Gravitational Waves from Critical Dynamics

Thomas Martin, Oliver Gould and Paul Saffin

University of Nottingham UK | CHINA | MALAYSIA

thomas.martin@nottingham.ac.uk oliver.gould@nottingham.ac.uk paul.saffin@nottingham.ac.uk

School of Physics & Astronomy, University of Nottingham, Nottingham, NG7 2RD, United Kingdom

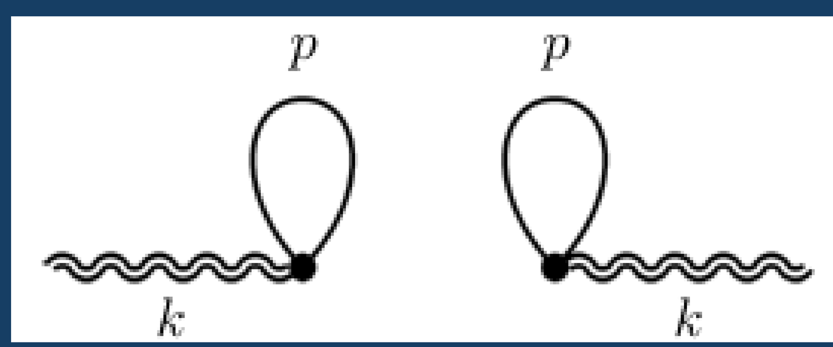
UKRI Science and Technology Facilities Council

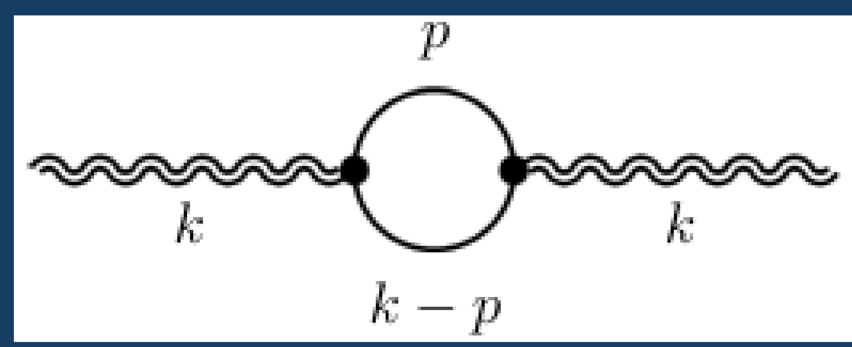
- Discussion of GWs from cosmological phase transitions limited to first order transitions [1], topological defects or post-symmetry-breaking phase ordering [2]
- We find additional emission from universal critical dynamics near second order transitions; universality classes in Hohenberg-Halperin (HH) catalogue [3], each with its own dynamic critical exponent, z, where $\tau_R \sim \xi^z$
- HH models are Langevin equations with Gaussian white noise, dissipative (symmetric) terms and non-dissipative, Poisson bracket (antisymmetric) terms
- GWs sourced by anisotropic stress correlation function [4], given by right-hand diagrams
- We use dimensional regularisation and the ε-expansion [5] to access critical properties
- Paper I limited to Model A (relaxational universality class) with unbroken time translation invariance (so in equilibrium with fluctuation-dissipation theorem obeyed)
- Model A has z=2+cη* [6], and GW spectrum amplitude inversely proportional to correlation length
- Non-trivial relation between energies (including temperature) in crit. dyn. EFTs and energies in gravitating theory – rescale by ξ_0^{z-1}

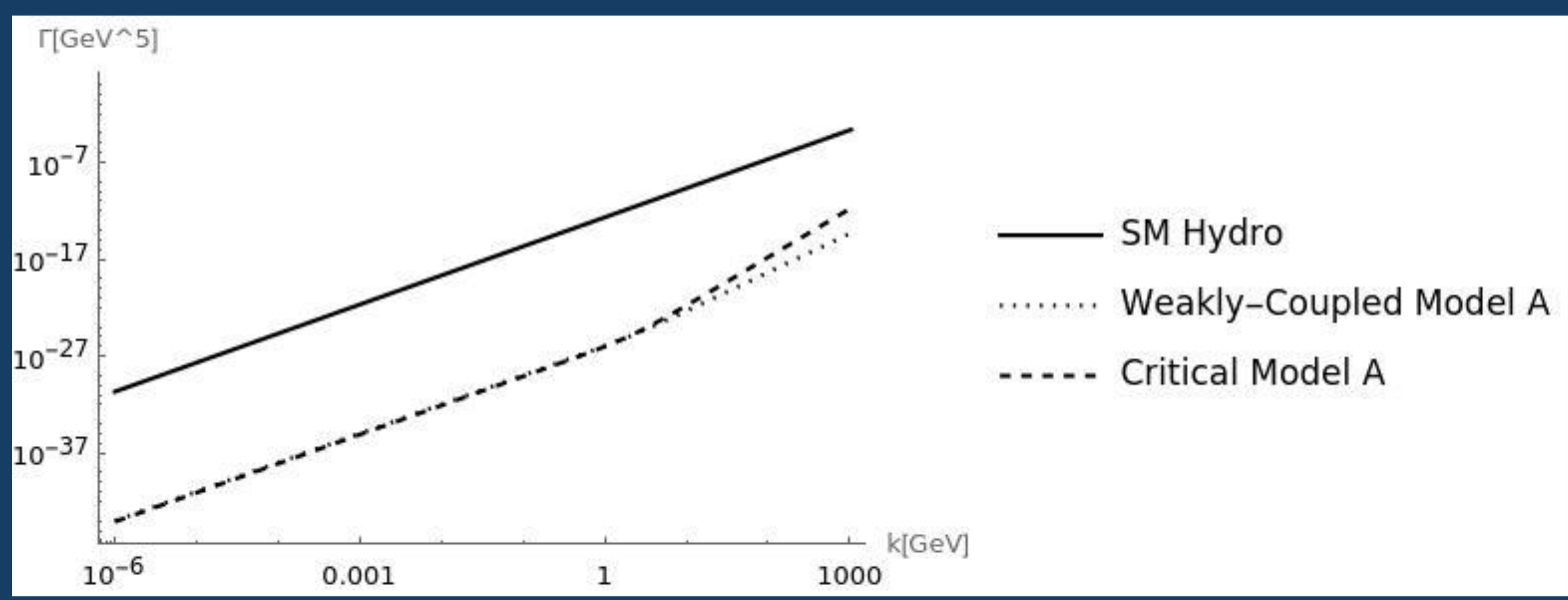
$$\frac{d\rho_{GW}}{d\ln(k)} = \frac{4G}{\pi}k^3\int_{t_i}^{t_f} dt_1 \int_{t_i-t_f}^{t_f-t_i} dt_- \cos(kt_-)\Delta^T(k,t_-)$$

$$\Delta^T(k^0,k) = \Lambda_{ijkl}\mathcal{F}^{k^0,k}_{t_-,x_-}[\langle\partial_i\phi(t_-,x_-)\partial_j\phi(t_-,x_-)\partial_k\phi(0,0)\partial_l\phi(0,0)\rangle]$$

$$\partial_t\phi = -\gamma((-\nabla^2+m^2)\phi + \frac{\lambda}{3!}\phi^3) + \sqrt{2\gamma}\zeta$$







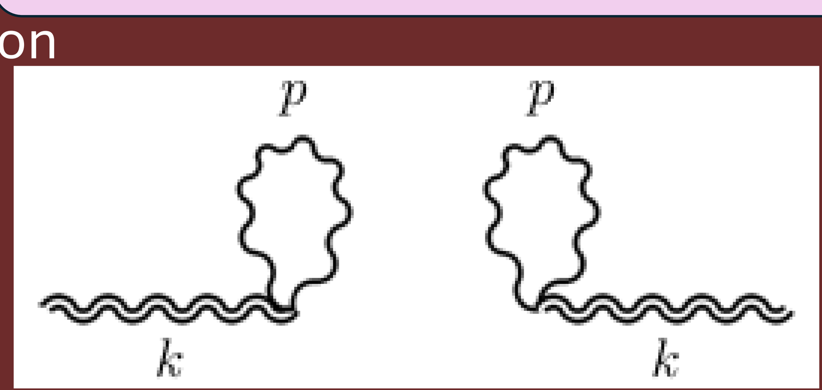
$$\Delta^T_A(k^0,k) = \frac{4(1-\frac{1}{d-1})\Gamma(\frac{d+3}{2})\Gamma(1-\frac{d}{2})T^2\xi^{z-d}}{\Gamma(\frac{d-1}{2})\gamma\xi^{-x_\gamma}}\int_0^{\frac{1}{2}}dx_3\int_0^{\frac{1}{2}}dx_4[1+(x_3+x_4)(1-x_3-x_4)(k\xi)^2+(x_3-x_4)(-i\frac{k^0\xi^z}{\gamma\xi^{-x_\gamma}})]^{\frac{d}{2}-1}$$

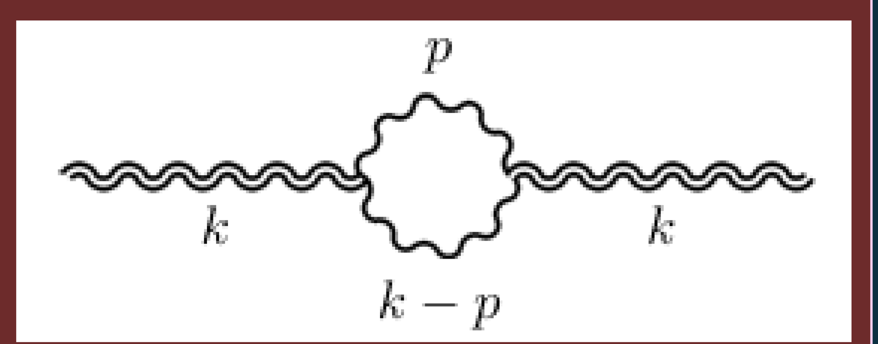
I

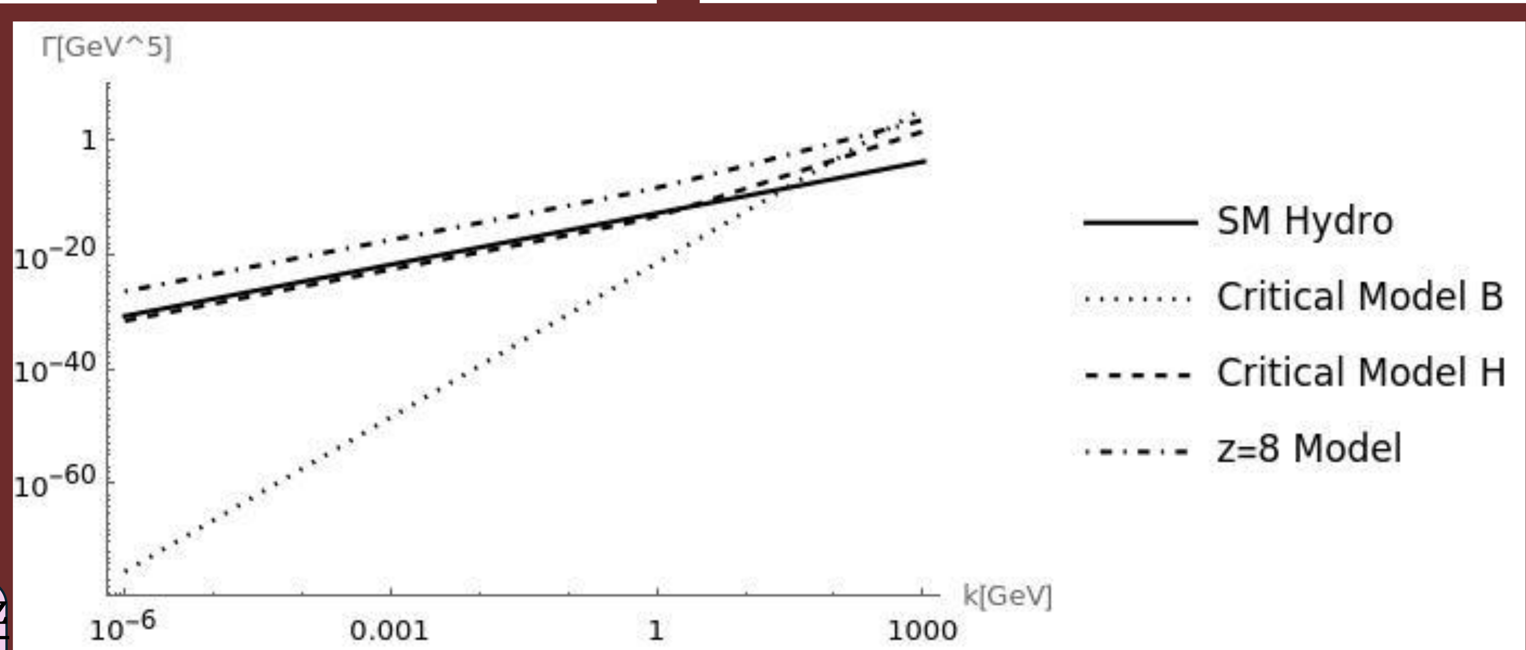
- Paper II discusses GW production from more models of crit. dyn.: diffusive (Model B, $z = 4 - \eta*$); advective (Model H, $z \sim d$); generic Gaussian ($z = z_0, \nu = 1/2$); generic Ising/Heisenberg ($z = z_0 - \eta*$, $\nu \approx 2/3$)
- Changing z produces qualitatively different spectra
- By the Kubo formula, corollary of anisotropic stress correlation function is shear viscosity correction (and critical exponent) due to order parameter fluctuations
- Leading order shear viscosity correction (hence cubic portion of GW spectrum up to a sign) exactly calculable in all of the above models, gives critical exponent x_η where $\eta \sim \xi^{x_\eta}$
- Finite momentum/frequency correlator requires $p\xi \lesssim 1$ approximation (p is loop momentum), then dimensional analysis (direct approx. calculation in Model B)
- Alternating structure in sign of the viscosity correction as z is increased
- Peak in spectrum at large z
- Model H also has transverse momentum density (vector field); dim. reg. gives non-local divergence matching cutoff-dependent shear viscosity in cutoff reg.

$$z = z_0 - x_\gamma \qquad x_\eta = z - d$$

$$\partial_t\phi = -\gamma(i\nabla)^{z_0-2}((-\nabla^2+m^2)\phi\{+\frac{\lambda}{3!}\phi^3\}) + \sqrt{2\gamma(i\nabla)^{z_0-2}}\zeta$$







$$\Delta^T_z(k^0,k)\sim\Gamma(\frac{6+d-z_0}{2})\Gamma(\frac{z_0-d}{2})T^2\xi^{z_0-d}\gamma^{-1}(1+(k\xi)^2+(-i\frac{k^0\xi^z}{\gamma\xi^{-x_\gamma}}))^{\frac{d+6-z}{2}}$$

II

- Paper III covers realistic cosmological GW spectra
- Cosmology (and temp. dependence of correlation length) break time translation invariance; can only use our time translation invariant results with high loop and external frequencies; gives leading term in expansion in Hubble parameter at T_c
- Universe adiabatically expands; must integrate high frequency production at each 'instantaneous' equilibrium over the cosmological history; production rate changes with t_1
- Use Kibble-Zurek analysis to find phase transition dynamics and domain size
- Long-lived, long-range anisotropic stress correlations give rise to substantial (and potentially detectable) signals!

III

Plot Details

T=10^6 GeV, $\frac{\xi}{\xi_0} = 10^3$, $\gamma = 1$

REFERENCES

1. C. Caprini *et al.*, JCAP 10 (2024) 020
2. J. T. Giblin Jr. *et al.*, JCAP 11 (2012) 006
3. P. C. Hohenberg & B. I. Halperin, Rev. Mod. Phys. **49** 435 (1977)
4. C. Caprini & D. G. Figueroa, Class. Quant. Grav. **35** 163001 (2018)
5. K. G. Wilson & J. B. Kogut, Phys. Rep. **12** (1974) 75
6. R. Folk & G. Moser, J. Phys. A: Math. Gen. **39** (2006) R207