

Precise hydrodynamics in cosmological phase transitions

Flynn Linton, William Searle, Csaba Balazs & Xiao Wang

My goals for today

1. *Where does hydrodynamics fit into cosmological phase transitions?*
2. *How accurate are simplified descriptions of the phase transition hydrodynamics?*

What is a cosmological phase transition?

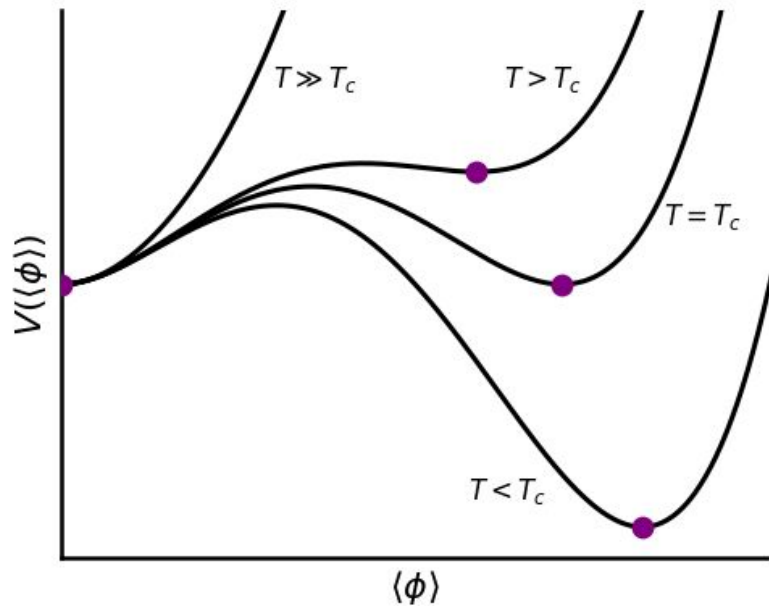
The shape of the vacuum potential

- Early universe was very hot
- Vacuum potential is temperature dependent (at 1-loop)

Vacuum potential develops extrema as the universe cools!

- Vacua are degenerate at the critical temperature $T = T_c$

Below the critical temperature, newly formed vacuum is energetically favourable!

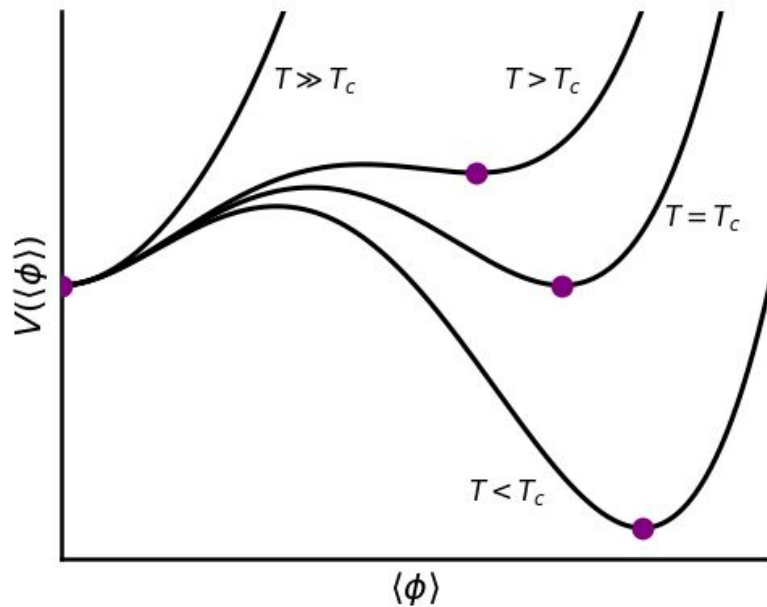
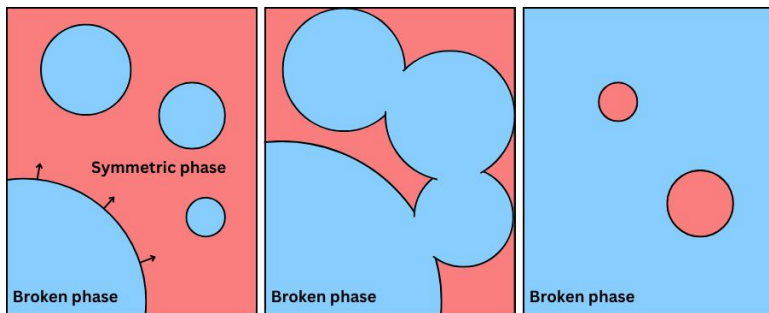


What is a cosmological phase transition?

The shape of the vacuum potential

- Latent heat $\rightarrow$ source of GWs
- Only second-order transition possible in the Standard Model

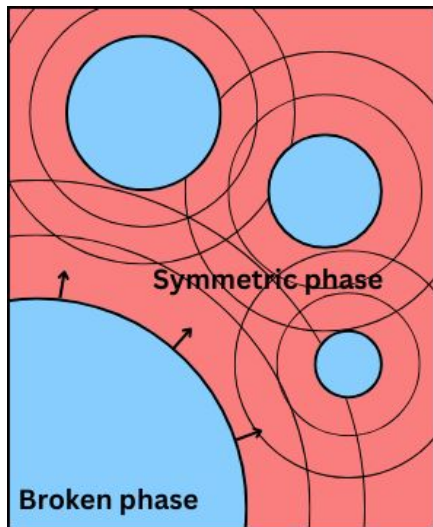
GWs from first-order phase transition corresponds to new physics!



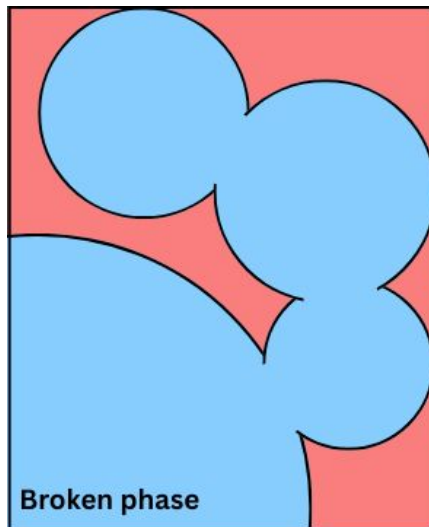
Gravitational waves from a first-order phase transition

Three dominant sources of GWs [1]:

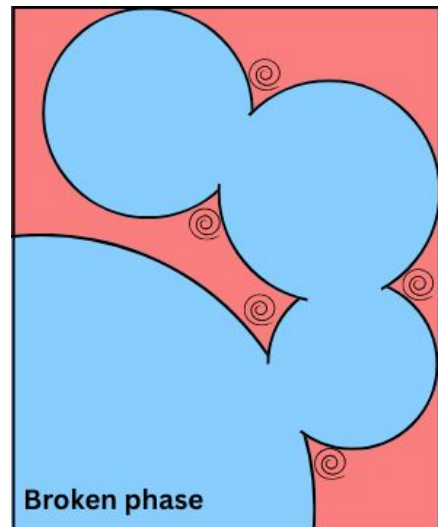
1. Sound waves



2. Bubble collisions



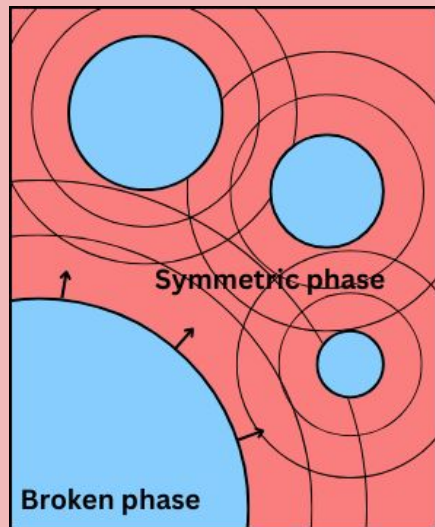
3. Non-linearities
(turbulence & vortices)



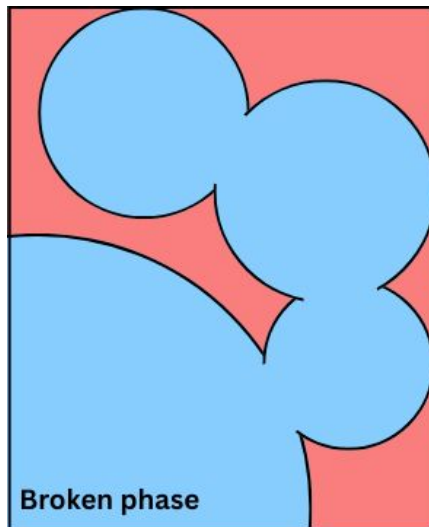
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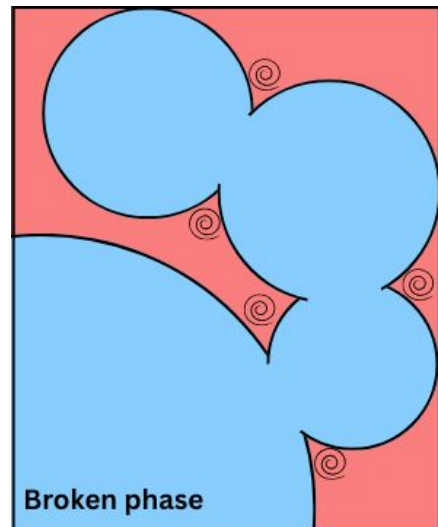
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From Lagrangians to gravitational waves

Lagrangian for a particle
physics model

$$\mathcal{L}$$

From Lagrangians to gravitational waves

Lagrangian for a particle
physics model

 $\mathcal{L}$ 

Construct effective
potential

 $V_{\text{eff}}(\phi, T)$

From Lagrangians to gravitational waves

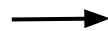
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$$(v_w, \beta, H_*, T_*)$$

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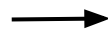
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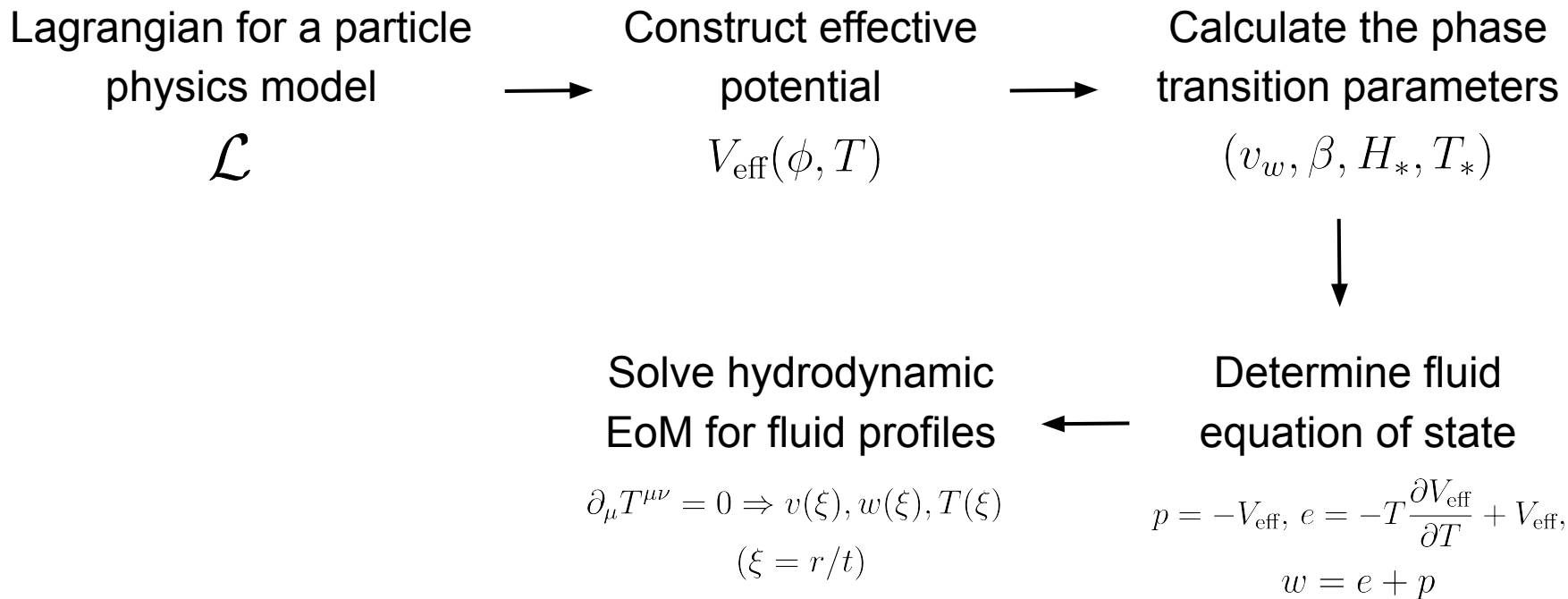


Determine fluid
equation of state

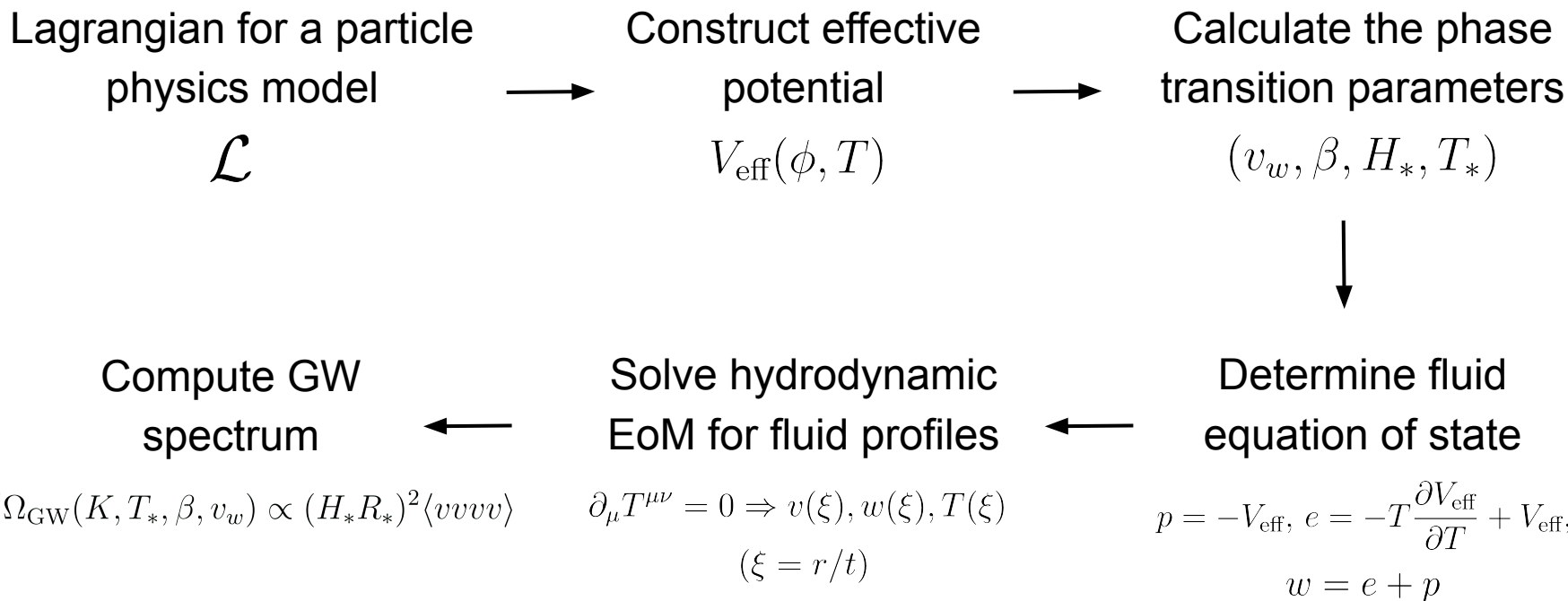
$$p = -V_{\text{eff}}, \quad e = -T \frac{\partial V_{\text{eff}}}{\partial T} + V_{\text{eff}},$$

$$w = e + p$$

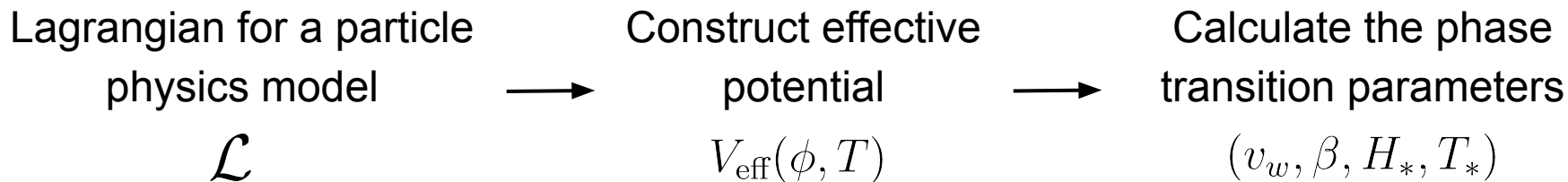
From Lagrangians to gravitational waves



From Lagrangians to gravitational waves



From Lagrangians to gravitational waves



Compute GW spectrum

$$h^2 \Omega_{\text{GW}}(K, T_*, \beta, v_w) \propto (H_* R_*)^2 \langle vvvv \rangle$$

Solve hydrodynamic EoM for fluid profiles

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow v(\xi), w(\xi), T(\xi)$$

$$(\xi = r/t)$$

Determine fluid equation of state

$$p = -V_{\text{eff}}, \quad e = -T \frac{\partial V_{\text{eff}}}{\partial T} + V_{\text{eff}},$$

$$w = e + p$$

Hydrodynamics of a first-order phase transition

The equation of state (EoS)

- Gives a thermodynamic description of the fluid during the transition from V_{eff}
- Needed to solve the fluid equation of motion: $\partial_\mu T^{\mu\nu} = 0$
- Defines how quickly sound waves that produce GWs can propagate

Exact EoS from the effective potential [2-7]:

- Hydrodynamics described entirely by the underlying particle physics model
- Sound speed in the fluid is $c_{s,\pm} = c_{s,s/b}(T_\pm)$ $c_{s,s/b}(T) = \left(\frac{dp_{s/b}}{de_{s/b}}\right)^{1/2}$

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No generic analytic form for
exact EoS



Everything must be solved numerically
(can be difficult!)

Hydrodynamics of a first-order phase transition

The equation of state (EoS)

Bag model [8-18]:

- Approximates temperature-dependent part of the effective potential by

$$V_{\text{eff}} \propto T^4$$

- Independent of the underlying particle physics model
- Assumes a radiation dominated universe, i.e. pressure = $(1/3) \times$ energy density
- Sound speed in the fluid is $c_{s,\pm} = \frac{1}{\sqrt{3}} \approx 0.57735$

$\mu\nu$ model [2, 3, 19-21]:

- Similar to bag model, but relaxes the radiation dominated assumption
- New degree of freedom*: constant sound speed in each vacuum phase

$$c_{s,\pm} \approx c_{s,s/b}(T_*)$$

$$c_{s,s/b}(T) = \left(\frac{dp_{s/b}}{de_{s/b}} \right)^{1/2}$$

T_* = percolation/nucleation
temperature

s = symmetric phase + = just in front of bubble wall
b = broken phase - = just behind bubble wall

Hydrodynamics of a first-order phase transition

Do the simplified equations of state give a sufficient hydrodynamic description of a first-order phase transition?...

Hydrodynamics of a first-order phase transition

Do the simplified equations of state give a sufficient hydrodynamic description of a first-order phase transition?...

...It depends.

An example: standard model + $\mathbb{Z}_2$ scalar singlet

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{\mu_s}{2} S^2 - \frac{\lambda_s}{4} S^4 - \frac{\lambda_{hs}}{2} H^\dagger H S^2$$

where $H = \frac{1}{\sqrt{2}} \begin{pmatrix} G^\pm \\ \phi_h + h + iG^0 \end{pmatrix}$

$$S = \phi_s + s$$

Tree-level scalar potential:

$$V(H, S) = -\mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2 - \frac{\mu_s}{2} S^2 + \frac{\lambda_s}{4} S^4 + \frac{\lambda_{hs}}{2} H^\dagger H S^2$$

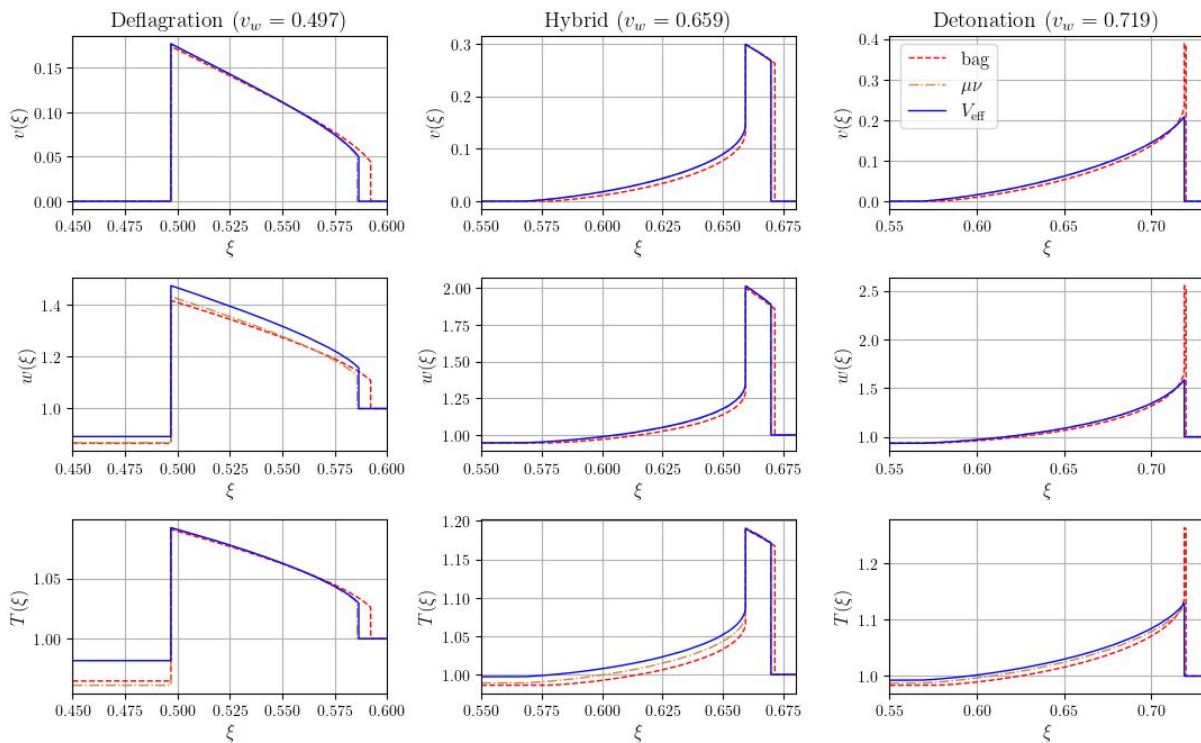
At 1-loop, temperature dependence
and multiple minima appear, allowing
for a thermal FOPT

An example: standard model + $\mathbb{Z}_2$ scalar singlet

Velocity, enthalpy & temperature profiles of the fluid

Bag/ $\mu\nu$ model: simplified hydrodynamics; radiation dominated universe;
independent of particle physics model (const. speed of sound)

Generic hydrodynamics (V_{eff}): hydrodynamics determined directly from particle physics model (temperature-dependent speed of sound)



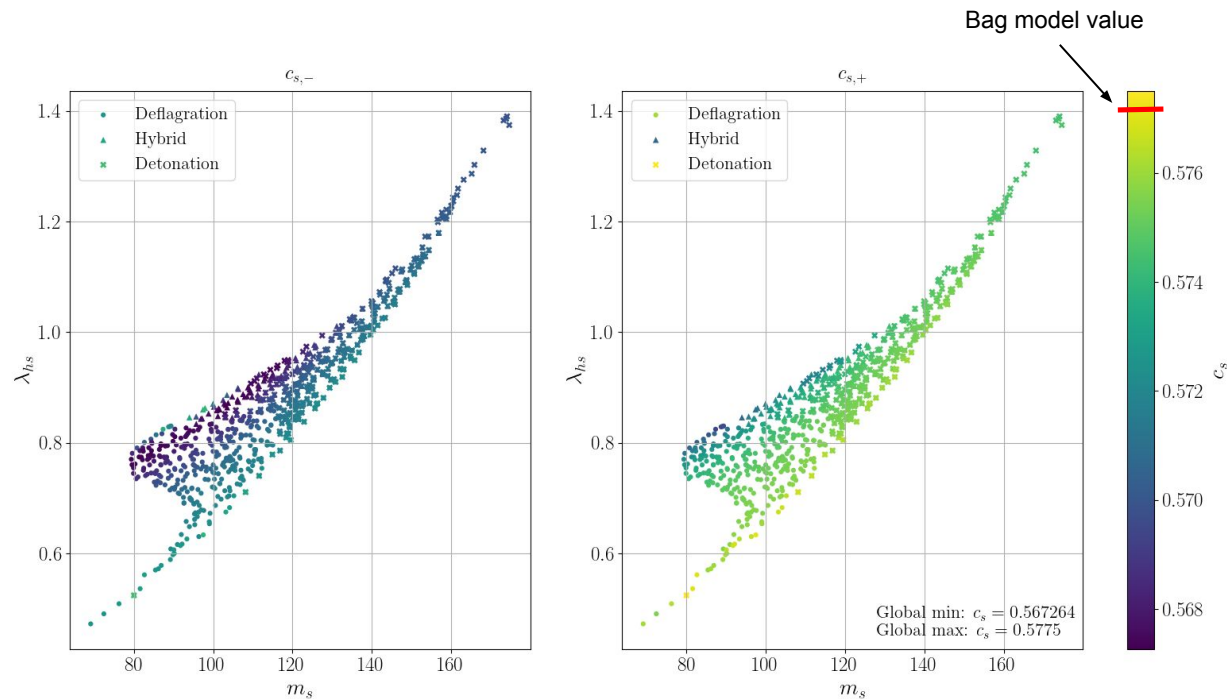
An example: standard model + $\mathbb{Z}_2$ scalar singlet

Where do the simplified (bag/ μ v) equations of state differ to the exact equation of state from the effective potential?

A BSM parameter scan

The speed of sound in both vacuum phases

- Deviation from the bag model largest in low-temperature/broken phase
- Full temperature dependence of sound speed only captured by utilising the true equation of state from V_{eff}



* $\mu\nu$ model sound speed generally agrees to $\sim 0.1\%$ of true value

A BSM parameter scan

Comparison of GW spectra

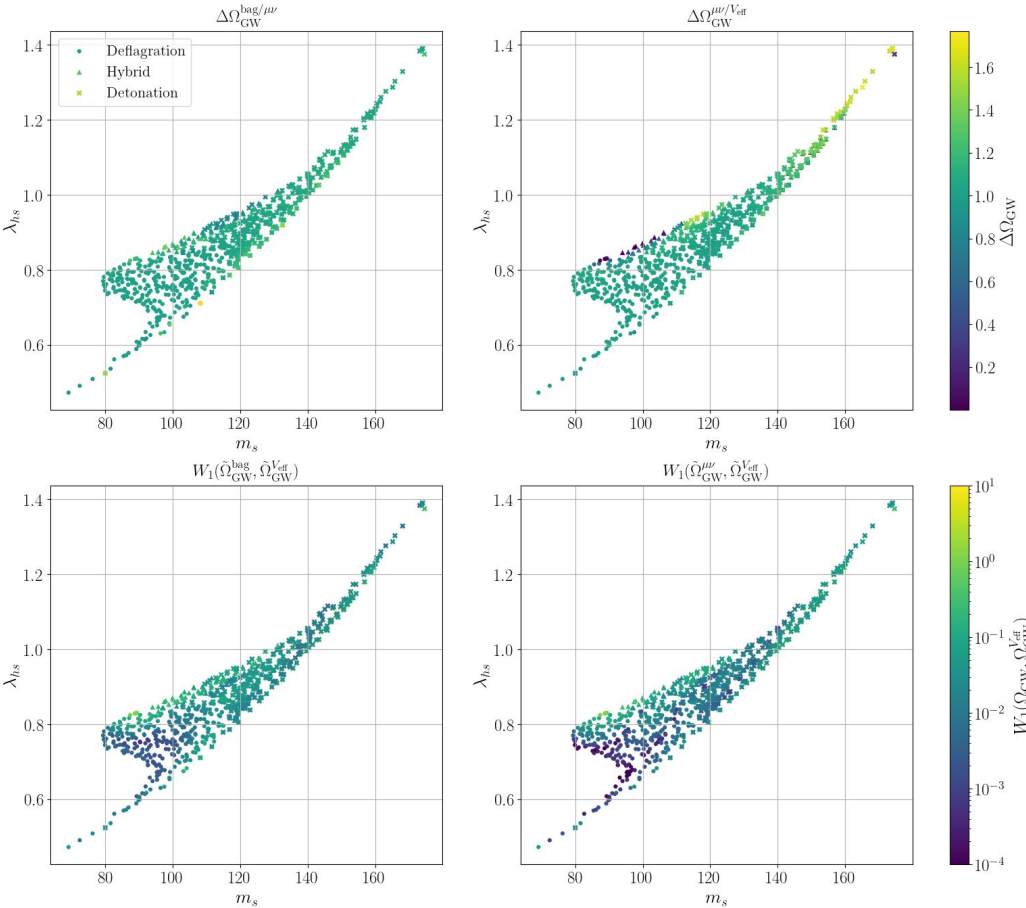
Top: Ratio of peak amplitudes

$$\Delta\Omega_{\text{GW}}^{\text{bag}/\mu\nu} = \frac{\Omega_{\text{GW}}^{\text{bag}}(f_{\text{pk}}^{\text{bag}})}{\Omega_{\text{GW}}^{\mu\nu}(f_{\text{pk}}^{\mu\nu})}; \quad \Delta\Omega_{\text{GW}}^{\mu\nu/V_{\text{eff}}} = \frac{\Omega_{\text{GW}}^{\mu\nu}(f_{\text{pk}}^{\mu\nu})}{\Omega_{\text{GW}}^{V_{\text{eff}}}(f_{\text{pk}}^{V_{\text{eff}}})}$$

f_{pk} = Peak frequency of GW spectrum Ω_{GW}

Bottom: Wassterstein distance
(difference in ‘shape’ between spectra)

$$W_1(\Omega_1, \Omega_2) = \int_{-\infty}^{\infty} df |F_1(f) - F_2(f)|$$
$$F_i(f) = \int_{-\infty}^f df' \Omega_i(f') \quad (\text{CDF of GW spectrum})$$



Summary

- Simplified equations of state ($\text{bag}/\mu\nu$) are *insensitive* to changes the sound speed due to reheating of the fluid
- Our hydrodynamic pipeline (HydroGrav) includes full temperature dependence in the sound speed when solving the fluid EoM

Precise treatment of
hydrodynamics of the phase
transition



Improved estimate of GW spectrum shape &
peak amplitude

**(needed to distinguish between FOPT and
other cosmological GW sources!)**

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Additional Slides

Further work

- Test other BSM models to identify parts of the parameter space where our accurate treatment of the hydrodynamics is needed to compute GW spectra
- Incorporate more subtle effects to hydrodynamics (e.g. GR corrections to hydrodynamics, non-spherical bubble dynamics)
- Incorporate semi-analytic contributions from sub-leading sources of GWs (bubble collisions & turbulence)

Additional Slides

Fluid equation of state

s = symmetric phase + = just in front of bubble wall
b = broken phase - = just behind bubble wall

	From effective potential	Bag model	$\mu\nu$ model
EoS	$p_s(T) = -V_{\text{eff}}(\phi_m^+(T)), T),$ $p_b(T) = -V_{\text{eff}}(\phi_m^-(T)), T),$ $e_s(T) = -T \frac{\partial V_{\text{eff}}}{\partial T} + V_{\text{eff}}(\phi_m^+(T)), T)$ $e_b(T) = -T \frac{\partial V_{\text{eff}}}{\partial T} + V_{\text{eff}}(\phi_m^-(T)), T)$ $p_+ = p_s(T_+), \quad e_+ = e_s(T_+)$ $p_- = p_b(T_-), \quad e_- = e_b(T_-)$	$p_+ = \frac{1}{3} a_+ T_+^4 - \epsilon,$ $p_- = \frac{1}{3} a_- T_-^4,$ $e_+ = a_+ T_+^4 + \epsilon$ $e_- = a_- T_-^4$	$p_+ = c_{s,+}^2 a_+ T_+^4 - \epsilon,$ $p_- = c_{s,-}^2 a_- T_-^4,$ $e_+ = a_+ T_+^4 + \epsilon$ $e_- = a_- T_-^4.$
Sound speed	$c_{s,s/b}(T) = \left(\frac{dp_{s/b}}{de_{s/b}} \right)^{1/2}$ $c_{s,\pm} = c_{s,s/b}(T_{\pm})$	$c_{s,\pm} = \frac{1}{\sqrt{3}} \approx 0.57735$	$c_{s,\pm} \approx c_{s,s/b}(T_*)$ $T_* = \text{percolation/nucleation temperature}$

Additional Slides

SM+SS effective potential at 1-loop

$$V_{\text{eff}}(\varphi, T) \equiv V_{\text{tree}}(\varphi) + V_{\text{CW}}(\varphi) + V_{\text{T}}(\varphi, T)$$

$$V_{\text{tree}}(\varphi_h, \varphi_s) = -\frac{1}{2}\mu_h^2\varphi_h^2 - \frac{1}{2}\mu_s^2\varphi_s^2 + \frac{\lambda_h}{4}\varphi_h^4 + \frac{\lambda_s}{4}\varphi_s^4 + \frac{1}{4}\lambda_{sh}\varphi_h^2\varphi_s^2.$$

$$V_{\text{CW}} = \sum_i \frac{n_i}{64\pi^2} \left\{ m_i^4(\varphi) \left(\ln \frac{m_i^2(\varphi)}{m_i^2(v_0)} - \frac{3}{2} \right) + 2m_i^2(\varphi)m_i^2(v_0) \right\} \quad (\text{Coleman-Weinberg potential})$$

$$V_{\text{T}}(\varphi, T) = \sum_F \frac{T^4}{2\pi^2} n_F J_F \left(\frac{m_F^2(\varphi)}{T^2} \right) + \sum_B \frac{T^4}{2\pi^2} n_B J_B \left(\frac{m_B^2(\varphi)}{T^2} \right) \quad (1\text{-loop thermal correction})$$

$$J_{B/F} = \int_0^\infty dx x^2 \log \left[1 \mp e^{-\sqrt{x^2 + m^2/T^2}} \right]$$

B = sum over bosons
F = sum over fermions

Additional Slides

Example benchmark points

Mode	λ_{hs}	m_s [GeV]	T_* [GeV]	β [s ⁻¹]	β/H_*	ξ_w	α_N^{bag}	$\alpha_N^{\mu\nu}$	$c_{s,+}$	$c_{s,-}$
Def.	0.826	88.75	39.78	9.52×10^{-12}	4077.5	0.497	0.074	0.072	0.568	0.588
Hyb.	0.881	106.49	62.63	4.38×10^{-12}	761.0	0.659	0.041	0.042	0.572	0.567
Det.	0.950	118.88	64.34	2.39×10^{-12}	393.5	0.719	0.046	0.047	0.573	0.567

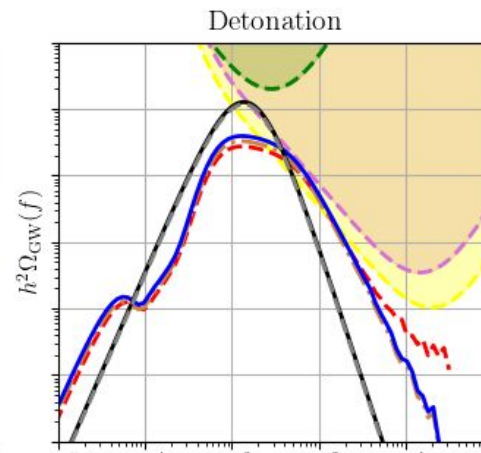
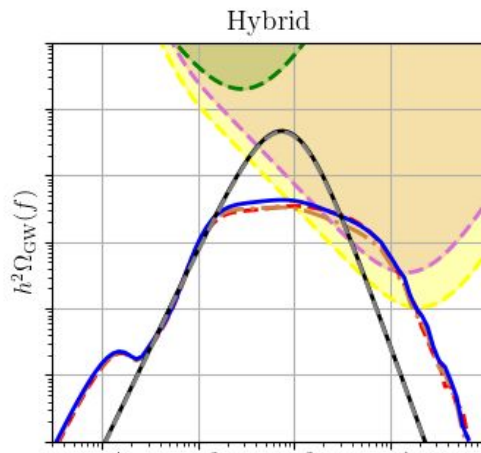
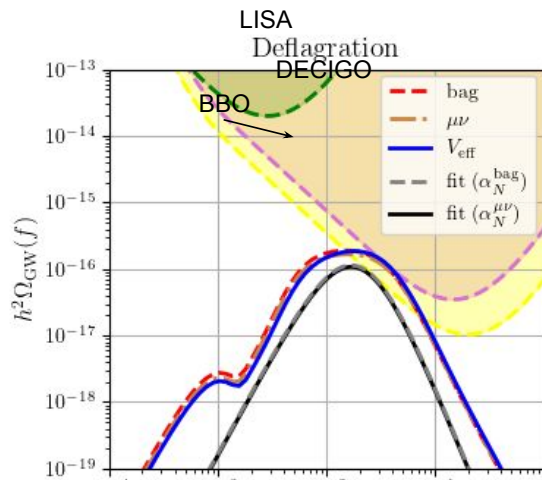
Table 2: Summary of the benchmark points presented in Figures 4 and 5 for a $\mathbb{Z}_2$ -symmetric extension of the Standard Model. The self-coupling of the scalar field is taken to be $\lambda_s = 1$. The phase transition strength parameters α_N^{bag} and $\alpha_N^{\mu\nu}$ are calculated using (4.20) and (4.23), respectively, and evaluated at the nucleation temperature $T_N \approx T_*$.

Additional Slides

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Generic hydrodynamics (V_{eff}): hydrodynamics determined directly from particle physics model (temperature-dependent speed of sound)



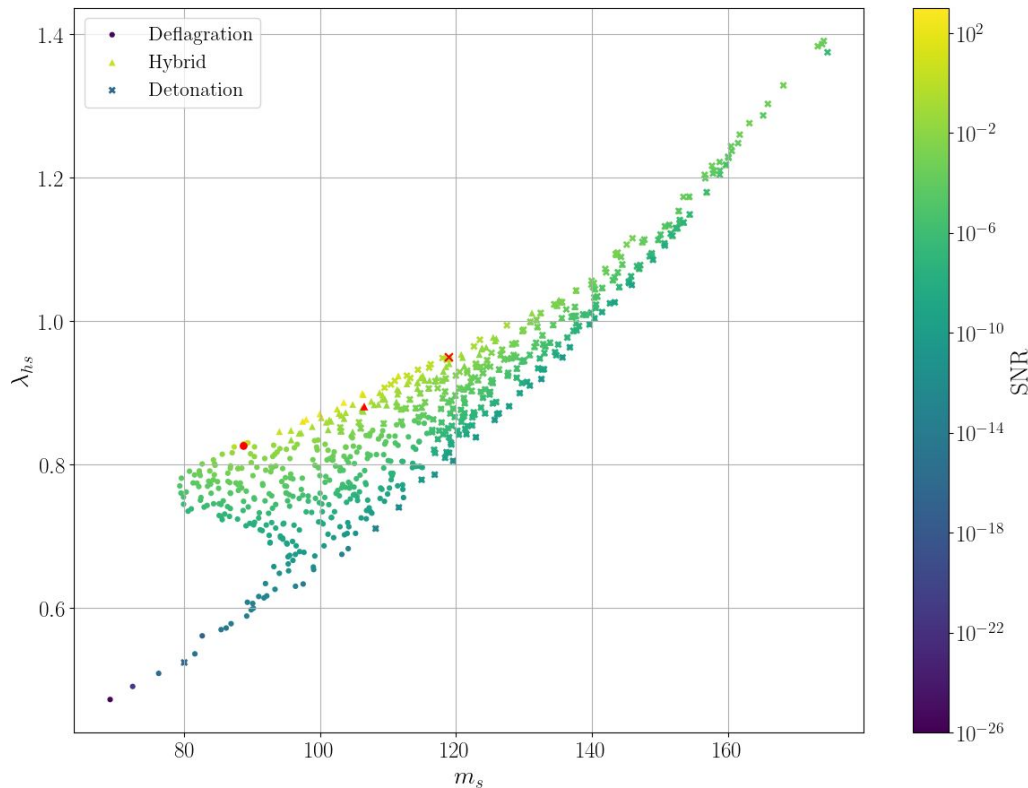
Additional Slides

SNR across the SM+SS parameter space

$$\text{SNR} = \sqrt{n_{\text{det}} t_{\text{obs}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{noise}}(f)} \right]^2}.$$

- Signal-to-noise ratio (SNR) falls within detectability window of LISA ($\text{SNR} \gtrsim 10$) in a small region of the parameter space

Caution: Choice of EFT construction & other theoretical uncertainties can change SNR by orders of magnitude



Additional Slides

Percolation temperature

- Temperature at which there is a connected cluster of bubbles that spans the entire Universe

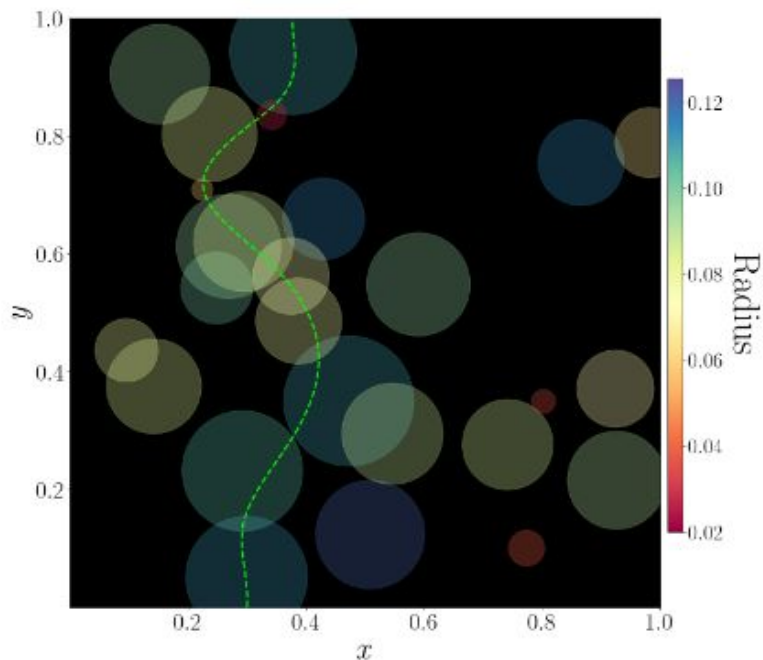


Figure taken from arXiv:2305.02357