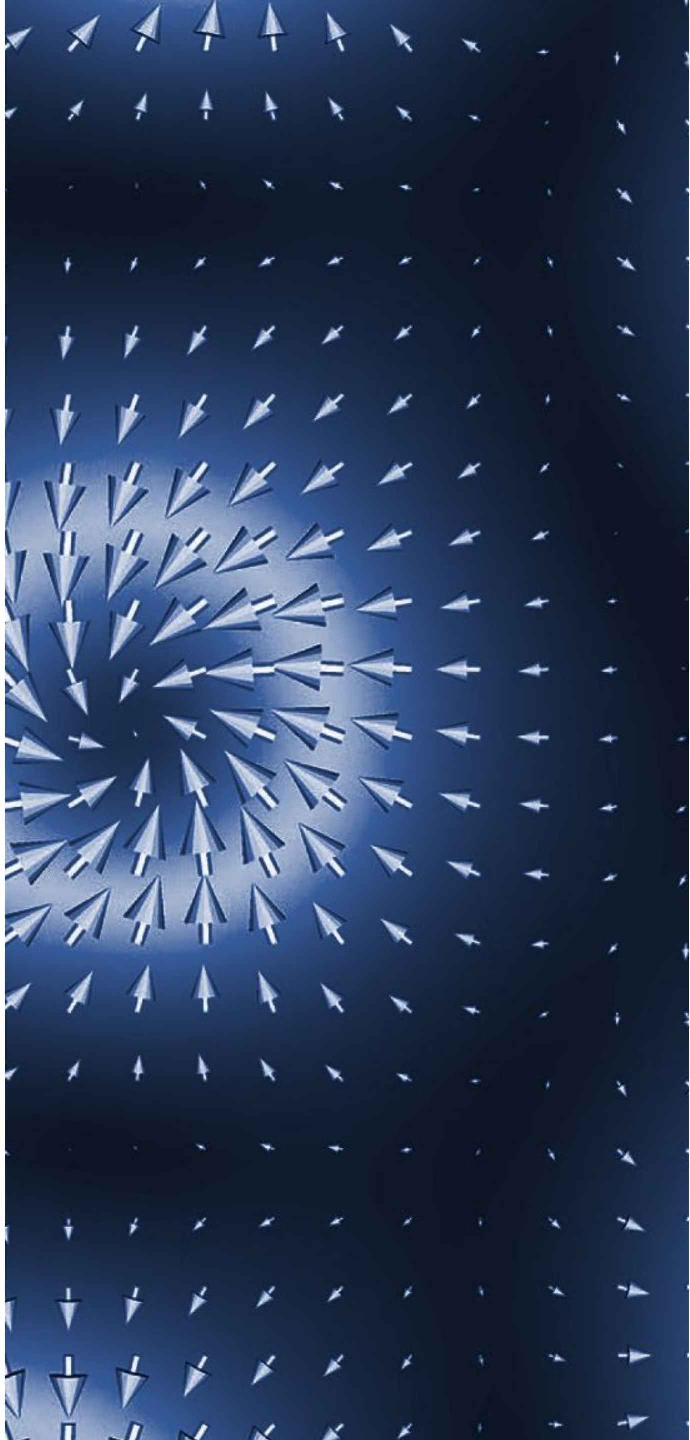
Thermoskyrmions

Strong and Electro-Weak Matter | August 19th 2026

Luis Gil (Universidad de Granada)

Based on:

M. Chala, J. C. Criado and LG **[2601.18873]**



FTAE High Energy Theory

Junta de Andalucía

GOBIERNO DE ESPAÑA | MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

Funded by the European Union

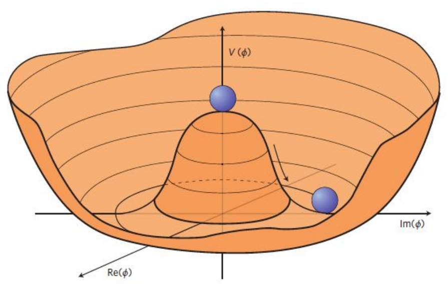
European Research Council
Established by the European Commission

Goal

- In cosmology, we can use **thermal QFT in equilibrium** to compute:

Critical parameters

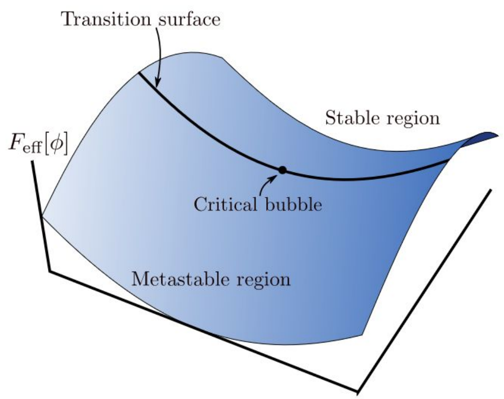
$$\left.\frac{dV_{\text{eff}}}{d\varphi}\right|_{\varphi=\varphi_{\text{vac}}} = 0$$



Nucleation rate

$$\Gamma \propto e^{-\beta F_{\text{eff}}[\varphi_b](T)}$$

$$\left.\frac{\delta F_{\text{eff}}}{\delta\varphi}\right|_{\varphi=\varphi_b} = 0$$

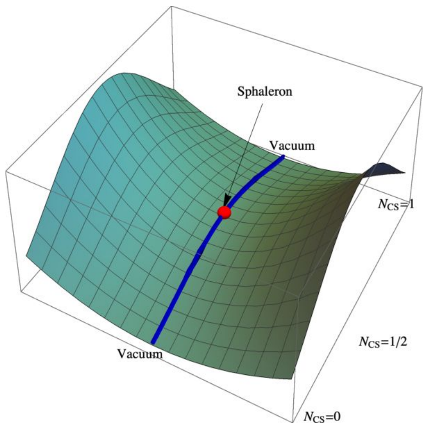


[Gould, Hirvonen - 2108.04377]

Sphaleron rate

$$\Gamma \propto e^{-\beta F_{\text{eff}}[\varphi_{\text{sph}},A_{\text{sph}}](T)}$$

$$\left.\frac{\delta F_{\text{eff}}}{\delta\Phi}\right|_{\Phi=\Phi_{\text{sph}}} = 0$$



[Sami, Gannouji - 2106.00843]

Goal

Q1

Are there other non-trivial minima of the free energy **different from the vacua**? **(Thermosolitons)**

Q2

Can we describe them using the same **high-temperature EFT techniques**?

Outline

Motivation

High-T setup

Results

Motivation

Pions and electroweak Goldstones

Localized, stable, finite energy field configuration

Fundamental fields		Solitons
Pions	⟺	Baryons
EW Goldstones	⟺	???

Motivation

Skyrme model

[Skyrme ‘61]

Pions $\Longleftrightarrow$ Baryons

$$\mathcal{L}_{\text{Skyrme}} = \frac{v^2}{4}\,\mathrm{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \frac{1}{32\,e^2}\,\mathrm{tr}\left[U^\dagger\partial_\mu U\,,\, U^\dagger\partial_\nu U\right]^2 \qquad U(x) = \exp\left(\frac{i\pi_a(x)\sigma_a}{v}\right) \in SU(2)\,, \qquad U^\dagger U = \mathbb{1}_2$$

Goal: Find static configurations of *U* that minimize the energy

Finite energy: *U(x)* must be continuous and constant at infinity, *U(∞)* = 1

$U : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \to SU(2) \cong S^3$ continuous mappings between 3-spheres

Motivation

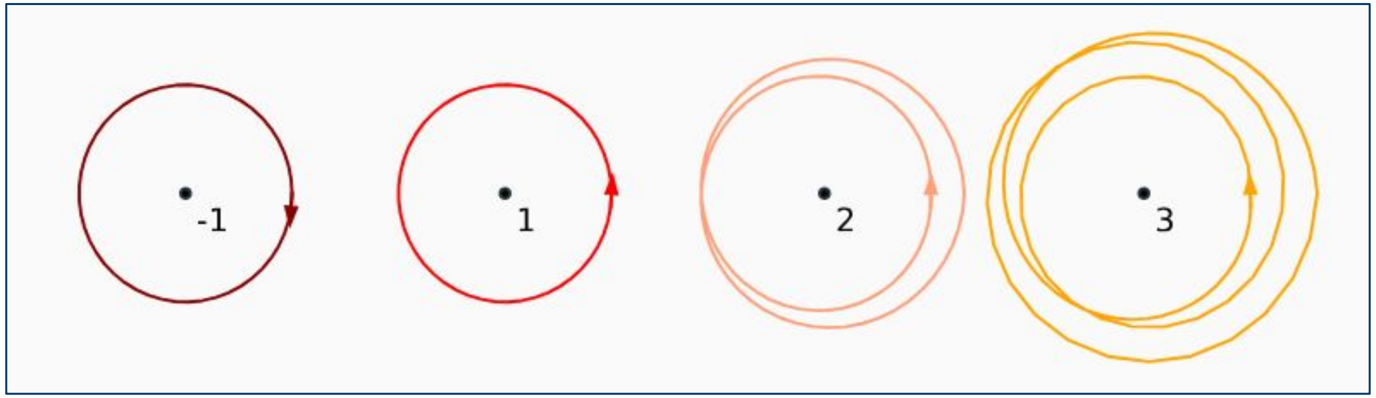
Skyrme model

[Skyrme ‘61]

Pions $\Longleftrightarrow$ Baryons

These mappings belong to different **homotopy classes**, which form a group $\pi_3(S^3) \cong \mathbb{Z}$

- **Example:** $S^1 \rightarrow S^1$



The **winding number**

$$n_U = \frac{1}{24\pi^2}\epsilon_{ijk}\int d^3x \, \langle (U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U)\rangle$$

is a **homotopy invariant** (conserved under continuous transformations)

e.g. time evolution

Motivation

Skyrme model

Pions $\Longleftrightarrow$ Baryons

Skyrmions = baryons ($n_U = 1 = B$)

[Skyrme ‘61] [Adkins, Nappi, Witten ‘83]

Motivation

Quick recap

Skyrmion:

Mapping between 3-spheres with unit winding number that defines a (meta)stable, localized field configuration with finite energy.

Motivation

Skyrmion recipe

EW Goldstones ⟷ ???

For there to be (semi-)classical skyrmions, we need:

1) **Non-trivial target space:** Fields whose target space is S^3
2) **Bosonic fields:** They are (semi-)classical configurations

and also

3) **Higher-derivative operators:** Stabilizers for extended configurations **(avoid Derrick’s no-go)**
4) **Good EFT convergence:** Higher-derivative operators cannot become too large

Motivation

Skyrmion recipe

EW Goldstones $\longleftrightarrow$???

For there to be (semi-)classical skyrmions, we need:

1) **Non-trivial target space:** Fields whose target space is S^3
2) **Bosonic fields:** They are (semi-)classical configurations

and also

3) **Higher-derivative operators:** Stabilizers for extended configurations **(avoid Derrick's no-go)**
4) **Good EFT convergence:** Higher-derivative operators cannot become too large

Natural in the high-temperature limit of SU(2)-symmetric QFTs!

Outline

Motivation

High-T
setup

Results

High-T setup

Global SU(2) Goldstone + fermions toy model

- Explore toy model **without classical skyrmions** and integrate out fermions at finite temperature

$$\mathscr{L}_{\text{bro, PG}} = \frac{v^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U \rangle + i\overline{\psi}_L \left(\partial\!\!\!/ + U \partial\!\!\!/ U^\dagger\right) \psi_L$$

Non-linear sigma model

$\Downarrow$

High-temperature dimensional reduction

(*) We actually put a ***g*** here and check different values

(*) Similar to

- Georgi-Manohar chiral quark-meson EFT **[Georgi, Manohar (1984)]**
- Pure gauge limit of broken SU(2) gauge theory coupled to massless fermions

High-T setup

Global SU(2) Goldstone + fermions toy model

No finite O(p⁴) contributions @ 1-loop

4D QFT at finite T

Bosonic 3D EFT to O(p⁶)

$$\mathscr{L} = \frac{v^2}{4}\left\langle \partial_\mu U^\dagger \partial^\mu U \right\rangle + \mathscr{L}_{\mathrm{kin}}^{\psi} - \mathscr{L}_{\mathrm{int}}$$

1-loop DR

$$\mathscr{O}_{53} = \langle [\partial_i\partial_j U]_- [\partial_i\partial_j U]_- [\partial_k U]_- [\partial_k U]_- \rangle\ , \qquad B_{53} = -\frac{7\zeta(3)g^2(1+2g)^2}{80\pi^4}\ ,$$

$$\mathscr{O}_{55} = \langle [\partial_i\partial_j U]_- [\partial_i\partial_k U]_- [\partial_k U]_- [\partial_j U]_- \rangle\ , \qquad B_{55} = -\frac{7\zeta(3)g^2(1+2g)^2}{240\pi^4}\ ,$$

$$\mathscr{O}_{100} = \langle [\partial_i U]_- [\partial_i U]_- [\partial_j U]_- [\partial_j U]_- [\partial_k U]_- [\partial_k U]_- \rangle \qquad B_{100} = \frac{7\zeta(3)(1+6g+8g^2)^2}{240\pi^4}\ ,$$

$$\mathscr{O}_2 = \left\langle \mathscr{O}^{(2)}_{\mathrm{EOM}} \mathscr{O}^{(2)}_{\mathrm{EOM}} [\partial_i U]_- [\partial_i U]_- \right\rangle\ , \qquad E_2 = -\frac{7\zeta(3)g^2(1+2g)^2}{240\pi^4}\ ,$$

...

	4D QFT at finite T	Bosonic 3D EFT to O(p⁶)
Bosonic fields (only):	✖	○
Higher-derivative operators:	✖	○

Outline

Motivation

High-T setup

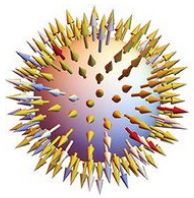
Results

Results

There are (thermo)skyrmions

1) Hedgehog ansatz

$$U(\mathbf{x}) = \exp\{i\eta(r)n_a\sigma_a\}$$

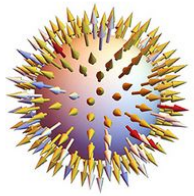
 $\overset{?}{\sim}$ 

Results

There are (thermo)skyrmions

1) Hedgehog ansatz

$$U(\mathbf{x}) = \exp\{i\eta(r) n_a \sigma_a\}$$

 $\overset{?}{\sim}$ 

2) Minimize energy with *n_U* = 1

Loss functional: $L[\eta] = \widetilde{S}_3[\eta] + \omega_{\mathrm{BC}} \sum_k \mathrm{BC}_k[\eta]^2$

Boundary cond.: $n_U = -\frac{2}{\pi}\int_0^\infty dr \partial_r \eta \sin^2(\eta) = \frac{1}{\pi}\eta(0) = 1$ $\qquad \eta(\infty) = 0$

Model solution as **neural network** and use **gradient descent** to minimize the loss functional.

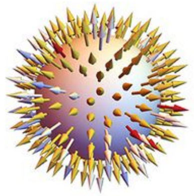
`Elvet` **[Araz, Criado, Spannowsky - 2103.14575]**

Results

There are (thermo)skyrmions

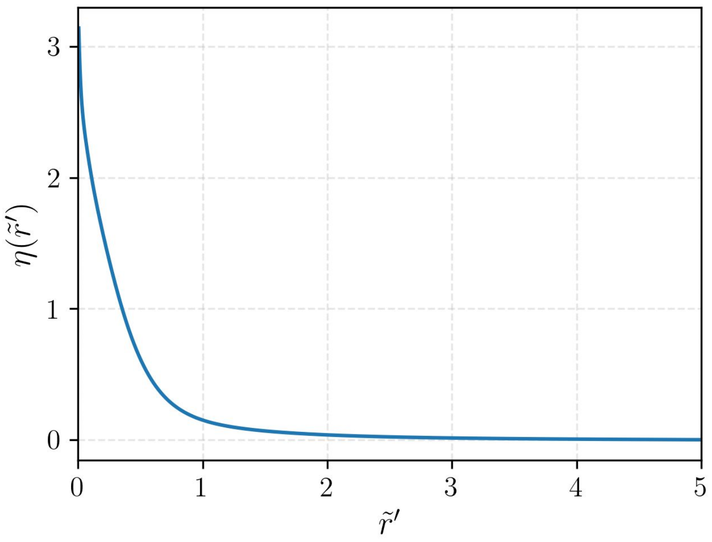
1) Hedgehog ansatz

$$U(\mathbf{x}) = \exp\left\{i\eta(r)n_a\sigma_a\right\}$$



$\overset{?}{\sim}$

2) Minimize energy with n_U = 1



We find a stable **(thermo)skyrmion** minimizing the 3D action with n_U **= 1!**

Results

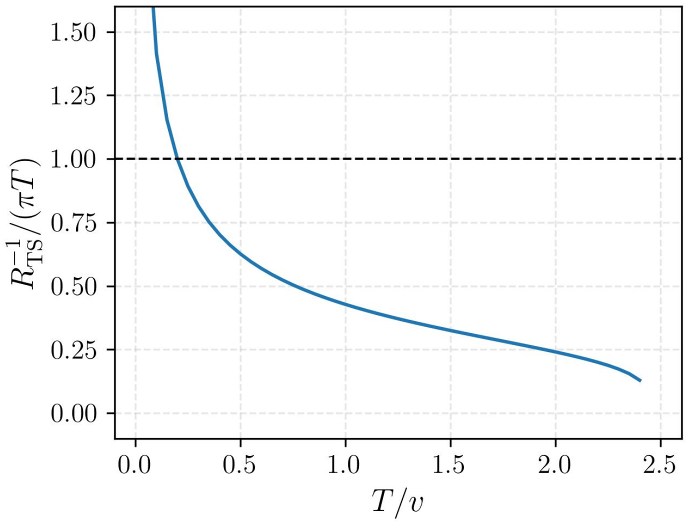
EFT validity

If thermoskyrmion size is **larger than**…

(1) inverse cutoff scale

⟹ valid 3D EFT

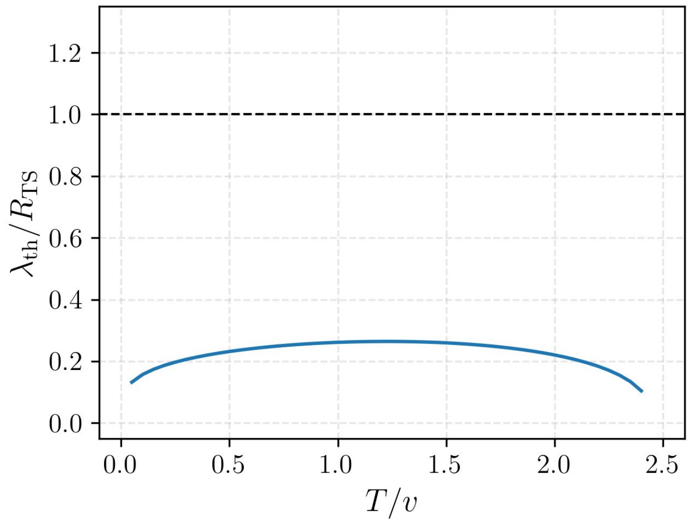
$$\frac{1}{R_{\mathrm{TS}}} \lesssim \pi T$$



(2) thermal de Broglie wavelength

⟹ classical configuration

$$R_{\mathrm{TS}} > \lambda_{\mathrm{th}} \sim \frac{1}{\sqrt{TE}}$$



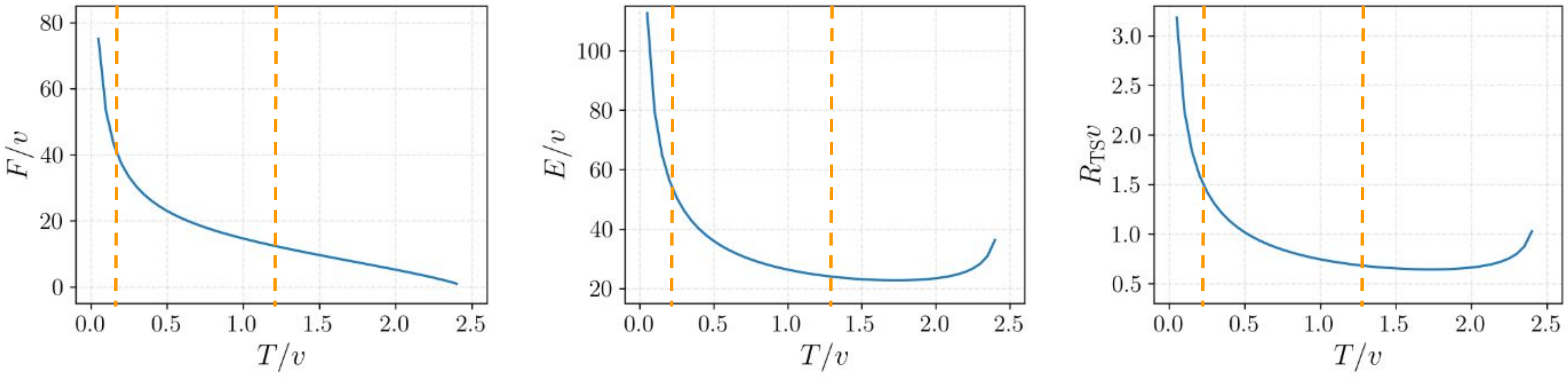
Results

Thermoskyrmion thermodynamics

Altogether, thermoskyrmions exist in the **conservative** range

$$\frac{1}{R_{\rm TS}} \lesssim \pi T \lesssim \sqrt{6}\pi v$$

and they are much **heavier** (energetic) than the thermal scale.



(*) Valid between - - -

***g* = 1**

Outlook

… and speculation

- **Thermoskyrmions** are topological collective excitations in a thermal plasma.

- They are **stabilized by fermions**, which appear in a derivative expansion because of the heavy T scale.

but…

- **Can they be stabilized in the EW sector by the top quark?**
  - (Fermionic) EW skyrmions not evident at T=0 in SM **[Farhi et al. (2003)]**
  - SMEFT/HEFT-like BSM theories contain bosonic EW skyrmions **[Criado et al. (2021)]**

- If found… Can they seed nucleation? Are they produced in FOPTs? How do they affect baryogenesis?

Stay tuned!

Thank you for your attention!

Backup

Dimensional reduction

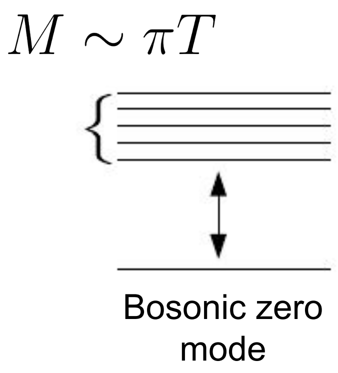
- **Generating functional** in thermal field theory (= Euclidean QFT with periodic time):

$$\mathcal{Z}_{\text{th}} = \text{Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q\ 0|e^{-\beta\mathcal{H}}|q\ 0\rangle = \mathcal{N} \int_{q(0)=q(-i\beta)} \mathcal{D}q \exp\left(-S_E\right)$$

- Fields decompose in tower of 3D **Matsubara modes** (~ Kaluza-Klein) with thermal masses:

Bosons: $\omega_n = 2\pi n T$

Fermions: $\omega'_n = 2\pi\left(n + \frac{1}{2}\right)T$



Expansion in **m/T**

3D EFT for bosonic zero modes

Backup

Dimensional reduction

- We determine the 3D EFT by matching off-shell correlators to the full theory in the **high-temperature limit**:

ex. $$\mathcal{L}_3 = \frac{K_3}{2}(\partial\varphi)^2 + \underbrace{\frac{1}{2}m_3^2\varphi^2}_{m_3^2 = m^2 + \#\lambda T^2 + \dots} + \kappa_3\varphi^3 + \lambda_3\varphi^4 + \underbrace{\textbf{higher orders in } \left(\frac{\mathbf{m}}{\mathbf{T}}\right)}_{\text{New effective operators } \textbf{(EO)}}$$

ex. $\alpha_6\varphi^6$ or $\beta_{61}\partial^2\varphi\partial^2\varphi$

Backup

Derrick's theorem

[Derrick '64]

Statement: Any stationary, localized solution to a nonlinear wave equation ensuing from a Hamiltonian with a kinetic + potential structure in *d* > 2 is unstable.

$$\mathscr{H}(\varphi, \partial\varphi) = \frac{1}{2}(\partial\varphi)^2 + V(\varphi)\,; \qquad E[\varphi] = \int \mathrm{d}^{\mathrm{d}}x \mathscr{H}(\varphi, \partial\varphi) = E_2[\varphi] + E_0[\varphi]$$

Backup

Derrick's theorem

[Derrick '64]

Statement: Any stationary, localized solution to a nonlinear wave equation ensuing from a Hamiltonian with a kinetic + potential structure in *d > 2* is unstable.

$$\mathscr{H}(\varphi, \partial\varphi) = \frac{1}{2}(\partial\varphi)^2 + V(\varphi)\,; \qquad E[\varphi] = \int d^d x \mathscr{H}(\varphi, \partial\varphi) = E_2[\varphi] + E_0[\varphi]$$

Rescale:

$$x \to \lambda x\,, \varphi(x) \to \varphi(\lambda x) \equiv \varphi_\lambda(x) \qquad E[\varphi_\lambda] = \lambda^{2-d} E_2[\varphi] + \lambda^{-d} E_0[\varphi]$$

Non-rescaled solution is minimal:

$$\left.\begin{aligned} 0 &= \frac{d}{d\lambda} E[\varphi_\lambda]\Big|_{\lambda=1} = (2-d)E_2[\varphi] - dE_0[\varphi] \\ 0 &< \frac{d^2}{d\lambda^2} E[\varphi_\lambda]\Big|_{\lambda=1} = (2-d)(2-d-1)E_2[\varphi] + d(d+1)E_0[\varphi] \end{aligned}\right\}$$

$$\implies (2-d)E_2[\varphi] > 0 \quad \text{!!}$$

Backup

Convergence

a) What if we had added even higher-derivative operators?

Ex.

Approximate $\partial \sim 1/R$:

$$E = \int d^3x \mathscr{H} \sim v^2 R^3 \left[\frac{1}{R^2} + \frac{c_4}{\Lambda^2}\frac{1}{R^4} + \frac{c_6}{\Lambda^4}\frac{1}{R^6} + \frac{c_8}{\Lambda^6}\frac{1}{R^8} + \dots \right]$$

Counterbalance 2- and 4-derivative terms:

$$\frac{d(E_2 + E_4)}{dR} = 1 - \frac{c_4}{\Lambda^2}\frac{1}{R^2} \sim 0 \implies E \sim \frac{v^2}{\Lambda}\left[\sqrt{c_4} + \sqrt{c_4} + \frac{c_6}{\sqrt{c_4^3}} + \frac{c_8}{\sqrt{c_4^5}} + \dots\right]$$

Can converge if EFT is **loop-level** generated!

Backup

Convergence

b) What if we had added higher loop orders?

At 2-loops, there are finite contributions to 4-derivative operators:

$$\frac{c_4^{(2)}}{T}\partial^4 \sim \frac{g^2 T}{v^2}\partial^4\,, \qquad \frac{c_6^{(1)}}{T^3}\partial^6 \sim \frac{g^2}{T^3}\partial^6$$

Are they more relevant than 1-loop 6-derivative? **No, within a certain regime**

$$\frac{1}{T^2}\frac{c_4^{(2)}\partial^4}{c_6^{(1)}\partial^6} \sim \frac{T^4}{v^2\partial^2} \sim \frac{T^4 R_{\mathrm{TS}}^2}{v^2} < 1 \ ?$$