Effective Field Theory for Thermal Phase Transitions

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Based on
2510.26878 (M. Chala, AD, G. Guedes)
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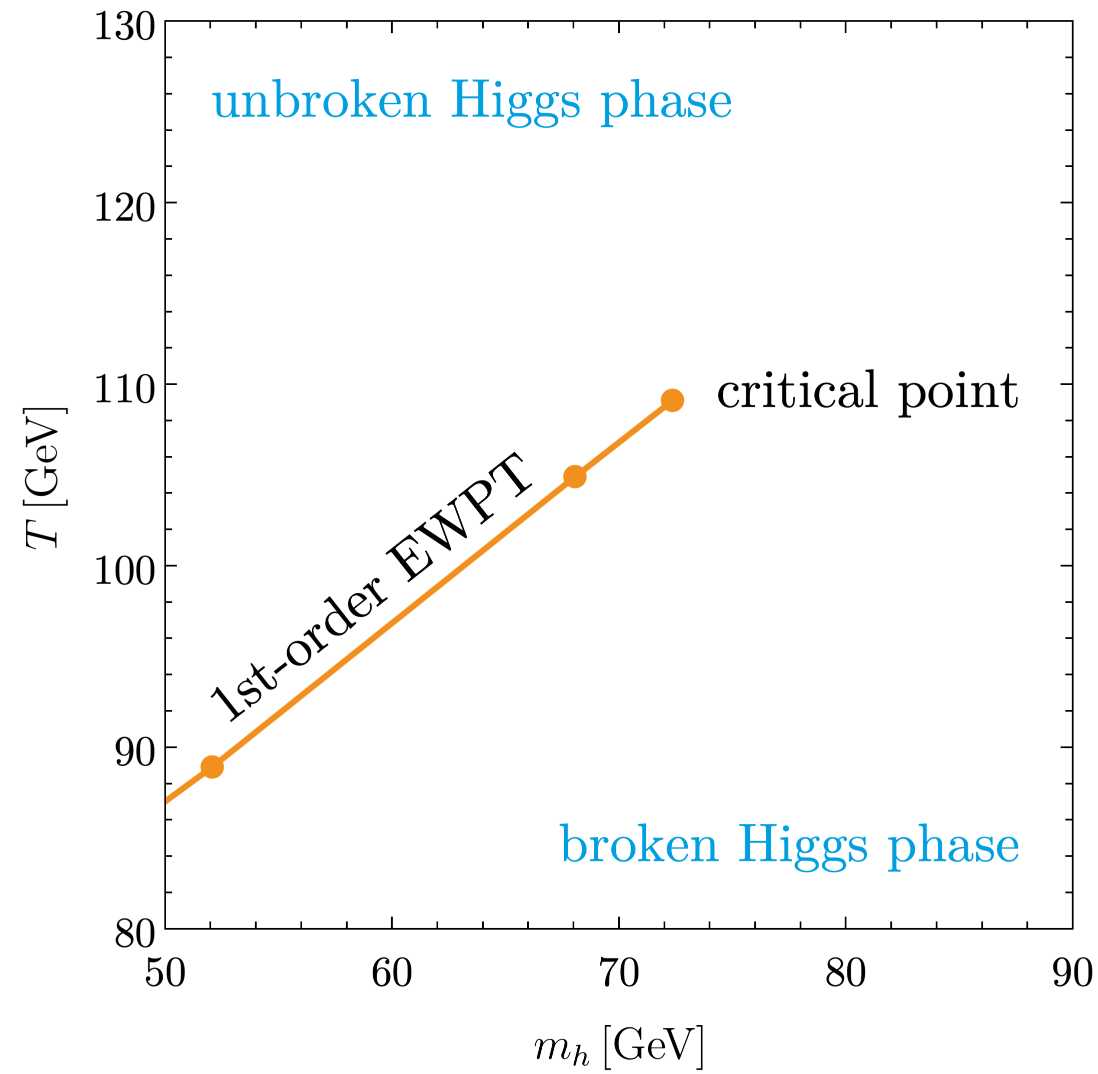
Electroweak Phase Transitions: Motivation

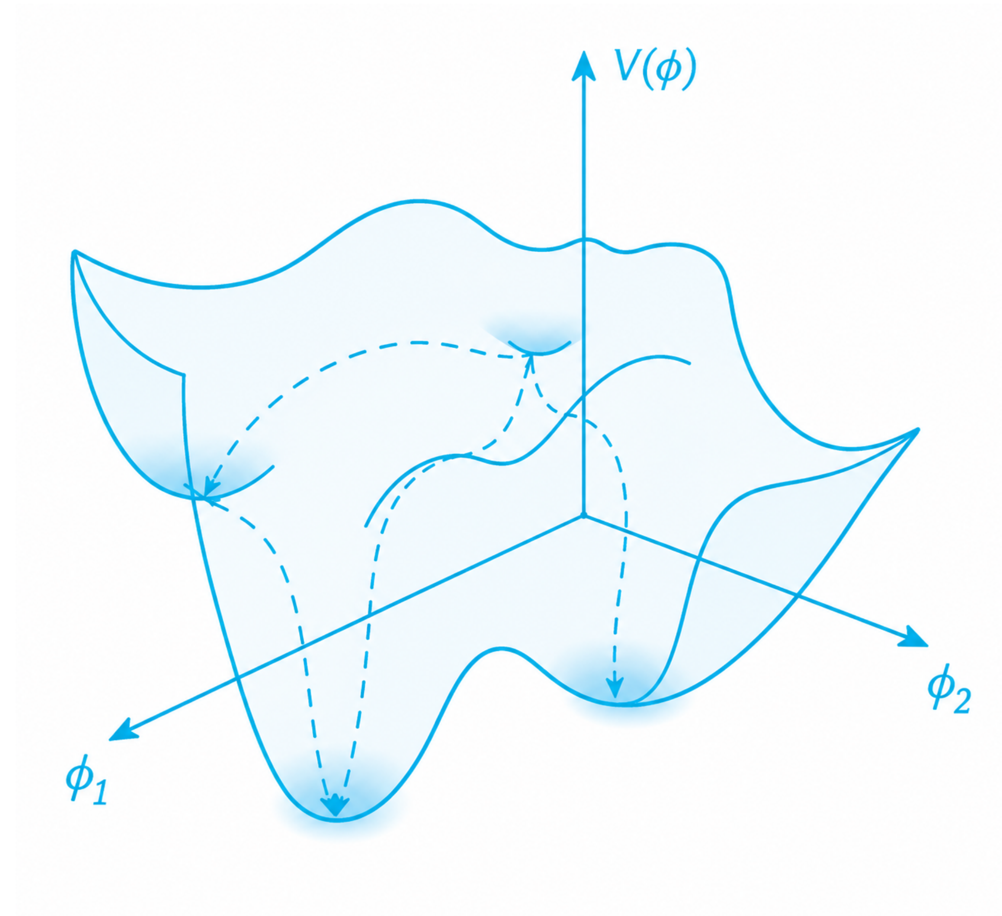
- Electroweak Phase Transition (EWPT) marks the breaking of $SU(2)_L \times U(1)_Y$
- Within the Standard Model (SM), EWPT is a smooth cross-over at $\sim 160\,\mathrm{GeV}$



[Laine et al., 0010275]

Electroweak Phase Transitions: Motivation

- Electroweak Phase Transition (EWPT) marks the breaking of $SU(2)_L \times U(1)_Y$
- Within the Standard Model (SM), EWPT is a smooth cross-over at $\sim 160\,\mathrm{GeV}$
- 1st order EWPT is natural in many extensions of the SM:
  extra scalars, SMEFT, SUSY,
  composite Higgs
- May result in detectable GWs and be responsible for baryon asymmetry of the Universe



[Adopted from K. Radchenko]

Electroweak Phase Transitions: General Pipeline

Our goal: do it with high precision and accuracy

Physical inputs

$(m_h, m_s, G_F, U(1)...)$

GW signal/Baryogenesis

$\Omega_{\mathrm{GW}}/\eta$

Electroweak Phase Transitions: Microscopic physics description

Physical inputs

$(m_h, m_s, G_F, U(1) ...)$

Model building,
relating physical inputs
to Lagrange parameters

Lagrangian

$\mathcal{L}_{\rm 4D}$

Thermal field theory,
equilibrium thermodynamics,
perturbative methods,
lattice simulations

Macroscopic observables

$(T_c, T_p, \alpha, v_w, ...)$

Relativistic hydrodynamics,
percolation simulations,
bubble collisions,
turbulence

GW signal/Baryogenesis

$\Omega_{\rm GW}/\eta$

Thermal scales at play

Separation of scales in weakly coupled thermal plasma

other modes

$m \sim \pi T$

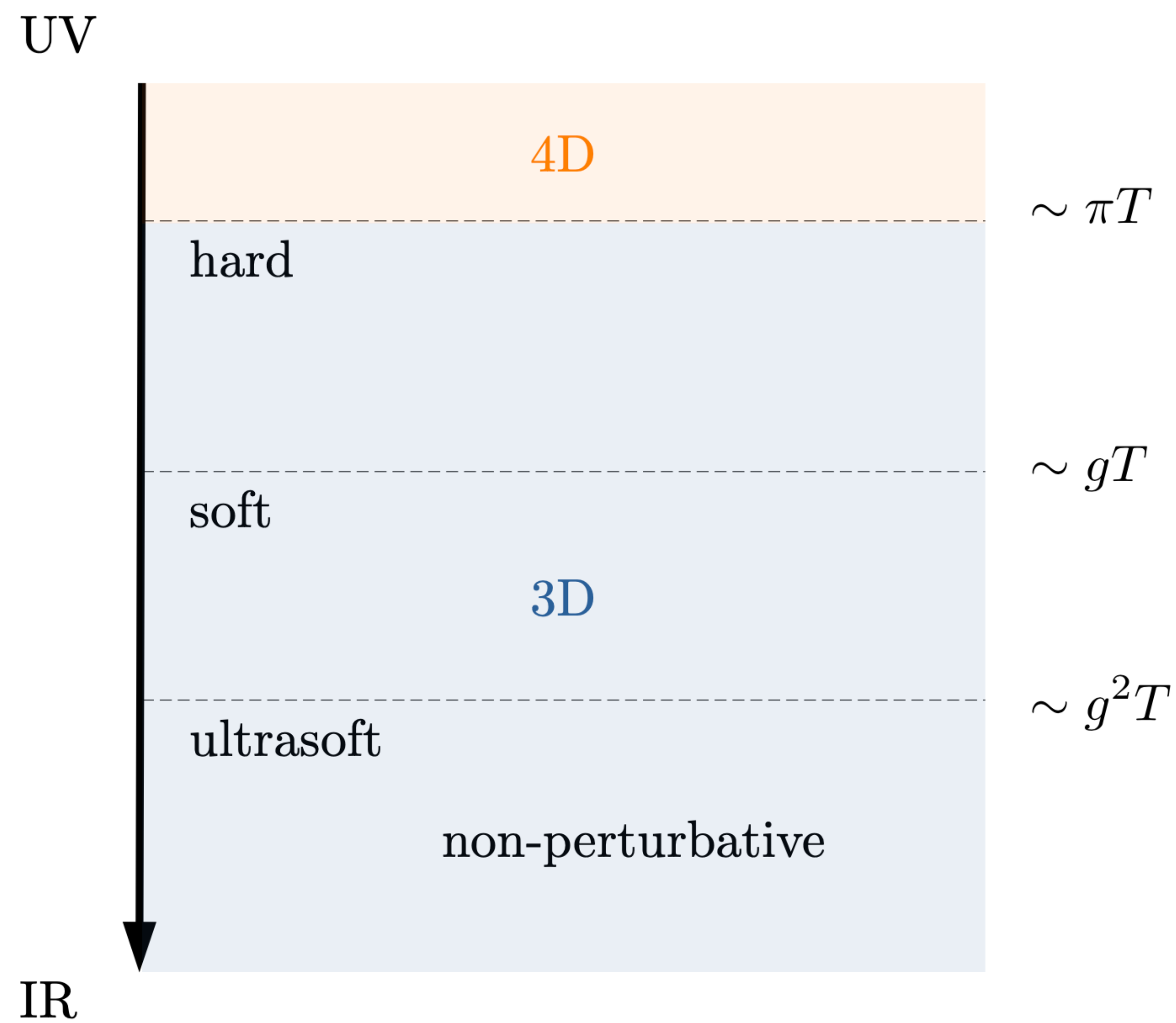
Bosons: $\omega_n = 2\pi n T$

Fermions: $\omega'_n = 2\pi\,(n + 1/2)\,T$

$m \ll \pi T$

$n = 0$

bosonic mode



- Dimensional reduction: construction an effective field theory (EFT) for nucleating bosonic zero modes — 3D EFT

- Additional fields can be integrated out if heavy, e.g., extra scalars

- Higher-order operators appear, formally suppressed by m/T

[Farakos et al., 9404201]

[Kajantie et al., 9508379]

First-order PT within SM(EFT)

$\tilde{g}, \Lambda$

BSM

$\mathcal{L}^{\mathrm{BSM}} = \dots$

$\Lambda \geq T, v$

SMEFT

Standard Model EFT (SMEFT) — new physics is encoded in new operators with SM fields

$$\mathscr{L}_4^{\mathrm{SMEFT}} = \mathscr{L}_4^{\mathrm{SM}} + \frac{1}{\Lambda^2}\Big\{ c_\phi (\phi^\dagger\phi)^3 + c_{\phi\Box}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + c_{\phi D}(\phi^\dagger D^\mu\phi)^*(\phi^\dagger D_\mu\phi)$$
$$+ c^{(1)}_{\phi\psi_L}(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\overline{\psi_L}\gamma^\mu\psi_L) + c^{(3)}_{\phi\psi_L}(\phi^\dagger i\overleftrightarrow{D}^I_\mu\phi)(\overline{\psi_L}\gamma^\mu\sigma^I\psi_L) + c_{\phi\psi_R}(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\overline{\psi_R}\gamma^\mu\psi_R)$$
$$+ \Big[c_{\phi ud}(\widetilde{\phi} i D_\mu\phi)(\overline{u_R}\gamma^\mu d_R) + c_{\psi_R\phi}(\phi^\dagger\phi)\overline{\psi_L}\widetilde{\phi}\psi_R + \mathrm{h.c.}\Big]\Big\}$$

$\psi^2 X\phi$	
$\mathscr{O}_{uG}$	$(\bar{q}T^A\sigma^{\mu\nu}u)\widetilde{\phi}G^A_{\mu\nu}$
$\mathscr{O}_{uW}$	$(\bar{q}\sigma^{\mu\nu}u)\sigma^I\widetilde{\phi}W^I_{\mu\nu}$
$\mathscr{O}_{uB}$	$(\bar{q}\sigma^{\mu\nu}u)\widetilde{\phi}B^I_{\mu\nu}$
$\mathscr{O}_{dG}$	$(\bar{q}T^A\sigma^{\mu\nu}d)\phi G^A_{\mu\nu}$
$\mathscr{O}_{dW}$	$(\bar{q}\sigma^{\mu\nu}d)\sigma^I\phi W^I_{\mu\nu}$
$\mathscr{O}_{dB}$	$(\bar{q}\sigma^{\mu\nu}d)\phi B^I_{\mu\nu}$
$\mathscr{O}_{eW}$	$(\bar{l}\sigma^{\mu\nu}e)\sigma^I\phi W^I_{\mu\nu}$
$\mathscr{O}_{eB}$	$(\bar{l}\sigma^{\mu\nu}e)\phi B_{\mu\nu}$

$\psi^2\phi^2$	
$\mathscr{O}^{(1)}_{\phi q}$	$\bar{q}\gamma^\mu q(\phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu\phi)$
$\mathscr{O}^{(3)}_{\phi q}$	$\bar{q}\sigma^I\gamma^\mu q(\phi^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu\phi)$
$\mathscr{O}_{\phi u}$	$\bar{u}\gamma^\mu u(\phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu\phi)$
$\mathscr{O}_{\phi d}$	$\bar{d}\gamma^\mu d(\phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu\phi)$
$\mathscr{O}_{\phi ud}$	$\bar{u}\gamma^\mu d(\widetilde{\phi}^\dagger \mathrm{i}D_\mu\phi)$
$\mathscr{O}^{(1)}_{\phi l}$	$\bar{l}\gamma^\mu l(\phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu\phi)$
$\mathscr{O}^{(3)}_{\phi l}$	$\bar{l}\sigma^I\gamma^\mu l(\phi^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu\phi)$
$\mathscr{O}_{\phi e}$	$\bar{e}\gamma^\mu e(\phi^\dagger \mathrm{i}\overleftrightarrow{D}_\mu\phi)$
$\psi^2\phi^3$	
$\mathscr{O}_{u\phi}$	$\bar{q}\widetilde{\phi}u(\phi^\dagger\phi)$
$\mathscr{O}_{d\phi}$	$\bar{q}\phi d(\phi^\dagger\phi)$
$\mathscr{O}_{e\phi}$	$\bar{q}\phi e(\phi^\dagger\phi)$

[Chala et al.,2507.16905]

Dimensional reduction for SM(EFT)

$\tilde{g}, \Lambda$

BSM

$$\mathcal{L}^{\mathrm{BSM}} = \ldots$$

$\Lambda \geq T, v$

SMEFT

$$\mathscr{L}_4^{\mathrm{SMEFT}} = \mathscr{L}_4^{\mathrm{SM}} + \frac{1}{\Lambda^2}\Big\{ c_\phi(\phi^\dagger\phi)^3 + c_{\phi\Box}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + c_{\phi D}(\phi^\dagger D^\mu\phi)^*(\phi^\dagger D_\mu\phi)$$
$$+ c^{(1)}_{\phi\psi_L}(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\overline{\psi_L}\gamma^\mu\psi_L) + c^{(3)}_{\phi\psi_L}(\phi^\dagger i\overleftrightarrow{D}^I_\mu\phi)(\overline{\psi_L}\gamma^\mu\sigma^I\psi_L) + c_{\phi\psi_R}(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\overline{\psi_R}\gamma^\mu\psi_R)$$
$$+ \Big[c_{\phi ud}(\widetilde{\phi} iD_\mu\phi)(\overline{u_R}\gamma^\mu d_R) + c_{\psi_R\phi}(\phi^\dagger\phi)\overline{\psi_L}\widetilde{\phi}\psi_R + \mathrm{h.c.}\Big]\Big\}$$

$T \gg m_h$

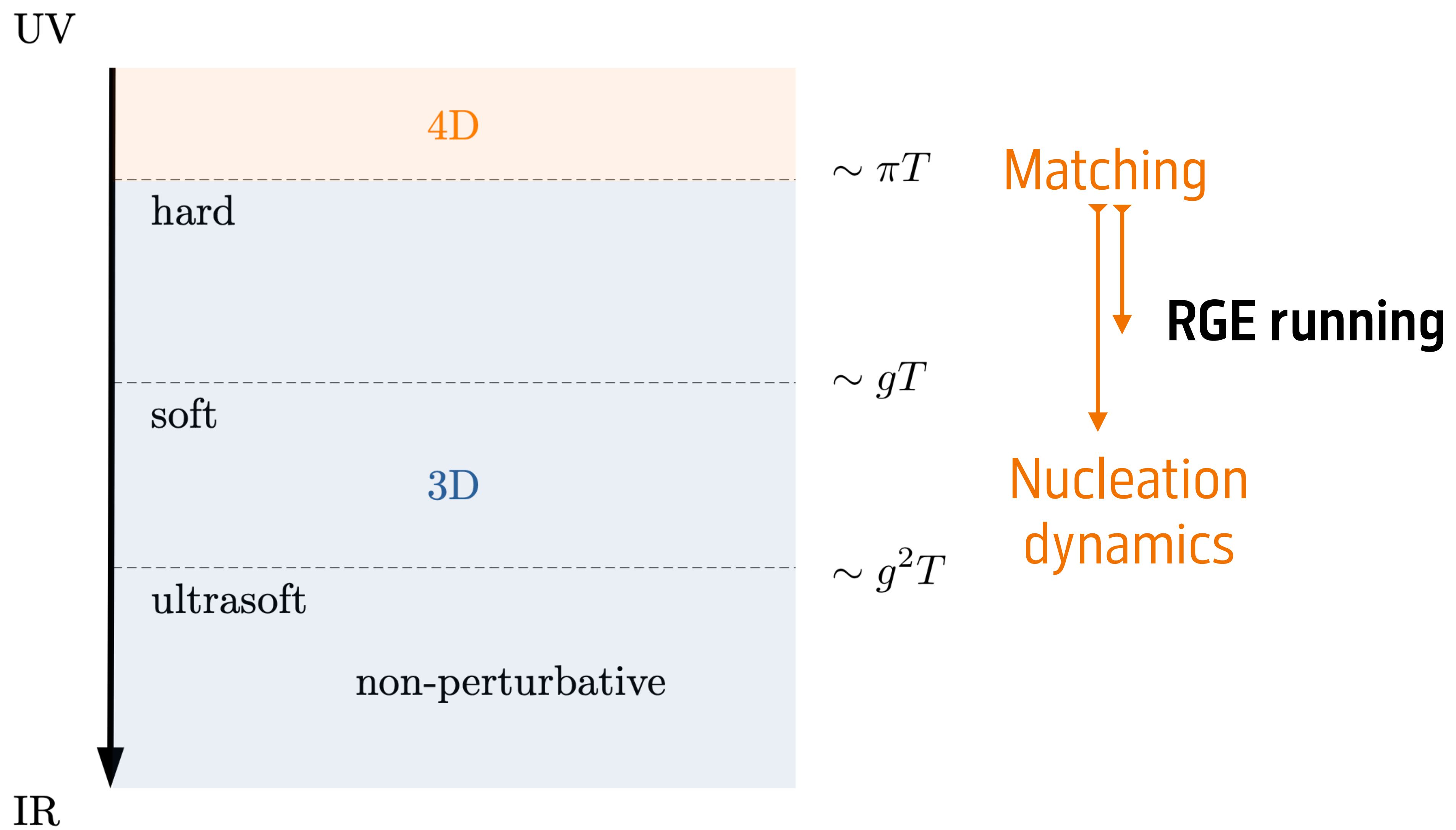
3D-EFT

$$\mathcal{L} = \frac{1}{4}W^I_{rs}W^I_{rs} + (D_r\phi)^\dagger(D_r\phi) + m_3^2|\phi|^2 + \lambda_3|\phi|^4 + c_{\phi^6}|\phi|^6$$
$$+ c^{(1)}_{\phi^4D^2}|\phi|^2\Box|\phi|^2 + c^{(2)}_{\phi^4D^2}(\phi^\dagger D_r\phi)^\dagger\phi^\dagger D_r\phi + r^{(3)}_{\phi^4D^2}|\phi|^2(D_r\phi)^\dagger D_r\phi$$
$$+ c_{\phi^8}|\phi|^8 + c_{\phi^2W^2}|\phi|^2W^I_{rs}W^I_{rs} + \ldots$$

Higher-dimensional operators

- Within an EFT, PTs can be studied: perturbatively or on the lattice

RGE evolution within an EFT



- Depending on the PT pattern (tree-level nucleation, radiative barrier, ...), nucleation occurs at different scale with different active d.o.f.s
- RGE evolution enables higher-precision perturbative calculations and is essential for lattice simulations: to relate lattice to continuum parameters

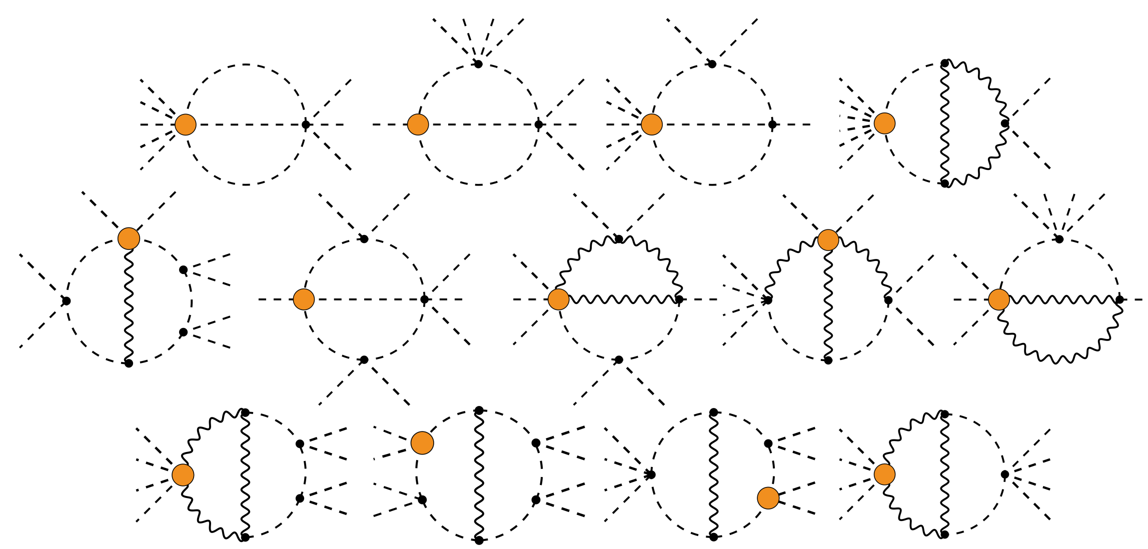
RGE evolution within an EFT

- Within 3D theory running starts at 2-loop level
- LO (super-renormalizable) running is exact

$$\mathcal{L}_{3D} = \overbrace{\frac{1}{4}W^I_{rs}W^I_{rs} + (D_r\Phi)^\dagger(D_r\Phi) + m_3^2|\Phi|^2 + \lambda_3|\Phi|^4}^{super-renorm.} + c_{\phi^6}|\Phi|^6$$
$$+ c_{\phi^8}|\Phi|^8 + c_{\phi^2W^2}|\Phi|^2W^I_{rs}W^I_{rs} + c^{(1)}_{\phi^4D^2}|\Phi|^2\Box|\Phi|^2 + c^{(2)}_{\phi^4D^2}(\Phi^\dagger D_r\Phi)^\dagger\Phi^\dagger D_r\Phi$$

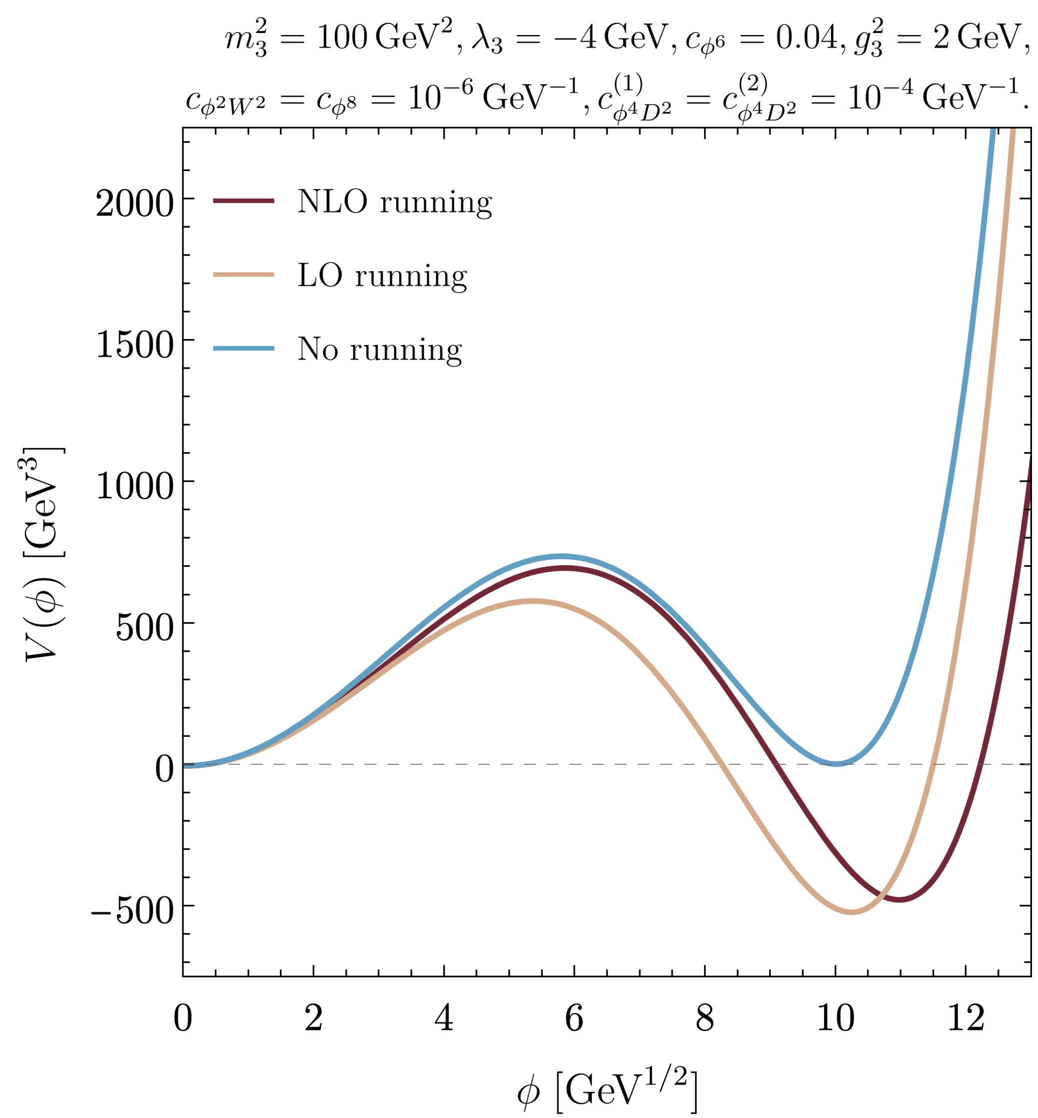
- NLO running can be parametrically of the same size as the LO running

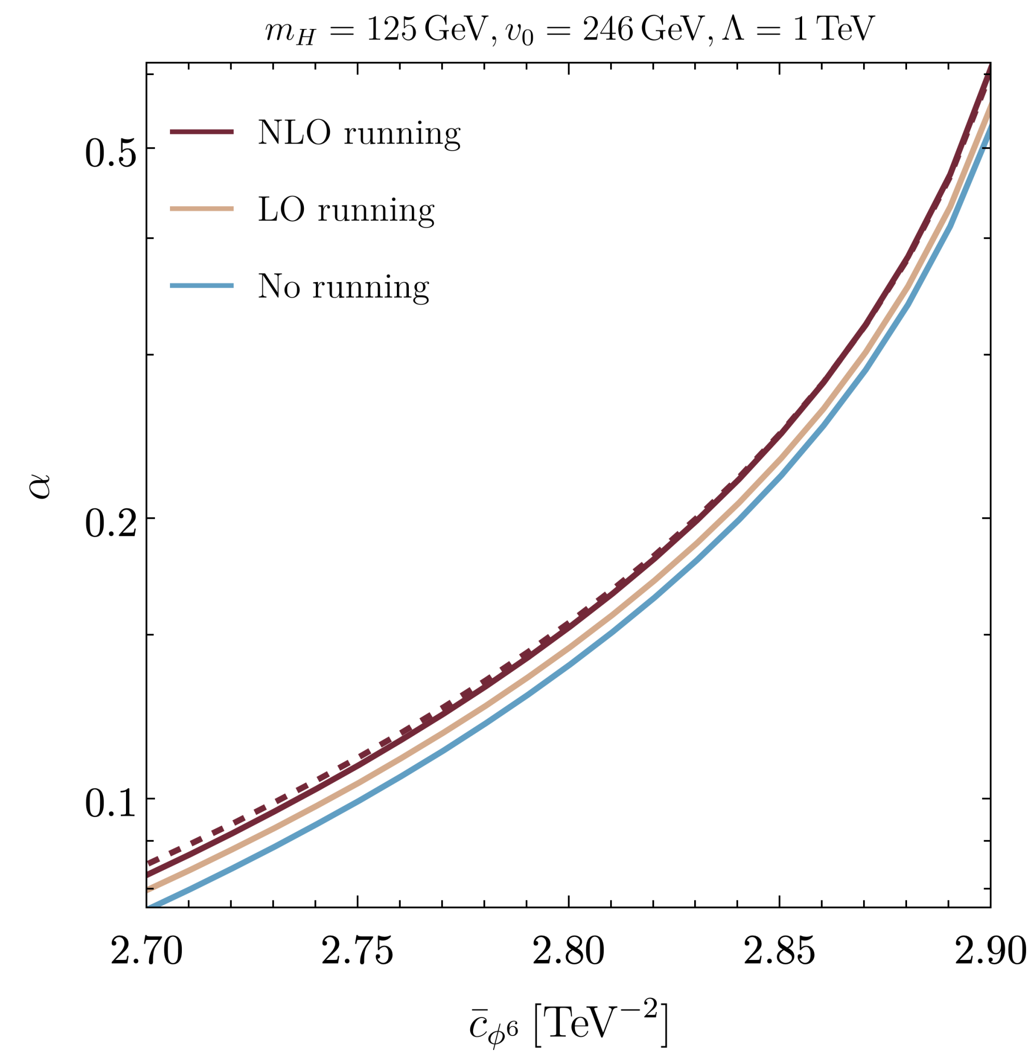
$$\frac{\partial\log m_3^2}{\partial\log\mu_{3D}} \sim \overbrace{g^2}^{\text{LO}} + g^3\frac{T}{\Lambda}\,,$$
$$\frac{\partial\log\lambda_3}{\partial\log\mu_{3D}} \sim \frac{\partial\log c_{\phi^6}}{\partial\log\mu_{3D}} \sim \frac{\partial\log c_{\phi^8}}{\partial\log\mu_{3D}} \sim g^2$$
$$\frac{\partial\log g_3^2}{\partial\log\mu_{3D}} \sim g^5\frac{T}{\Lambda}\,.$$



Tree-level barrier case

RGE evolution: SMEFT





LO running — dim<3 operators running
NLO running — dim≤4 operators running

Phase transition strength α

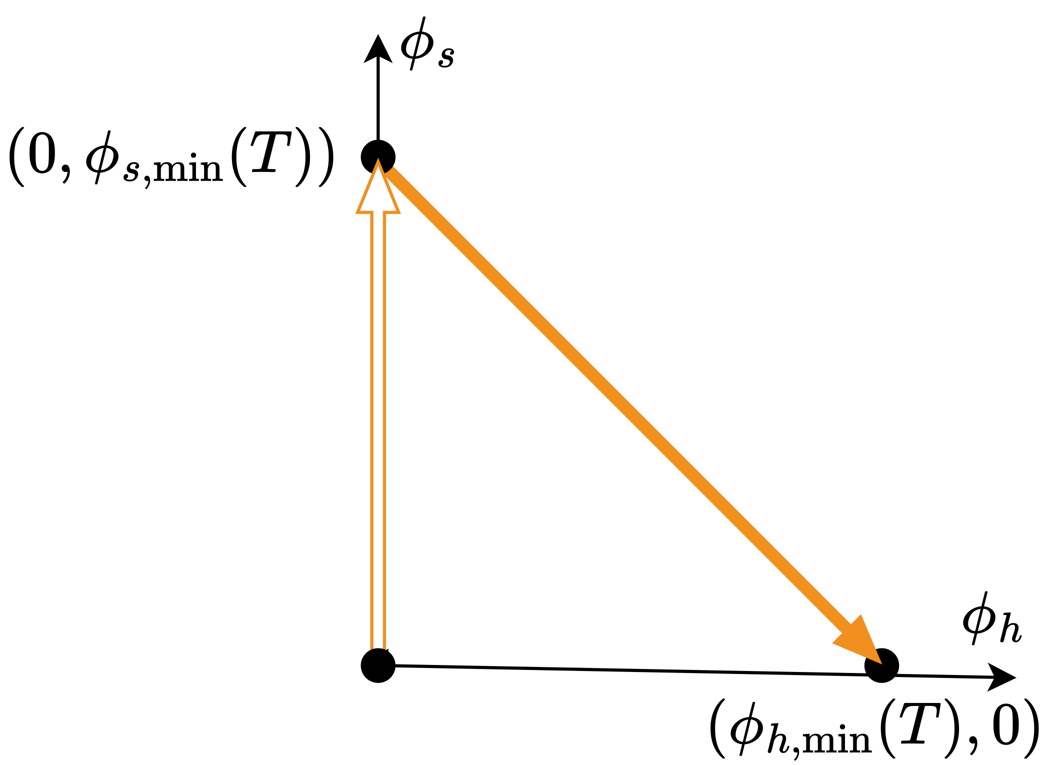
SM + singlet

$$V(\Phi, S) = \mu_\Phi^2(\Phi^\dagger\Phi) + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \lambda_{\Phi S}(\Phi^\dagger\Phi)S^2$$

Dimensional reduction

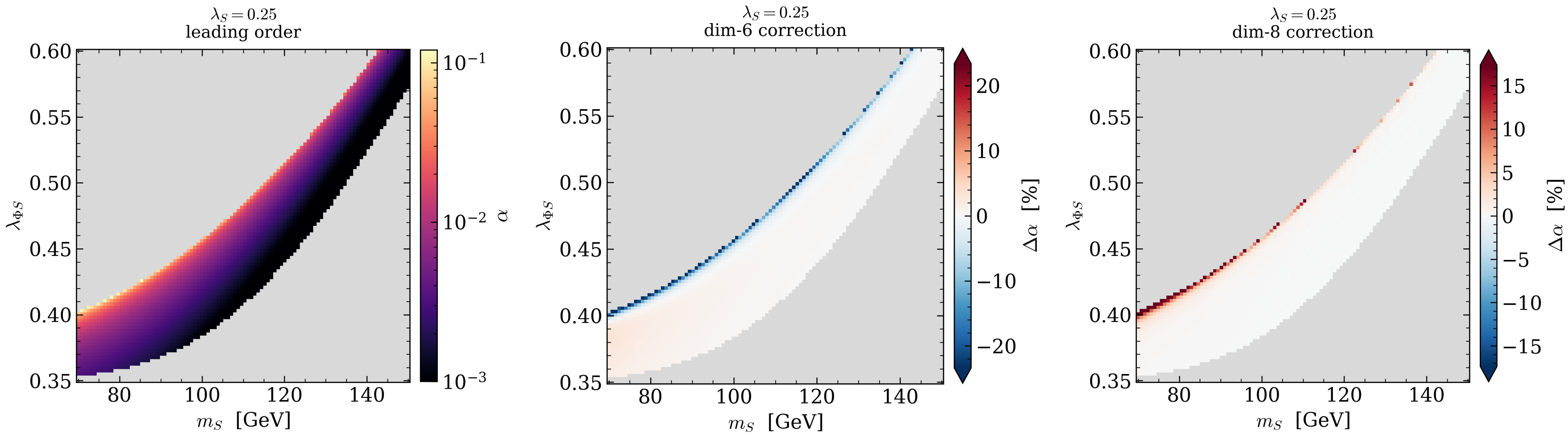
$$\begin{aligned}\mathcal{L}_3^{(6),Green} &= \Phi^6 + \Phi^4 S^2 + \Phi^2 S^4 + S^6 \\ &+ \mathbf{4} \times \Phi^4 D^2 + \mathbf{4} \times \Phi^2 S^2 D^2 + S^4 D^2 \\ &+ \Phi^2 D^4 + S^2 D^4\end{aligned}$$

$$\begin{aligned}\mathcal{L}_3^{(8),Green} &= \Phi^8 + \Phi^6 S^2 + \Phi^4 S^4 + \Phi^2 S^6 + S^8 \\ &+ \mathbf{4} \times \Phi^6 D^2 + \mathbf{7} \times \Phi^4 S^2 D^2 + \mathbf{4} \times \Phi^2 S^4 D^2 + S^6 D^2 \\ &+ \mathbf{13} \times \Phi^4 D^4 + \mathbf{13} \times \Phi^2 S^2 D^4 + \mathbf{3} \times S^4 D^4 \\ &+ \Phi^2 D^6 + S^2 D^6\end{aligned}$$



- Matching generated a tower of higher-dimensional operators, which must be accounted for to assess the validity of the EFT approach

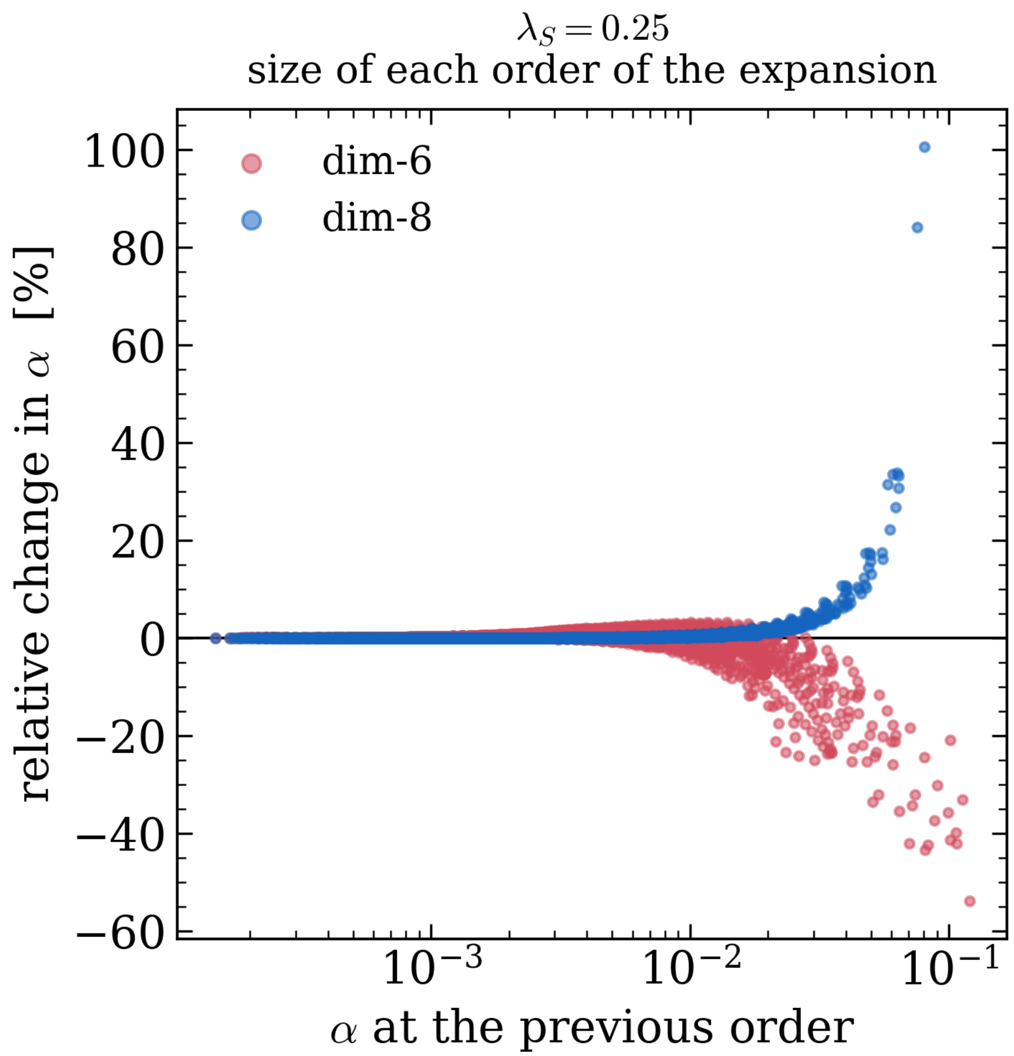
SM + singlet

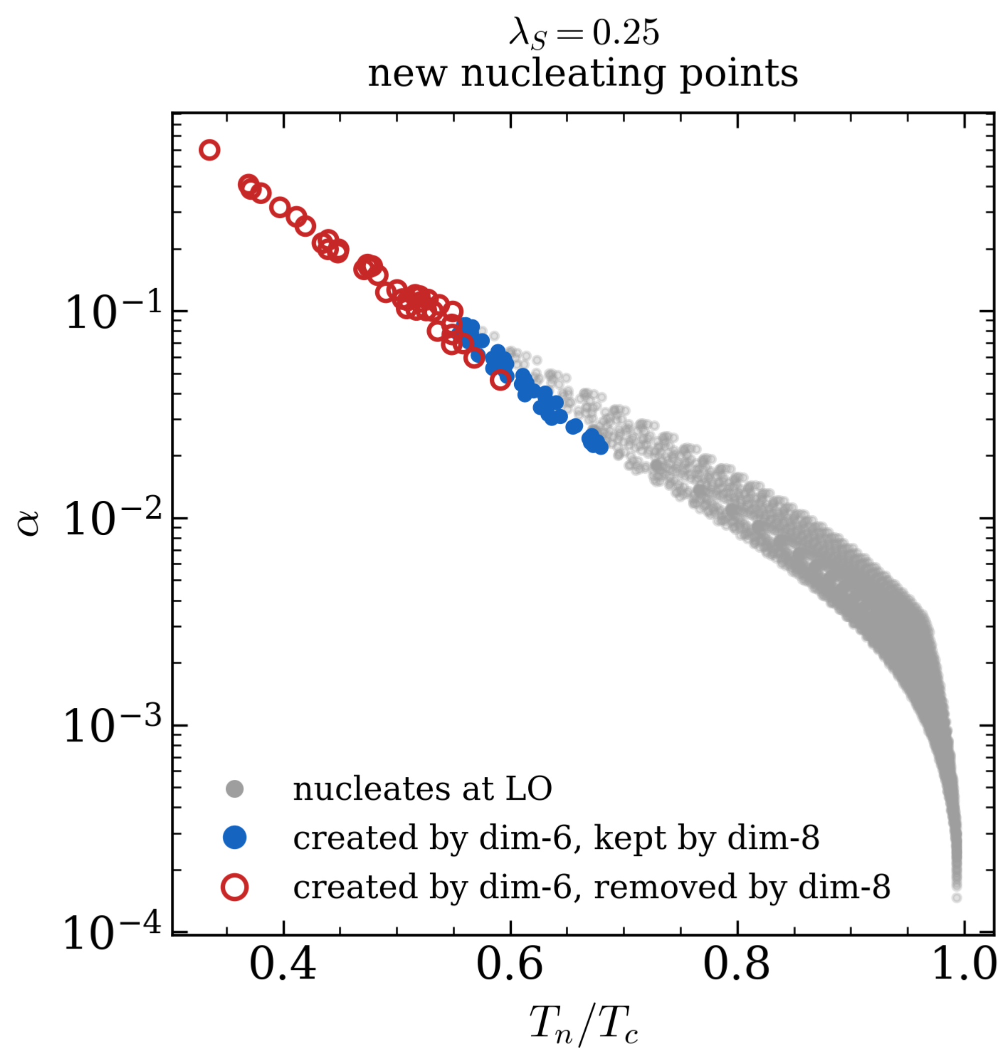


- Higher-dimensional operators are especially relevant for strong PTs: may result in a big correction (up to tens of %).

  Dim-6 operators increase nucleation temperature/decrease strength;
  Dim-8 — decrease nucleation temperature/increase strength.

SM + singlet





- Higher-dimensional operators open new parameter points for strong nucleation.
- Only "trustworthy" points are shown: $\frac{\left|S_3^{(1)}\right|}{S_3^{(0)}} < 0.5, \ \frac{\left|S_3^{(2)}\right|}{\left|S_3^{(1)}\right|} < 0.5$

Take-home messages

- Thermally driven phase transitions introduce hierarchy between scales at play, which requires modification of the usual perturbation theory — EFT techniques may be exploited.

- Higher-dimensional operators become important for strong PTs and are needed to assess the validity of the EFT approach.

- RGE running within the EFT is essential for precision perturbative calculations and lattice simulations. Effects of beyond-super-renormalizable running might parametrically be of the leading order.

SMEFT PT patterns

$$V(\phi) = m_3^2\phi^2 + \lambda_3\phi^4 + c_{\phi^6}\phi^6 - \frac{(g_3^2\phi^2)^{3/2}}{16\pi}$$

***Radiative symmetry breaking**

Tree-level barrier

$m_\phi \sim g^{3/2}T \ll m_{\mathrm{D}}$

Radiative barrier

$m_\phi \sim gT \sim m_{\mathrm{D}}$

dynamical d.o.f. ······ $\{\phi, W_i\}$ $\{\phi, W_i, W_0\}$

$$\mathcal{L} = \frac{1}{4}W^I_{rs}W^I_{rs} + (D_r\phi)^\dagger(D_r\phi) + m_3^2|\phi|^2 + \lambda_3|\phi|^4 + c_{\phi^6}|\phi|^6$$
$$+c^{(1)}_{\phi^4D^2}|\phi|^2\Box|\phi|^2 + c^{(2)}_{\phi^4D^2}(\phi^\dagger D_r\phi)^\dagger\phi^\dagger D_r\phi + r^{(3)}_{\phi^4D^2}|\phi|^2(D_r\phi)^\dagger D_r\phi$$
$$+c_{\phi^8}|\phi|^8 + c_{\phi^2W^2}|\phi|^2W^I_{rs}W^I_{rs} + \ldots$$
$$+\frac{1}{2}(D_rW^I_0)(D_rW^I_0) + \frac{1}{2}m_D^2W^I_0W^I_0 + \lambda_{W_0^4}(W^I_0W^I_0)^2 + \lambda_{\phi^2W_0^2}|\phi|^2W^I_0W^I_0 + \ldots$$

Perturbative bounce expansion

$$\ddot{\varphi}_i^B + \frac{2}{r}\dot{\varphi}_i^B = \left.\frac{\partial V_3}{\partial \varphi_i}\right|_{\varphi_i^B}$$

$$\dot{\varphi}_i^B(0) = 0$$

$$\varphi_i^B(\infty) = \varphi_{\mathrm{FV},i}$$

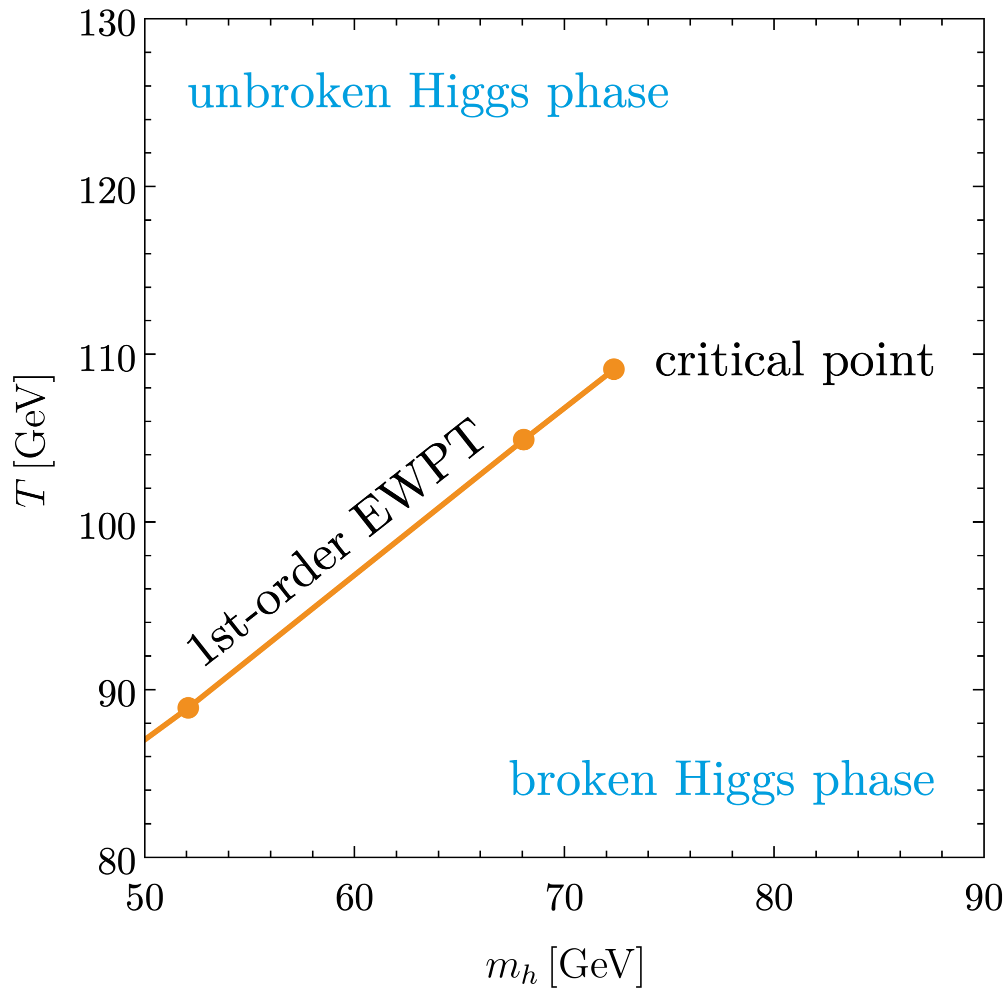
$$\varphi_i^B = \varphi_i^{B,(0)} + \epsilon\varphi_i^{B,(1)} + \epsilon^2\varphi_i^{B,(2)} + \cdots$$

$$S_3 = S_3^{(0)} + \epsilon S_3^{(1)} + \epsilon^2 S_3^{(2)} + \cdots$$

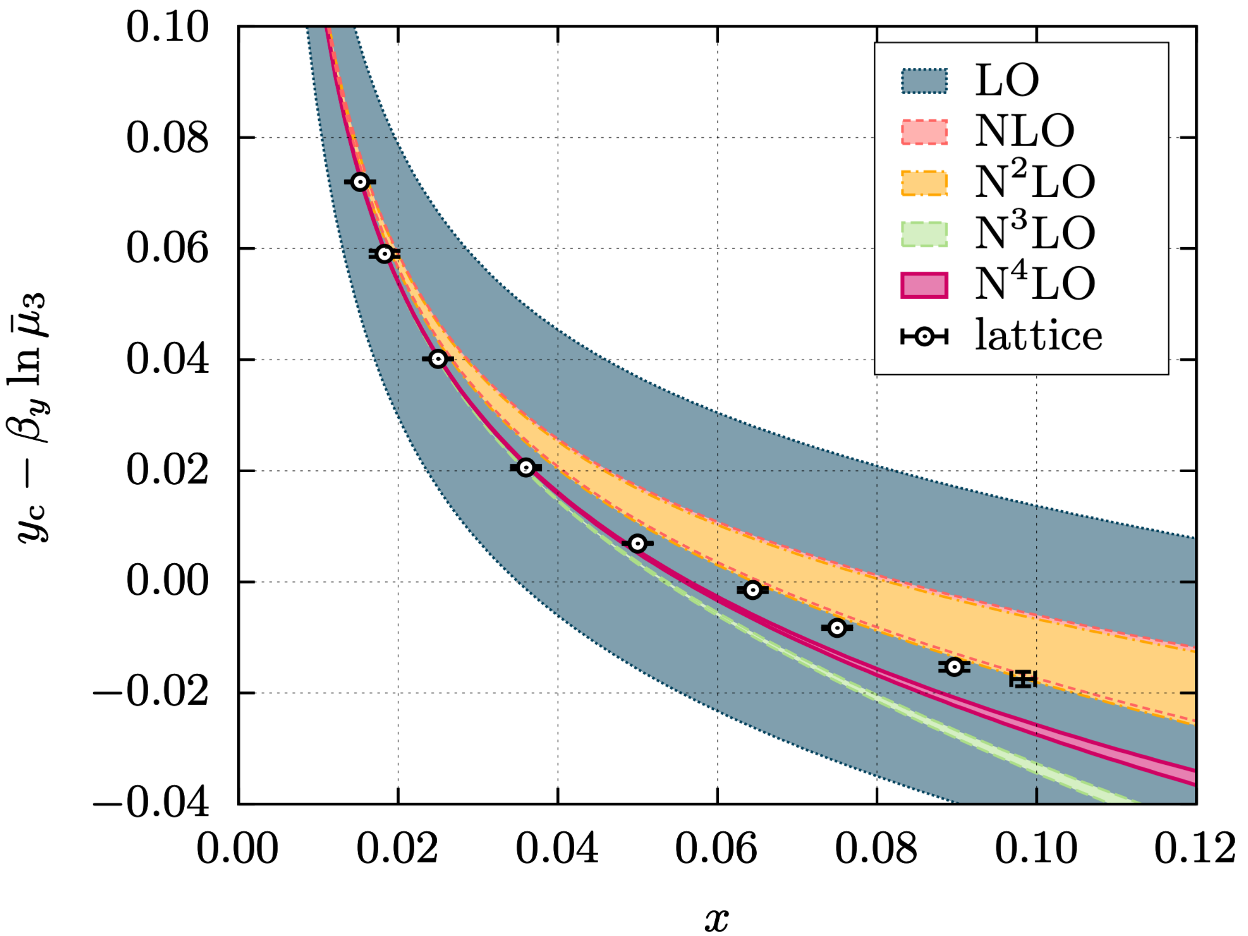
$$S_3[\varphi_i^B] = S_3^{(0)}[\varphi_i^{B,(0)}] + \epsilon S_3^{(1)}[\varphi_i^{B,(0)}]$$

$$+\epsilon^2\left\{S_3^{(2)}[\varphi_i^{B,(0)}] + 2\pi\int_0^\infty dr r^2 \varphi_i^{B,(1)} \left.\frac{\delta S_3^{(1)}}{\delta\varphi_i}\right|_{\varphi_i^{B,(0)}}\right\}$$

Lattice simulations of EWPT



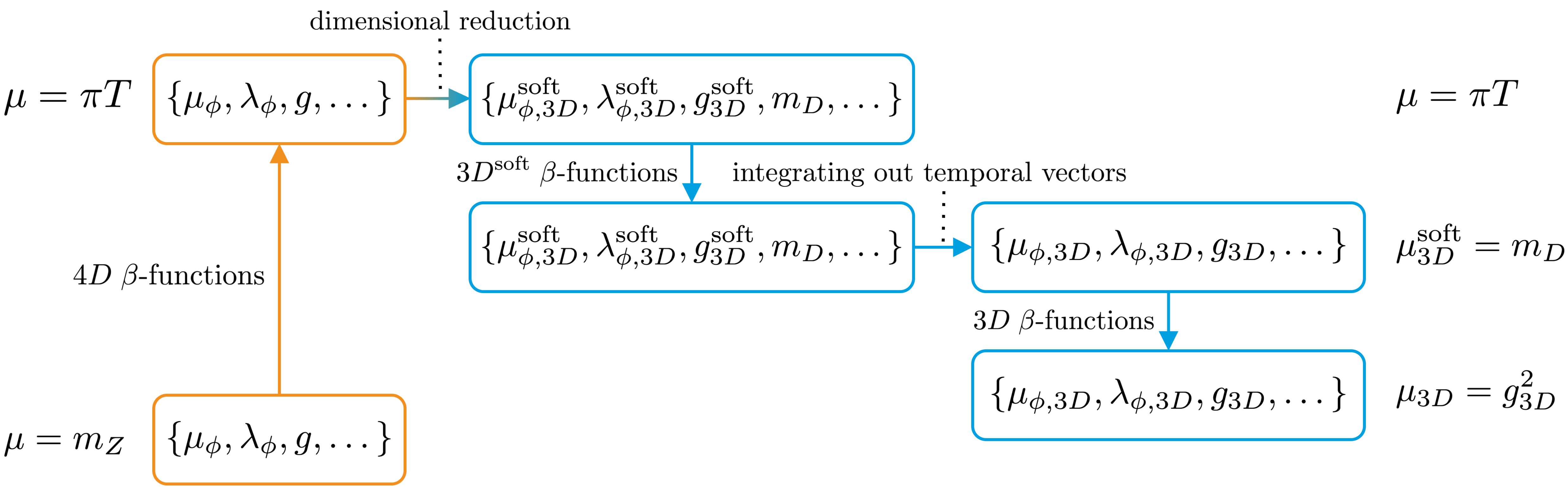
[Laine et al., 0010275]



[Ekstedt et al., 2405.18349]

- Effects of higher-dimensional operators within the high-T EFT?
- Precision for the SM and open possibilities for the SMEFT

xSM: dimensional reduction



[Biekötter, AD et al., 2511.14831]