Thermal axion production at strong coupling

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to appear soon on the arxiv!

Strong and Electroweak Matter, University of Helsinki, 19.8.26

Outline

1. Hot axions in the early universe
2. Interaction rate at strong coupling
3. Results for ΔN_{eff}
4. Conclusion and Outlook

Hot axions in the early universe

QCD axion

Axion is hypothetical particle beyond the Standard Model (SM)

- Solves the Strong CP problem through Peccei-Quinn Mechanism
- Holds promise as potential dark matter candidate

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Here we rather assume **hot axion**, a, was in thermal equilibrium with the primordial plasma at some temperature $T_{\max}$, to later freeze out around T_{min} (see talk by **Guy Moore**, Mon, $18\,00$)

Frozen-out abundance constrained by N_{eff}, number of light degrees of freedom present in early universe
(see talk by **Greg Jackson**, Wed, $9\,00$)

$\Rightarrow$ Hopeful that next generation of CMB experiments will be able to further constrain axion model parameters through

$$N_{\mathsf{eff}} - N_{\mathsf{eff}}^{\mathsf{SM}} = \Delta N_{\mathsf{eff}} \sim \frac{e_a(T_{\mathsf{CMB}})}{e_\gamma(T_{\mathsf{CMB}})}$$

Interaction rate

Assume KSVZ axion model, where at energies far below **Peccei-Quinn scale**, $f_{PQ} \gtrsim 10^8$GeV, only relevant coupling is to gluons

$$\mathcal{L}_{gga} = -\frac{\alpha_s}{8\pi f_{PQ}} a F^b_{\mu\nu} \tilde{F}^{\mu\nu,b}$$

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[2] Weldon, H. (1983), Phys. Rev. D, 28:2007-2015

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How do we calculate $e_a(T_{\mathsf{CMB}})$?

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How do we calculate $e_a(T_{\text{CMB}})$?

QCD axion interaction rate is major source of uncertainty for freeze-out calculations[1], and is given by[2]

$$\Gamma(k) = \frac{1}{k} \mathrm{Im} \Pi^R_a(k) \tag{1}$$

where $\Pi^R_a(k)$ is **retarded axion self-energy**, evaluated on light cone (neglecting axion mass)

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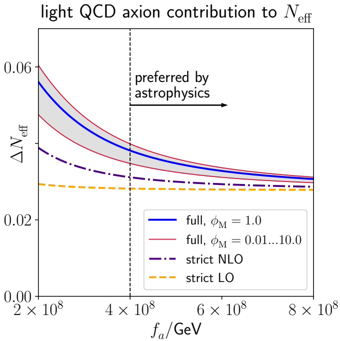
[2] Weldon, H. (1983), Phys. Rev. D, 28:2007-2015

State of the art at weak-coupling



- Recently, $\Gamma(k)$ for QCD axion has been computed using a combination of perturbative methods and classical lattice simulations[3] (see figure).

[3] Bouzoud, K. et al. (2026), JHEP, 05:034
[4] Notari, A. et al., (2023), Phys. Rev. Lett. 1:011004
[5] D'eramo, F. et al. (2021), JHEP, 10:224

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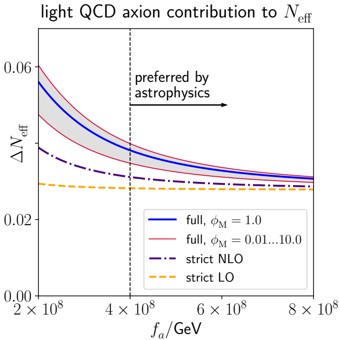

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Tracks production from above electroweak crossover, down to regime of QCD crossover ($T_{\text{min}} = 0.2\,\text{GeV}$), where **weak-coupling approximation should not hold**.
See[4] for study of hot axion production below QCD crossover and[5] for analysis across thresholds.

[3] Bouzoud, K. et al. (2026), JHEP, 05:034
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Interaction rate at strong coupling

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Reminder:

$$\Gamma(k) = \frac{1}{k}\mathrm{Im}\,\Pi_a^R(K) \tag{2}$$

valid to leading order in axion coupling to gluons and to all orders internal QCD coupling.

[6] Caron-Huot, S. et al. (2006), JHEP, 12:015

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$\Rightarrow$ Consider axion weakly coupled to strongly coupled plasma, calculate correlator using **holography** (similar setup as for photon/dilepton[6] and gravitational wave production[7])

$\Rightarrow$ For AdS/CFT, we need $\lambda = g^2 N_c \gg 1$ but for Eq. (2), $\frac{\alpha_s T}{f_{\mathsf{PQ}}} \ll 1$

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Reasonable for $f_{PQ} \gtrsim 10^8$ GeV (depending on temperature of course)

[6] Caron-Huot, S. et al. (2006), JHEP, 12:015
[7] Castells-Tiestos, L. and Casalderrey-Solana, J. (2022), JHEP, 10:049

Disclaimer

Consider axion weakly coupled to strongly coupled plasma, calculate correlator using **holography**:

- Map between $\mathcal{N}=4$ Supersymmetric Yang-Mills (SYM), a strongly coupled **conformal field theory** (CFT) and **classical supergravity** on AdS_5

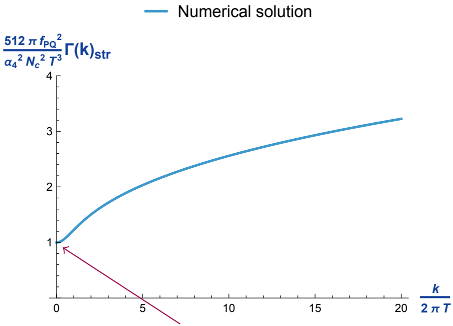
- $\mathcal{N}=4$ SYM is an example of a **non-Abelian gauge theory**, but differences with respect to QCD (supersymmetry, no confinement, fermions in adjoint rep, additional scalars in adjoint rep...)

  $\Rightarrow$ Can provide qualitative insight into strong-coupling behaviour of $\Gamma(k)$

Holographic setup

- Relevant holographic setup is provided by Son and Starinets[8], where Ramond-Ramond scalar, ϕ, is dual to axion
- Calculating $\Pi_a^R(K)$ reduces to solving mode equation (same one solved in[7]) for linearized ϕ perturbations on top of AdS background (see backup slides if interested)
- Our computation of $\Pi_a^R(K)$ generalizes mode equation solution from[8] to arbitrary momentum on the lightcone

[7] Castells-Tiestos, L. and Casalderrey-Solana, J. (2022), JHEP, 10:049
[8] Son, D.T. and Starinets, A.O. (2002), JHEP, 09:042



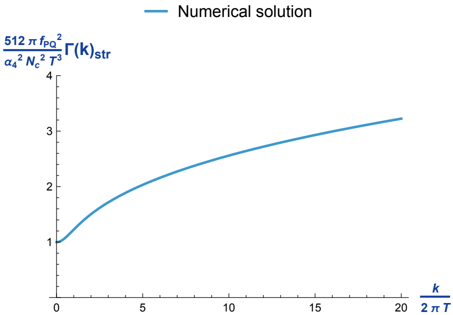
$\mathcal{N} = 4$ Sphaleron rate

$$\Gamma_{\mathsf{sph}} = 32\, T f^2_{\mathsf{PQ}} \Gamma(k \to 0)_{\mathsf{str}}$$

recovered at small momenta

[8] Son, D.T. and Starinets, A.O. (2002), JHEP, 09:042

Strongly coupled rate for $\mathcal{N} = 4$ plasma



Fit: $\Gamma(k)_{\mathsf{str}} = \frac{\alpha_4^2 N_c^2 T^3}{512\pi f_{\mathsf{PQ}}^2}\left(1 + \left(\frac{\gamma^3 k}{2\pi T}\right)^{2\beta}\right)^{\frac{1}{6\beta}}$ accurate to within 0.01%

with $\gamma \approx 1.19$ and $\beta = \frac{6}{5}$

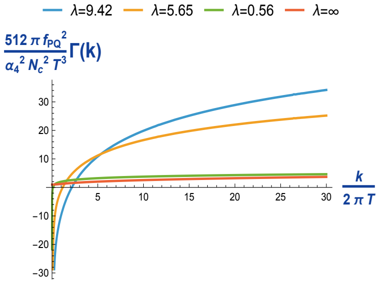
[8] Son, D.T. and Starinets, A.O. (2002), JHEP, 09:042

Compare strongly and weakly coupled (LO) rates for $\mathcal{N}=4$ SYM
For $k \gg T$:

$$\Gamma(k)_{\text{str}} \sim \frac{\alpha_4^2 N_c^2 T^3}{512\pi f_{\text{PQ}}^2}\left[\frac{\gamma^3 k}{2\pi T}\right]^{\frac{1}{3}}$$

$$\Gamma(k)_{\text{wk}} \overset{N_c\gg 1}{\sim} \frac{\alpha_4^3 N_c^3 T^3}{8\pi^2 f_{\text{PQ}}^2} \ln\left(\frac{k^2}{m^2_{D,\mathcal{N}=4}}\right)$$

where $m^2_{D,\mathcal{N}=4} = 2\lambda T^2$.



[7] Castells-Tiestos, L. and Casalderrey-Solana, J. (2022), JHEP, 10:049

Weak- and strong- coupling $\mathcal{N}=4$ rates

Compare strongly and weakly coupled (LO) rates for $\mathcal{N}=4$ SYM
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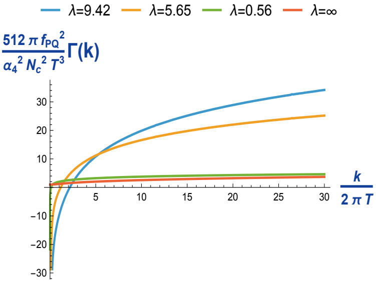
where $m_{D,\mathcal{N}=4}^2 = 2\lambda T^2$.



Rates approximately equal for $\lambda \sim 0.5\,(\alpha_4 \sim 0.01)$, similar to what was observed for thermal gravitational wave production[7].

[7] Castells-Tiestos, L. and Casalderrey-Solana, J. (2022), JHEP, 10:049

Results for ΔN_{eff}

In heavy-ion collisions, **quark-gluon plasma** (QGP) is created at temperatures of a few hundred MeV

Examples of quantities whose determination from AdS/CFT match experimental measurements better than perturbative QCD counterparts, especially for "dynamical quantities" (i.e, $\hat{q}$, η/s)

[9] Liu, H. et al., (2007), JHEP, 03:066

Extrapolation to strongly coupled QCD

In heavy-ion collisions, **quark-gluon plasma** (QGP) is created at temperatures of a few hundred MeV

Examples of quantities whose determination from AdS/CFT match experimental measurements better than perturbative QCD counterparts, especially for "dynamical quantities" (i.e, $\hat{q}$, η/s)

$\Rightarrow$Despite obvious differences between microscopic descriptions, assume QGP is approximately conformal

Then, from conjecture[9] (done there for $\hat{q}$), can estimate QCD rate by multiplying $\mathcal{N}=4$ one by ratio of entropy densities for two theories

$$\nu \approx \left(\frac{s_{\text{QCD}}}{s_{\mathcal{N}=4}}\right)^2 = \left(\frac{16+\frac{21}{2}N_f}{120}\right)^2 \approx \begin{cases} 0.16, & T = 0.3\,\text{GeV} \\ 0.23, & T = 3\,\text{GeV} \\ 0.33, & T = 52\,\text{GeV} \\ 0.43, & T = 180\,\text{GeV} \end{cases}, \tag{3}$$

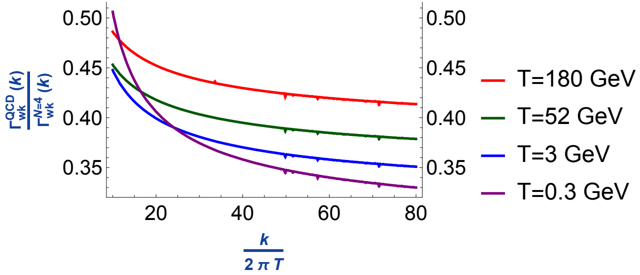
Note: power below comes from the fact that $\Gamma(k)_{\text{str}} \sim \lambda^2$

[9] Liu, H. et al., (2007), JHEP, 03:066

Estimating QCD rate at strong-coupling

$$\nu \approx \left(\frac{s_{\text{QCD}}}{s_{\mathcal{N}=4}}\right)^2 = \left(\frac{16+\frac{21}{2}N_f}{120}\right)^2 \approx \begin{cases} 0.16, & T = 0.3\,\text{GeV} \\ 0.23, & T = 3\,\text{GeV} \\ 0.33, & T = 52\,\text{GeV} \\ 0.43, & T = 180\,\text{GeV} \end{cases}, \qquad (4)$$

Should hold regardless of coupling so can test against our results at weak-coupling – **not bad!**



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To calculate ΔN_{eff}, need to solve **Boltzmann equation**

$$\partial_t f_a(k) - Hk\partial_k f_a(k) = \Gamma(k)\Big[n_{\mathrm{B}}(k) - f_a(k)\Big]$$

- $f_a(k)$, axion phase space density, $n_{\mathrm{B}}(k)$ Bose-Einstein distribution
- H, Hubble rate

[10] Laine, M. et al., (2019), Phys. Rev. D, 101:023532

Towards ΔN_{eff} extraction

To calculate ΔN_{eff}, need to solve **Boltzmann equation**

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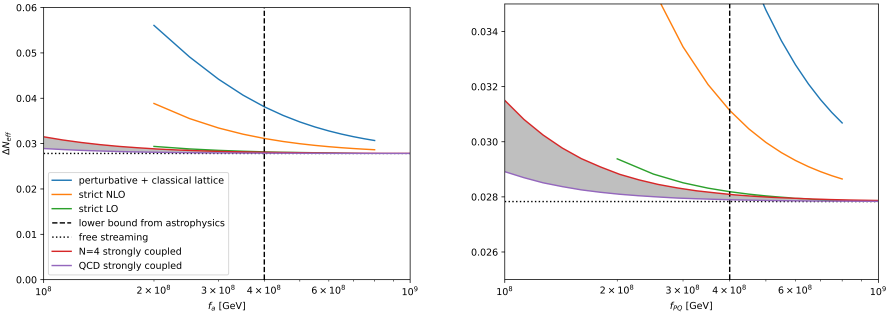
- $f_a(k)$, axion phase space density, $n_{\rm B}(k)$ Bose-Einstein distribution
- H, Hubble rate

Evolve system from $T_{\text{max}} = 10^4$ GeV down to $T_{\text{min}} = 200$ MeV, after which we assume free streaming

$\Rightarrow \; \Delta N_{\text{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}} \frac{s^{\frac{4}{3}}(T_{\text{CMB}})}{s^{\frac{4}{3}}(T_{\text{min}})} \frac{e_a(T_{\text{min}})}{e_\gamma(T_{\text{CMB}})}$,

Use weak-coupling determinations from[10] for thermodynamic variables as well as running of the coupling (see outlook/backup)

[10] Laine, M. et al., (2019), Phys. Rev. D, 101:023532



Clear message: Strong-coupling effects should pull curves closer to free-streaming value

$\Rightarrow$ Need to do some matching between strong- and weak-coupling rates to be able give more quantitative statement

Conclusion and Outlook

Conclusion

- Computed thermal axion production rate from strongly coupled $\mathcal{N} = 4$ SYM plasma using AdS/CFT

$$\Gamma(k)_{\mathsf{str}} \approx \frac{\alpha_4^2 N_c^2 T^3}{512\pi f_{\mathsf{PQ}}^2}\left(1 + \left(\frac{\gamma^3 k}{2\pi T}\right)^{2\beta}\right)^{\frac{1}{6\beta}}$$

- Saw that strongly coupled and weakly coupled $\mathcal{N} = 4$ rates agree quantitatively for $\alpha_4 \sim 0.01$
- Demonstrated that strongly-coupling effects should reduce axion contribution to $\mathrm{N}_{\mathrm{eff}}$

Conclusion

- Computed thermal axion production rate from strongly coupled $\mathcal{N}=4$ SYM plasma using AdS/CFT

$$\Gamma(k)_{\mathsf{str}} \approx \frac{\alpha_4^2 N_c^2 T^3}{512\pi f_{\mathsf{PQ}}^2}\left(1+\left(\frac{\gamma^3 k}{2\pi T}\right)^{2\beta}\right)^{\frac{1}{6\beta}}$$

- Saw that strongly coupled and weakly coupled $\mathcal{N}=4$ rates agree quantitatively for $\alpha_4 \sim 0.01$
- Demonstrated that strongly-coupling effects should reduce axion contribution to $\mathrm{N}_{\mathsf{eff}}$
  $\Rightarrow$ Hopefully should appear on the **arxiv** in the coming weeks!

Outlook

- Would like to find a satisfactory way to match perturbative or lattice and holographic determinations of $\Gamma(k)$ (similar to neutron star eos situation?)
- Deform gravity theory from AdS_5 (e.g, by introducing dilaton to break conformality) to produce more "realistic" theory of QCD[11,12] $\Rightarrow$ Would be very interesting to see how momentum dependence in rate changes
- Eventually incorporate high-temperature thermodynamics from the lattice[13] (see talk by **Matteo Bresciani** on Tuesday at 9)

[11] Erlich, J. et al., (2005), Phys. Rev. Lett., 95:261602
[12] Gursoy, U. et al., (2011), Lect. Notes. Phys. Rev. D, 79:146
[13] Bresciani, M. et al., (2026), Phys. Rev. D., 113:034506

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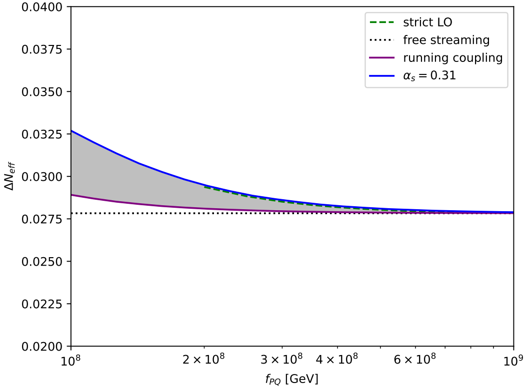
Appendix

Effect of running the coupling



Difference between running coupling and setting it to a constant value used in heavy-ion, yields noticeable but not huge difference

Holographic computation

GKP-Witten relation

$$\left\langle e^{\int_{\partial M}\phi_0\mathcal{O}}\right\rangle = e^{-S_{\mathsf{cl}}[\phi]}$$

For us, axion corresponds to ϕ_0 whereas $\mathcal{O} = F^b_{\mu\nu}\tilde{F}^{b,\mu\nu}$

To evaluate LHS:

1. Write action S for scalar field that is dual to axion (dilaton action in our case)
2. Obtain S_{cl} by solving the bulk configurations $\phi(X,u)$ for a fixed boundary condition $\phi(X,u)|_{u=0} \equiv a(X)$
3. Calculate variational derivative of RHS with respect to $a(X)$ to get $\langle OO\rangle^R \sim \Pi^R_a(K)$

Holographic computation

Linearized equation for ϕ

$$\frac{1}{\sqrt{g}}\partial_z\left(\sqrt{-g}g^{zz}\partial_z\phi\right)+g^{\mu\nu}\partial_\mu\partial_\nu\phi=0$$

where

$$\phi(X,u)=\int\frac{d^4K}{(2\pi)^4}e^{iK\cdot X}f_k(u)\phi_0(K)$$

with $\phi_0(K)$ determined from the boundary condition

$$\phi(X,u)=\int\frac{d^4}{(2\pi)^4}e^{iK\cdot X}\phi_0(K)$$

Computing $\Pi^R_a(K)$ then reduces to solving Heun differential equation for $f_k(u)$, since

$$\Pi^R_a(K)\sim\lim_{u\to0}\frac{1-u^2}{u}f^*_k(u)\partial_u f_k(u)$$

Comparing QCD and $\mathcal{N}=4$ weak-coupling rates

Calculate hard piece with kinetic theory and soft piece with HTL resummation

$$\Gamma_{\mathsf{QCD}}(k) \sim 2N_c\mathcal{B}(k) + 2N_fT_f\mathcal{F}(k) + m^2_{D,\mathsf{QCD}}\ln\left(\frac{4k^2}{m^2_{D,\mathsf{QCD}}}\right) \tag{5}$$

where $m^2_{D,\mathsf{QCD}} = \frac{g^2T^2}{3}\Big(N_c + N_fT_f\Big)$, $T_f = 1/2$

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where $m^2_{D,\mathsf{QCD}} = \frac{g^2T^2}{3}\Big(N_c + N_fT_f\Big)$, $T_f = 1/2$
In contrast,

$$\Gamma_{\mathcal{N}=4}(k) \sim (2+6)N_c\mathcal{B}(k) + 4N_c\mathcal{F}(k) + m^2_{D,\mathcal{N}=4}\ln\left(\frac{4k^2}{m^2_{D,\mathcal{N}=4}}\right) \qquad (6)$$

where $m^2_{D,\mathcal{N}=4} = 2g^2T^2N_c$.