Automated thermal field theory with `AutoTherm`

The gravitino example

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SUBATECH

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Contents of the presentation

Based on the two upcoming papers

- *"Automated Thermal Field Theory rates for cosmology"*
- *"Thermal gravitino and axino rate from AutoTherm"*

with Greg Jackson and Jacopo Ghiglieri

Introduction

Current understanding of the Universe based on the following **Standard Models (SM)**

Particle physics

- 17 **elementary particles**
- 3 **fundamental interactions**
- 19 **free parameters**

Cosmology (ΛCDM)

- **Expanding** Universe
- 5% **baryonic matter** + 27% **dark matter (DM)** + 68% **dark energy**
- 6 free parameters

Some issues remain however, *e.g.*

Baryon asymmetry of the Universe (BAU)

More matter than anti-matter **but** not enough $\mathcal{CP}$ violation in the SM

Nature of DM

ΛCDM makes no assumptions about the nature of DM (boson or fermion, ...)

Most solutions invoke the existence of **new** particle degrees of freedom in the early Universe, new particles **beyond the Standard Model (BSM)**

Thermodynamical functions characterizing a particle ϑ (*e.g.* number density n_ϑ, energy density e_ϑ, ...) depend on its **phase space distribution (PSD)** f_ϑ

Boltzmann equation

$$\Big(\underbrace{\partial_t}_{t/T\ \text{evolution}} - \underbrace{Hk\partial_k}_{\text{Hubble term/expansion}}\Big) f_\vartheta = \underbrace{\Gamma_\vartheta \left(f_{\text{eq}} - f_\vartheta\right)}_{\text{Creation/destruction of }\vartheta\text{ particles}} \tag{1}$$

Γ_ϑ: **production rate** of ϑ

Remark

$f_{\text{eq}} \gg f_\vartheta$ case: sometimes this is called the production rate

$$(\cdots)\, f_\vartheta = \Gamma_\vartheta f_{\text{eq}} \tag{2}$$

Here: will call Γ_ϑ the **equilibration/interaction/production rate** interchangeably

- Γ_ϑ characterizes the departure from **thermal equilibrium**: relevant for BAU because of the **Sakharov conditions**
- Particle production is efficient for $\Gamma_\vartheta \gg H$

In this talk, we will consider the **gravitino** $\tilde{G}$ as the particle of interest ϑ to:

- illustrate the strengths of `AutoTherm`,
- showcase **new results** that can be obtained with it.

Quick introduction

$\tilde{G}$ appears in **supergravity (SUGRA)** [local version of **supersymmetry (SUSY)**] as the **superpartner** of the **graviton** G.

Characteristics:

- **Massive** spin-$3/2$ particle
  - Can be decomposed between a $m_s = \pm 1/2$ component χ and a $m_s = \pm 3/2$ component ψ

    V. S. Rychkov, A. Strumia, *Phys. Rev. D 75 (2007) 075011*
- Coupled to other particles via a $d = 5$ interaction
  - Coupling $\kappa \propto m_{\mathrm{Pl}}^{-1}$

Confronting theory and experiment

What do we want from ϑ models?

1. The problem is actually solved

DM example

Correct current-day abundance generated, ϑ is stable on cosmological times, ...

2. We don't break anything else

DM example *(cont.)*

Avoid **overclosure**, ...

These restrictions allow us to **constrain** the parameter space of ϑ models $\longrightarrow$ We want **precise** calculations, especially with next-generation detectors on the way

To summarize, the flowchart for phenomenological computations looks like this:

$$\boxed{\mathcal{L}} \longrightarrow \boxed{\textbf{Perturbative methods}} \longrightarrow \boxed{\Gamma_\vartheta} \longrightarrow \boxed{\textbf{Boltzmann equation}} \longrightarrow \boxed{f_\vartheta} \longrightarrow \boxed{\int} \longrightarrow \boxed{n_\vartheta, e_\vartheta, \cdots}$$

One small issue: there are **a lot** of ϑ models

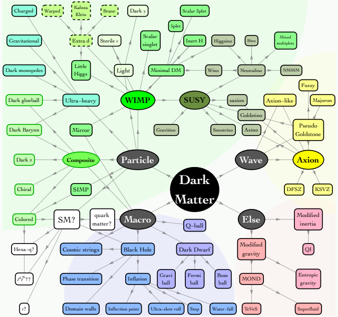


Figure: Tentative mindmap of all DM models (from `2406.01705`)

However, the computation always goes through the **same steps**.

Automation is interesting to:

- save time,
- reduce the risk of errors.

Previously on SEWM...

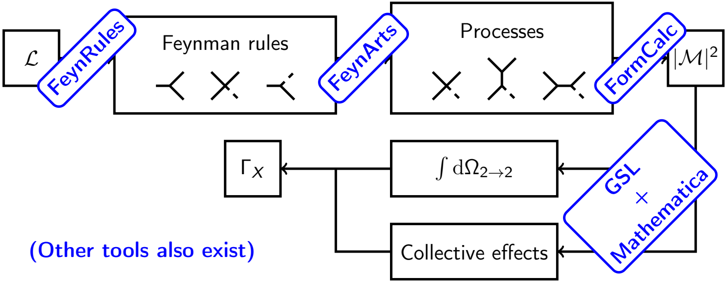


Figure: Flowchart of the production rate computation process

Introducing: the `AutoTherm` code

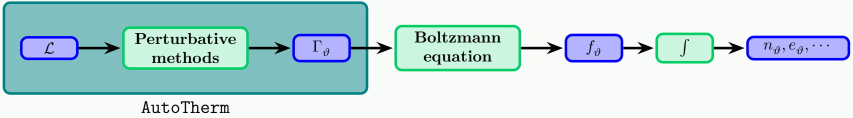
You push the button and we'll do the rest

— "The World's First Ever Monster Truck Front Flip" by Arctic Monkeys

- Developped by Jacopo Ghiglieri, Greg Jackson and myself
- To be released alongside the manual *"Automated Thermal Field Theory rates for cosmology"*

The `AutoTherm` code

`AutoTherm` is a code to automate the computation of the **thermal production rate** Γ_ϑ of a particle ϑ.



Γ_ϑ is computed for $k \gg T$ at **leading order (LO)**.

Current limitations

Only:

- **Massless** or **ultrarelativistic (UR)** particles (*i.e.* $m \ll T$),
- **Single production** (only $\cdots \leftrightarrow \cdots + \vartheta$ processes),
- $2 \leftrightarrow 2$ processes.

Remark

It can be shown that the $2 \leftrightarrow 2$ component of the rate is:

- **part** of the LO for $d = 4$ interactions,
- **all** of the LO for $d \geq 5$ interactions.

$1 \leftrightarrow 2$ processes are also important for $d = 4$ interactions but they have different physics than what we will go over in this talk.

So our starting point is:

Definition 1 — ($2 \leftrightarrow 2$ component of the production rate)

$$\Gamma_\vartheta(k)\Big|_{2\leftrightarrow 2} \equiv \frac{1}{2}\frac{1}{2k}\frac{1}{d_\vartheta} \underbrace{\int \mathrm{d}\Omega_{2\leftrightarrow 2}}_{2 \leftrightarrow 2 \text{ phase space}} \underbrace{\sum_{a,b,c} \left|\mathcal{M}_{a+b\leftrightarrow c+\vartheta}\right|^2}_{2 \leftrightarrow 2 \text{ scatterings}} \underbrace{\frac{f_a(p_1) f_b(p_2)\left[1 \pm f_c(k_1)\right]}{f_{\text{eq}}(k)}}_{\text{Thermal distributions}} \tag{3}$$

Computation process

Step 1

Generating the Feynman rules

Step 2

Generating all processes

Step 3

Computing the matrix elements squared

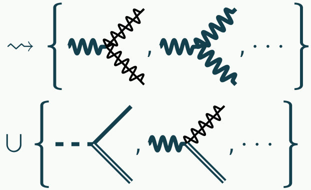
Step 4

Doing the phase space integration

Starting point

$$\mathcal{L}_{\text{tot}} = \underbrace{\mathcal{L}_{\text{med}}}_{\text{Bath} \leftrightarrow \text{bath}} + \overbrace{\mathcal{L}_{\vartheta}}^{\vartheta \text{ kinetic term}} + \underbrace{\mathcal{L}_{\text{int}}}_{\text{Bath} \leftrightarrow \vartheta} \tag{4}$$

$\mathcal{L}_{\text{tot}} \rightsquigarrow$



Computation process

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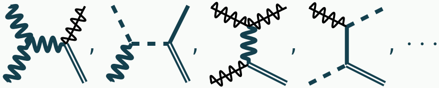

Remark

`AutoTherm` can't handle spin-$3/2$ particles.

<u>Solution:</u> It can be shown that...

$$\Gamma_\chi\Big|_{\text{gauge}} \propto \Gamma_\psi\Big|_{\text{gauge}} \;\&\; \Gamma_\chi\Big|_{\text{top}} \propto \Gamma_\psi\Big|_{\text{top}}$$

V. S. Rychkov, A. Strumia, *Phys. Rev. D 75 (2007) 075011*

...so we just do the computation for χ.

Computation process

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Computation process

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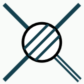
Doing the phase space integration

$$\Gamma \sim \int \mathrm{d}\Omega_{2\leftrightarrow 2} \sum_{a,b,c} \Big|$$



$$\Big|^2 \frac{f_a f_b\,[1 \pm f_c]}{f_{\mathrm{eq}}}$$

$\int \mathrm{d}\Omega_{2\leftrightarrow 2}$ gives only **4** integrals, because of **energy-momentum conservation** and **rotational invariance**

Remaining integrals

- q_0, q: **exchanged momentum**
- p_i: one momentum
- φ: one angle

Computation process

Step 1

Generating the Feynman rules

Step 2

Generating all processes

Step 3

Computing the matrix elements squared

Step 4

Doing the phase space integration

Strategy

For $d = 4$ or 5, $|\mathcal{M}|^2$ can be **decomposed** on a **basis**. *E.g.* for $d = 5$:

$$|\mathcal{M}|^2 = a_1 \frac{s^2 + u^2}{t} + a_2 t + b_1 \frac{t^2}{s} + b_2 s \quad (5)$$

J. Ghiglieri *et al.*, *JHEP 07 (2020) 092*

The $\int \mathrm{d}p_i \int \mathrm{d}\varphi$ can then be done **analytically**

Computation process

Step 1

Generating the Feynman rules

Step 2

Generating all processes

Step 3

Computing the matrix elements squared

Step 4

Doing the phase space integration

There is one issue however: diagrams of these types...

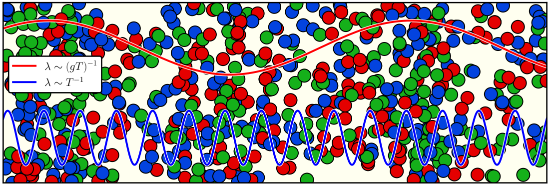
$d = 4$ $d = 5$

...lead to a **logarithmic divergence** due to the **massless mediator**

$$\int \mathrm{d}\Omega_{2\to 2} \frac{1}{t} \sim \int_{\Lambda}^{k} \frac{\mathrm{d}q}{q} \sim \ln\left(\frac{k}{\Lambda}\right) \tag{6}$$

Why is that so?

The $q \to 0$ limit corresponds to a mediator with $\lambda \to +\infty$, which doesn't resolve **individual particles** in the thermal bath but **collective oscillations**:



Our working formula for the rate isn't wrong but doesn't account for collective effects.

The Hard Thermal Loop (HTL) theory

Describes collective dynamics within the thermal bath.

E. Braaten, R.D. Pisarski, *Nucl.Phys. B337 (1990) 569* + *Phys. Rev. D45 (1992) 1827*

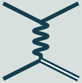
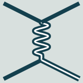
The scale of the oscillations is set by the **thermal mass** of the mediator. For gauge bosons (relevant for the $\tilde{G}$ case), it's the **Debye mass** $m_{\mathrm{D}} \propto gT$

Computation schemes

Strict LO scheme

E. Braaten, T. C. Yuan, *Phys. Rev. Lett. 66 (1991) 2183*

Introduce **cutoff** $gT \ll q^\star \ll T$ on exchanged momentum and use HTL **only** for $q < q^\star$

$\int_{q>q^\star} \mathrm{d}q \,\Big|$  $\Big|^2 + \int_{q<q^\star} \mathrm{d}q \,\Big|$  $\Big|^2$

+ Solves the divergence

– **Unphysical** $\Gamma < 0$ for $k \lesssim m_\mathrm{D}$

Subtracted scheme

J. Ghiglieri *et al.*, *JHEP 07 (2020) 092*

Subtract divergent part and add HTL contribution computed **analytically** with **sum rules**

P. Aurenche, F. Gelis, H. Zaraket, *JHEP 0205 (2202) 043*

S. Caron-Huot, *Phys.Rev. D79 (2009) 065039*

+ **Reduces** Γ negativity

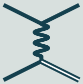
– Still $\Gamma < 0$ for $k \lesssim m_\mathrm{D}$

Computation schemes

Tuned mass scheme

M. C. A. York, A. Kurkela, E. Lu, G. D. Moore, *Phys.Rev. D89 (7) (2014) 074036*

Regulate gauge boson propagator with "mass" ξm_D and **analytically** determine ξ to reproduce the strict LO results for $g \ll 1$

$$\int \mathrm{d}q \left|$$

$$\right|^2 \times \frac{q^4}{\left(q^2 + \xi^2 m_\mathrm{D}^2\right)^2}$$

$+$ **Positive-definite** scheme

All schemes require computing thermal masses

How AutoTherm does it

The thermal mass of a particle P can be written as...

$$m^2 \sim \sum_\mathrm{X} \underbrace{\mathcal{M}_{\mathrm{P+X\to P+X}}}_{\text{Forward scattering}} \overbrace{H(\sigma_\mathrm{X})}^{T\text{-dependent factor}} \tag{7}$$

Analogous to dLTD (see Kaapo's talk yesterday)

...so we can the same tools we used to compute Γ.

Results

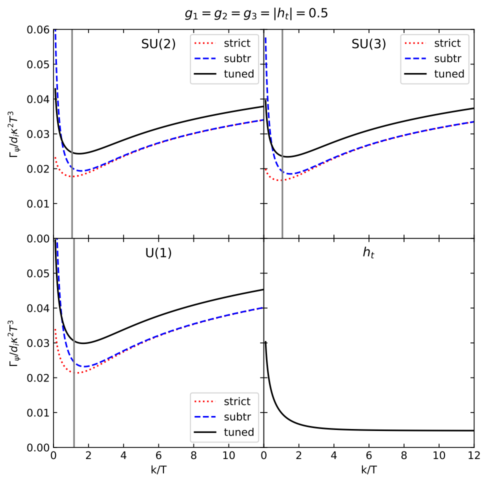


Figure: Small coupling

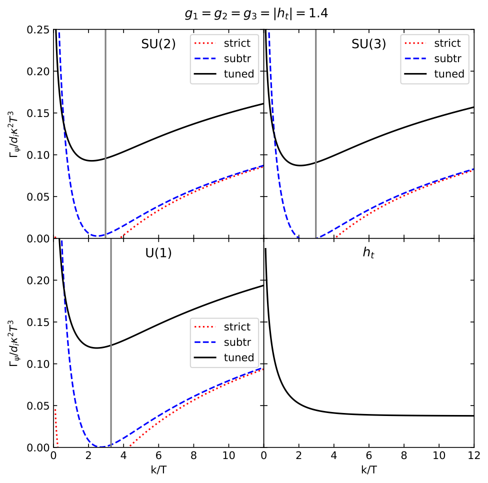
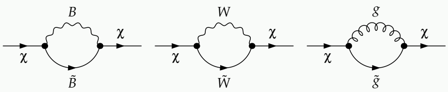


Figure: Large coupling

Gray line: $k = m_\mathrm{D}$

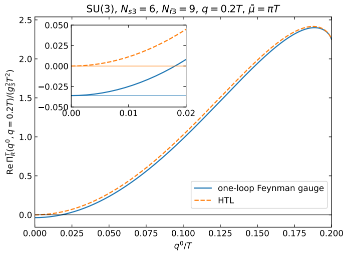
V. S. Rychkov and A. Strumia, *Phys. Rev. D 75 (2007) 075011* introduced a scheme to compute the $\tilde{G}$ **self-energy** ($\propto \Gamma_{\tilde{G}}$) using **gauge-dependent, one-loop, 1PI resummed propagators**



Rychkov-Strumia use the **Feynman gauge** specifically.

To reproduce gauge boson mediated scattering (divergent part), the gauge boson must be **space-like** ($Q^2 < 0$). In this region, for $q^0 \to 0$, the gauge boson self-energy behaves as:

- $\mathrm{Re}[\Pi_\mathrm{T}^\mathrm{full}(q^0 \to 0, q)] < 0$
- $\mathrm{Re}[\Pi_\mathrm{T}^\mathrm{HTL}(q^0 \to 0, q)] \to 0$



This causes a **divergence** to appear. If we write the real part as...

$$\mathrm{Re}[\Pi_\mathrm{T}(q^0 \to 0, q \ll T)] = -aq + bq^2 + \mathcal{O}\left(\frac{g^2 q^3}{T}\right) \quad (8)$$

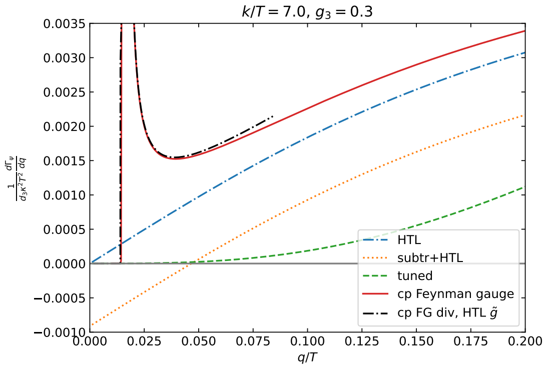
...one can show that:

$$\Gamma_\psi \sim \int_{\mathcal{O}(g^2T)} \mathrm{d}q \frac{q^2}{\left|q - \frac{a}{1+b}\right|} \tag{9}$$

- HTL: $a = b = 0 \to$ no divergence
- Full: Divergence for $q \approx a$ (when $a > 0$)

$a \sim g^2 T$: **chromomagnetic/ultrasoft scale**

but ultrasoft gauge fields can't be described perturbatively (**Linde problem**)



a depends on the content of the gauge group. In the MSSM:

- $\mathrm{U}(1)$: Abelian gauge groups don't have an ultrasoft scale $\to$ no divergence
- $\mathrm{SU}(2)$: Interestingly, $a < 0 \to$ no divergence (in the Feynman gauge)
- $\mathrm{SU}(3)$: $a > 0 \to$ divergence

Unknown why numerical results in the literature are **finite**: they are **untrustworthy**

Conclusion

AutoTherm

- AutoTherm: new code for **automatically** computing **thermal particle production rates**, starting from the **Lagrangian** of the model
- Novelty: automated implementation of **medium effects**, using **HTL theory**
- Different computation schemes: better idea of the **theory uncertainty**
- AutoTherm was tested on a variety of models and can **successfully reproduce** results from the literature

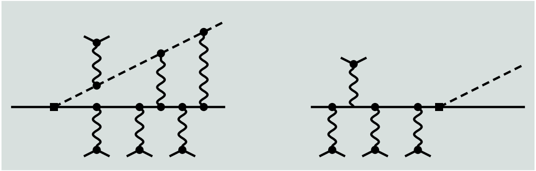
Gravitino rate

- $\tilde{G}$: particle of interest in cosmology
- **Gauge-dependent resummation scheme** (previous state of the art) was recently discovered to be **problematic** (**divergence issue**)
- **Tuned scheme results**: current best theoretical determination of $\Gamma_{\tilde{G}}$

Outlook

Next possible improvement for AutoTherm

- $1 \leftrightarrow 2$ processes can be very relevant in some cases (mainly $d = 4$ interactions)
- Need to account for **Landau-Pomeranchuk-Migdal (LPM) effect**: **interference** between terms with multiple scatterings during particle formation.



However the algorithm for performing **LPM resummation** is **known**

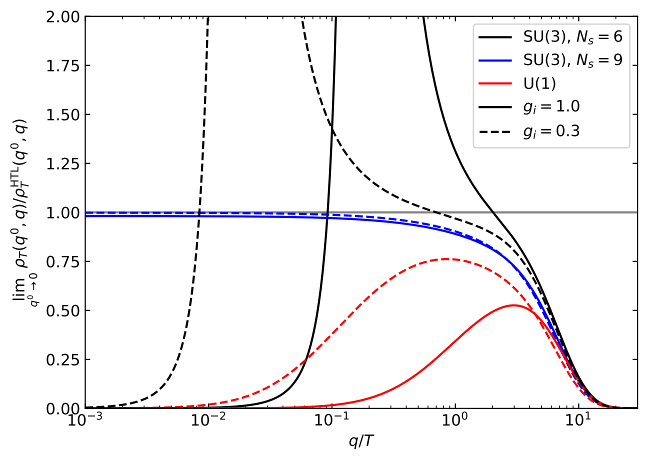
See *e.g.* P. Arnold, G. D. Moore, L. G. Yaffe, *JHEP 0206 (2002) 030* + J. Ghiglieri, M. Laine, *JHEP 01 (2022) 173* +...

Could easily build upon the $2 \to 2$ code

Thank you for your attention

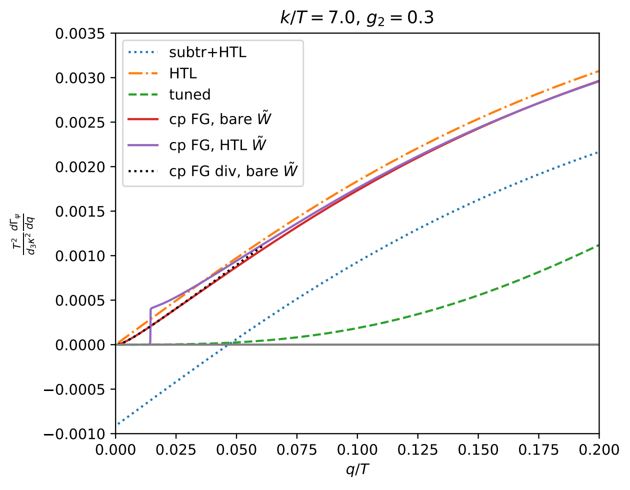
Backup slides

FG *vs.* HTL spectral functions

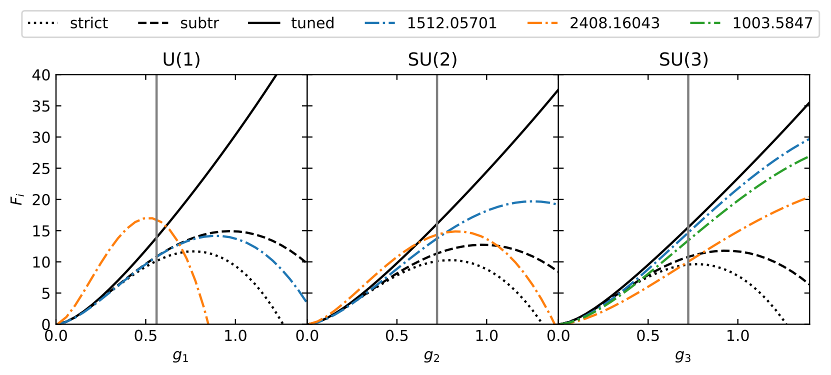


- Visible divergence for $\mathrm{SU}(3)$
- No divergence for $\mathrm{U}(1)$
- In a **hypothetical** $\mathrm{SU}(3)$ with more scalars ($N_s = 9$ blue curve), the divergence is **lifted** (in Feynman gauge)

Differential $\tilde{G}$ rate $(\mathrm{SU}(2))$ component)



Momentum-averaged rate

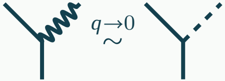


Gray line: GUT coupling

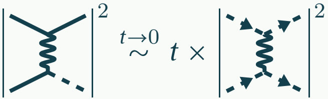
$$\gamma_{\tilde{G}} \equiv 2\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left[\Gamma_\psi(k)+\Gamma_\chi(k)\right]n_\mathrm{F}(k) \sim \sum_i \kappa^2 T^6 F_i(g_i) + [\text{top part}] \tag{10}$$

Vertex eikonalization

It is known that, for $d = 4$ vertices



For $d = 5$ (gauge scattering) vertices, can show that something similar to $d = 4$ happens but an extra factor of t appears in $|\mathcal{M}|^2$. Thus, we could say that



(11)

Important conclusion

When doing the HTL resummation, the **model dependence** is ENTIRELY absorbed in a **multiplicative factor**

More information about AutoTherm

General information

- Python+Mathematica-based code
- Building upon tools from the literature: FeynRules, FeynArts & FormCalc [+ others like SympPy, Cython, ...]

Inputs

- **FeynRules model file**
- **AutoTherm configuration file**
  - Which particle is ϑ
  - What is the thermal bath composed of
  - ...

The AutoTherm release includes some ready-made model files (including the $\tilde{G}$ one!) + a SM model file for $T \geq 160\ \mathrm{GeV}$ (**symmetric phase**) to use as a **base**

Intermediate outputs

- Total $|\mathcal{M}|^2$ split by (a, b, c) **statistics** [*e.g.* (boson,fermion,fermion)]
- Total $|\mathcal{M}|^2$ split by processes
- Feynman diagrams for all processes

More information about `AutoTherm` (*cont.*)

Adaptations to usual `Mathematica` tools

`FormCalc` has no native options for:

- Spin-2 polarization sum
- $\mathrm{SU}(2)_\mathrm{L}$ indices summation

- `AutoTherm` can also be used as a **library** (*e.g.* to just compute thermal masses)
- **Quick execution**: generally around $\lesssim 1\ \mathbf{min}$ [slower for $\tilde{G}$ ($\sim 10\ \mathbf{min}$) due to large number of vertices]

Running `AutoTherm`

Example run

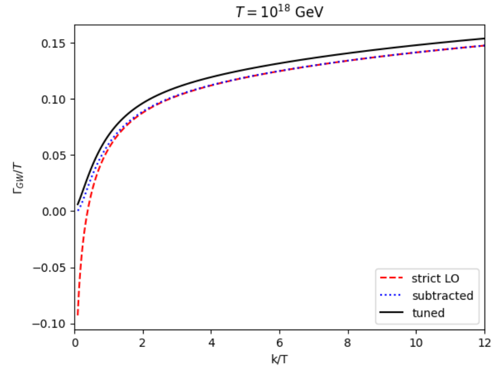
Reproducing the results of J. Ghiglieri, G. Jackson, M. Laine, Y. Zhu, *JHEP 07 (2020) 092* for graviton production in the SM:

- Requires few lines of code,
- Is relatively fast ($\lesssim 5\ \mathrm{min}$).

```
testGW=analytical_pipeline("analytical/grav.cfg")
```

```
GWrate=iterate_decomposition(testGW[0],testGW[1],testGW[2],2)
```

```
k=numpy.logspace(numpy.log10(0.1),numpy.log10(12.),100)
GWtestrate = GWrate(k,numpy.sqrt(32*numpy.pi),(numpy.sqrt(0.172942),numpy.sqrt(0.255890),numpy.sqrt(0.235984),\
                                             numpy.sqrt(0.152066),0.),0)
```



Graviton production in the MSSM

We also computed the **graviton** production rate in the MSSM.

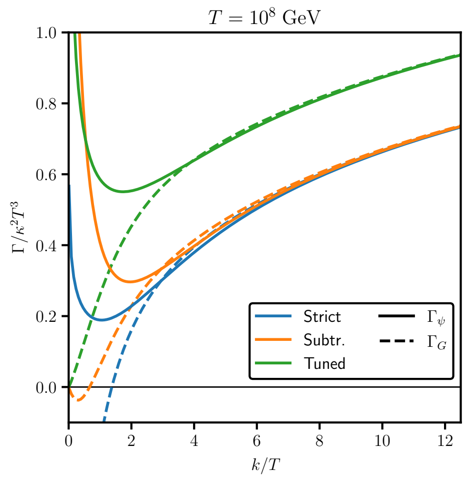
Remark

$$\Gamma_\psi(k \gg T) \approx \Gamma_G(k \gg T)$$

This agrees with the theoretical arguments of S. Caron-Huot, *Phys.Rev.D 79 (2009) 125002*

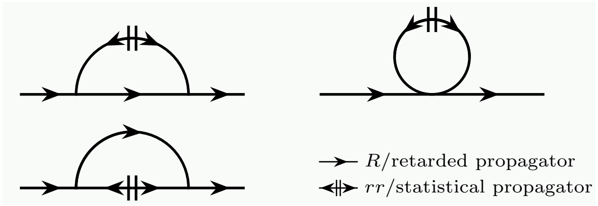
The reason

- $|\mathcal{M}|^2$, summed over supermultiplets, are SUSY-invariant
- For $k \gg T$, $\mathrm{BE} \approx \mathrm{FD} \approx \mathrm{MB}$



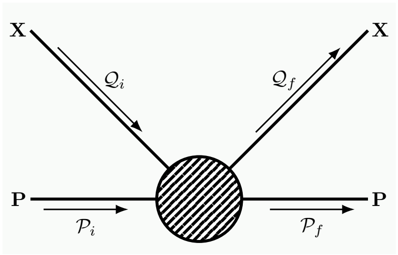
More on the computation of the thermal masses

One-loop diagrams in the r/a basis



Remark

$\mathbf{P}$ and $\mathbf{X}$ in thermal bath so $\mathbf{P} \neq \vartheta$ & $\mathbf{X} \neq \vartheta$



No tadpole: either **zero color factor** or **cancellation** between particle and anti-particle for $\mu = 0$

rr propagator in each diagram $\rightarrow$ puts the line **on-shell** $\rightarrow$ **cut** the loop into a **tree-level** diagram

- $\mathbf{P}$: particle whose thermal mass we are computing
- $\mathbf{X}$: particle in the cut line

$\mathcal{Q}_i = \mathcal{Q}_f$ (cut line) $\Rightarrow \mathcal{P}_i = \mathcal{P}_f \rightarrow$