



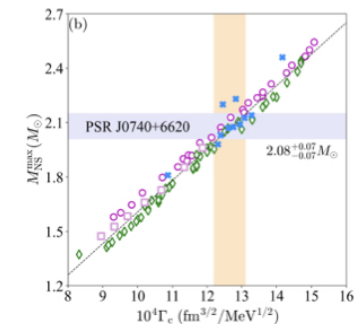
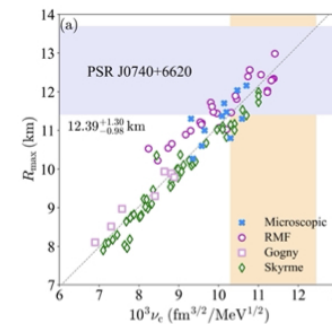
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Observable Scalings and Equation of State of Cold Dense Matter in Neutron Star Cores

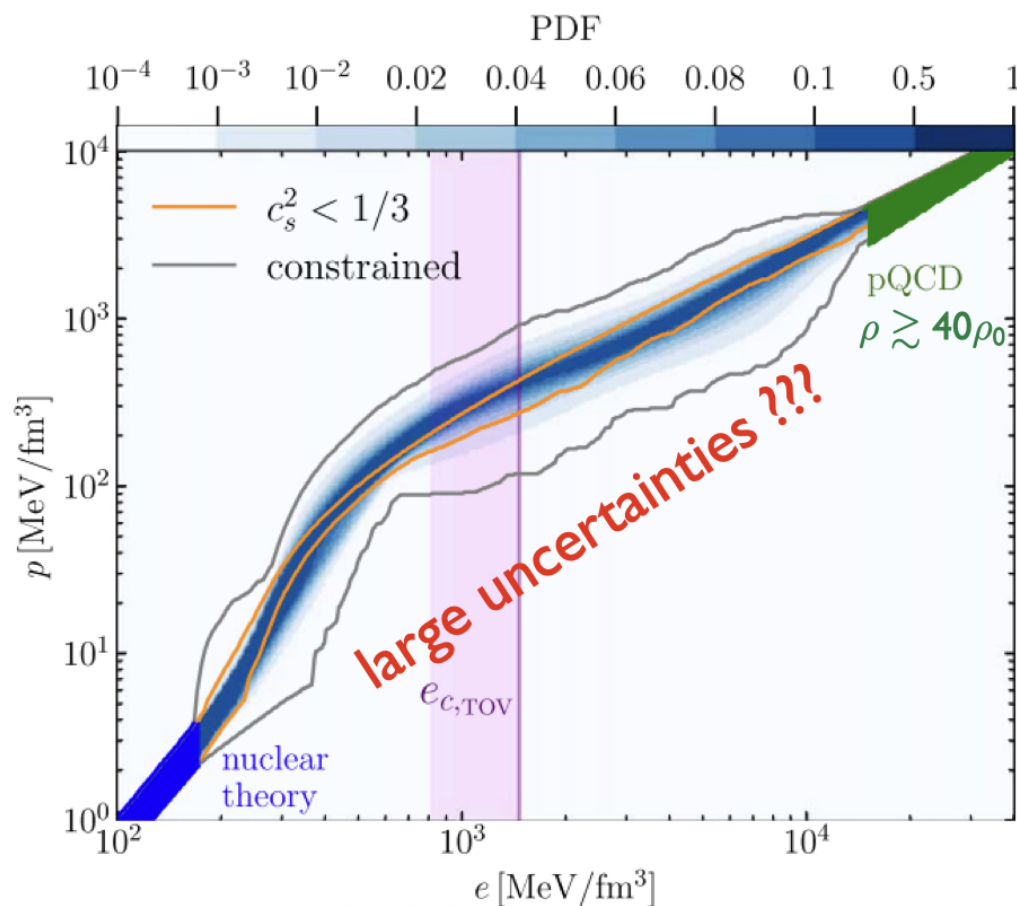
Bao-Jun Cai
Fudan University, 8/21/2026

Collaborators: Bao-An Li, Yu-Gang Ma, Jian-Hao Shi, Zhen Zhang

arXiv:2306.08202; arXiv:2307.15223
arXiv:2406.05025; arXiv:2501.18676
arXiv:2601.02980; arXiv:2606.21402



Cold dense matter equation of state (EOS)



Altiparmak et al., ApJL, 2022

- (1) low-density nuclear many-body theories
- (2) pQCD at extremely high densities
- (3) observational data (GW170817, NICER, ...)
- (4) heavy-ion collision data
- (5) inference algorithms (Bayesian, ML, NN, ...)
- (6) others

EOS-parameter $\phi = P/\varepsilon$
(w in cosmology)

dimensionless trace anomaly $\Delta = 1/3 - \phi$

$T = 0 : \text{EOS} \rightarrow P = P(\varepsilon)$

Tolman-Oppenheimer-Volkoff (TOV) equations



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$$\frac{dP}{dr} = \underbrace{-\frac{GM\epsilon}{r^2}}_{\text{Newtonian}} \underbrace{\left(1 + \frac{P}{\epsilon c^2}\right)}_{\text{matter correction}} \underbrace{\left(1 + \frac{4\pi r^3 P}{Mc^2}\right)}_{\text{matter-geometry coupling}} \underbrace{\left(1 - \frac{2GM}{rc^2}\right)^{-1}}_{\text{geometry correction}}; \quad \frac{dM}{dr} = 4\pi r^2 \epsilon / c^2$$

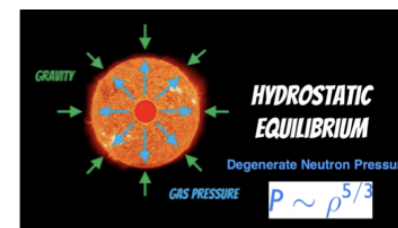
total matter+geometry corrections: $\gg 1$

same as Newtonian

$$\nabla_\mu T^\mu_\nu = 0$$

$$8\pi G T_{\mu\nu} = R_{\mu\nu} - 2^{-1} g_{\mu\nu} R$$

spherical metric



Tolman, PR, 1939; Oppenheimer and Volkoff, PR, 1939

gravity in GR is stronger than Newtonian's

Newtonian

$$R \sim \sqrt{\frac{P}{\epsilon^2}} \cdot \underbrace{\vartheta(\phi = P/\epsilon)}_{\text{dimensionless}}$$

$\phi = P/\epsilon$: dimensionless

strong gravity in GR: < 1

conventionally, one adopts a model for dense matter, solves the general-relativistic structural equations, and infers the properties of the matter

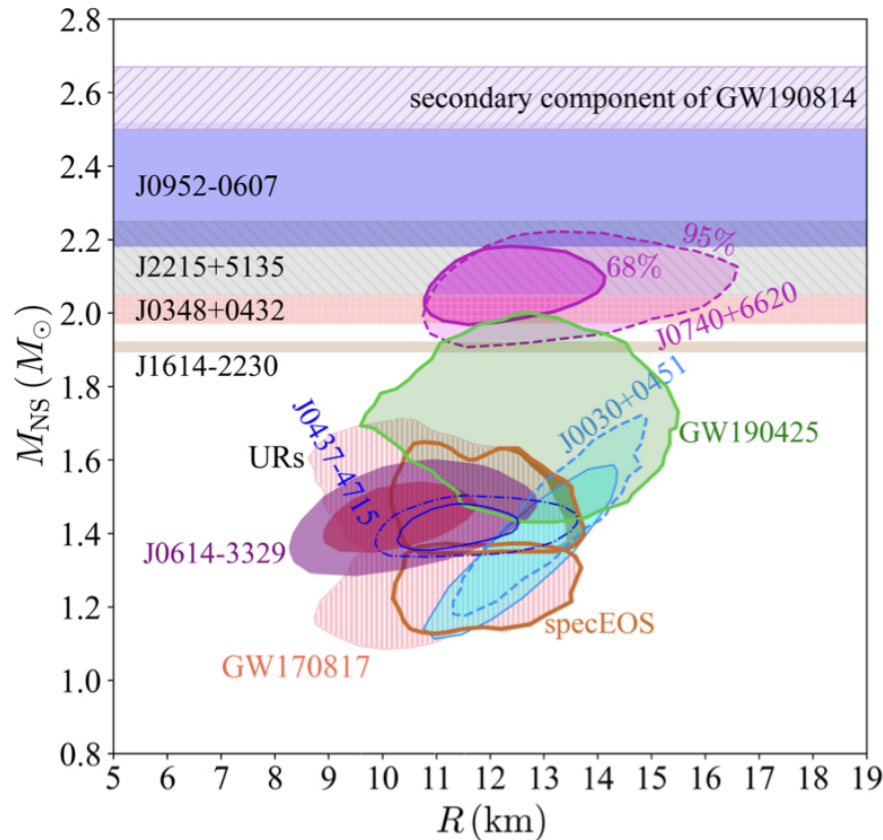
$$\vartheta \rightarrow M_{\text{TOV}}$$

Inverse problem is very difficult

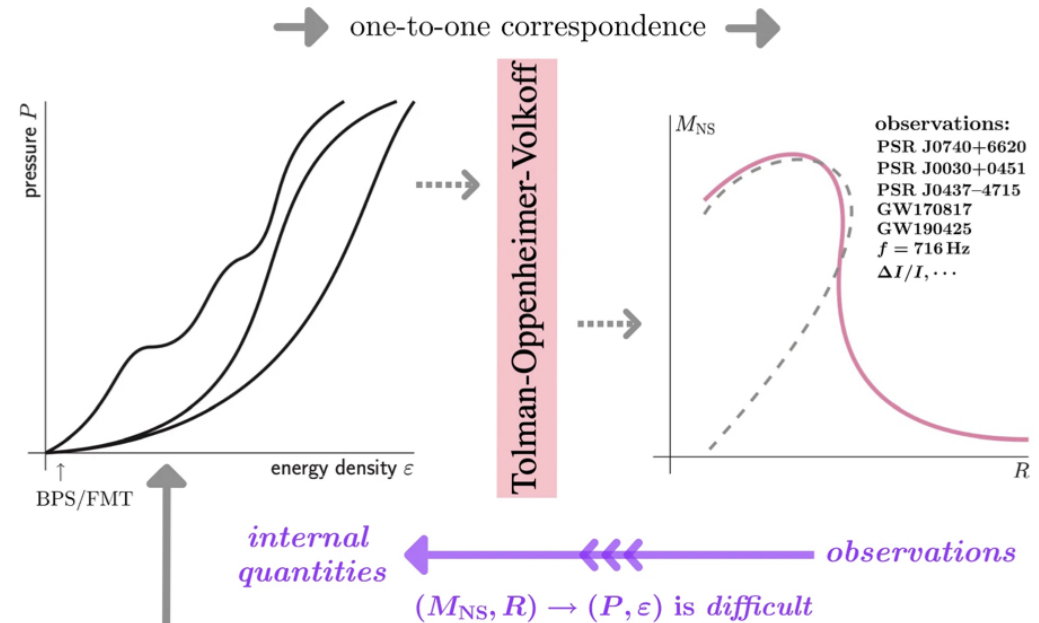
BLINDNESS/DEGENERACY OF TOV EQUATIONS



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limited observational data on masses and radii of several NSs

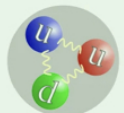


model construction for $P(\epsilon)$:

- neutrons/protons
- quarks, gluons
- heavy-ion collision data
- exotics: $\Lambda, \Sigma^{0,\pm}, \Delta^{0,\pm,++}, \dots$
- dark matter (DM), $\dots$
- phase transitions (PTs), $\dots$
- continuous crossover, $\dots$

Example: limitation of the bottom-up approach (EOS of an URFG)

$$\left. \begin{array}{l} P = \epsilon/3 \rightarrow \gamma = 1 \\ \text{TOV Eqs.} \rightarrow \gamma \gtrsim 4/3 \end{array} \right\} \leftrightarrow \text{inconsistent}$$



Frontiers of multi-messenger astrophysics



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Extreme gravity and fundamental physics

by Bangalore Sathyaprakash, Alessandra Buonanno, Luis Lehner, Chris Van Den Broeck, P. Ajith, Archisman Ghosh, Katerina Chatziioannou, Paolo Pani, Michael Puerrer, Thomas Sotiriou, Salvatore Vitale, Nicolas Yunes, K.G. Arun, Enrico Barausse, Masha Baryakhtar, Richard Brito, Andrea Maselli, Tim Dietrich, William East, Ian Harry, Tanja Hinderer, Geraint Pratten, Lijing Shao, Nicola Tamanini, Maaretn van de Meent, Vijay Varma, Justin Vines, Huan Yang, and Miguel Zumalacarregui

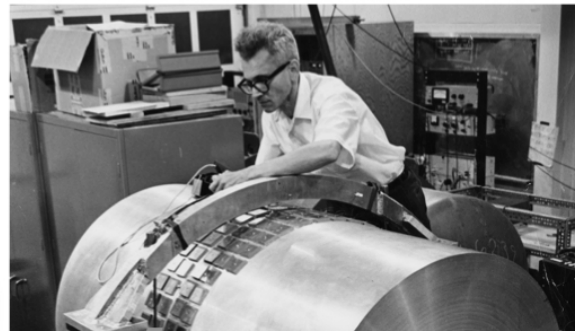
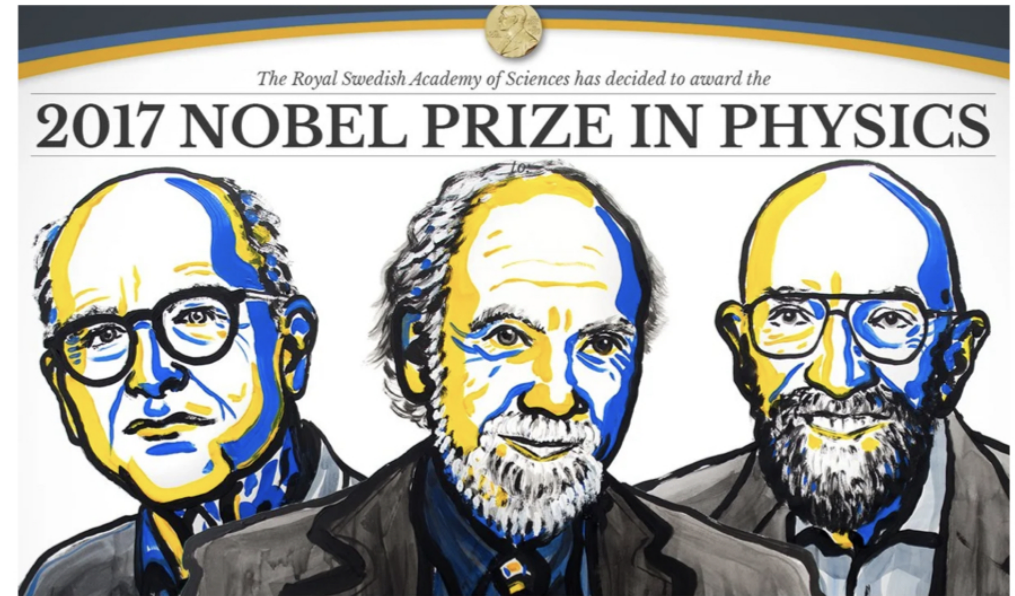
Future gravitational-wave observations will enable unprecedented and unique science in *extreme gravity and fundamental physics*, that form the core topics of the Thematic Area 7 of Astro-2020 decadal survey.

- **The nature of gravity.** Can we prove Einstein wrong? What building-block principles and symmetries in nature invoked in the description of gravity can be challenged?
- **The nature of dark matter.** Is dark matter composed of particles, dark objects or modifications of gravitational interactions?
- **The nature of compact objects.** Are black holes and neutron stars the only astrophysical extreme compact objects in the Universe? **What is the equation of state of densest matter?**

Sathyaprakash et al., 2019

- What is the nature of gravity?
- What is the nature of dark matter?
- What is nature of compact objects

What is the EOS of densest matter?



Zel'dovich 1961: EOS at the highest densities



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SOVIET PHYSICS JETP

VOLUME 14, NUMBER 5

MAY, 1962

THE EQUATION OF STATE AT ULTRAHIGH DENSITIES AND ITS RELATIVISTIC LIMITATIONS

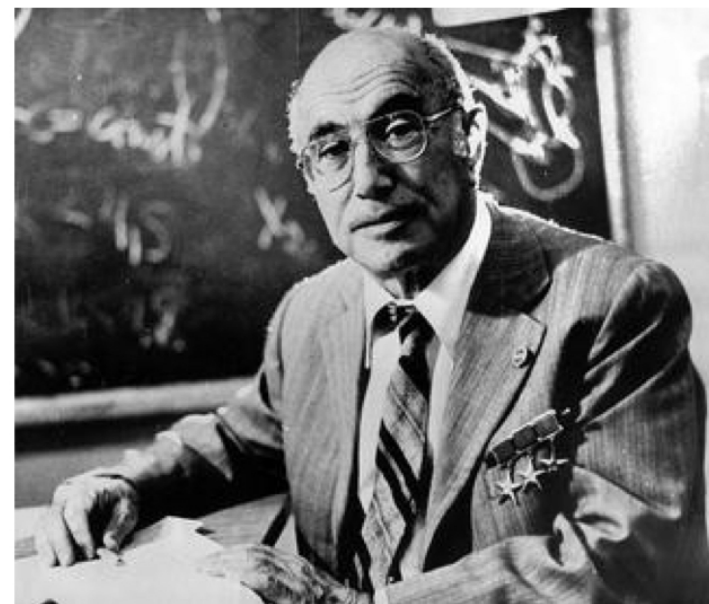
Ya. B. ZEL'DOVICH

Submitted to JETP editor May 31, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 1609-1615 (November, 1961)

vector field

The most rigid equation of state compatible with the requirements of relativity theory is $p = \epsilon \sim n^2$, $D \rightarrow c$, where p is the pressure, ϵ the volume density of energy, n the density of baryons, D the speed of sound, and c the speed of light. This differs from the previously proposed asymptotic behavior $3p = \epsilon \sim n^{4/3}$, $D \rightarrow 3^{-1/2}c$. The case of interaction of the baryons through a vector field is considered and it is shown (both by considering the interaction of pairs of baryons and by using the stress tensor of the field) how in this case the equation $p = \epsilon \sim n^2$ is realized and how the transition to the equation $3p = \epsilon$ occurs as the mass of the field quanta goes to zero.



Zel'dovich (1961)

example: ω -meson in Walecka model

(at ultra-high densities)

$$P/\epsilon \rightarrow 1; s^2 \rightarrow 1$$

$$\mathcal{L}_\omega = \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - g_\omega\omega_\mu\bar{\psi}\gamma^\mu\psi \rightarrow \epsilon \sim P \sim \frac{1}{2}m_\omega^2\omega^2 \sim \rho^2$$

Bjorken 1983: ultra-relativistic Fermi gas EOS



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Highly relativistic nucleus-nucleus collisions: The central rapidity region (approximate conformal symmetry of quark matter)

J. D. Bjorken

Fermi National Accelerator Laboratory, * P.O. Box 500, Batavia, Illinois 60510

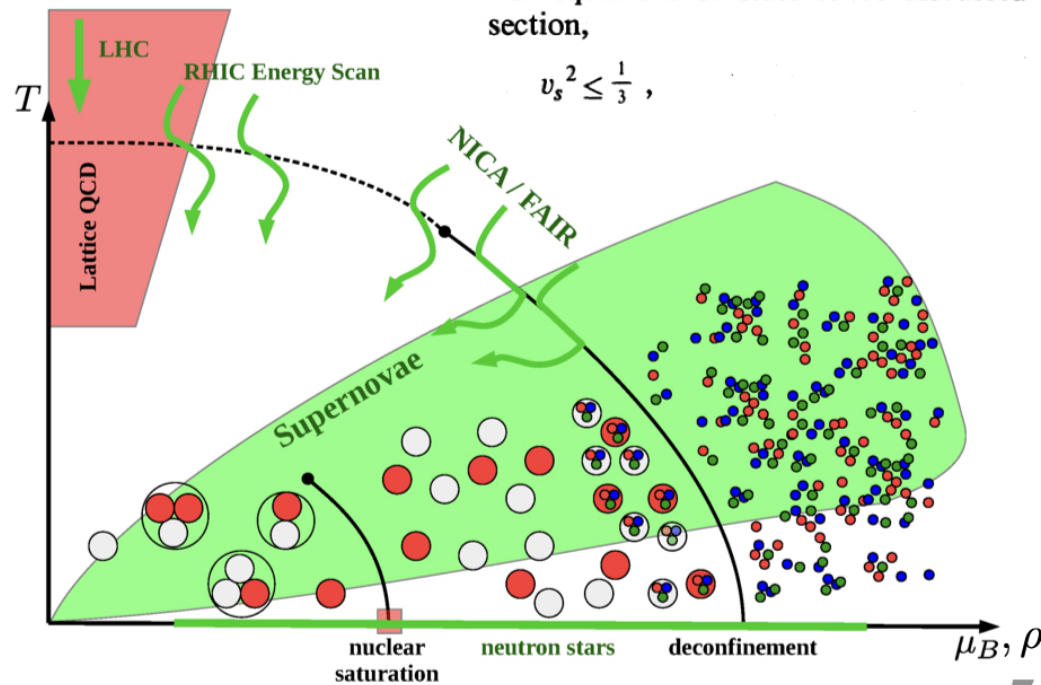
(Received 13 August 1982)

ideal ultra-relativistic Fermi gas:

very large densities : $\frac{P}{\epsilon} = \frac{1}{3} \rightarrow s^2 = \frac{dP}{d\epsilon} = \frac{1}{3}$

For equations of state to be discussed in the next section,

$$v_s^2 \leq \frac{1}{3},$$



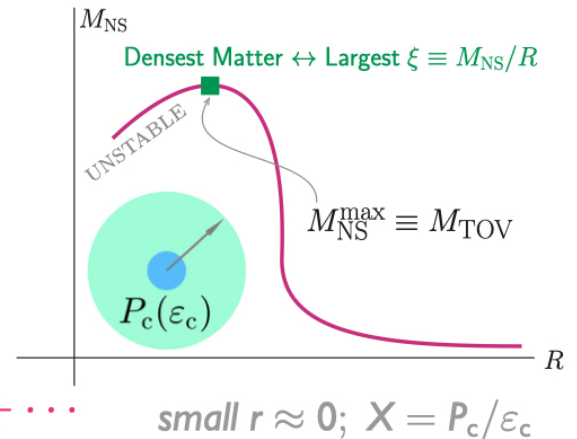
The method: perturbation around NS center



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$$\text{length scale: } Q = \frac{1}{\sqrt{4\pi\epsilon_c}} \approx 10 \cdot \left(\frac{\epsilon_c \text{ in MeV/fm}^3}{600} \right)^{-1/2} \text{ km} \sim \text{typical NS radius}$$

$$\frac{d\hat{P}}{d\hat{r}} = -\frac{\hat{M}\hat{\epsilon}}{\hat{r}^2} \left(1 + \frac{\hat{P}}{\hat{\epsilon}} \right) \left(1 + \frac{\hat{r}^3\hat{P}}{\hat{M}} \right) \left(1 - \frac{2\hat{M}}{\hat{r}} \right)^{-1}; \quad \frac{d\hat{M}}{d\hat{r}} = \hat{r}^2\hat{\epsilon}; \quad (c = G = 1)$$



$$\hat{P} = X + b_2\hat{r}^2 + b_4\hat{r}^4 + \dots; \hat{\epsilon} = 1 + a_2\hat{r}^2 + a_4\hat{r}^4 + \dots; \hat{M} = c_3\hat{r}^3 + c_5\hat{r}^5 + \dots$$

$$\rightarrow R \sim \nu_c \equiv \frac{X^{1/2}}{\sqrt{\epsilon_c}} \underbrace{\left(\frac{1}{1 + 3X^2 + 4X} \right)^{1/2}}_{\equiv \vartheta(X) < 1}; \quad \underbrace{M_{\text{NS}} \sim R^3 \epsilon_c}_{\text{macroscopic observations}} \sim \underbrace{\Gamma_c \equiv \frac{1}{\sqrt{\epsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{3/2}}_{\text{microscopic combination (EOS)}}$$

$P(R) = 0$

(strong gravity reduces the radius)

$$s_c^2 \leq 1 \leftrightarrow \phi = P/\epsilon \leq X \lesssim 0.374$$

$$\text{densest matter} \leftrightarrow \text{peak position} \leftrightarrow \frac{dM_{\text{NS}}}{dR} = 0 \rightarrow s_c^2 = X \left(1 + \frac{1}{3} \frac{1 + 3X^2 + 4X}{1 - 3X^2} \right) \leftarrow \text{nonlinear}$$

$$s_c^2 = \frac{dP_c}{d\epsilon_c}$$

8

* $P = \epsilon/3$ is inconsistent with NSs

IPAD-TOV: Intrinsic and Perturbative Analyses of Dimensionless TOV Equations

$$\frac{d}{d\hat{r}} \hat{P} = -\frac{\hat{\varepsilon} \hat{M}}{\hat{r}^2} \frac{(1 + \hat{P}/\hat{\varepsilon})(1 + \hat{r}^3 \hat{P}/\hat{M})}{1 - 2\hat{M}/\hat{r}}; \quad \frac{d}{d\hat{r}} \hat{M} = \hat{\varepsilon} \hat{r}^2 \quad E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}}(\rho) \delta^2$$

How to perturb?

top-down
(strong-field gravity in GR)
(nuclear model independent)

$$\frac{M_{\text{NS}}}{R} \leftrightarrow \frac{P}{\varepsilon} \leftrightarrow \frac{dP}{d\varepsilon} \leftrightarrow \frac{d \ln P}{d \ln \varepsilon}$$

$$\hat{P} = P/\varepsilon_c \approx X + b_2 \hat{r}^2 + b_4 \hat{r}^4 + \dots$$

$$\hat{\varepsilon} = \varepsilon/\varepsilon_c \approx 1 + a_2 \hat{r}^2 + a_4 \hat{r}^4 + \dots$$

$$\hat{M} \approx \frac{1}{3} \hat{r}^3 + \frac{1}{5} a_2 \hat{r}^5 + \frac{1}{7} a_4 \hat{r}^7 + \dots$$

$$X \equiv P_c/\varepsilon_c, \quad \mu \equiv \hat{\varepsilon} - 1$$

$$\mathcal{U}/\mathcal{U}_c \approx 1 + \sum_{i+j \geq 1} u_{ij} X^i \mu^j$$

(double-element expansion)

★ ultimate goal: core/central EOS in NSs

numerical

input dense matter EOS

others,...

inference algorithms

astrophysical data

heavy ion collision data

transitions, exotic particles

nuclear structure quantities

nuclear model selected

CEFT (χ EFT): low-density

physical ingredients
mechanisms on EOS

bottom-up

(based on model construction)

- ◆ **core/central EOS**
- ◆ **trace anomaly Δ**
- ◆ **sound speed s^2**
- ◆ sharp PTs?
- ◆ conformality?
- ◆ maximum NS mass
- ◆ Newtonian limit
- ◆ NS binding energy
- ◆ moments of inertia
- ◆ **tidal deformability**
- ◆ slope of MR curve

[arXiv:2501.18676](https://arxiv.org/abs/2501.18676)

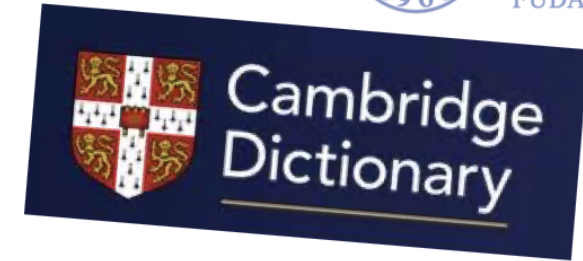
IPAD-TOV: analogy to Apple's IPAD



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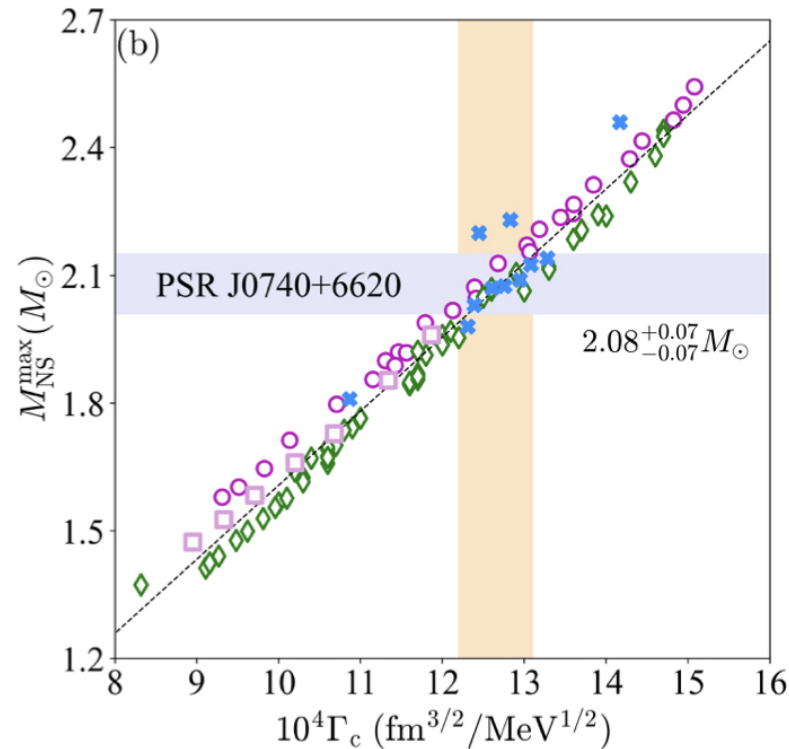
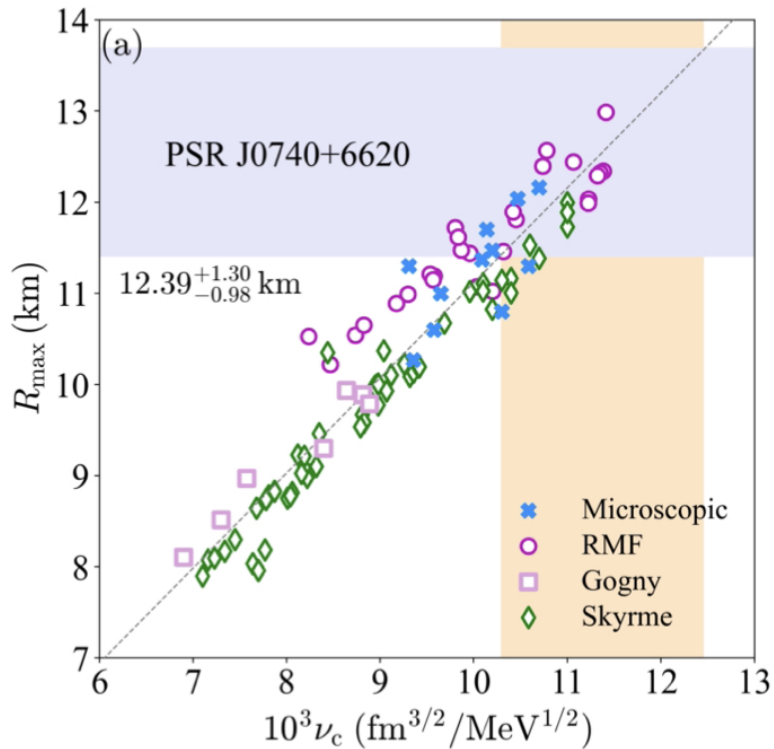
Steven Paul Jobs (2/24/1955-10/5/2011)



English meaning of Apple's iPad from Cambridge Dictionary: a brand name for a tablet (aka, small computer) that is controlled by touch rather than having a keyboard.

In our IPAD-TOV approach, certain properties of supra-dense NS matter governed by the TOV equations are probed perturbatively without using a specific input model EOS.

Mass and radius scalings from IPAD-TOV



EOS lists (#=104):

- * Skyrme, Gogny, RMF
- * APR (2N+3N)
- * DBHF (MPA1, ENG)
- * chiral mean fields (CMFs)
- * hyperons (DD2, H4)
- * hybrid (ALF2)
- * strange quark stars (SQM3)

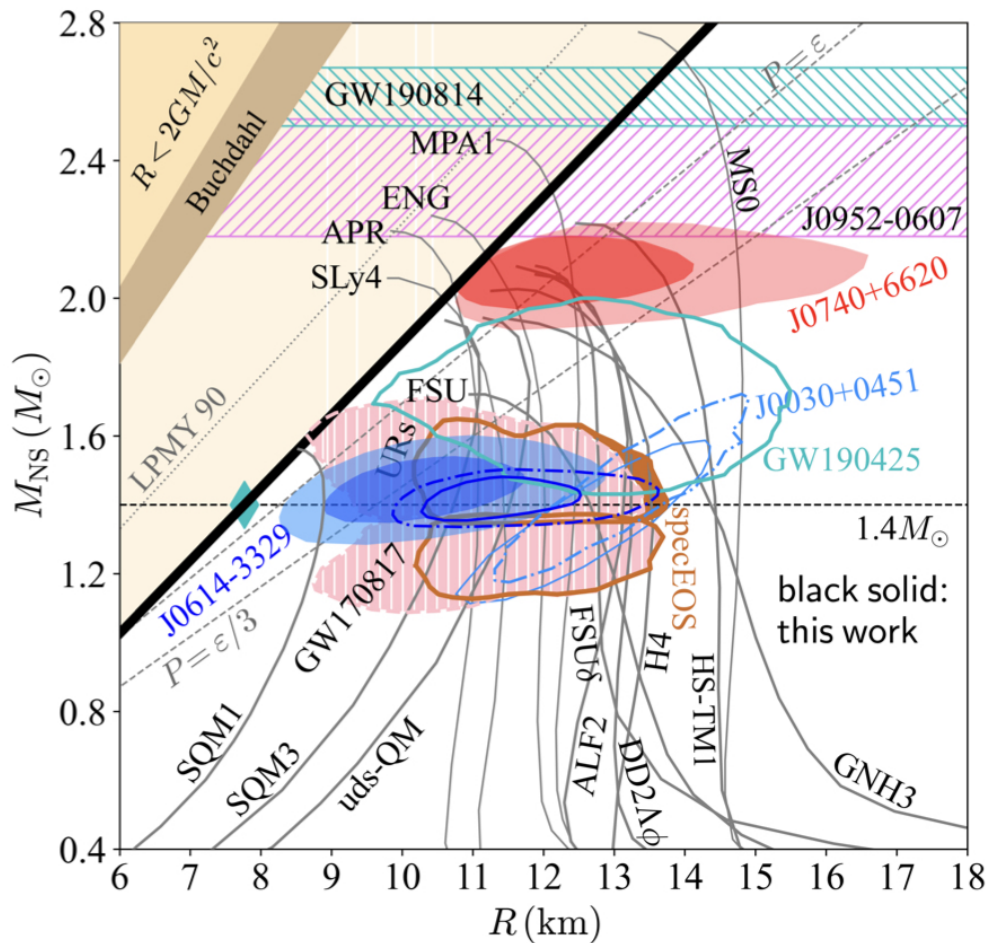
TOV with anisotropies

$$\nu_c = \frac{1}{\sqrt{\varepsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{1/2} \quad X = P_c / \varepsilon_c \rightarrow P_c(\varepsilon_c) \quad \Gamma_c = \frac{1}{\sqrt{\varepsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{3/2}$$

Implication: can a massive NS have $R \lesssim 10\text{km}$?



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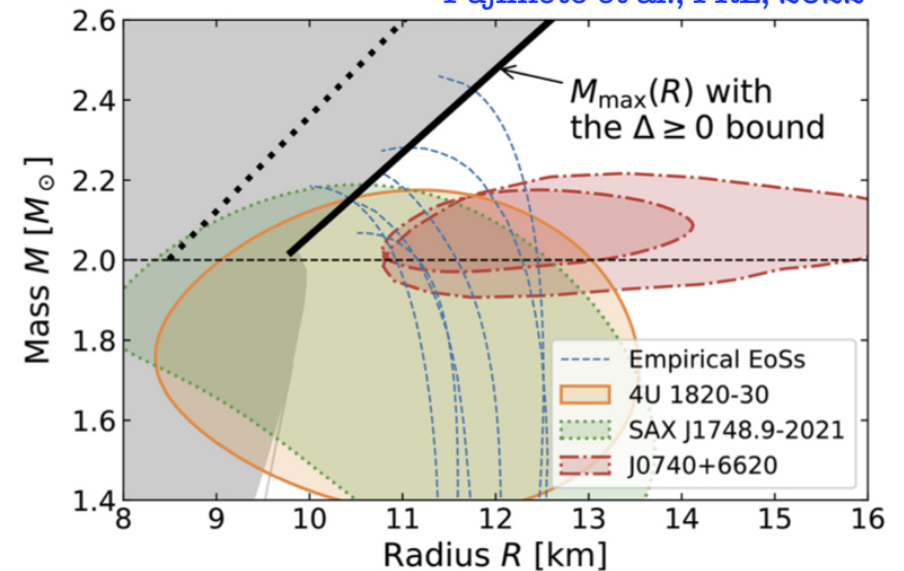


this boundary SHOULD be checked
by observational NS data/GW signals

$$\leftarrow R_{\text{max}}/\text{km} \gtrsim 4.73 M_{\text{NS}}^{\text{max}}/M_{\odot} + 1.14$$

$$M_{\text{NS}}^{\text{max}}/M_{\odot} \approx 2 \rightarrow R_{\text{max}} \gtrsim 10.6 \text{ km}$$

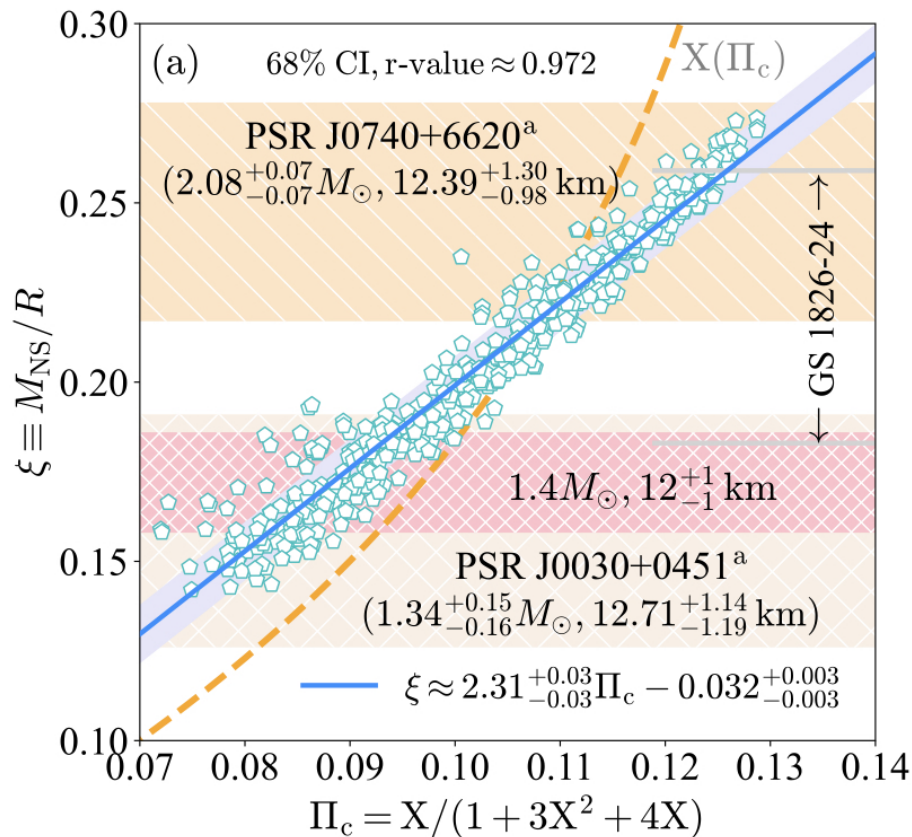
Fujimoto et al., PRL, 2022



Compactness probes the trace anomaly Δ



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$$M_{\text{NS}} \sim \left(\frac{6}{\pi G^3} \right)^{1/2} \cdot \frac{1}{\sqrt{\epsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{3/2}$$

$$R \sim \left(\frac{3}{2\pi G} \right)^{1/2} \cdot \frac{1}{\sqrt{\epsilon_c}} \left(\frac{X}{1 + 3X^2 + 4X} \right)^{1/2}$$

$$\Delta_c = \frac{1}{3} - X \quad s^2 = dP/d\epsilon = \overbrace{-\epsilon d\Delta/d\epsilon}^{\text{derivative}} + \underbrace{3^{-1} - \Delta}_{\text{non-derivative}}$$

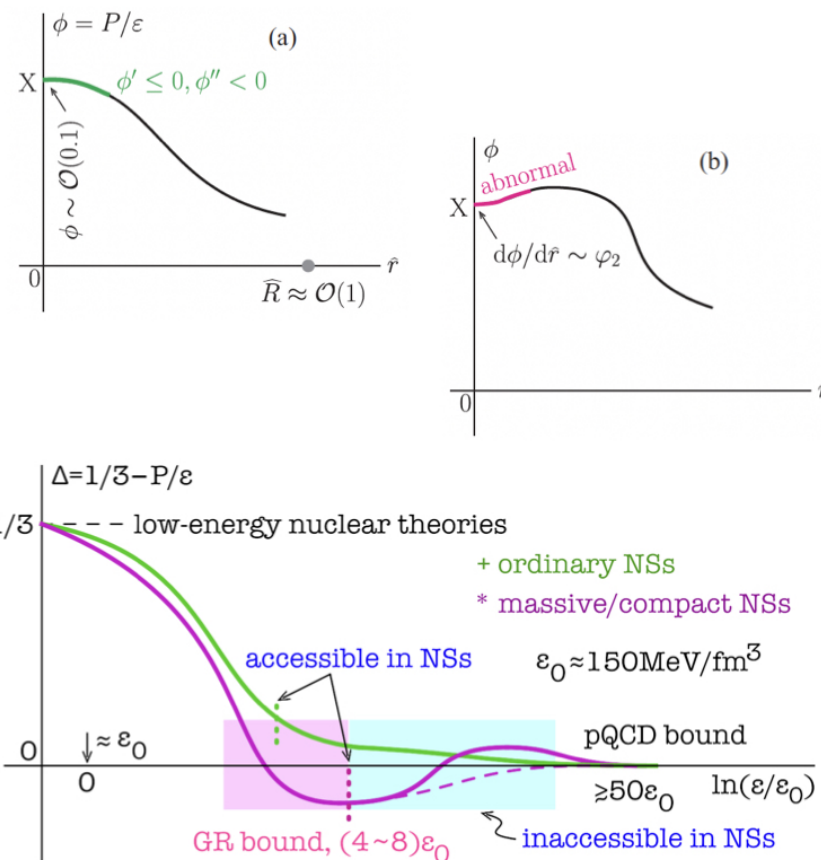
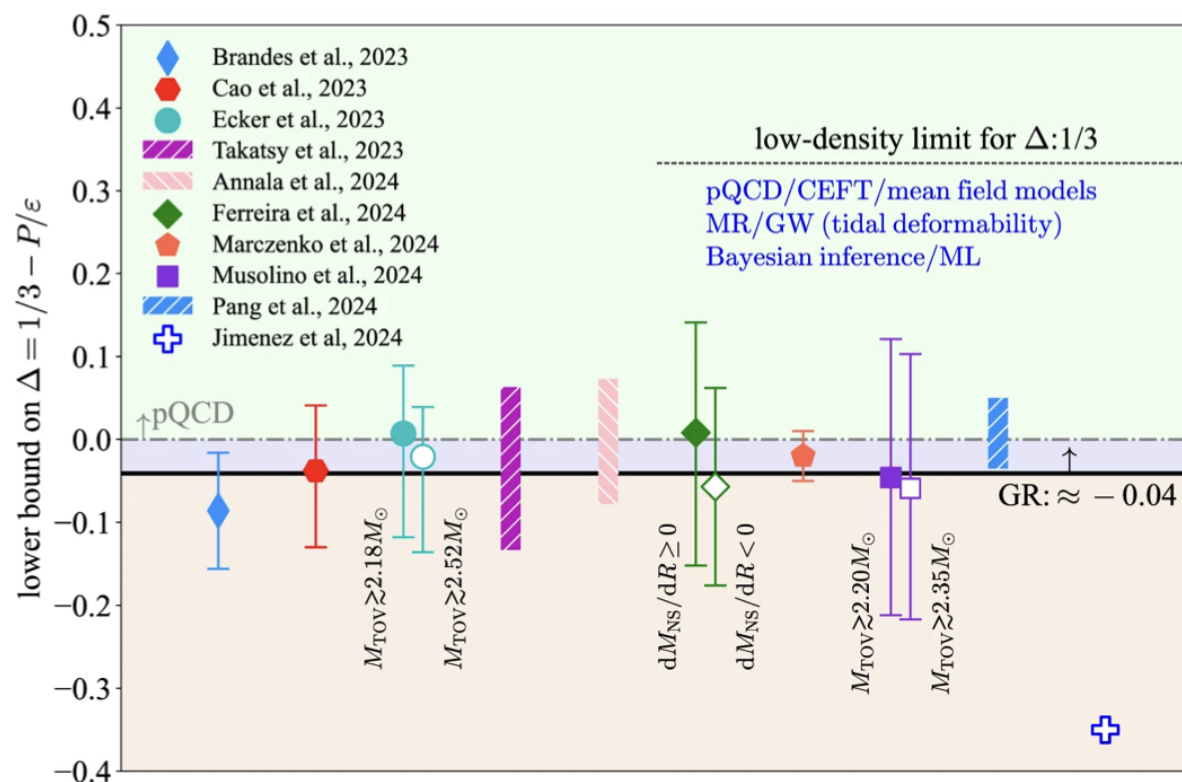
central EOS-parameter $X = P_c/\epsilon_c$

$$\xi = \frac{M_{\text{NS}}}{R} = \frac{2X}{1 + 3X^2 + 4X} \sim \frac{X}{1 + 3X^2 + 4X} \equiv \Pi_c$$

the dimensionless trace anomaly Δ can be directly extracted from the compactness

GR refines the lower bound for Δ

$$\phi = P/\varepsilon \lesssim 0.374 \leftrightarrow \Delta = 3^{-1} - P/\varepsilon \gtrsim -0.04$$



Scaling for dimensionless tidal deformability

scaling factor $D(X, \Psi) = 2k_2(X, \Psi)/3\xi^5$

$$k_2(X, \Psi) \approx \frac{256}{5} \frac{\Pi_c^5 (1 - 4\Pi_c)^2}{H_2/H_1 + 3(1 - 4\Pi_c)^2 \ln(1 - 4\Pi_c)}$$

$$\Psi = \frac{2d \ln M_{\text{NS}}}{d \ln \varepsilon_c} \geq 0$$

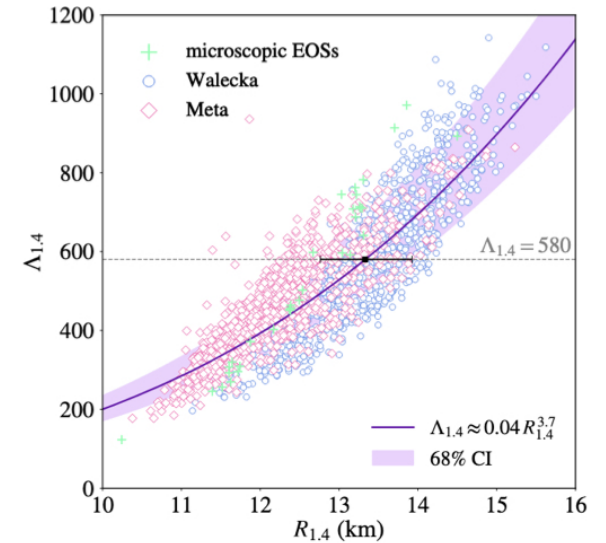
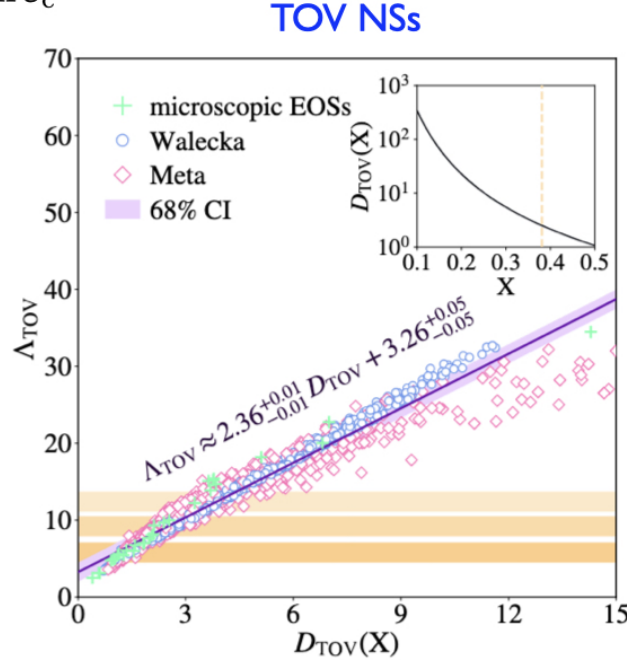
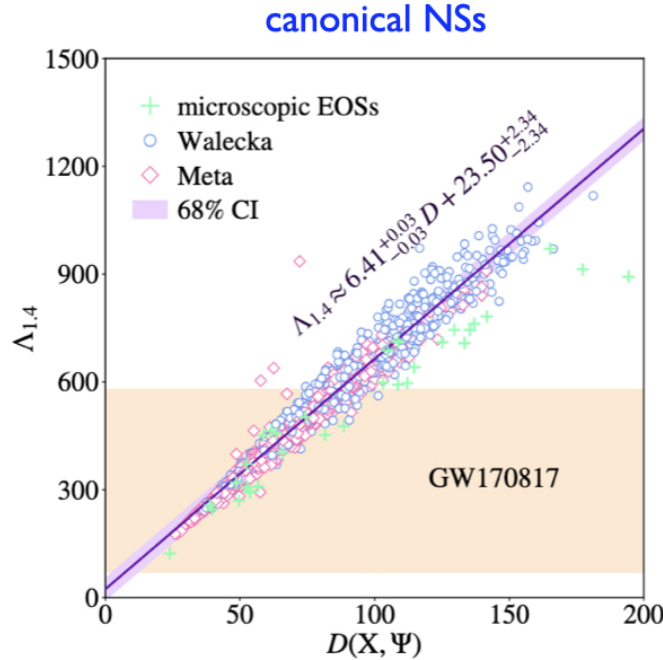
$$\xi \approx 2\Pi_c$$

$$H_1 = 2 - Y + 4\Pi_c(Y - 1)$$

$$H_2 = 12\Pi_c [2 - Y + 2\Pi_c(5Y - 8)] + 32\Pi_c^3 H$$

$$Y = 2 - \frac{2}{7} \frac{X}{1 + 3X} \left(\frac{1 + 36X + 99X^2}{1 + 4X + 3X^2} + \frac{3}{s_c^2} \right)$$

$$H = 13 - 11Y + 2\Pi_c(3Y - 2) + 8\Pi_c^2(1 + Y)$$



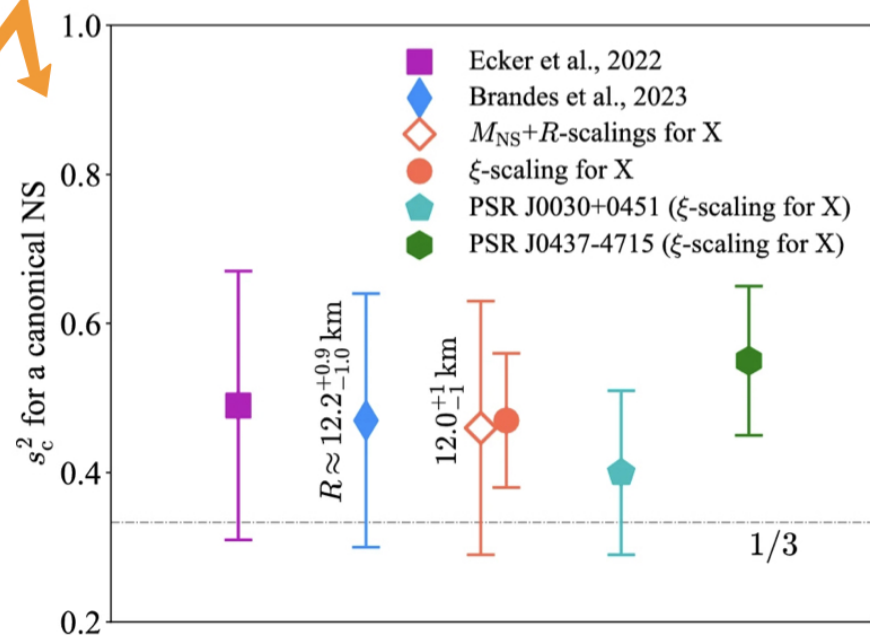
Previously, Λ is mainly used to constrain the $P(2\rho_0)$. 15

Example: central sound speed in canonical NSs



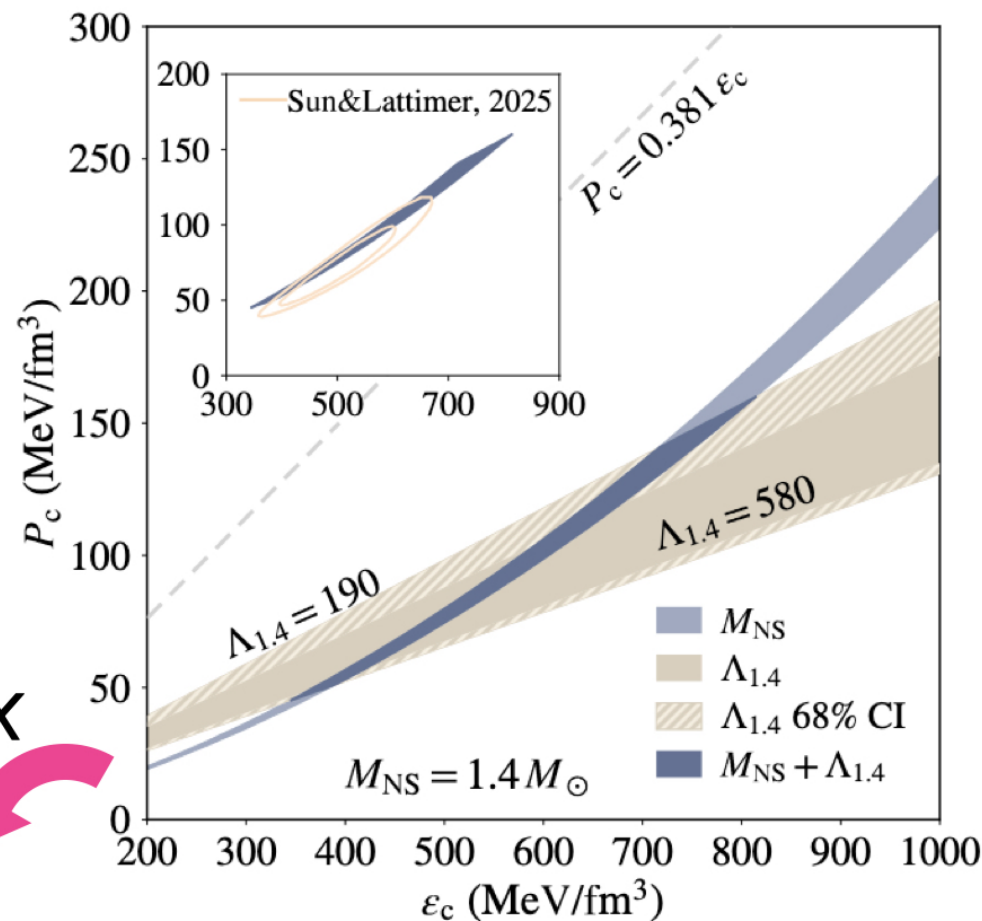
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mass/radius/compactness $\rightarrow s_c^2 \approx 0.47 \pm 0.09$



$190 \lesssim \Lambda_{1.4} \lesssim 580 \rightarrow 0.39^{+0.01}_{-0.01} \lesssim s_c^2 \lesssim 0.62^{+0.06}_{-0.04}$

X

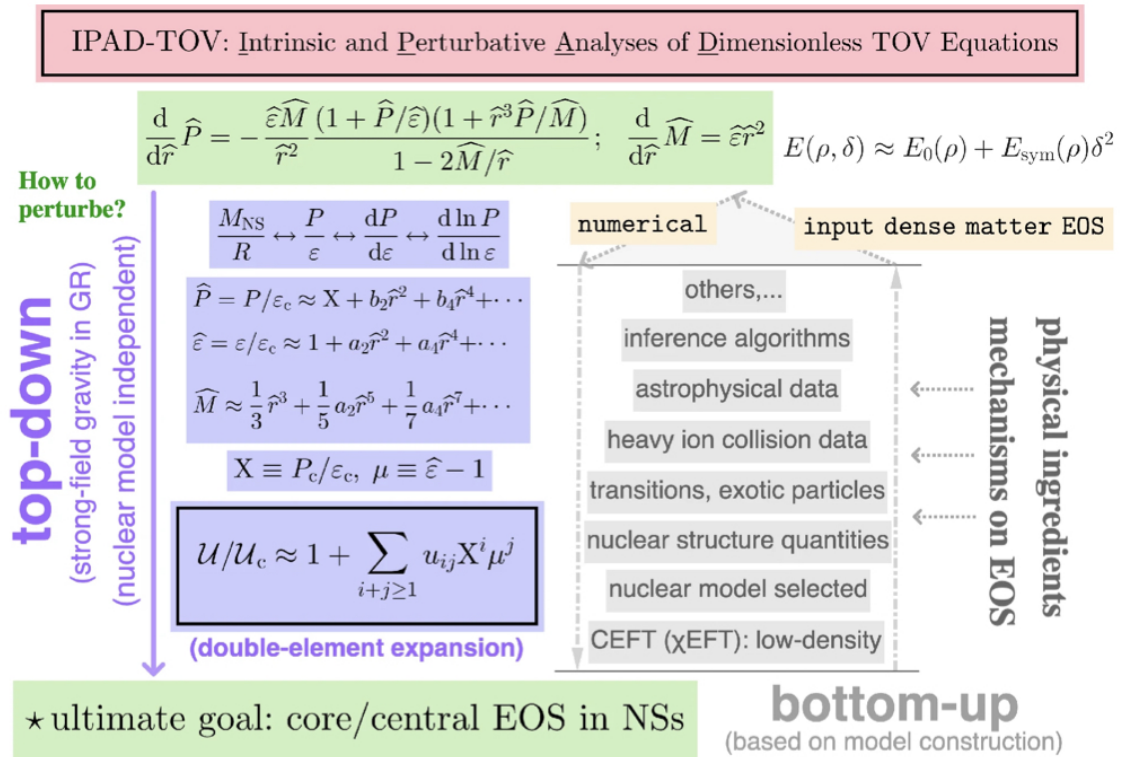




Summary of my talk

- (1) $\overbrace{(M_{\text{NS}}, R, \xi, \Lambda)}^{\text{observables}} \leftrightarrow \underbrace{P_c, \varepsilon_c, X = P_c/\varepsilon_c, s_c^2, \dots}_{\text{EOS-related quantities}}$
- (2) $\phi = P/\varepsilon \lesssim 0.374 \leftrightarrow \Delta \equiv 1/3 - \phi \gtrsim -0.04$
- (3) boundary: $R/\text{km} \gtrsim 4.73 M_{\text{NS}}/M_{\odot} + 1.14$
- (4) ξ or $\Lambda \rightarrow s_c^2 \gtrsim 0.4$ (for canonical NSs)
- (5) Λ encodes the core EOS information

Thank you!



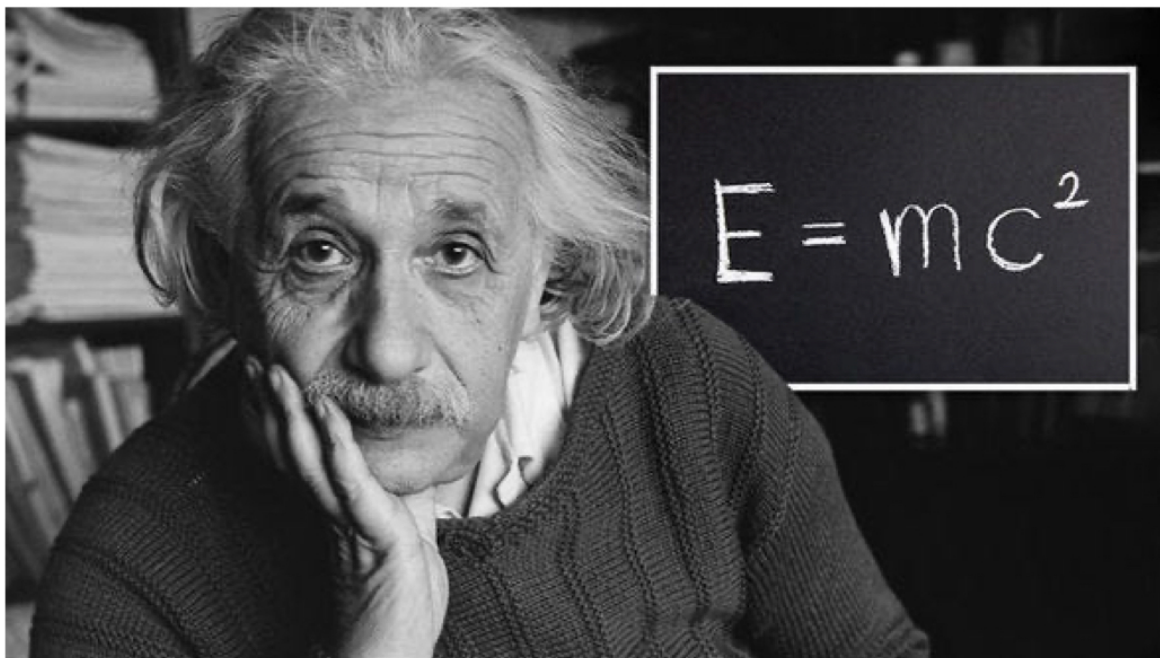
Related: is “ $\text{pressure} \leq \text{energy density}$ ” sufficient?



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speed of light is the upper limit of any signal (special relativity/causality):

$$s^2 = dP/d\varepsilon \leq 1 \Leftrightarrow P \leq \varepsilon$$



(1) Can strong-field gravity in General Relativity (GR) tell us a more refined upper limit on P/ε ?

(2) What's this upper limit?

This bound tells vividly how gravity influences strongly-interacting dense matter!

Stellar radius scaling: dimensional analysis

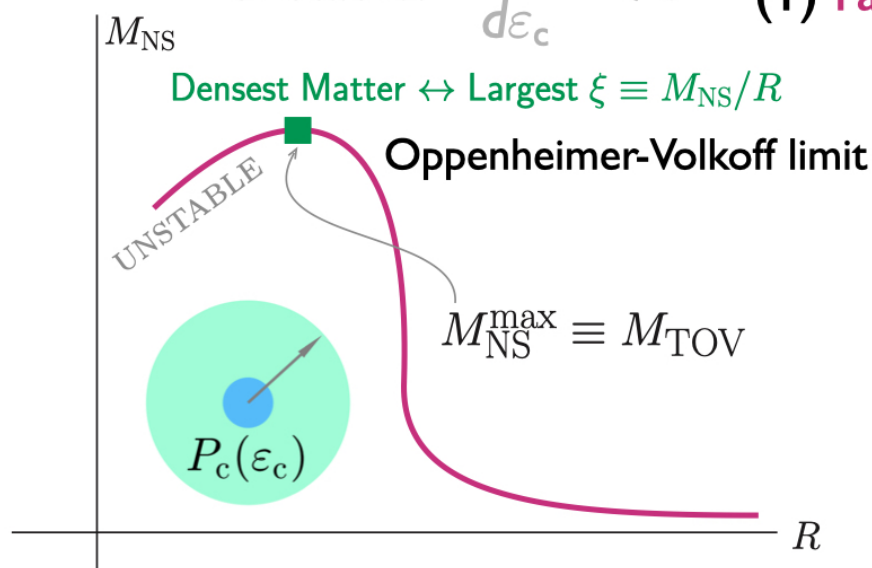
$$\underbrace{\frac{dP}{dr} = -\frac{GM\epsilon}{r^2}}_{\text{Newtonian gravity}} \rightarrow P \sim Gr^2\epsilon^2 \text{ since } M \sim \epsilon r^3 \rightarrow R \sim \sqrt{\frac{P}{\epsilon^2}} \sim \sqrt{\frac{P}{\epsilon \epsilon}} \sim \sqrt{\frac{\phi}{\epsilon}}$$

$$\phi = P/\epsilon \sim \text{dimensionless}$$

as energy density increases:

$$\text{unstable: } \frac{dM_{\text{NS}}}{d\epsilon_c} < 0$$

(1) **radius decreases**; (2) **while mass increases**



$$\text{compactness : } \xi = GM_{\text{NS}}/Rc^2 = M_{\text{NS}}/R$$

larger $\xi \leftrightarrow$ denser; densest $\leftrightarrow$ maximum configuration

$$M_{\odot} \approx 1.5 \text{ km} \rightarrow \xi_{\text{NS}} \sim \mathcal{O}(0.1)$$

$$\text{black holes : } M = 2R \rightarrow \xi_{\text{BH}} = 1/2$$

$$\text{white dwarfs : } \xi \sim 10^{-4}; \text{ solar core : } \xi \sim 10^{-6}$$

****core of NSs at the TOV configuration contains the densest visible matter existing in our universe**

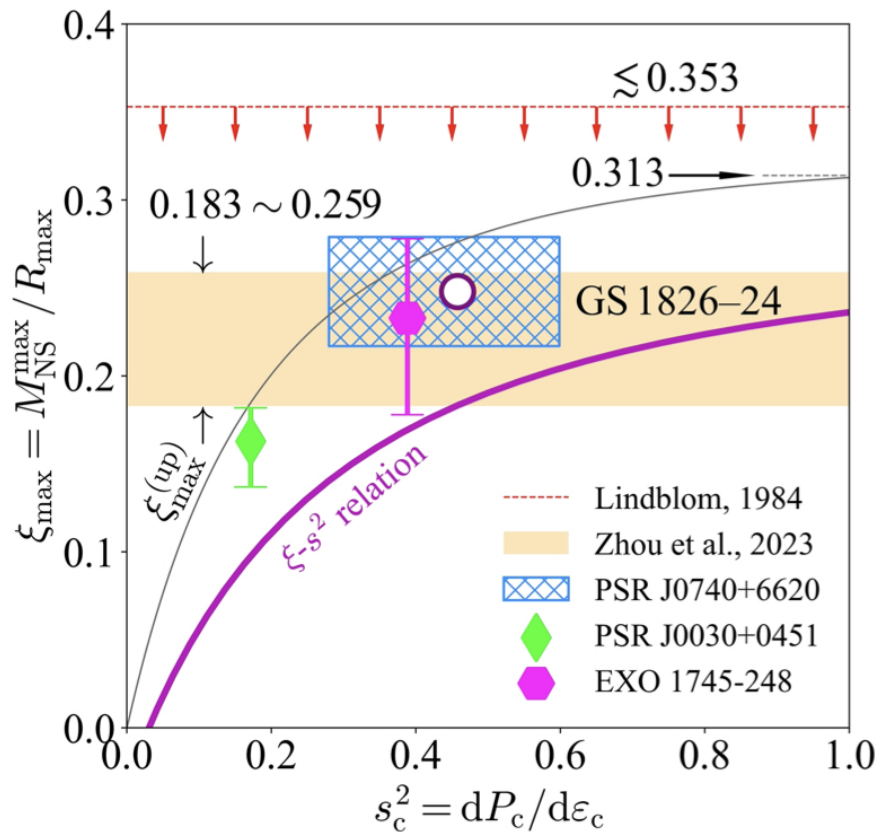
IMP mass measurements: strong implications



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IMP: $s_c^2 \gtrsim 0.4 \rightarrow$ exceeding the conformal bound

1/3



nature physics

$0.183 \lesssim \xi \lesssim 0.259$



Article

<https://doi.org/10.1038/s41567-023-02034-2>

Mass measurements show slowdown of rapid proton capture process at waiting-point nucleus ^{64}Ge

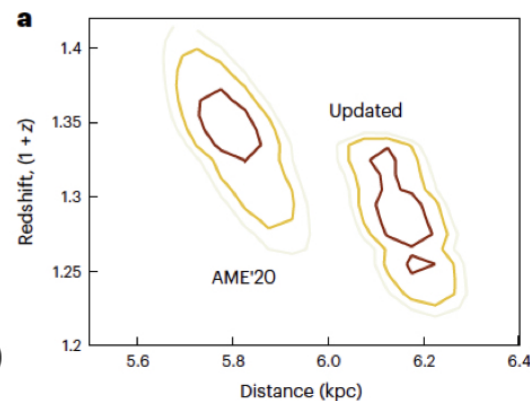
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X-ray bursts are among the brightest stellar objects frequently observed in the sky by space-based telescopes. A type-I X-ray burst is understood as a violent thermonuclear explosion on the surface of a neutron star, accreting matter from a companion star in a binary system. The bursts are powered by a nuclear reaction sequence known as the rapid proton capture process (rp process), which involves hundreds of exotic neutron-deficient nuclides. At so-called waiting-point nuclides, the process stalls until a slower β^+ decay enables a bypass. One of the handful of rp process waiting-point nuclides is ^{64}Ge , which plays a decisive role in matter flow and therefore the produced X-ray flux. Here we report precision measurements of the masses of ^{63}Ge , $^{64,65}\text{As}$ and $^{66,67}\text{Se}$ —the relevant nuclear masses around the waiting-point ^{64}Ge —and use them as inputs for X-ray burst model calculations. We obtain the X-ray burst light curve to constrain the neutron-star compactness, and suggest that the distance to the X-ray burster GS 1826–24 needs to be increased by about 6.5% to match astronomical observations. The nucleosynthesis results affect the thermal structure of accreting neutron stars, which will subsequently modify the calculations of associated observables.

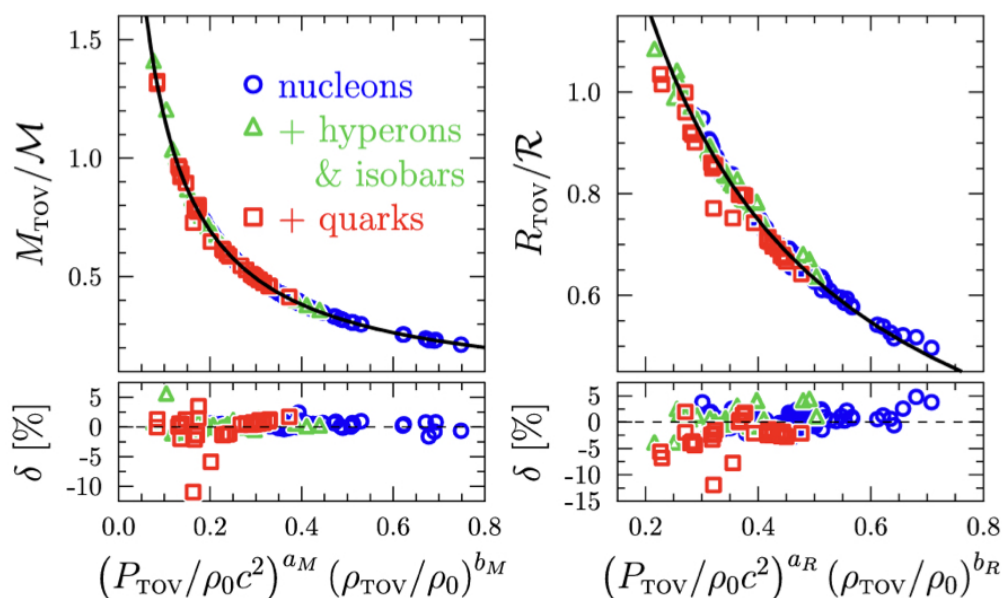
Zhou et al., Nat. Phys., 2023

Extracting central NS EOS via similar idea(s)

Ofengeim et al., PRD, 2024

$$M_{\text{NS}}^{\text{max}} = \sqrt{\frac{P_c^3}{\epsilon_c^4} \frac{1}{f_M(P_c, \epsilon_c)}}, \quad R_{\text{NS}}^{\text{max}} = \sqrt{\frac{P_c}{\epsilon_c^2} \frac{1}{f_R(P_c, \epsilon_c)}}$$

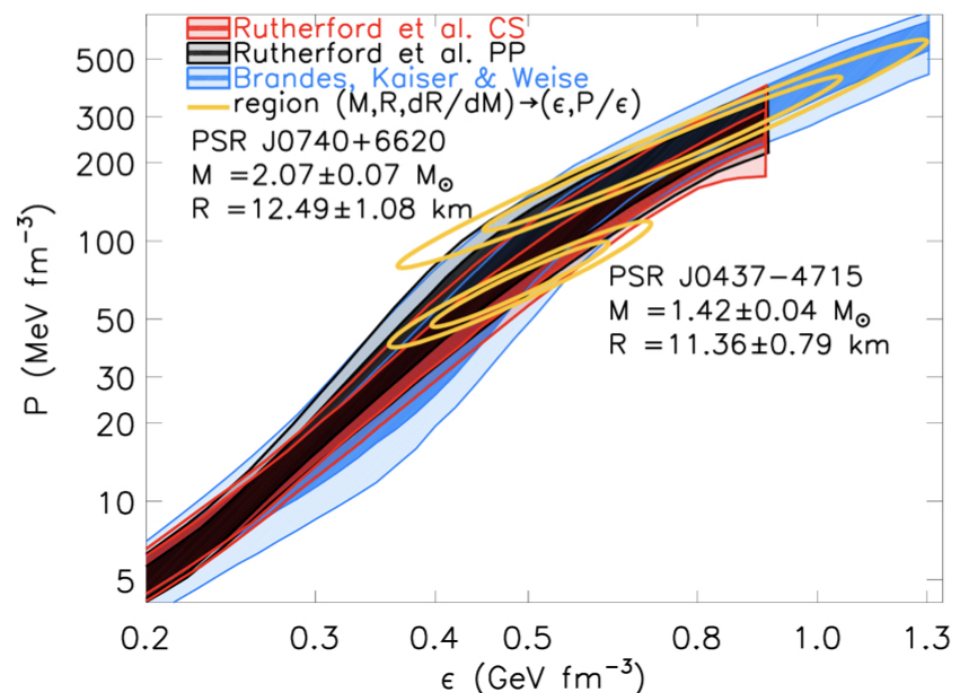
f_M, f_R : fitting functions with 3 parameters for each



Sun and Lattimer, ApJ, 2025

power law fitting: $\mathcal{P} = A_P P_c^{B_P} \epsilon_c^{C_P}$ for $\mathcal{P} = M_{\text{NS}}^{\text{max}}, R_{\text{NS}}^{\text{max}}$

A_P, B_P, C_P : fitting parameters, RMS accuracy $\lesssim 0.5\%$



NS mass distribution



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J1614-2230 : $1.91 \pm 0.02 M_{\odot}$

J0348+0432 : $2.01 \pm 0.04 M_{\odot}$

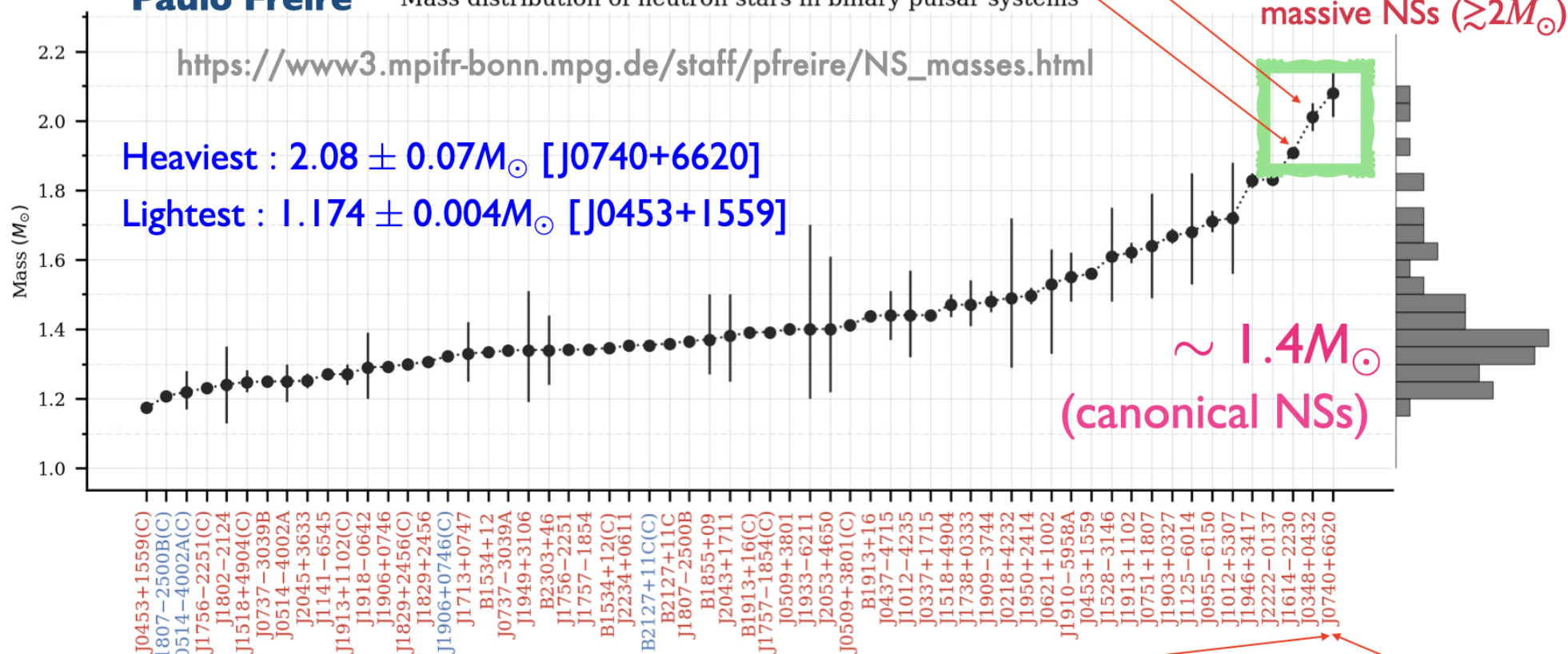
Paulo Freire

Mass distribution of neutron stars in binary pulsar systems

https://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Heaviest : $2.08 \pm 0.07 M_{\odot}$ [J0740+6620]

Lightest : $1.174 \pm 0.004 M_{\odot}$ [J0453+1559]



massive NSs: soft EOSs are basically excluded

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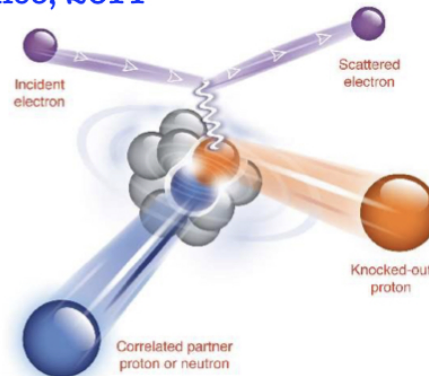
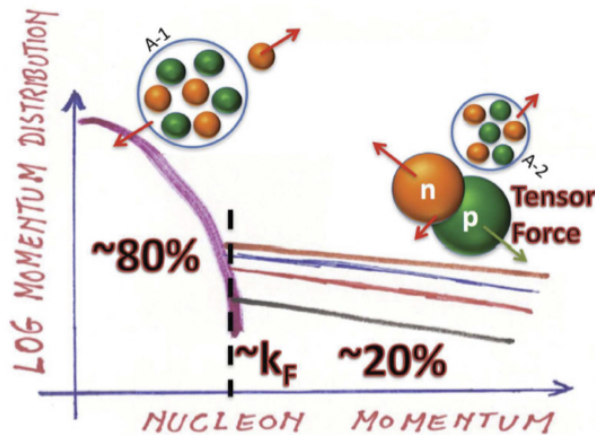
$$M \approx 2.08^{+0.07}_{-0.07} M_{\odot}; R \approx 12.39^{+1.30}_{-0.98} \text{ km}$$

Summary for $n(k)$ with (isospin-dependent) HMT



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Hen et al., RMP, 2017; Hen et al., Science, 2014



$$n_k^I(\rho, \delta) = \begin{cases} \Delta_J + \beta_J(|\mathbf{k}|/k_F)^2, & 0 < |\mathbf{k}| < k_F \\ C_J(k_F/|\mathbf{k}|)^4, & k_F < |\mathbf{k}| < \phi_J k_F \end{cases}$$

$$\mathcal{P}_J = \mathcal{P}_0 \left(1 + \tau_3^J \mathcal{P}_J \delta \right)$$

25% HMT: SNM
1%-2% HMT: PNM } → strong isospin dependence

analogy: angle-resolved photoemission spectroscopy (ARPES) → electron wf

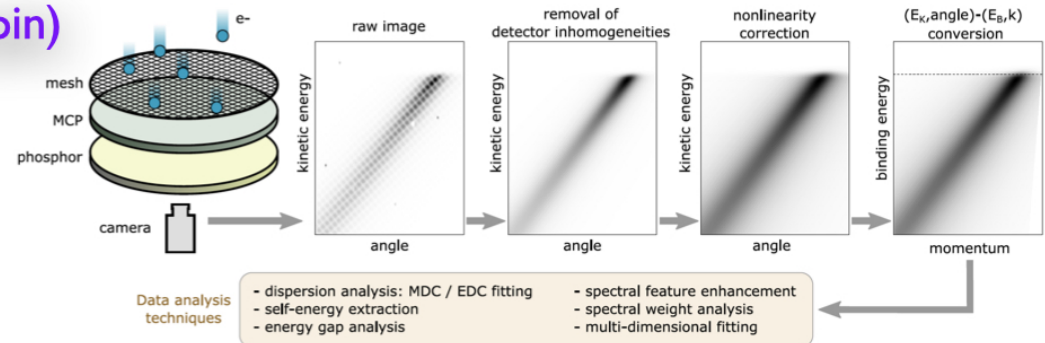
nuclear interaction is more complicated ($\rho, \delta, \mathbf{k}$, spin)

spectral function: $\mathcal{A}(\mathbf{k}, \omega) = 2\text{Im} G(\mathbf{k} - i0^+) \sim d\sigma$

hole

$$\mathcal{A}(\mathbf{k}, \omega)/2\pi = \overbrace{Z_{\mathbf{k}} \delta(\omega - \epsilon_{\mathbf{k}})}^{\epsilon_{\mathbf{k}} = k^2/2M - \mu} + \text{continuum}$$

$$n_{\mathbf{k}} = \int_{-\infty}^0 \frac{d\omega}{2\pi} \mathcal{A}(\mathbf{k}, \omega) = Z_{\mathbf{k}} \Theta(-\epsilon_{\mathbf{k}}) + \frac{C}{|\mathbf{k}|^4} \Theta(\epsilon_{\mathbf{k}})$$



Physical origin: why a maximum mass must exist for stable NSs?

(1) self-gravitating: $M_{\text{NS}} \sim \frac{1}{\sqrt{\epsilon_c}} \overbrace{\left(\frac{X}{1 + 3X^2 + 4X} \right)^{3/2}}^{\text{upper bounded}} \rightarrow \epsilon_c \sim M_{\text{NS}}^{-2} \quad Y_+$

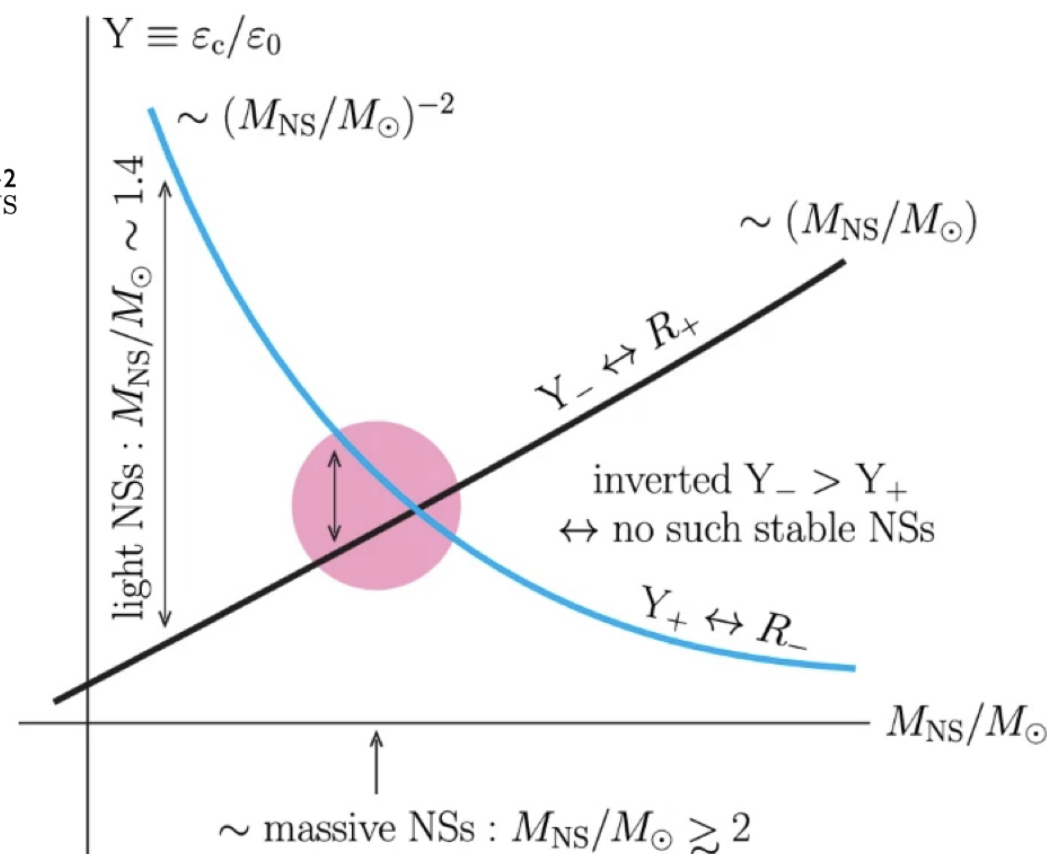
(2) stability condition: $dM_{\text{NS}}/d\epsilon_c \geq 0 \rightarrow \epsilon_c \text{ increases with } M_{\text{NS}} \quad Y_-$

$$\frac{\epsilon_c}{\epsilon_0} \lesssim \frac{21.71}{(M_{\text{NS}}/M_{\odot} + 0.08)^2} \equiv Y_+ \leftrightarrow R_-$$

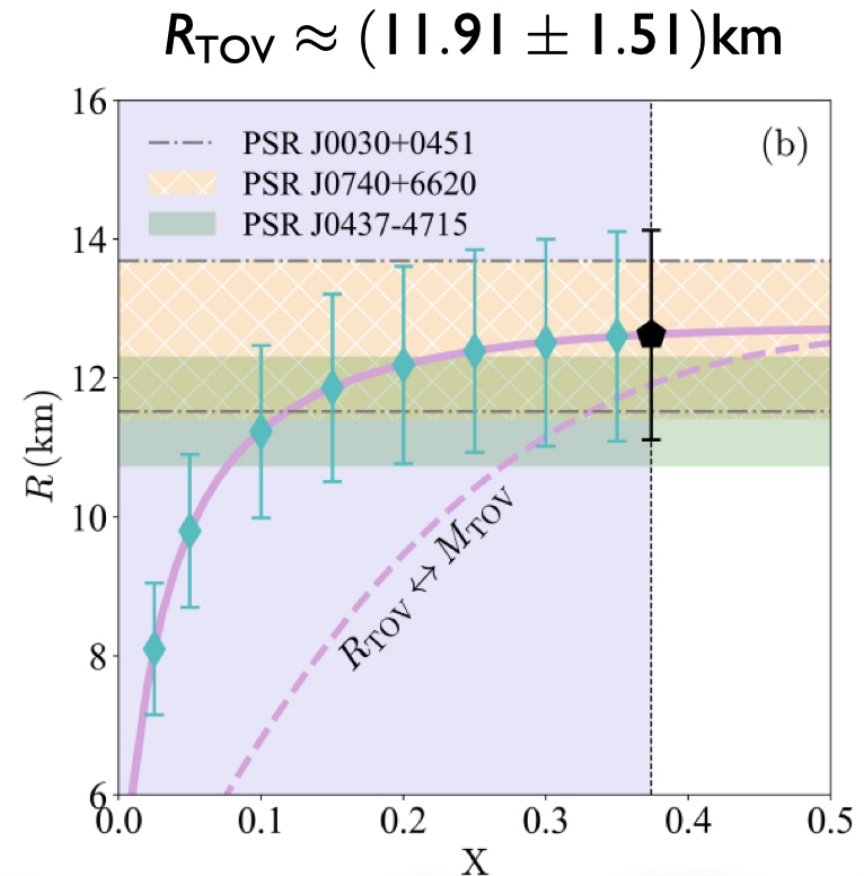
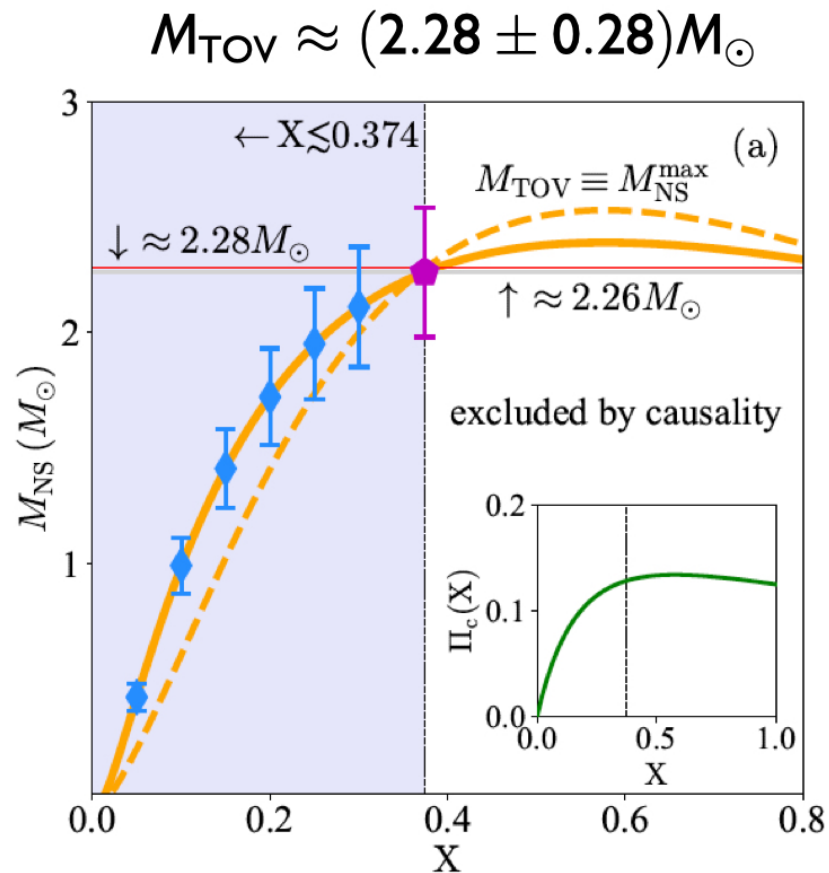
canonical NSs $M_{\text{NS}}/M_{\odot} \approx 1.4$: $\epsilon_c/\epsilon_0 \lesssim 9.9$

massive NSs $M_{\text{NS}}/M_{\odot} \approx 2.3$: $\epsilon_c/\epsilon_0 \lesssim 3.8$

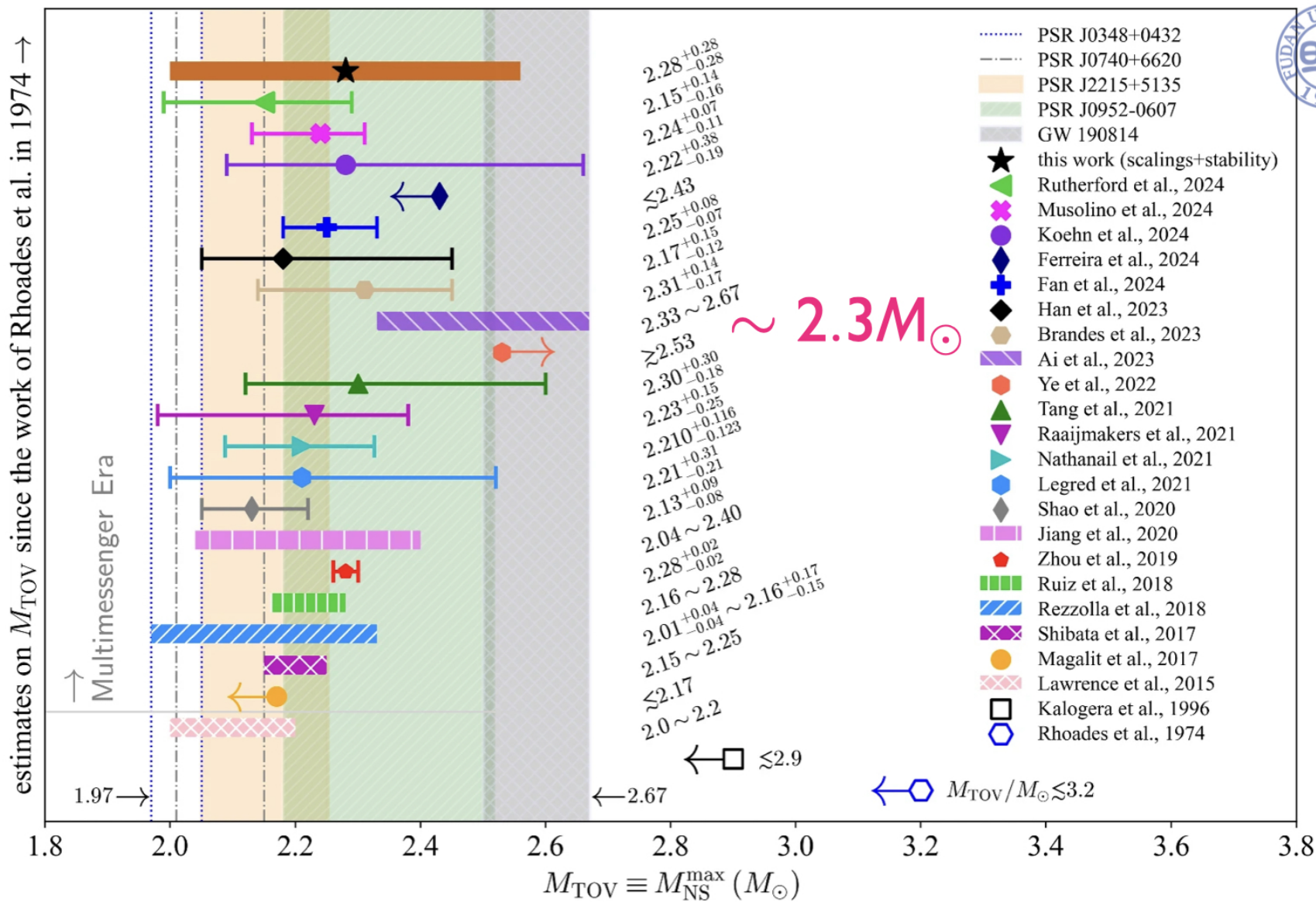
$\epsilon_0 \approx 150 \text{ MeV/fm}^3$ **strange??**



Extraction directly from TOV equations



without using specific input EOS models



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consistent
but no surprise