Precise hydrodynamics in cosmological phase transitions

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Work based on arXiv:2606.27775

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My goals for today

1. *Where does hydrodynamics fit into cosmological phase transitions?*

2. *How accurate are simplified descriptions of the phase transition hydrodynamics?*

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What is a cosmological phase transition?

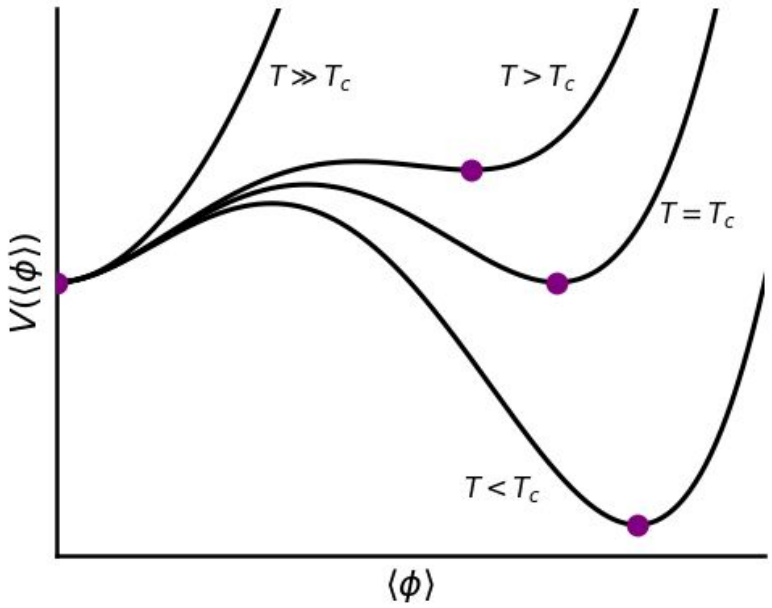
The shape of the vacuum potential

- Early universe was very hot
- Vacuum potential is temperature dependent (at 1-loop)

Vacuum potential develops extrema as the universe cools!

- Vacua are degenerate at the critical temperature $T = T_c$

Below the critical temperature, newly formed vacuum is energetically favourable!



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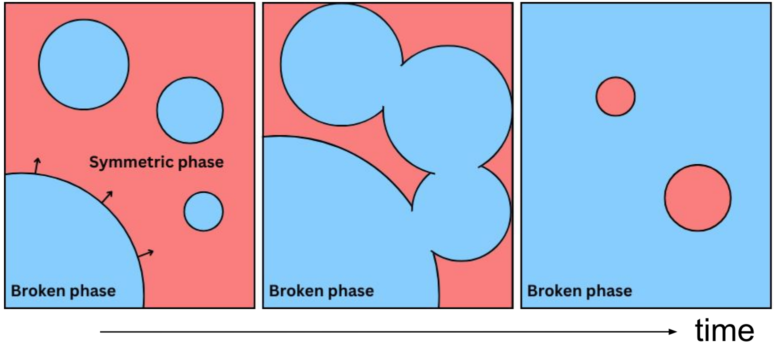
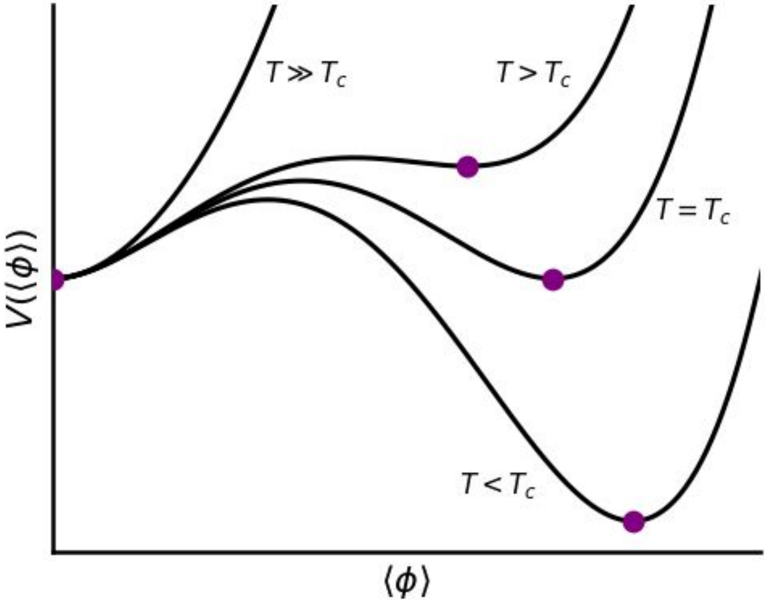
What is a cosmological phase transition?

The shape of the vacuum potential

- Latent heat -> source of GWs
- Only second-order transition possible in the Standard Model

GWs from first-order phase transition corresponds to new physics!





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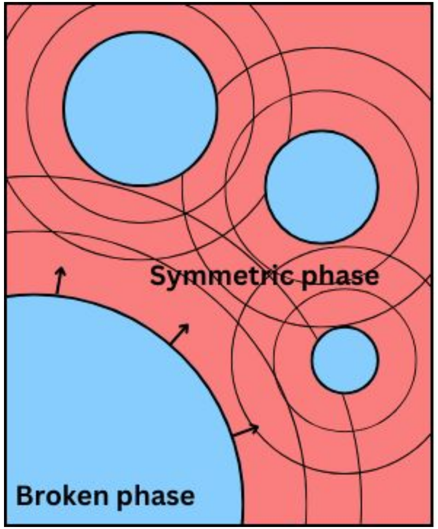
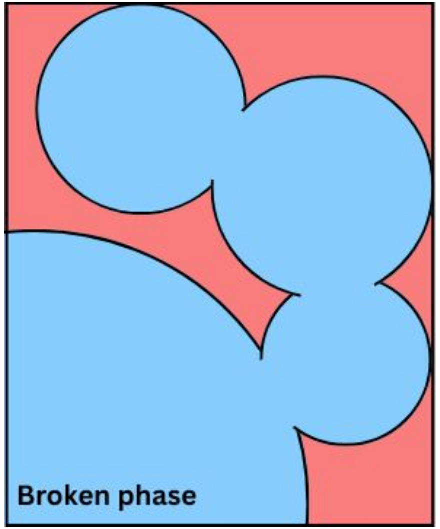
Gravitational waves from a first-order phase transition

Three dominant sources of GWs:

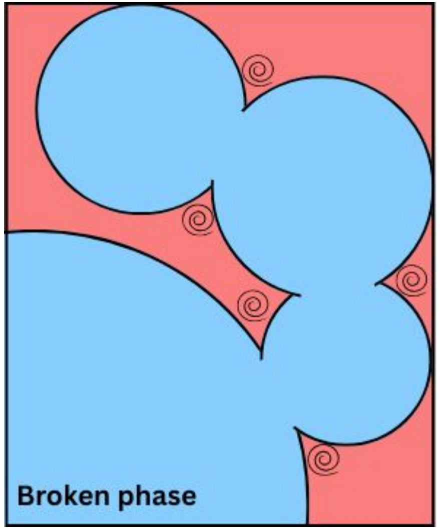
1. Sound waves



2. Bubble collisions



3. Non-linearities (turbulence & vortices)



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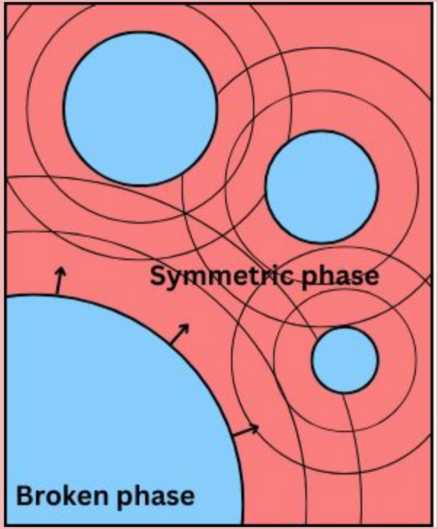

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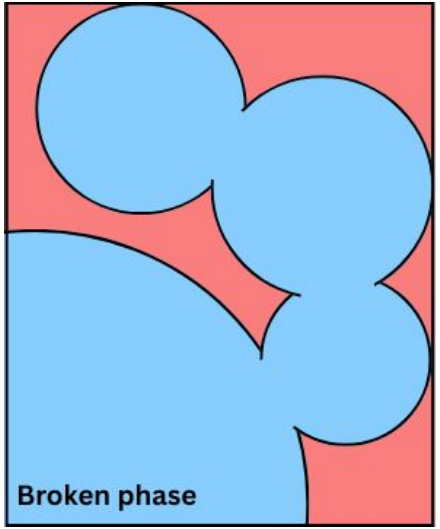

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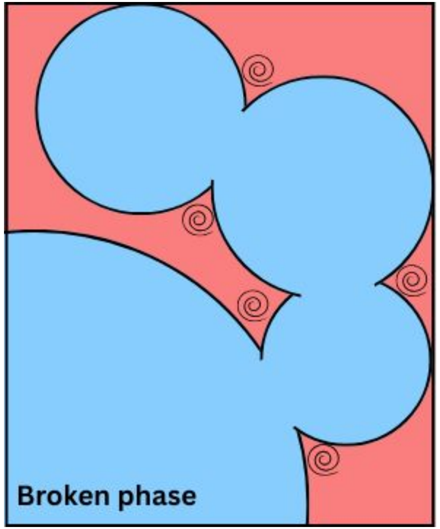

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From Lagrangians to gravitational waves

Lagrangian for a particle physics model

$$\mathcal{L}$$

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From Lagrangians to gravitational waves

Lagrangian for a particle physics model $\longrightarrow$ Construct effective potential

$\mathcal{L}$ $\qquad$ $V_{\text{eff}}(\phi, T)$

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From Lagrangians to gravitational waves

Lagrangian for a particle physics model	$\longrightarrow$	Construct effective potential	$\longrightarrow$	Calculate the phase transition parameters
$\mathcal{L}$		$V_{\text{eff}}(\phi, T)$		(v_w, β, H_*, T_*)

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From Lagrangians to gravitational waves

Lagrangian for a particle physics model

$\mathcal{L}$

→

Construct effective potential

$V_{\text{eff}}(\phi, T)$

→

Calculate the phase transition parameters

(v_w, β, H_*, T_*)

↓

Determine fluid equation of state

$$p = -V_{\text{eff}},\ e = -T\frac{\partial V_{\text{eff}}}{\partial T} + V_{\text{eff}},$$

$$w = e + p$$

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Solve hydrodynamic EoM for fluid profiles

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow v(\xi), w(\xi), T(\xi)$$

$$(\xi = r/t)$$

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Compute GW spectrum

$$h^2\Omega_{\text{GW}}(K, T_*, \beta, v_w) \propto (H_* R_*)^2 \langle vvvv \rangle$$

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Hydrodynamics of a first-order phase transition
The equation of state (EoS)

- Gives a thermodynamic description of the fluid during the transition from V_{eff}
- Needed to solve the fluid equation of motion: $\partial_\mu T^{\mu\nu} = 0$
- Defines how quickly sound waves that produce GWs can propagate

Exact EoS from the effective potential:

- Hydrodynamics described entirely by the underlying particle physics model
- Sound speed in the fluid is $c_{s,\pm} = c_{s,s/b}(T_\pm)$ $\quad c_{s,s/b}(T) = \left(\frac{\mathrm{d}p_{s/b}}{\mathrm{d}e_{s/b}}\right)^{1/2}$

s = symmetric phase
b = broken phase

+ = just in front of bubble wall
- = just behind bubble wall

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No generic analytic form for exact EoS → Everything must be solved numerically (can be difficult!)

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Hydrodynamics of a first-order phase transition

The equation of state (EoS)

Bag model:

- Approximates temperature-dependent part of the effective potential by

$$V_{\text{eff}} \propto T^4$$

- *Independent* of the underlying particle physics model
- Assumes a radiation dominated universe, i.e. $\text{pressure} = (1/3) \times \text{energy density}$
- Sound speed in the fluid is $c_{s,\pm} = \frac{1}{\sqrt{3}} \approx 0.57735$

μν model:

- Similar to bag model, but relaxes the radiation dominated assumption
- *New degree of freedom:* constant sound speed in each vacuum phase

$$c_{s,\pm} \approx c_{s,s/b}(T_*)$$

$$c_{s,s/b}(T) = \left(\frac{\mathrm{d}p_{s/b}}{\mathrm{d}e_{s/b}}\right)^{1/2}$$

$T_* =$ percolation/nucleation temperature

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Hydrodynamics of a first-order phase transition

Do the simplified equations of state give a sufficient hydrodynamic description of a first-order phase transition?...

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Hydrodynamics of a first-order phase transition

Do the simplified equations of state give a sufficient hydrodynamic description of a first-order phase transition?...

…It depends.

An example: standard model + $\mathbb{Z}_2$ scalar singlet

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{\mu_s}{2}S^2 - \frac{\lambda_s}{4}S^4 - \frac{\lambda_{hs}}{2}H^\dagger H S^2$$

where

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} G^\pm \\ \phi_h + h + iG^0 \end{pmatrix}$$

$$S = \phi_s + s$$

Tree-level scalar potential:

$$V(H,S) = -\mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2 - \frac{\mu_s}{2}S^2 + \frac{\lambda_s}{4}S^4 + \frac{\lambda_{hs}}{2}H^\dagger H S^2$$

At 1-loop, temperature dependence and multiple minima appear, allowing for a thermal FOPT

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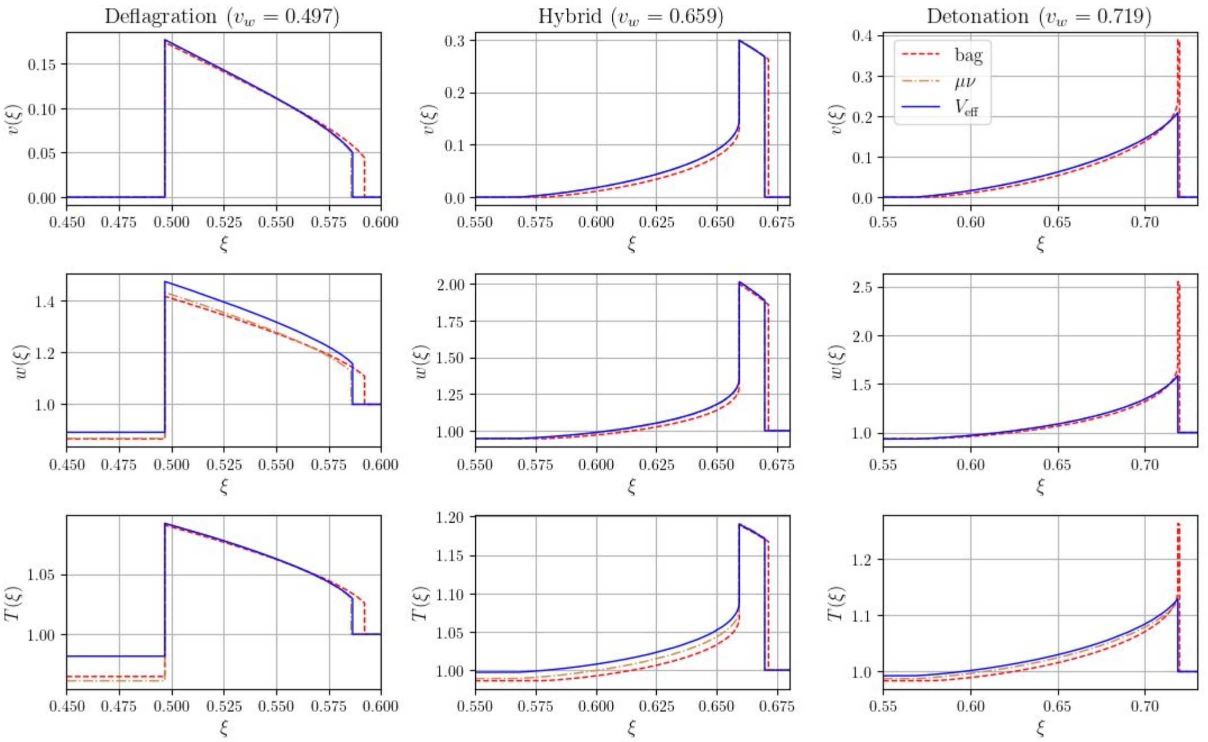
An example: standard model + $\mathbb{Z}_2$ scalar singlet

Velocity, enthalpy & temperature profiles of the fluid

Bag/μν model: simplified hydrodynamics; radiation dominated universe; *independent* of particle physics model (*const. speed of sound*)

Generic hydrodynamics (Veff): hydrodynamics determined directly from particle physics model (*temperature-dependent speed of sound*)



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An example: standard model + $\mathbb{Z}_2$ scalar singlet

Where do the simplified (bag/μν) equations of state differ to the exact equation of state from the effective potential?

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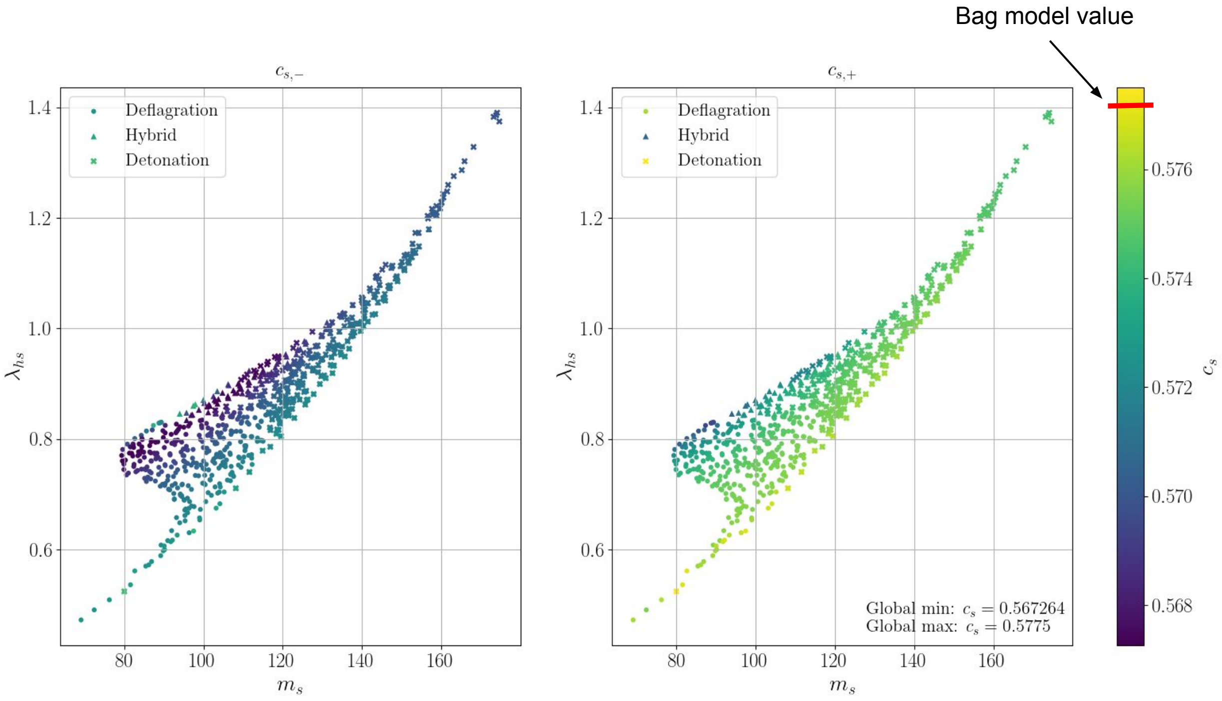
A BSM parameter scan
The speed of sound in both vacuum phases

- Deviation from the bag model largest in low-temperature/broken phase
- Full temperature dependence of sound speed only captured by utilising the true equation of state from V_{eff}



*μv model sound speed generally agrees to ~0.1% of true value

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A BSM parameter scan

Comparison of GW spectra

Top: Ratio of peak amplitudes

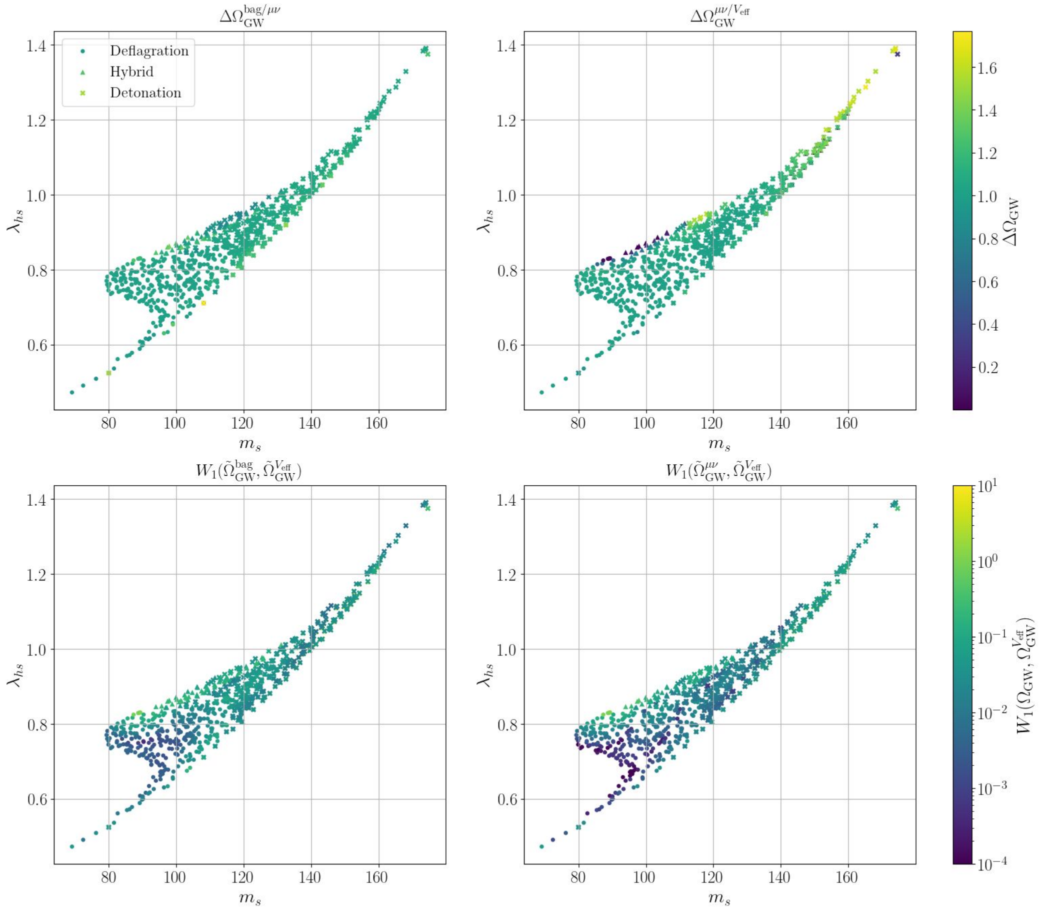
$$\Delta\Omega_{\rm GW}^{{\rm bag}/\mu\nu} = \frac{\Omega_{\rm GW}^{\rm bag}(f_{\rm pk}^{\rm bag})}{\Omega_{\rm GW}^{\mu\nu}(f_{\rm pk}^{\mu\nu})}; \quad \Delta\Omega_{\rm GW}^{\mu\nu/V_{\rm eff}} = \frac{\Omega_{\rm GW}^{\mu\nu}(f_{\rm pk}^{\mu\nu})}{\Omega_{\rm GW}^{V_{\rm eff}}(f_{\rm pk}^{V_{\rm eff}})}$$

$f_{\rm pk}$ = Peak frequency of GW spectrum $\Omega_{\rm GW}$

Bottom: Wassterstein distance (difference in 'shape' between spectra)

$$W_1(\Omega_1, \Omega_2) = \int_{-\infty}^{\infty} {\rm d}f\, |F_1(f) - F_2(f)|$$

$$F_i(f) = \int_{-\infty}^{f} {\rm d}f'\, \Omega_i(f') \qquad \text{(CDF of GW spectrum)}$$



Summary

- Simplified equations of state (bag/μν) are *insensitive* to changes the sound speed due to reheating of the fluid
- Our hydrodynamic pipeline (HydroGrav) includes full temperature dependence in the sound speed when solving the fluid EoM

Precise treatment of hydrodynamics of the phase transition → Improved estimate of GW spectrum shape & peak amplitude

(needed to distinguish between FOPT and other cosmological GW sources!)

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Additional Slides
Further work

- Test other BSM models to identify parts of the parameter space where our accurate treatment of the hydrodynamics is needed to compute GW spectra
- Incorporate more subtle effects to hydrodynamics (e.g. GR corrections to hydrodynamics, non-spherical bubble dynamics)
- Incorporate semi-analytic contributions from sub-leading sources of GWs (bubble collisions & turbulence)

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Additional Slides

Fluid equation of state

s = symmetric phase
b = broken phase

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	From effective potential	Bag model	μv model
EoS	$p_s(T) = -V_{\rm eff}(\phi_m^+(T)), T),$ $p_b(T) = -V_{\rm eff}(\phi_m^-(T)), T),$ $e_s(T) = -T\frac{\partial V_{\rm eff}}{\partial T} + V_{\rm eff}(\phi_m^+(T)), T)$ $e_b(T) = -T\frac{\partial V_{\rm eff}}{\partial T} + V_{\rm eff}(\phi_m^-(T)), T)$ $p_+ = p_s(T_+), \quad e_+ = e_s(T_+)$ $p_- = p_b(T_-), \quad e_- = e_b(T_-)$	$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon,$ $p_- = \frac{1}{3}a_-T_-^4,$ $e_+ = a_+T_+^4 + \epsilon$ $e_- = a_-T_-^4$	$p_+ = c_{s,+}^2 a_+ T_+^4 - \epsilon,$ $p_- = c_{s,-}^2 a_- T_-^4,$ $e_+ = a_+T_+^4 + \epsilon$ $e_- = a_-T_-^4.$
Sound speed	$c_{s,s/b}(T) = \left(\frac{\mathrm{d}p_{s/b}}{\mathrm{d}e_{s/b}}\right)^{1/2}$ $c_{s,\pm} = c_{s,s/b}(T_\pm)$	$c_{s,\pm} = \frac{1}{\sqrt{3}} \approx 0.57735$	$c_{s,\pm} \approx c_{s,s/b}(T_*)$ $T_* =$ percolation/nucleation temperature

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Additional Slides

SM+SS effective potential at 1-loop

$$V_{\text{eff}}(\varphi, T) \equiv V_{\text{tree}}(\varphi) + V_{\text{CW}}(\varphi) + V_{\text{T}}(\varphi, T)$$

$$V_{\text{tree}}(\varphi_h, \varphi_s) = -\frac{1}{2}\mu_h^2\varphi_h^2 - \frac{1}{2}\mu_s^2\varphi_s^2 + \frac{\lambda_h}{4}\varphi_h^4 + \frac{\lambda_s}{4}\varphi_s^4 + \frac{1}{4}\lambda_{sh}\varphi_h^2\varphi_s^2.$$

$$V_{\text{CW}} = \sum_i \frac{n_i}{64\pi^2}\left\{m_i^4(\varphi)\left(\ln\frac{m_i^2(\varphi)}{m_i^2(v_0)} - \frac{3}{2}\right) + 2m_i^2(\varphi)m_i^2(v_0)\right\}$$ (Coleman-Weinberg potential)

$$V_{\text{T}}(\varphi, T) = \sum_F \frac{T^4}{2\pi^2} n_F J_F\left(\frac{m_F^2(\varphi)}{T^2}\right) + \sum_B \frac{T^4}{2\pi^2} n_B J_B\left(\frac{m_B^2(\varphi)}{T^2}\right)$$ (1-loop thermal correction)

$$J_{B/F} = \int_0^\infty dx\, x^2 \log\left[1 \mp e^{-\sqrt{x^2+m^2/T^2}}\right]$$

B = sum over bosons
F = sum over fermions

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Additional Slides

Example benchmark points

Mode	λ_{hs}	m_s [GeV]	T_* [GeV]	β [s^{-1}]	β/H_*	ξ_w	α_N^{bag}	$\alpha_N^{\mu\nu}$	$c_{s,+}$	$c_{s,-}$
Def.	0.826	88.75	39.78	9.52×10^{-12}	4077.5	0.497	0.074	0.072	0.568	0.588
Hyb.	0.881	106.49	62.63	4.38×10^{-12}	761.0	0.659	0.041	0.042	0.572	0.567
Det.	0.950	118.88	64.34	2.39×10^{-12}	393.5	0.719	0.046	0.047	0.573	0.567

Table 2: Summary of the benchmark points presented in Figures 4 and 5 for a $\mathbb{Z}_2$-symmetric extension of the Standard Model. The self-coupling of the scalar field is taken to be $\lambda_s = 1$. The phase transition strength parameters α_N^{bag} and $\alpha_N^{\mu\nu}$ are calculated using (4.20) and (4.23), respectively, and evaluated at the nucleation temperature $T_N \approx T_*$.

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Additional Slides
Example benchmark points

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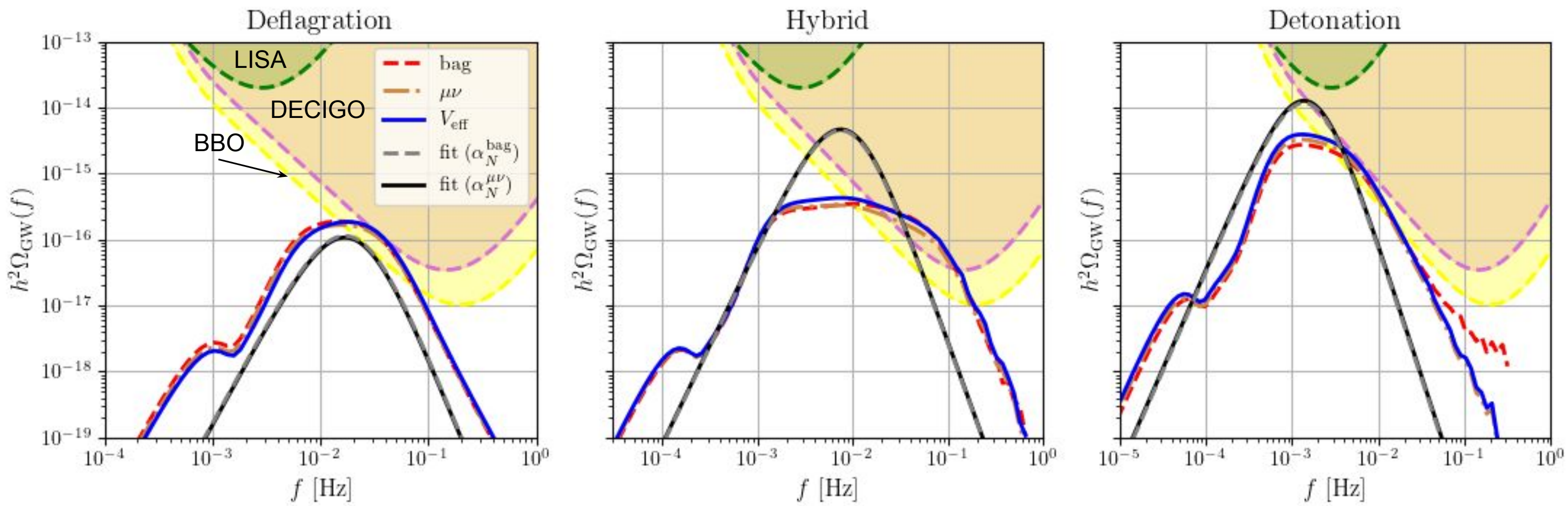


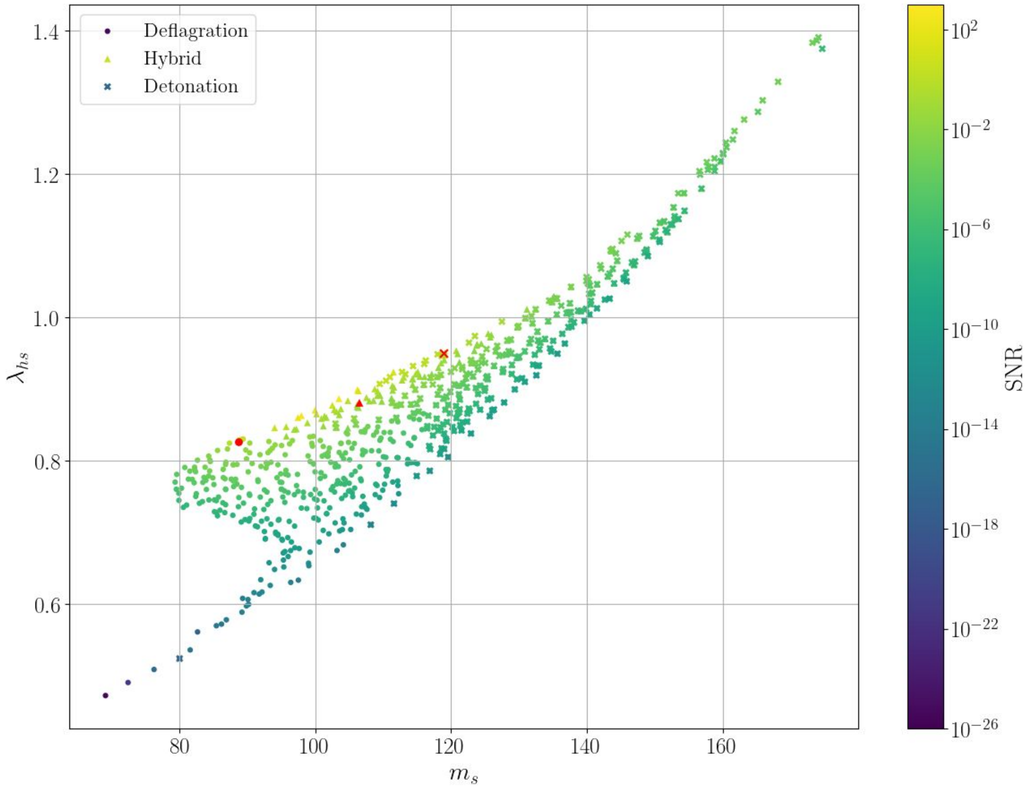
Additional Slides

SNR across the SM+SS parameter space

$$\mathrm{SNR} = \sqrt{n_{\mathrm{det}} t_{\mathrm{obs}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \mathrm{d}f \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\mathrm{noise}}(f)}\right]^2}.$$

- Signal-to-noise ratio (SNR) falls within detectability window of LISA (SNR$\gtrsim$10) in a small region of the parameter space

Caution: Choice of EFT construction & other theoretical uncertainties can change SNR by orders of magnitude



Additional Slides
Percolation temperature

- Temperature at which there is a connected cluster of bubbles that spans the entire Universe

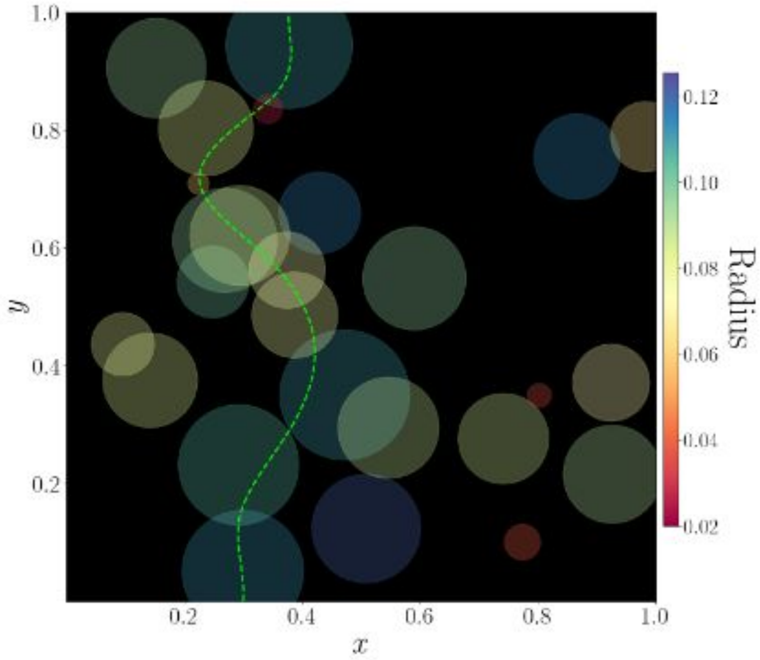


Figure taken from arXiv:2305.02357