Bubble Wall Velocity from Entropy Production

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Based on upcoming work with
Marcela Carena, Aurora Ireland, Tong Ou and Isaac R. Wang

Outline

1. First-order phase transitions: A brief review

2. Bubble wall velocity: General framework

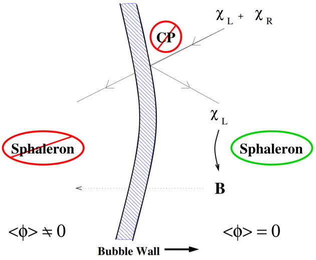
3. Wall velocity from entropy production

First-order phase transitions: A brief review

Why are FOPTs interesting?

Baryogenesis:

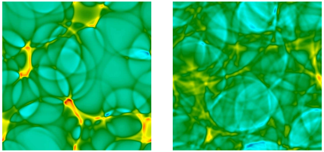
- FOPTs offer a mechanism to explain the observed baryon asymmetry of the Universe (e.g. electroweak baryogenesis),
- Baryogenesis requires a departure from equilibrium, which can be provided by the bubble wall.



Morrissey and Ramsey-Musolf (1206.2942).

Gravitational waves:

- When bubbles collide, spherical symmetry is broken and GWs can be generated,
- In the main mechanism, the energy in the wall is converted into sound waves which are then dissipated as GWs,
- These GWs could be detected by future space-based experiments such as LISA.



Hindmarsh, Huber, Rummukainen and Weir (1304.2433)

GW sensitivity on the PT parameters

The GW spectrum depends on a limited number of parameters:

- Strength of PT: $\alpha \approx \frac{\Delta V}{\rho_\gamma}$,
- PT temperature T_*,
- Inverse PT duration: $\beta = \frac{(8\pi)^{1/3} v_w}{R}$,
- Wall velocity: v_w.

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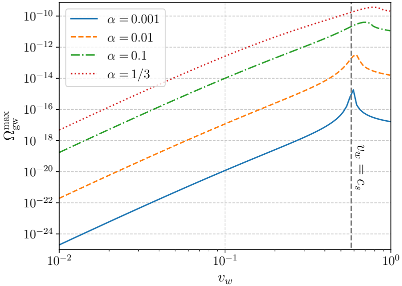
For *slow* walls:

$$\Omega_{\rm gw}^{\rm max} \propto \left(\frac{\beta}{H}\right)^{-2} \alpha^3 v_w^5, \qquad f_{\rm p,0} \propto \frac{\beta}{H} T_* v_w^{-1}$$



Bubble wall velocity: General framework

Conservation equations

In general, we have 5 unknown quantities:

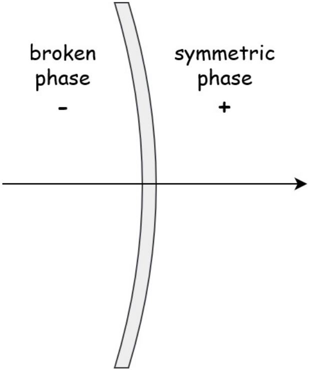
- Plasma temperatures $T_{\pm}$,
- Plasma velocities $v_{\pm}$,
- Wall velocity v_w.

2 are known from **boundary conditions**, and 2 can be obtained from **conservation of energy-momentum**

$$\partial_\mu T^{\mu\nu} = 0$$

$$\Rightarrow w_+ \gamma_+^2 v_+ = w_- \gamma_-^2 v_-$$

$$w_+ \gamma_+^2 v_+^2 + p_+ = w_- \gamma_-^2 v_-^2 + p_-$$



W. Ai, BL and J. van de Vis (2303.10171)

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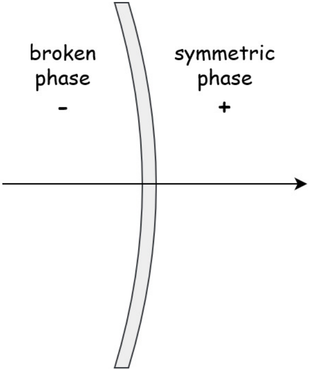

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We still need one final equation!

Pressure on the wall

The typical solution consists in solving the static EOM (wall frame)

$$-\partial_z^2\phi + \frac{\partial V_0(\phi)}{\partial\phi} + \frac{1}{2}\sum_i \frac{\partial m_i^2}{\partial\phi}\int\frac{d^3p}{(2\pi)^3 E_i}\, f_i = 0.$$

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Integrating with respect to $\int dz\,\partial_z\phi$:

$$-\Delta V_0 = \frac{1}{2}\sum_i \int dz\,\frac{\partial m_i^2}{\partial z}\int\frac{d^3p}{(2\pi)^3E_i}\, f_i,$$

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Computing δf_i is challenging because it requires solving **Boltzmann equations** which need a good semiclassical description of the plasma in the wall.

$$\left(p_z\partial_z - \frac{1}{2}\partial_z(m_i^2)\partial_{p_z}\right) f_i + \mathcal{C}_i[f] = 0.$$

Python package

How fast does the `WallGo`?

A package for computing wall velocities in first-order phase transitions

Andreas Ekstedt[a,*], Oliver Gould[b,†], Joonas Hirvonen[b,c,‡], Benoit Laurent[d,§],
Lauri Niemi[c,¶], Philipp Schicho[e,‖], Jorinde van de Vis[f,††]

Alternative approaches

Bounds on v_w can be found by considering 2 opposite limits:

Ai, BL, van de Vis (2411.13641)

Ballistic

- $\mathcal{C}[f] \to 0$,
- Particles follow ballistic trajectories (no plasma-plasma interactions),
- Pressure known analytically:

$$\mathcal{P} = \sum_i \int \frac{d^3p}{(2\pi)^3} \Delta p_z \frac{p_z}{E_\pm^{(i)}} f_{\mathrm{eq},\pm}^{(i)},$$

- Completely described in terms of

$$\alpha, \quad m_\pm^{(i)}$$

- Lower bound on v_w

Local thermal equilibrium

- $\mathcal{C}[f] \to \infty$,
- Plasma in local equilibrium everywhere

$$f = f_{\mathrm{eq}}(T(z), v(z)),$$

- Entropy conservation:

$$s_+\gamma_+v_+ = s_-\gamma_-v_-,$$

- Completely described in terms of

$$\alpha, \quad \psi = \frac{w_-}{w_+} \approx \frac{g_-^\star}{g_+^\star},$$

- Upper bound on v_w

Wall velocity from entropy production

From entropy production to wall velocity

Away from equilibrium, the LTE method can be generalized to include a *source* of entropy:

$$s_-\gamma_-v_- - s_+\gamma_+v_+ = s_+\gamma_+v_+\sigma,$$

with $\sigma \geq 0$ the entropy production fraction.

From entropy production to wall velocity

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The LTE equations remain the same with the substitution

$$\psi \to \psi_{\text{eff}} = \frac{\psi}{(1+\sigma)^4}, \qquad (\psi = w_-/w_+)$$

which leads to the formal relation

$$v_w^{\text{out}}(\alpha, \psi, \sigma) = v_w^{\text{LTE}}(\alpha, \psi_{\text{eff}}).$$

Evaluation of σ

σ represents the entropy production needed to satisfy the equations of motion. Neglecting the gradient in T and $v_{\rm pl}$, one can show it is

$$\sigma = \frac{1}{w_+}\sum_i \int dz\, \partial_z m_i^2 \int \frac{d^3p}{(2\pi)^3 E_i}(f_i - f_{{\rm eq},i})$$
$$= \frac{\mathcal{P}_{\rm plasma} - \mathcal{P}_{\rm eq}}{w_+} = \frac{\mathcal{P}^{\rm out}_{\rm friction}}{w_+}$$

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Strategy:

- Evaluate σ in the near-LTE limit using a Chapman-Enskog expansion,
- Evaluate σ in the ballistic limit using the known $\mathcal{P}_{\rm ball}$,
- Interpolate between the two regimes requiring $0 < \sigma < \sigma_{\rm ball}$.

Near-LTE limit

We wish to evaluate δf in the limit where $\mathcal{C}[\delta f]$ is large but finite:

- Boltzmann equation:

$$\mathcal{L}[\delta f] + \frac{1}{\epsilon}\mathcal{C}[\delta f] = \mathcal{S},$$

  with $\mathcal{S} = -\mathcal{L}[f_{\text{eq}}]$ and ϵ an expansion parameter,

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- For $N = 1$,

$$\delta f \approx \mathcal{C}^{-1}[\mathcal{S}].$$

Relaxation time approximation

We approximate the collision operator by the relaxation time approximation:

$$\mathcal{C}[\delta f] \to \frac{E}{\tau}\delta f,$$

with τ the relaxation time (homogeneous perturbations decay like $\delta f \sim e^{-t/\tau}$).

The NLTE solution becomes

$$\delta f_{\mathrm{RT}} = \frac{\tau}{E}\mathcal{S} \approx \tau \frac{\gamma_{\mathrm{pl}} v_{\mathrm{pl}}}{2ET} \partial_z m^2 f'_{\mathrm{eq}}.$$

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We find τ by matching the pressures $\mathcal{P}[\delta f] = \mathcal{P}[\delta f_{\mathrm{RT}}]$:

$$\Rightarrow \tau = \frac{\int_{\mathbf{p}} \mathcal{C}^{-1}[f'_{\mathrm{eq}}]}{\int_{\mathbf{p}} \frac{f'_{\mathrm{eq}}}{E}}.$$

Which can be computed by `WallGo`.

$$\text{With only QCD interactions}: \tau_q \approx 59/T$$

Limits and interpolation

The regime in which we are is now controlled by τ:

Small-τ limit: NLTE

$$\sigma(\tau) \approx \sigma_{\mathrm{NLTE}}(\tau) = \frac{\tau}{4w_+} \int dz \, \frac{\gamma_{\mathrm{pl}} v_{\mathrm{pl}}}{T} (\partial_z m^2)^2 \int \frac{d^3p}{(2\pi)^3} \frac{|f'_{\mathrm{eq}}|}{E^2}$$

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Large-τ limit: Ballistic

$$\sigma(\tau) \approx \sigma_{\mathrm{ball}} = \frac{\mathcal{P}_{\mathrm{ball}} - \mathcal{P}_{\mathrm{eq}}}{w_+} = \frac{1}{w_+} \left(\int \frac{d^3p}{(2\pi)^3}\, \Delta p_z \frac{p_z}{E_\pm} f_{\mathrm{eq},\pm} - \Delta p_T \right)$$

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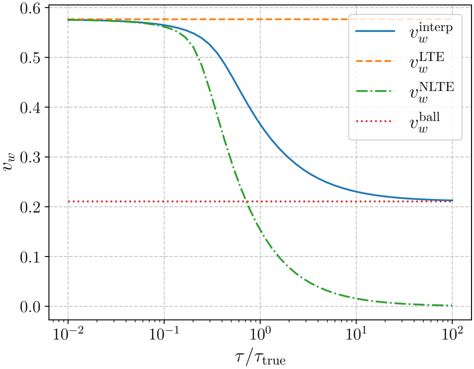
Intermediate regime: Padé approximant

$$\sigma(\tau) \approx \frac{\sigma_{\mathrm{NLTE}}(\tau) \sigma_{\mathrm{ball}}}{\sigma_{\mathrm{NLTE}}(\tau) + \sigma_{\mathrm{ball}}}$$

Parameters needed: α, ψ, $m_\pm$ and τ.

Result: Interpolated velocity

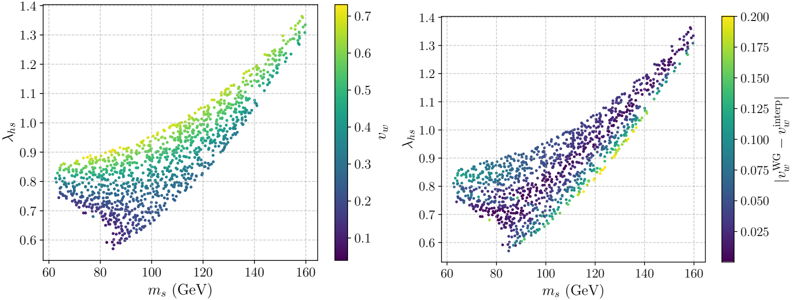
Effect of varying τ on v_w:



Result: xSM scan and WallGo comparison

As an example, we scan the parameter space of the singlet scalar extension (with $\lambda_s = 1$) and compare the results to `WallGo`:

$$V_{\text{tree}}(h,s) = \frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_{hs} h^2 s^2.$$



	LTE	Ballistic	Interpolated entropy
Median $\lvert v_w^{\mathrm{WG}} - v_w \rvert$	0.21	0.067	0.027
90th percentile	0.39	0.20	0.097

Summary

- Cosmological first-order PTs can generate GWs through sound waves in the plasma, its amplitude is proportional to v_w^5,
- The wall velocity can be bounded with the LTE ($\mathcal{C} \to \infty$) and ballistic ($\mathcal{C} \to 0$) approximations,
- The entropy production fraction σ can be used to go beyond these simple approximations using

$$v_w^{\mathrm{out}}(\alpha, \psi, \sigma) = v_w^{\mathrm{LTE}}\left(\alpha, \frac{\psi}{(1+\sigma)^4}\right),$$

- σ is estimated by interpolating between LTE and ballistic using a Chapman-Enskog expansion; $N = 1$ already shows good agreement with `WallGo`,
- Straightforward to generalize to higher N.