Nucleation in supercooled cosmological phase transitions

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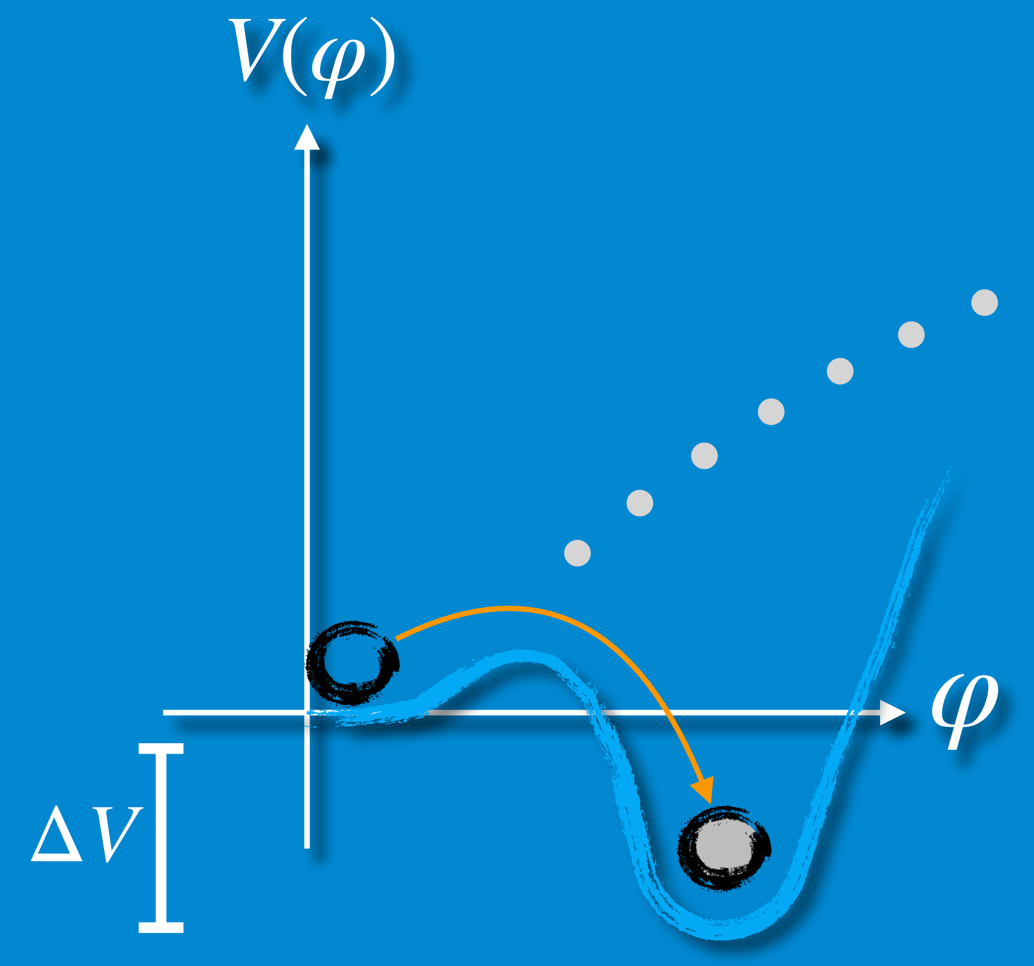
Based on: 2503.13597, 2506.15496, 2607.18233


Why study supercooled PTs?
Strong GW signal!
PTA data
DM/pBHs production
LISA is coming
We need robust framework

Key quantity for PT dynamics

Field theory

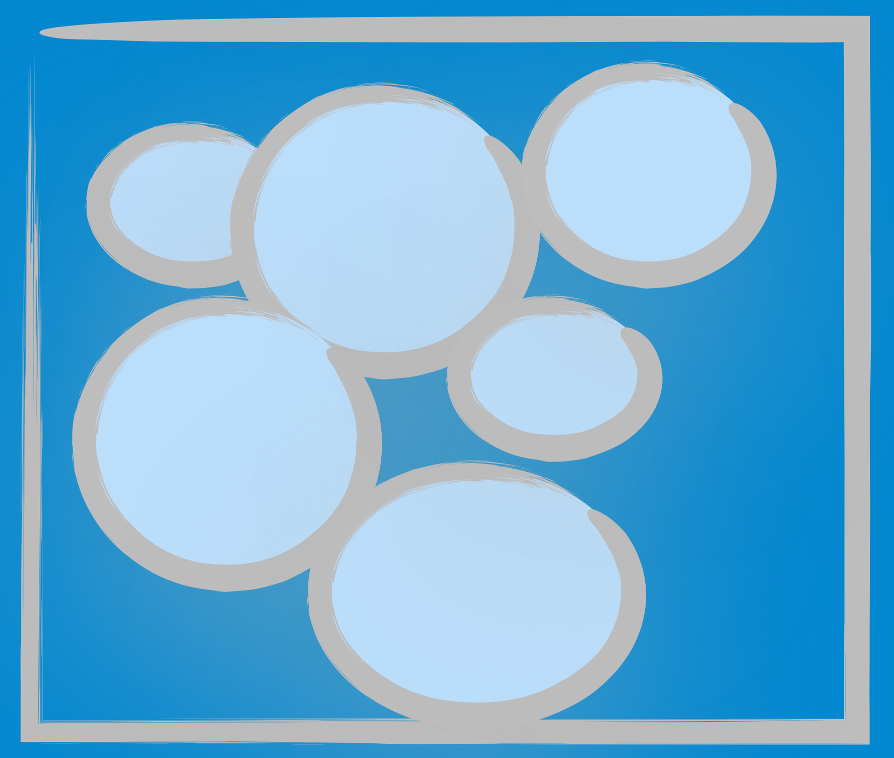


T_p

Bubble nucleation rate

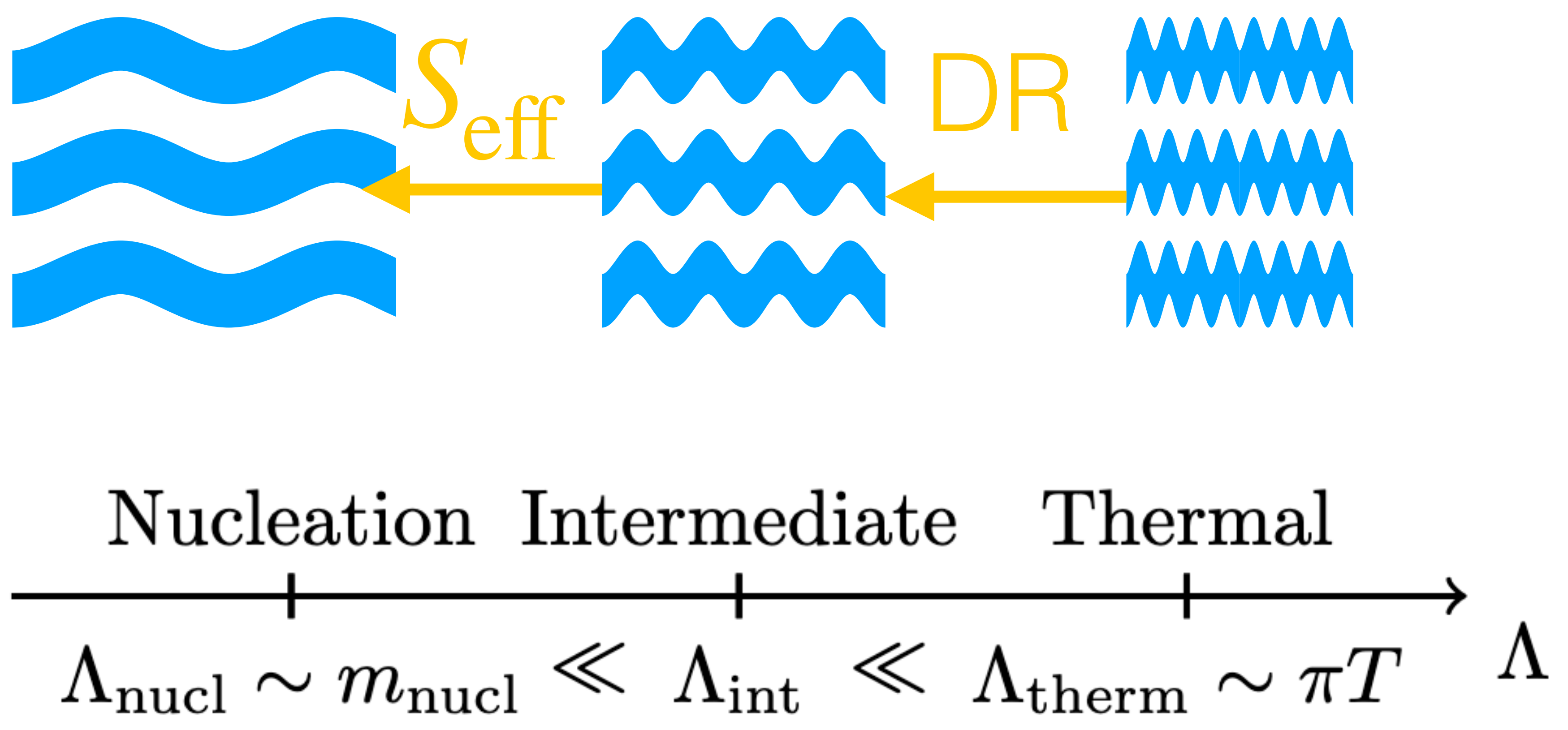
$$\frac{\Gamma}{H^4}$$

Cosmology



Idea: use EFT for long-range modes

Building blocks of field ϕ



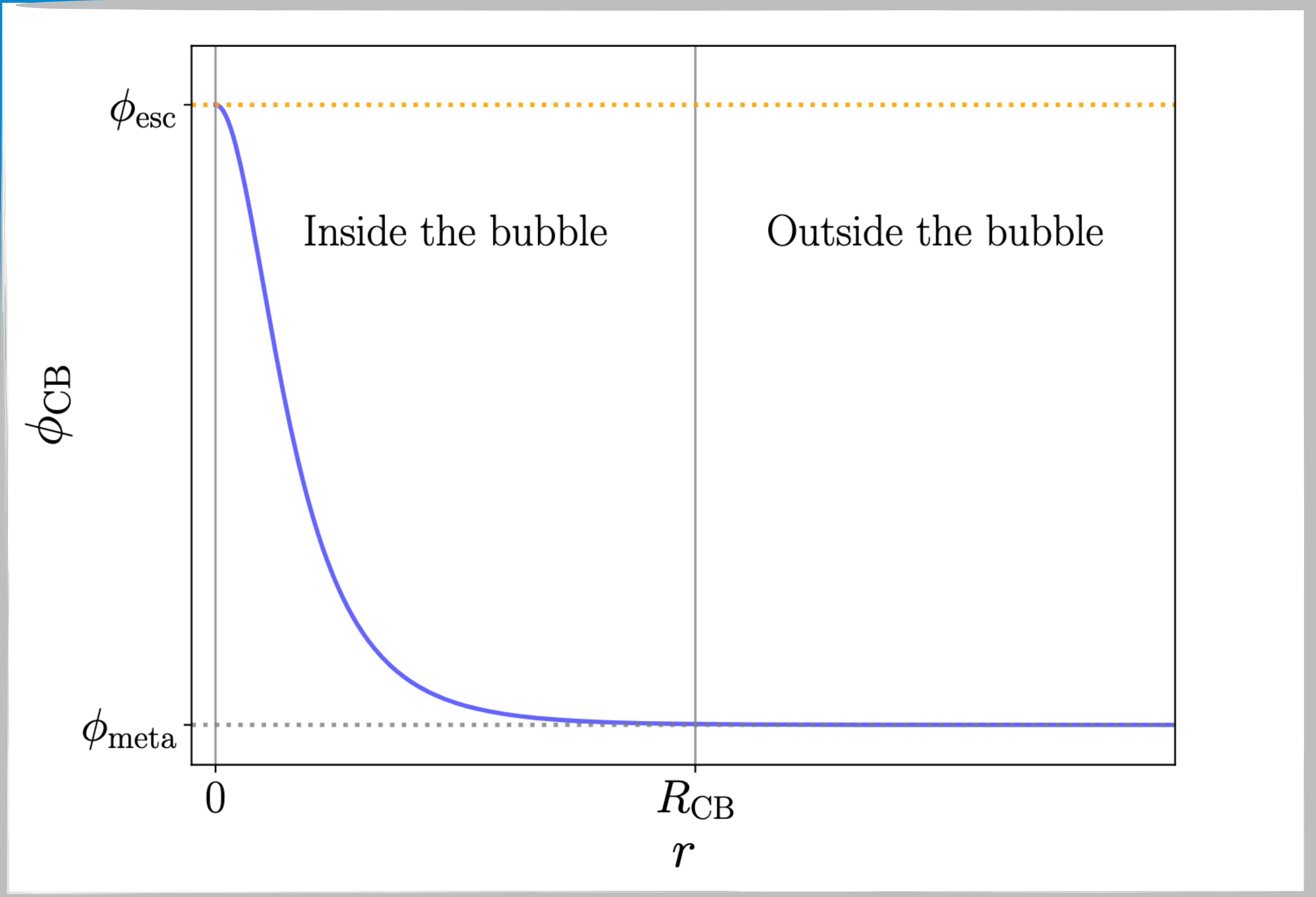
Take quantum fields, integrate out UV modes, and obtain nucleation EFT

$$F_{\rm eff} \longleftrightarrow S_{\rm nucl}$$

$$F''_{\rm eff} \longleftrightarrow S''_{\rm nucl}$$

See also: J.Hirvonen, O. Gould, 2108.04377, J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331

Idea: use EFT for long-range modes



We expand around the O(3)-symmetric stationary point of the action

$$A_{\mathrm{stat}} = \underbrace{\mathscr{V} \left| \frac{\det' S''_{\mathrm{nucl}}[\phi_{\mathrm{CB}}]}{\det S''_{\mathrm{nucl}}[\phi_{\mathrm{meta}}]} \right|^{-\frac{1}{2}}}_{\text{modes } E<\Lambda_{\mathrm{nucl}}} \underbrace{e^{-\Delta S_{\mathrm{nucl}}[\phi_{\mathrm{CB}}]}}_{\text{modes } E>\Lambda_{\mathrm{nucl}}}$$

See also: J.Hirvonen, O. Gould, 2108.04377, J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331

Computing effective action - 3D toy model

$$\mathscr{L}_3 = \frac{1}{2}(\partial_i\Phi)^2 + \frac{1}{2}(\partial_i\chi)^2 + \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa\chi^2\Phi^2 + \frac{1}{2}M^2\chi^2$$

Expand field on its background

$$\Phi \to \phi_b(r) + H(x)$$

At 1-loop we need quadratic terms only

$$\mathscr{L} \supset \chi \underbrace{\left(-\nabla^2 + M^2 + \kappa\phi_b^2\right)}_{\mathscr{M}_\chi} \chi$$

Performing the $\int \mathscr{D}\chi$

$$e^{iS_{\text{eff}}[\phi_b]} \sim \left[\det\left(-\nabla^2 + M^2 + \kappa\phi_b^2\right)\right]^{-\frac{1}{2}} e^{-S_3[\phi_b]}$$

One-loop contributions to nucleation rate

$$\Gamma = A_{\mathrm{dyn}} \underbrace{\mathrm{Det}_S}_{E<\Lambda} \underbrace{\mathrm{Det}_{X_0}\, \mathrm{Det}_{XG}\, e^{-\Delta S_3[\phi_{\mathrm{b}}]}}_{E>\Lambda}$$

Derivative expansion of gauge-Goldstone det

$$\boxed{\frac{m_3}{m_X} \ll 1}$$

$$\delta Z = -\frac{11}{16\pi}\frac{g_3}{\phi_b}$$

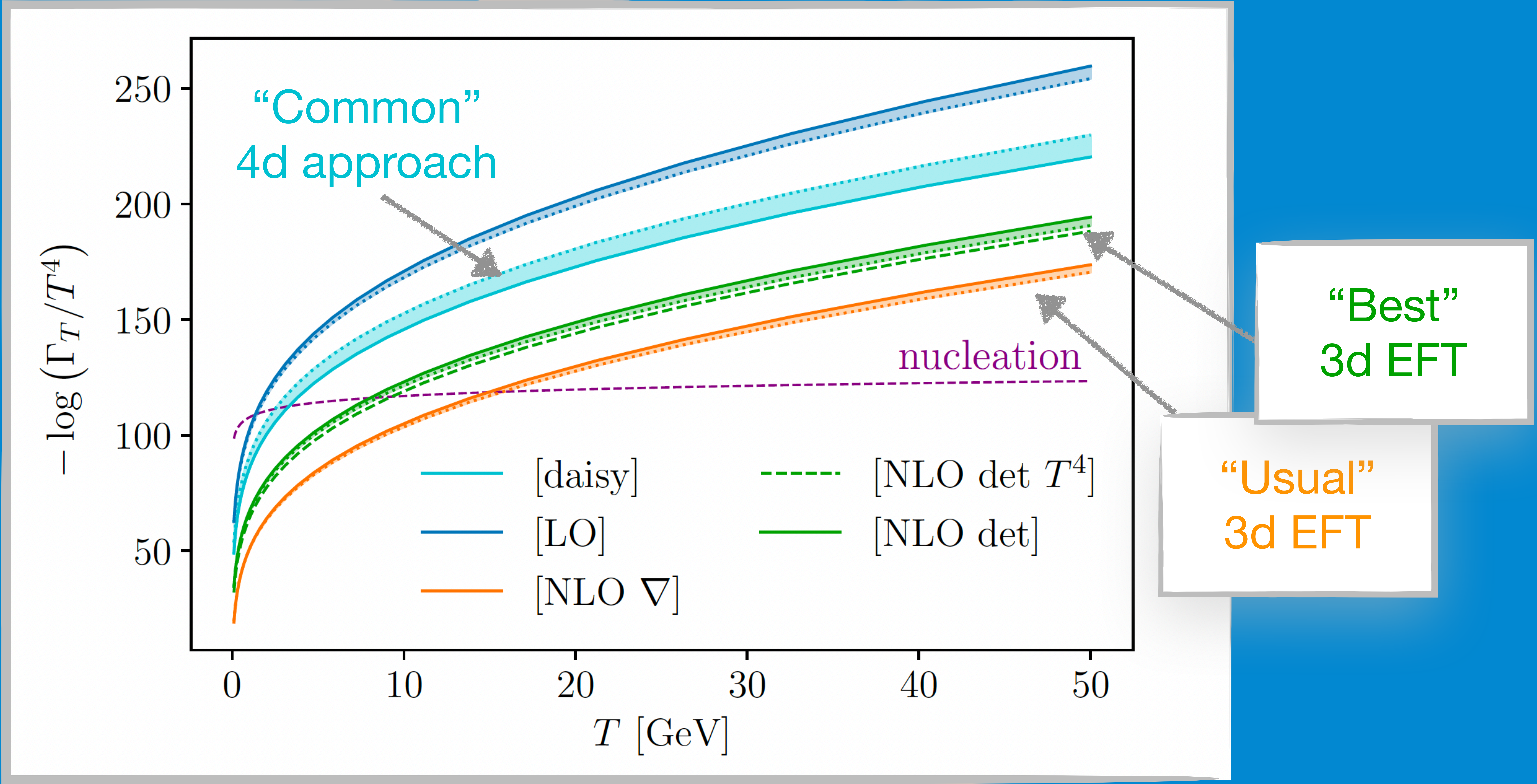
$$-\log \mathrm{Det}_{XG} \simeq \underbrace{\int_x \frac{-3}{48\pi} g^3 \phi_b^3}_{\mathcal{O}(g^3)\ \mathrm{LO}} + \underbrace{\int_x \frac{1}{2}\delta Z[\phi_b](\partial\phi_b)^2}_{\mathcal{O}(g^4)\ \mathrm{NLO}} + \ldots$$

Barrier generating term

Terms with higher powers in derivatives

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804, J. Hirvonen et al 2112.08912, MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

Results: nucleation rate from different "recipes"



Note:
$A_{\mathrm{dyn}} \det{}_S \simeq T^4$
everywhere
except [NLO det]

[2503.13597, with P. Schicho, B. Świeżewska, T.V.I. Tenkanen and J. van de Vis,]

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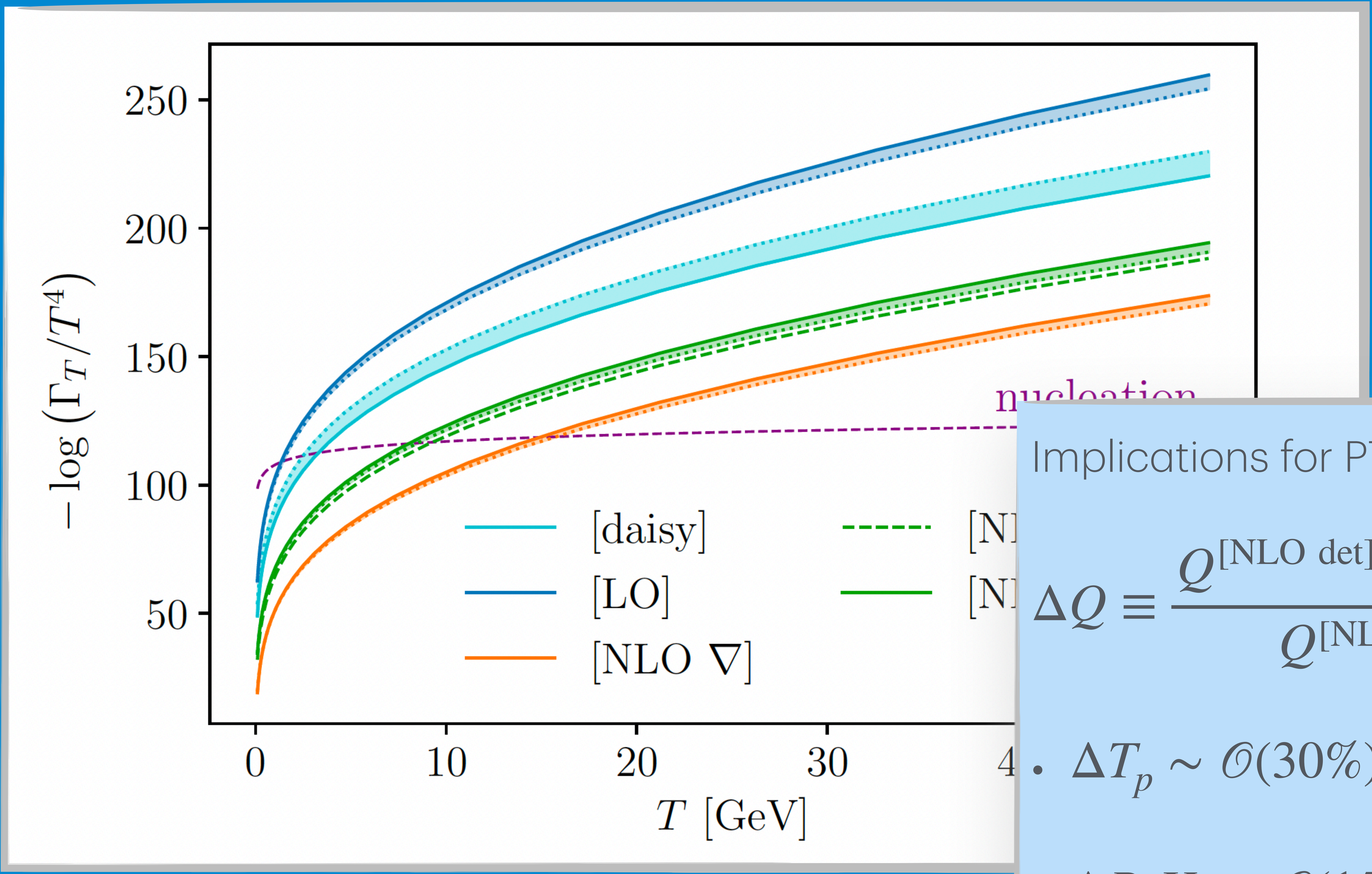

Implications for PT parameters:

$$\Delta Q \equiv \frac{Q^{[\text{NLO det}]} - Q^{[\text{daisy}]}}{Q^{[\text{NLO det}]}}$$

- $\Delta T_p \sim \mathcal{O}(30\%) - \mathcal{O}(80\%)$
- $\Delta R_* H_* \sim \mathcal{O}(15\%) - \mathcal{O}(20\%)$

Note:
$A_{\text{dyn}} \det_S \simeq T^4$
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How do dim-6 operators affect the nucleation rate?

$$\mathcal{L}_1 = \alpha_{\phi^2F^2} F_{ij}F_{ij}\phi^\dagger\phi + \alpha_{D^2\phi^4}\left(D_i\phi^\dagger D_i\phi\right)\left(\phi^\dagger\phi\right)$$
$$+\alpha_{D^2\phi^2B_0^2}\left(D_i\phi^\dagger D_i\phi\right)B_0^2 + \alpha_{B_0^2\phi^4}B_0^2\left(\phi^\dagger\phi\right)^2 + \alpha_{\phi^6}\left(\phi^\dagger\phi\right)^3$$

Bounce sol
gets modified

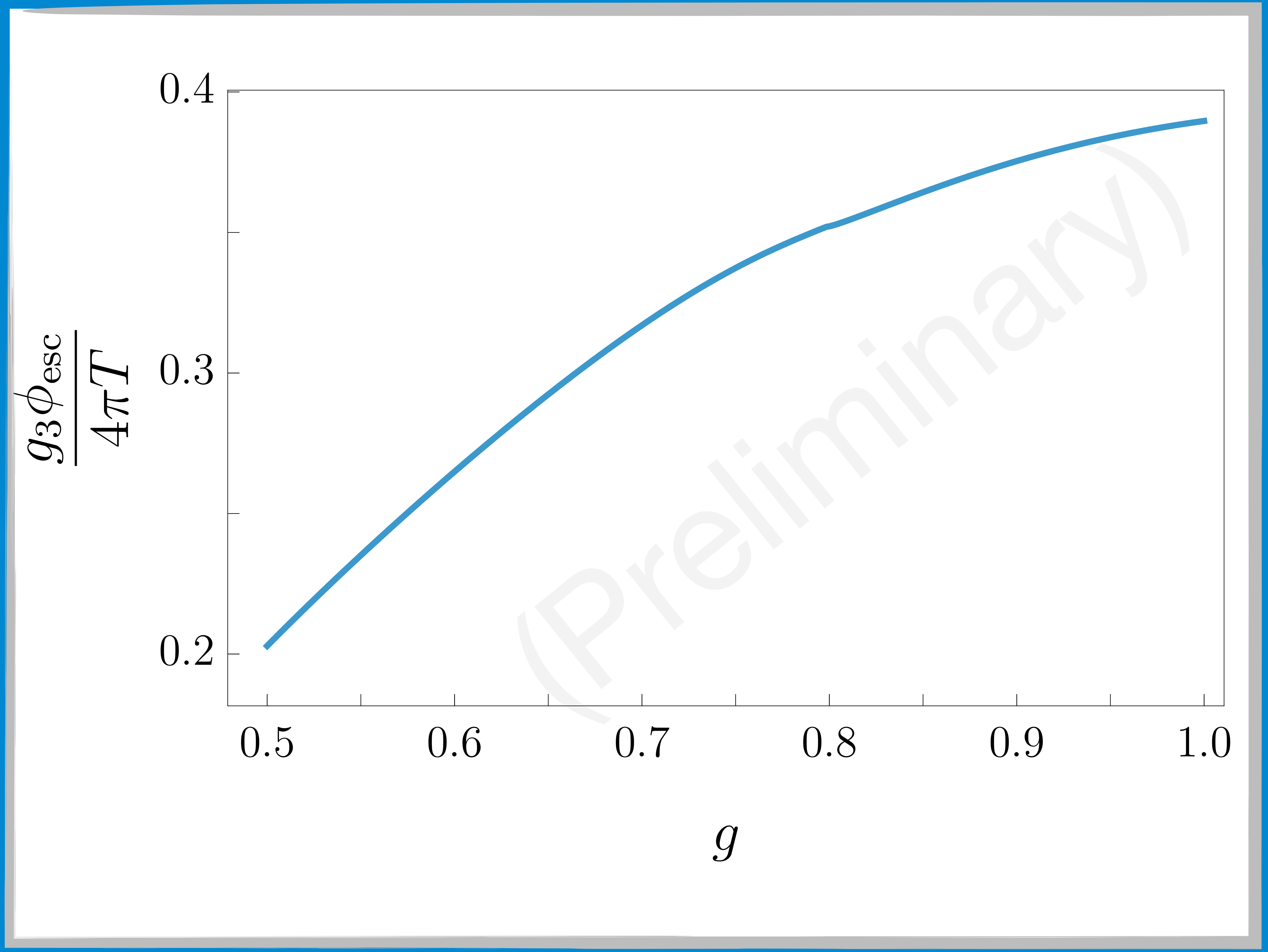
Quadratic terms
get modified

Example:

$$\mathrm{Det}_{X_0} = \mathrm{Det}\left(\partial^2 + m_{X_0}^2 + \frac{1}{2}\alpha_{\phi^4B_0^2}\phi_b^4 + \alpha_{D^2\phi^2B_0^2}\phi_b'^2\right)$$

F. Bernardo, MK, N. Leister, P. Schicho, P. Schwaller, 2609.xxxxx

HT validity => small effect of dim-6 operators



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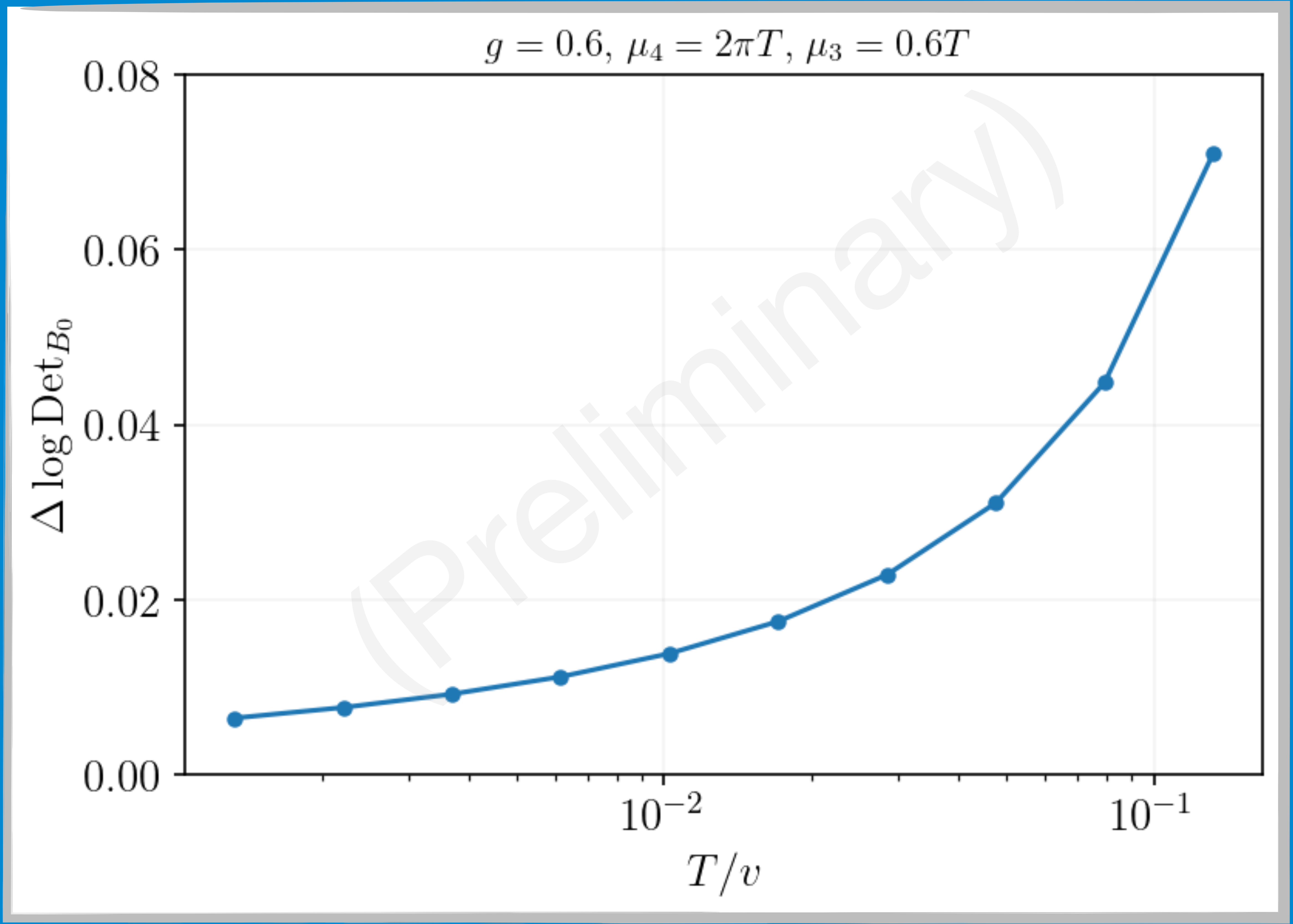
The relative difference for Det_{X_0} is up to 7%



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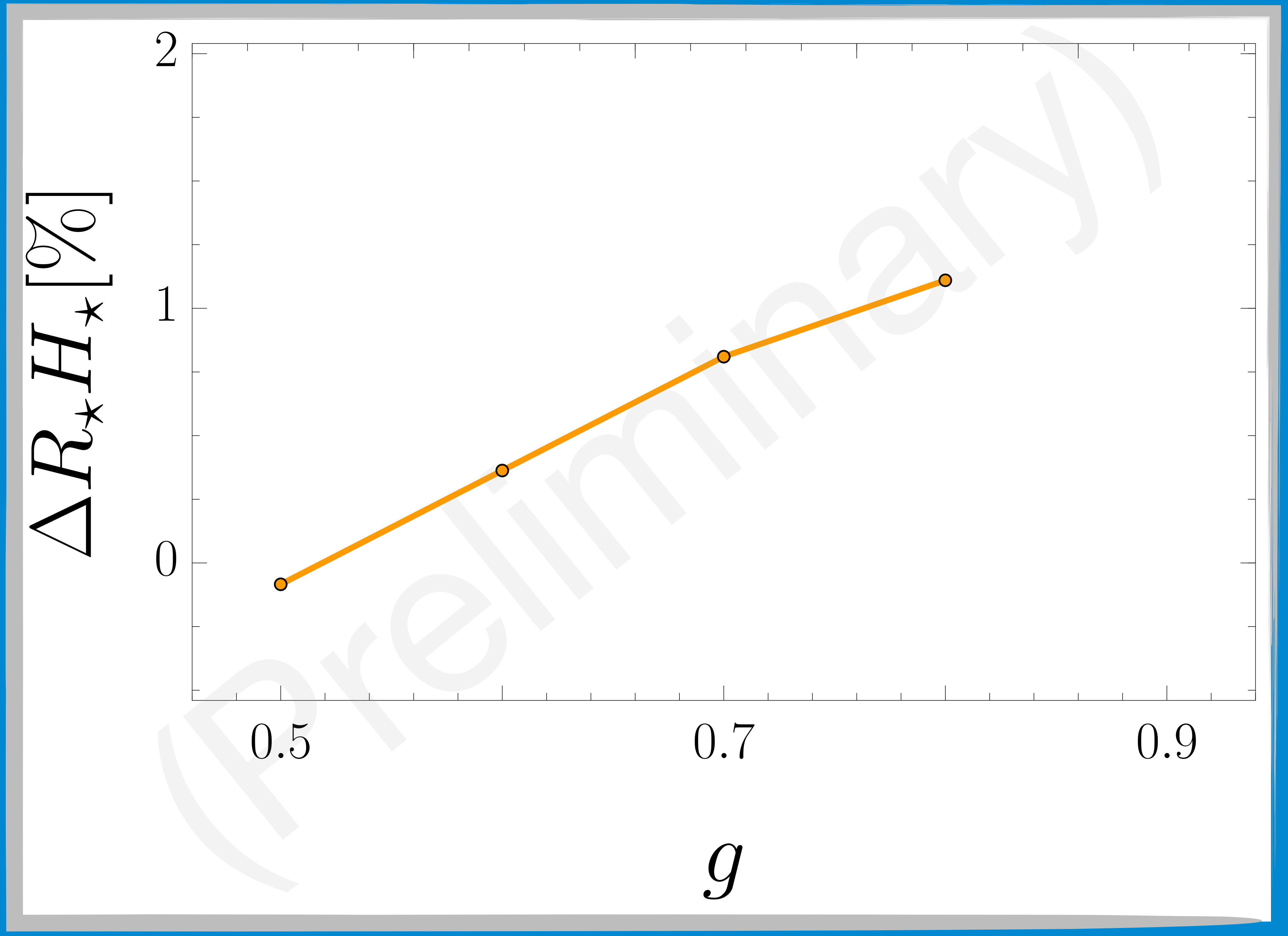
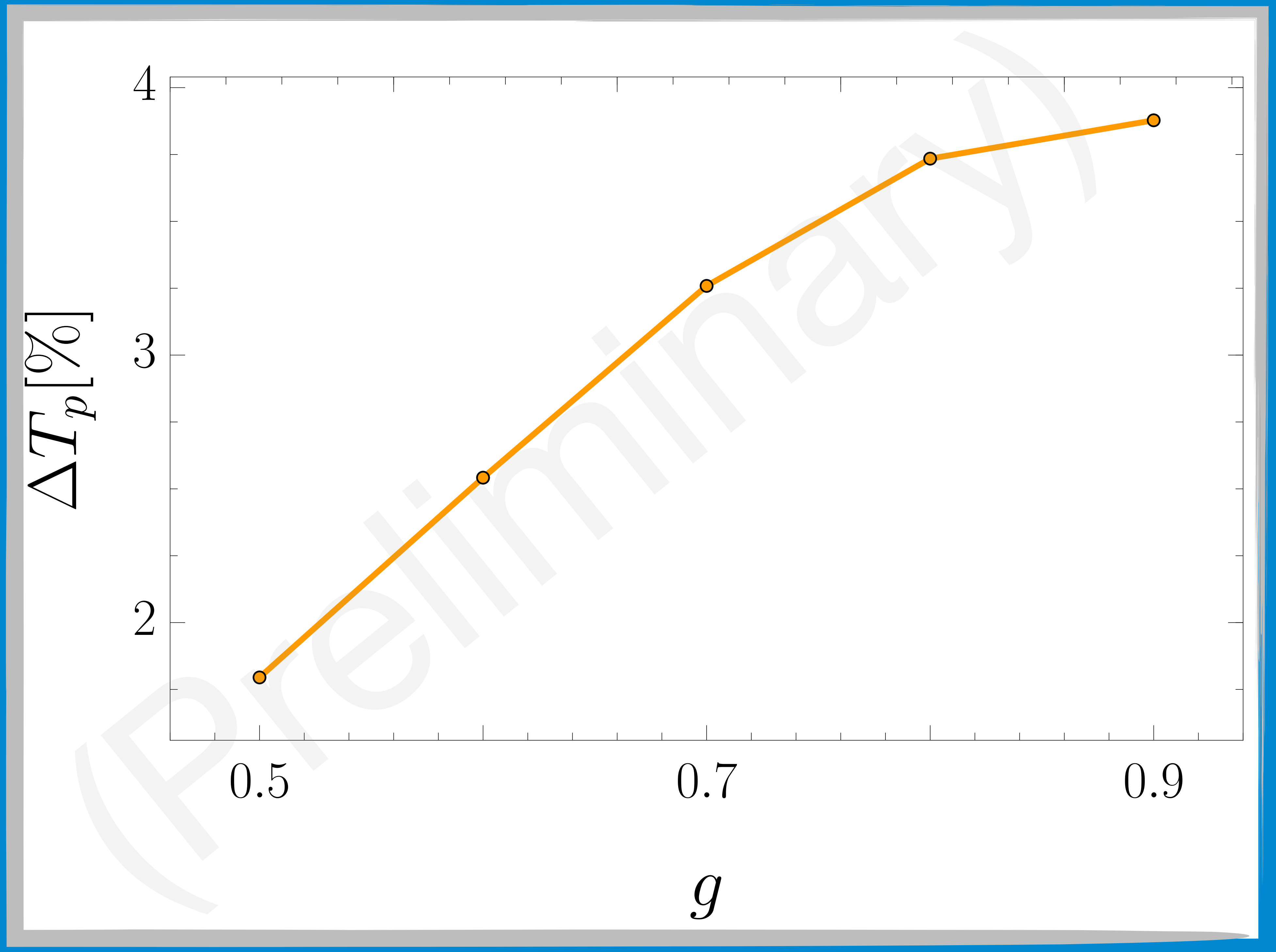
T_p changes up to 4%, while $R_* H_*$ up to 1%

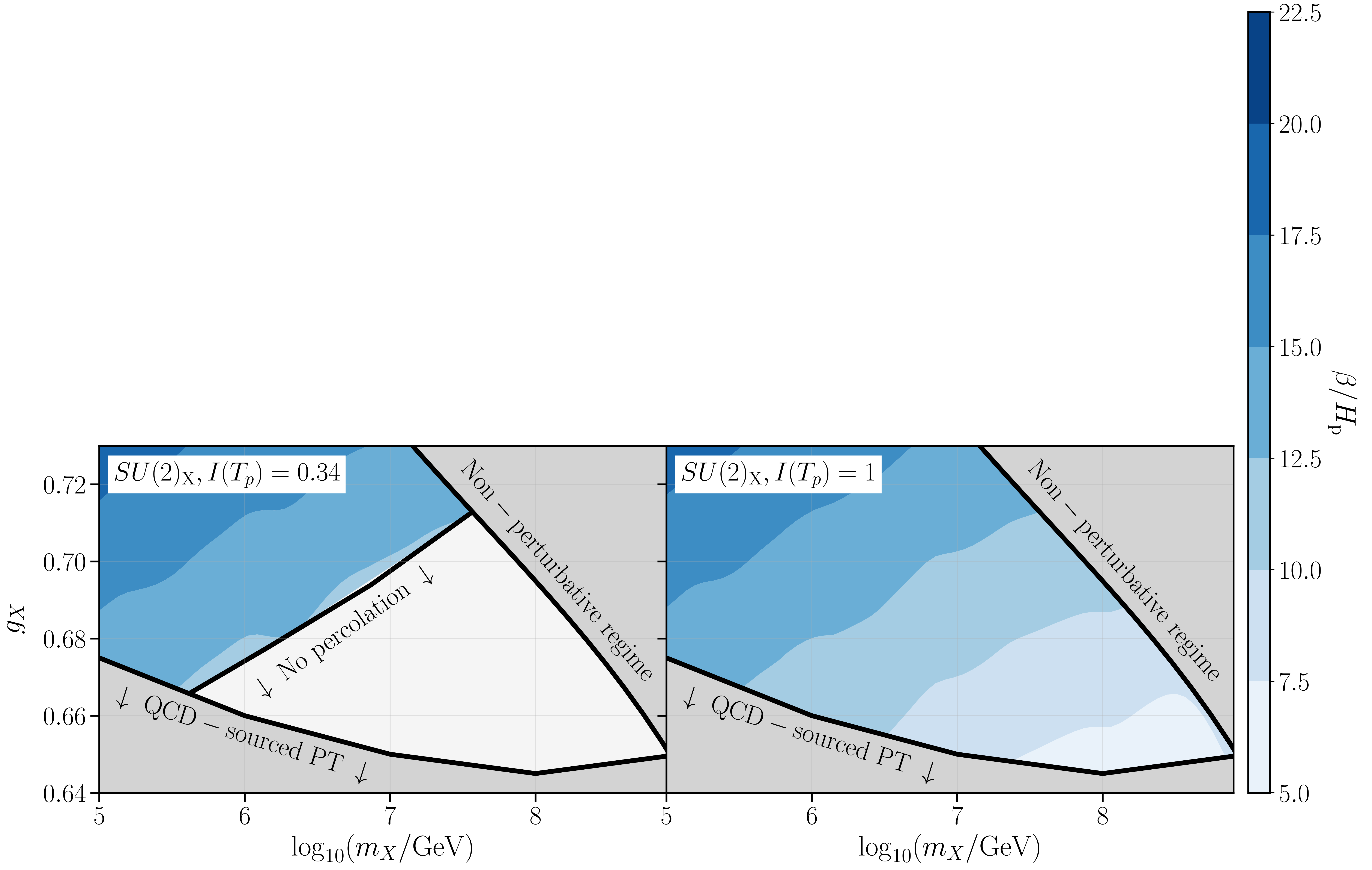




(Plots due to courtesy of N. Leister)

F. Bernardo, MK, N. Leister, P. Schicho, P. Schwaller, 2609.xxxxx

What can happen for extreme supercooling?



MK, N. Ramberg, P. Schicho, D. Schmitt, 2506.15496

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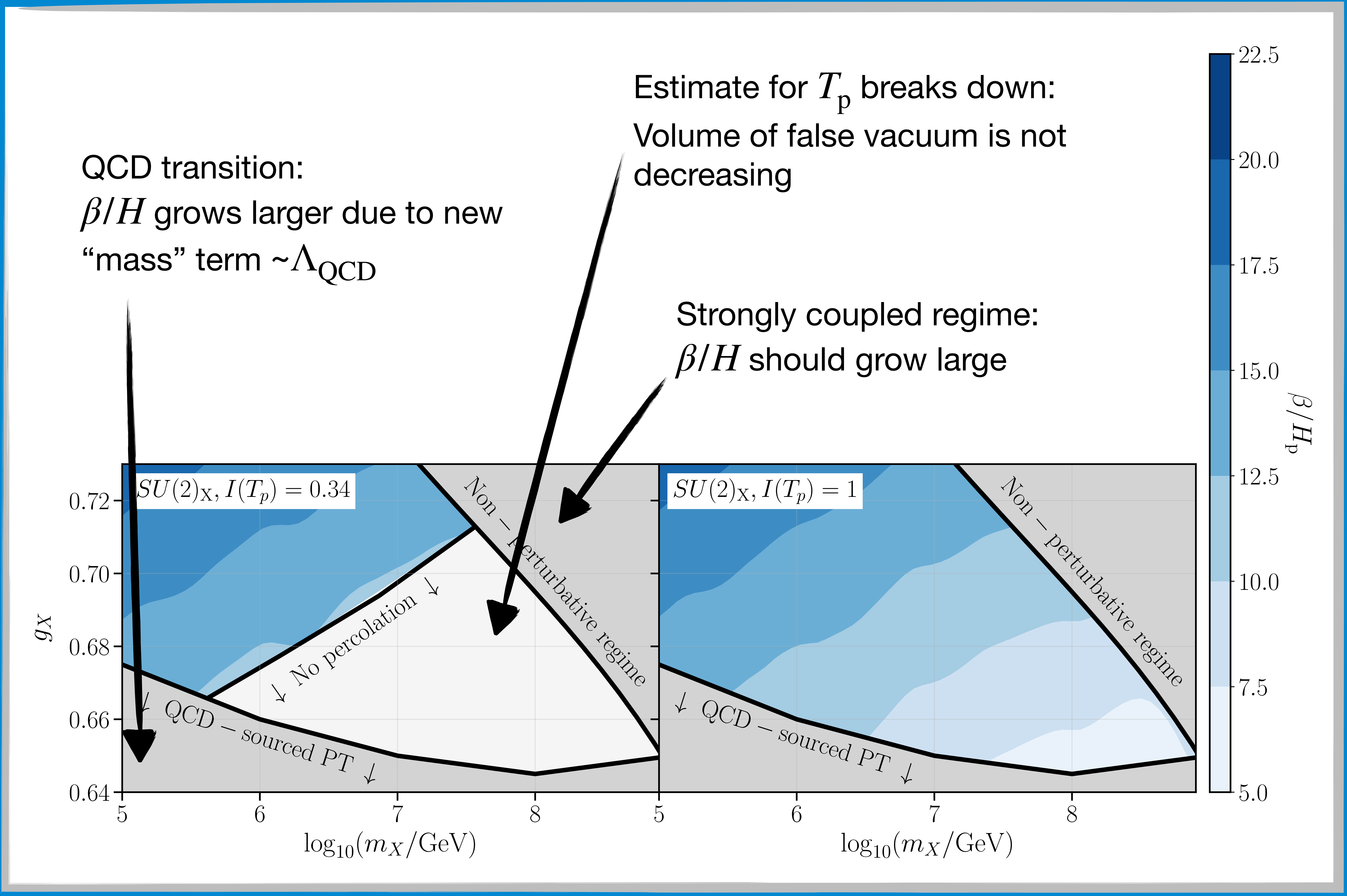

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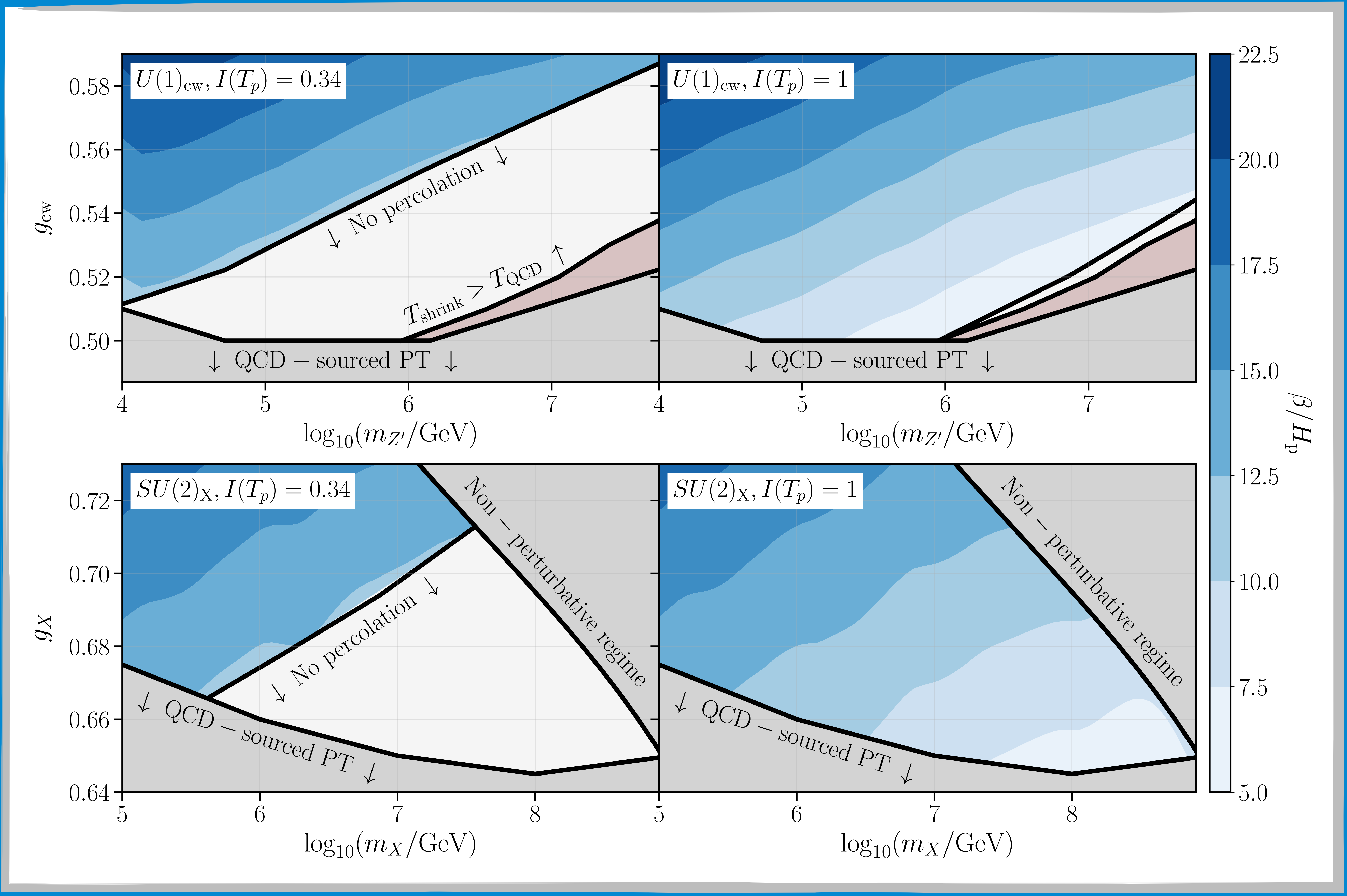
It's unlikely to go below $\beta/H \sim 5$



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Summary

Dim-6 operators seem to have **little effect on nucleation**

Derivative expansion for **gauge modes** introduces significant **errors**.

NLO corrections are **mandatory** for reliable pheno predictions

Min value of $\beta/H \simeq 5$

Outlook

Go to NNLO to assert convergence of perturbative expansion

It'd be nice to have a comparison to nucl. rate on a lattice!

Precise predictions for PTA data

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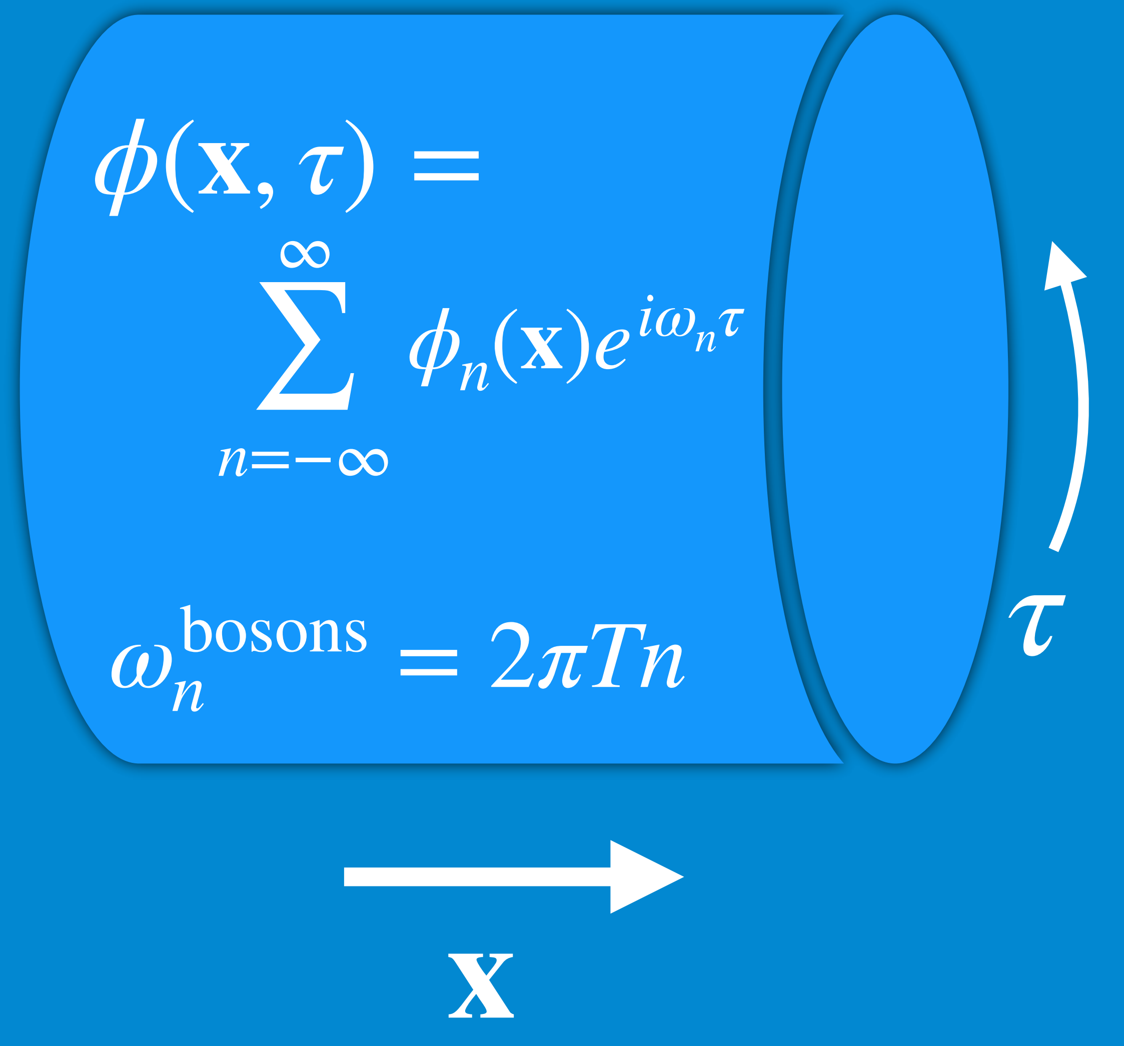
Precise predictions for PTA data

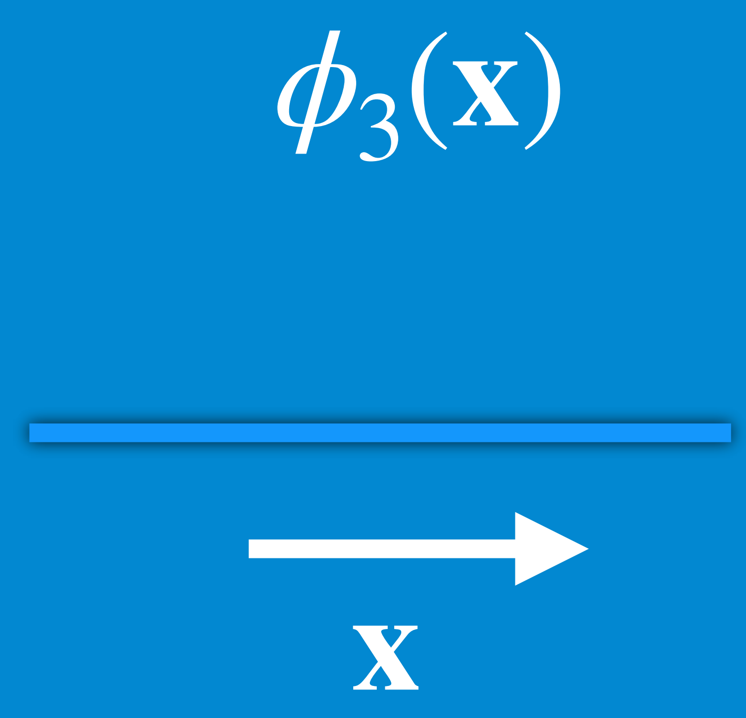
Thank you!

High-temperature Dimensional Reduction (DR)



$\longrightarrow$

Integrating out $\omega_{n\neq 0}$



$$Z = \int D\phi \exp\left(-\int_0^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3\mathbf{r}\ \mathscr{L}_E \right) \longrightarrow Z_3 = \int D\phi_{n=0} \exp\left(-\frac{1}{T} \int \mathrm{d}^3\mathbf{r}\ \mathscr{L}_3 \right)$$

Fields: ϕ, X | Fields: $\phi_3, X_3, X_{0,3}$

Couplings: g, λ | Couplings: $m_3(T), g_3(T), \lambda_3(T)$