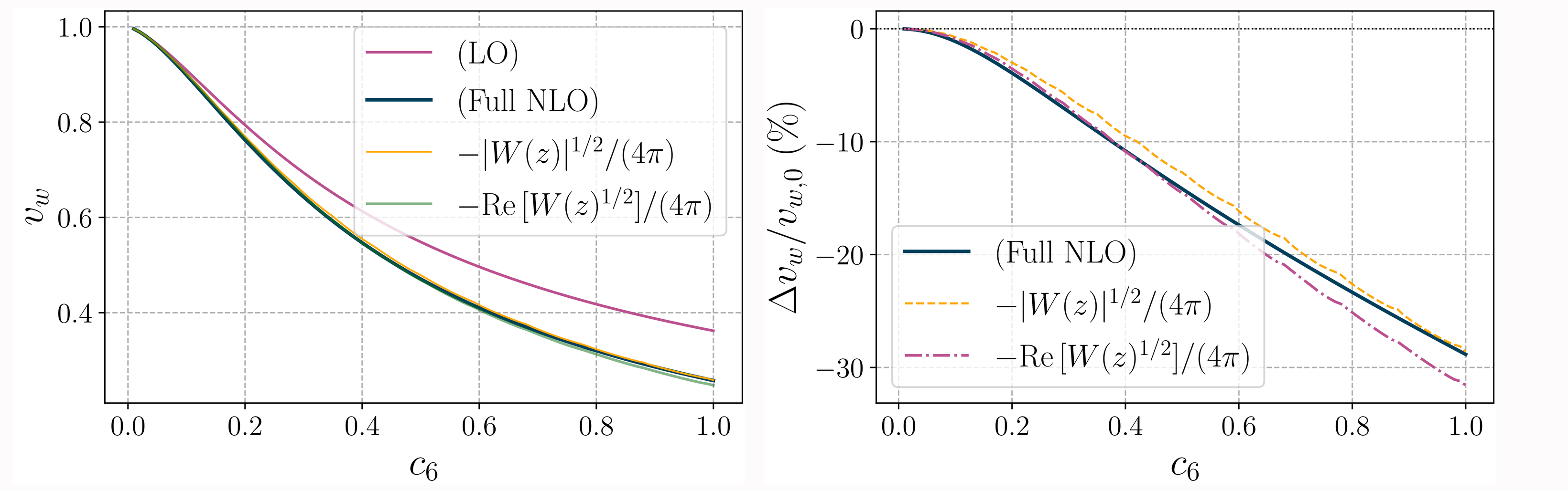

Figure 3: *Left: scan of the wall velocity over the coupling c_6, using the full propagator and the WKB approximations. Right: deviation from the LO wall velocity.*

▶ Soft fluctuations slow the wall, more so at larger c_6; $v_{w,\text{LO}} \approx 0.61$, $v_{w,\text{NLO}} \approx 0.55$ at $c_6 = 0.40$.
With $x = \lambda_{3d}/g_{3d}^2$, $y = m_{3d}^2/g_{3d}^4$, we compare:

$$V_{\text{eff}}(\phi) = \frac{y}{2}\phi^2 - \frac{1}{6}\phi^3 + \frac{x}{4}\phi^4, \tag{4}$$
$$V_{\text{eff}}(\phi) = \frac{y}{2}\phi^2 - \frac{1}{6}\phi^3 + \frac{x}{4}\phi^4 + \boldsymbol{\frac{c_6}{6}\phi^6}. \tag{5}$$

and we track v_w along the Higgs mass m_H in SU(2)+Higgs model: small m_H is a strong transition, large m_H a weak one [7].

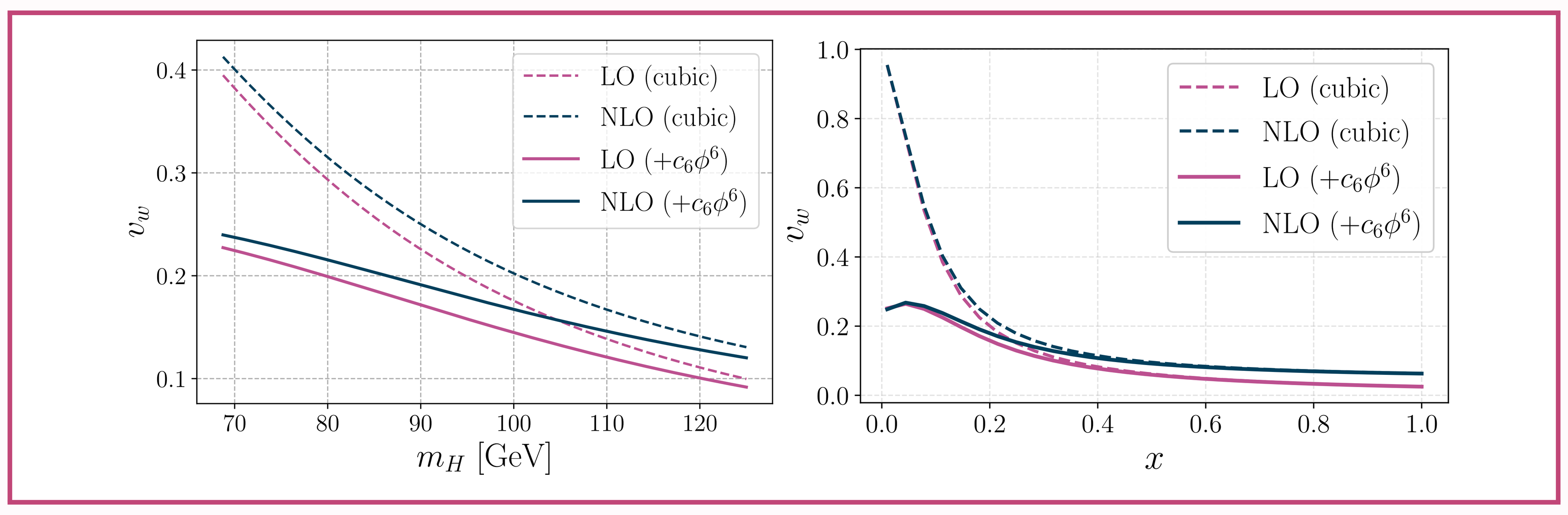

Figure 4: *SU(2)+Higgs using eq.(5) with $c_6 = 0.005$; v_w versus x (left) and versus m_H (right).*

▶ In SU(2)+Higgs, the ϕ^6 operator dominates at small m_H, and is more important than soft fluctuations.

Conclusion

- Soft fluctuations shift v_w by up to $\sim$30%, not negligible for GW or baryogenesis predictions.
- Hard vs. soft dominate in complementary regimes: the ϕ^6 operator wins for strong transitions, soft fluctuations for weak ones.
- Both hard and soft mode-effects decrease v_w.

References

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