Neutrino interactions and decoupling in the early universe[1,2]

Greg Jackson

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1 based on collaboration w/ M. Escudero, M. Laine, S. Sandner

2 supported by the ANR under grant No. 22-CE31-0018

1. motivation

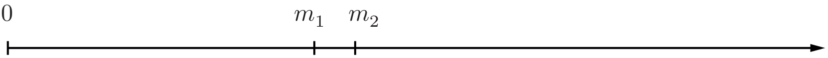
neutrino masses $\Rightarrow$ SM is not enough

much oscillation data to respect: Esteban, *et al.* (NuFit-6.0) [2410.05380]

$$\underbrace{\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}}_{\text{flavour eigenstate}} = \underbrace{\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}\,e^{-i\delta} \\ & 1 & \\ -s_{13}\,e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}}_{\equiv U_{\text{PMNS}}} \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}_{\text{mass eigenstate}}$$

$(c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij})$

$$\Delta m^2_{21} = (7.59 \pm 0.19) \times 10^{-5}\ \text{eV}^2\ ; \quad \theta_{12}/^\circ = 33.68^{+0.73}_{-0.70}$$



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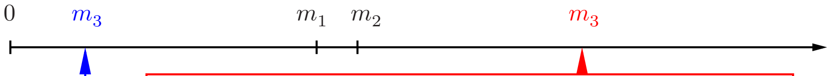


normal hierarchy (NH): $\Delta m^2_{31} = (2.513^{+0.021}_{-0.019}) \times 10^{-3}\ \text{eV}^2$

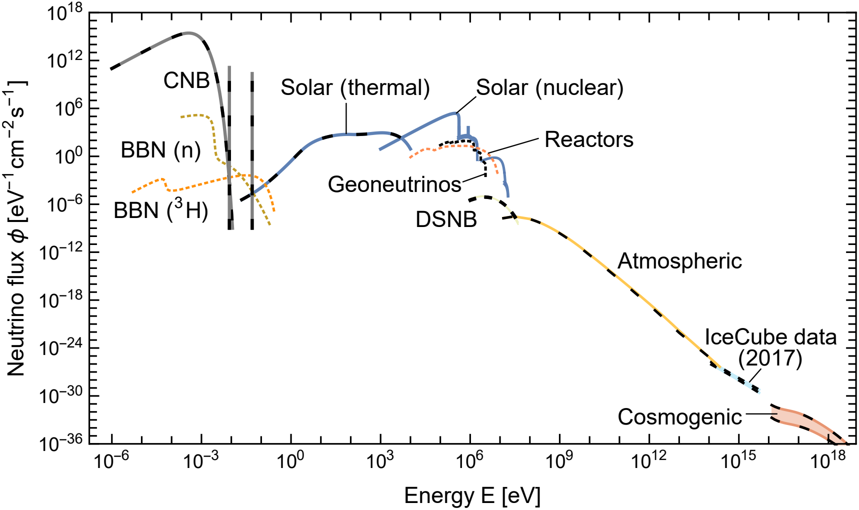
$\theta_{23}/^\circ = 43.3^{+1.0}_{-0.8}$; $\theta_{13}/^\circ = 8.56 \pm 0.11$; $\delta/^\circ = 212^{+26}_{-41}$

inverted hierarchy (IH): $\Delta m^2_{32} = (-2.484 \pm 0.020) \times 10^{-3}\ \text{eV}^2$

$\theta_{23}/^\circ = 47.9^{+0.7}_{-0.9}$; $\theta_{13}/^\circ = 8.59 \pm 0.11$; $\delta/^\circ = 274^{+22}_{-25}$

many sources in nature, including **cosmological**

cf. spectrum on earth: Vitagliano, Tamborra, Raffelt [1910.11878]



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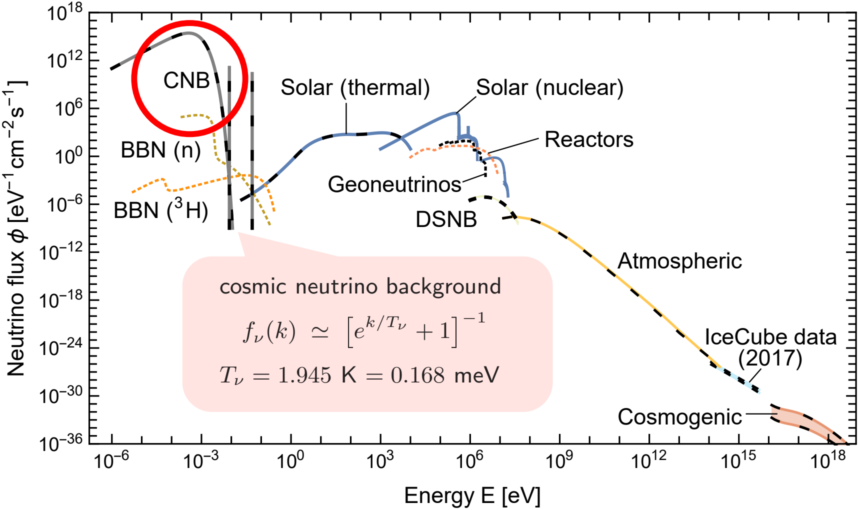


cosmic neutrino background

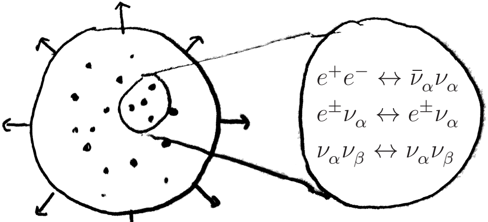
$$f_\nu(k) \simeq \left[e^{k/T_\nu}+1\right]^{-1}$$

$T_\nu = 1.945$ K $= 0.168$ meV

akin to CMB: $f_\gamma(k) \simeq \left[e^{k/T_\gamma}-1\right]^{-1}$, $T_\gamma = 2.725$ K $\simeq 1.4\,T_\nu$

Why do $\nu, \bar{\nu}$ decouple?

global expansion
vs.
microscopic processes



(α, β are flavour indices)

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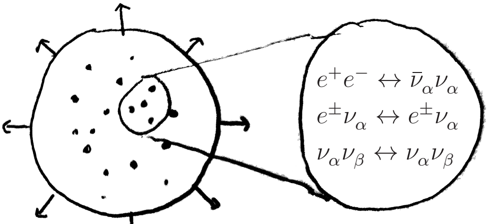
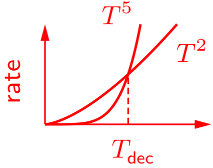


(α, β are flavour indices)

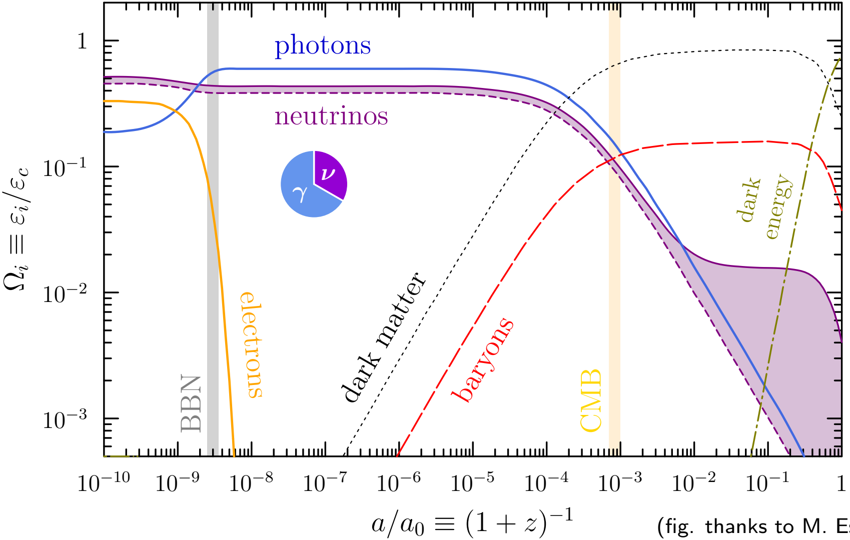
- interaction rate $\Gamma \;\sim\; \int \mathrm{d}\Omega \, |\mathcal{M}|^2 \left(n_{\rm F} \, n_{\rm F} \;-\; \ldots \right) \;\sim\; G_{\rm F}^2 \, T^5$
- Hubble rate $H = \dfrac{\dot{a}}{a} \sim \dfrac{T^2}{m_{\rm Pl}}$ where $m_{\rm Pl} \sim 10^{19}$ GeV (Planck mass)



$$\Rightarrow \quad T_{\text{dec}} \sim \left(\frac{1}{m_{\rm Pl} \, G_{\rm F}^2} \right)^{1/3} \sim 2 \text{ MeV}$$

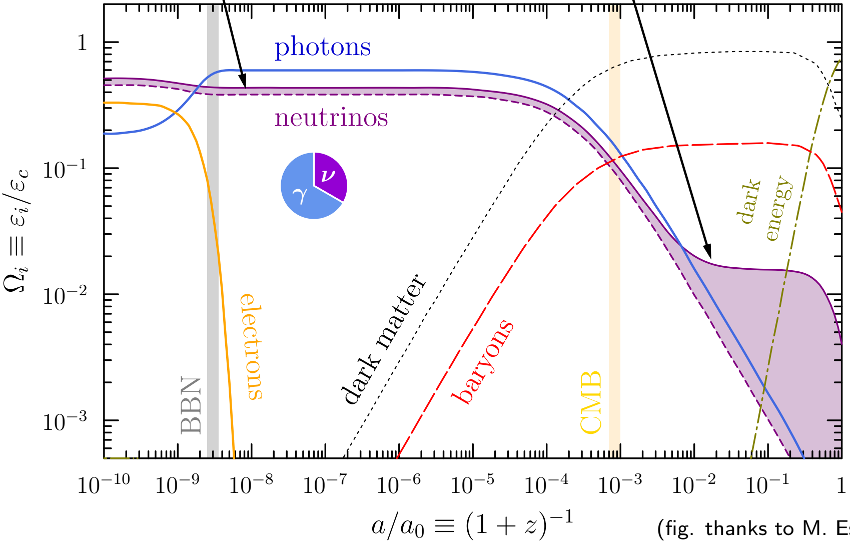
[i.e. $t_{\text{CNB}} \sim 1$ sec]

Neutrinos are always relevant in the universe's evolution!



(fig. thanks to M. Escudero!)

$N_{\text{eff}} = 3.0 \pm 0.3$ $\sum m_\nu < 0.2$ eV



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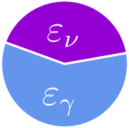
early radiation energy budget:

$$N_{\text{eff}} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\varepsilon_{\mathsf{rad}} - \varepsilon_\gamma}{\varepsilon_\gamma}$$



(zeroth order approx. $N_{\text{eff}} \simeq 3 =$ number of neutrino species)

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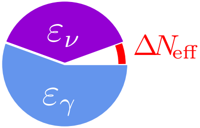

observation[3]

$N_{\rm eff}^{\rm BBN} = 2.93 \pm 0.08$ Yeh, *et al.* [2601.22239]

$N_{\rm eff}^{\rm CMB} = 2.82 \pm 0.12$ SPT+PI+ACT [2506.20707]

theory

$N_{\rm eff}^{\rm SM} = 3.044 \pm 0.001$ $\Rightarrow$ focus of this talk

(hints at $N_{\rm eff} < 3$... new physics? Escudero, *et al.* [2601.22239])

[3] Simons Obs will target ± 0.045 accuracy $\Rightarrow$ $\Delta N_{\rm eff}$ probes BSM physics!

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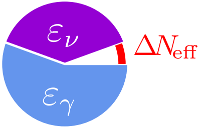

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at late times $\nu, \bar{\nu}$ non-rel: $\Omega_\nu \simeq \dfrac{\sum_\nu m_\nu}{3} \dfrac{\sum_\alpha (n_{\nu_\alpha} + n_{\bar{\nu}_\alpha})}{\varepsilon_{\sf crit}}$

$\sum_\nu m_\nu < 0.048$ eV SPT+PI+ACT+DESI [2506.20707]

[3] Simons Obs will target ± 0.045 accuracy $\Rightarrow$ $\Delta N_{\rm eff}$ probes BSM physics!

2. interaction rates at $T \sim$ MeV

theoretical setup

- consider $e^{\pm}, \gamma, \nu, \bar{\nu}$ plasma at temperatures $T \sim$ MeV
  (all hadrons, μ, τ leptons suppressed, so ν_μ & ν_τ basically degenerate)
- QED interactions are "fast" ($\exists$ temperature T_γ, pressure P_γ, ...)
- as $T_\gamma(t)$ drops, neutrinos fall out of equilibrium: $T_\nu \neq T_\gamma$

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$\Rightarrow$ quantum kinetic equations: [Sigl, Raffelt (1993)]

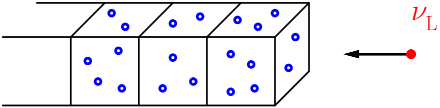
$$\dot{\rho} \;\simeq\; -i\big[\mathcal{H}, \rho\big] + \mathcal{C}[\rho] \quad , \quad \rho_{\alpha\beta} = \Big\langle \hat{a}^\dagger_{\alpha \boldsymbol{q}} \hat{a}_{\beta \boldsymbol{q}} \Big\rangle$$

where $(\dot{...}) \equiv (\partial_t - H\, q\, \partial_q)(...)$; α, β are flavour indices,

$$\mathcal{H} \;\simeq\; U \frac{1}{2q} \begin{pmatrix} \; & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} U^\dagger - 2\sqrt{2} G_{\mathrm{F}} \frac{q}{m_W^2} \begin{pmatrix} \varepsilon_\gamma + p_\gamma & & \\ & \ddots & \\ & & \; \end{pmatrix} + \ldots$$

and $\mathcal{C}$ is a "collisional operator" (in general $\rho \neq \bar{\rho}$, hence also $\dot{\bar{\rho}} = ...$)

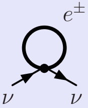
start with a simpler problem: interaction rate for "tagged" particle



calculate the rate in thermal field theory Bödeker, *et al.* [1510.06742]

$$\Gamma \;=\; \frac{1}{2k}\,\mathrm{Tr}\left[\not{\mathcal{K}}\,\mathrm{Im}\,\Sigma(\mathcal{K})\right]\,, \quad \mathcal{K} = (k + i0^+, \boldsymbol{k})$$

$\Sigma(\mathcal{K}) \;=$



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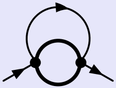

$+\;\mathcal{O}(G_{\mathrm{F}}^{3})$

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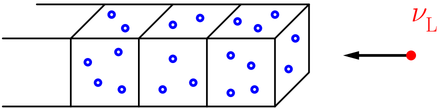

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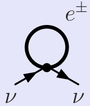

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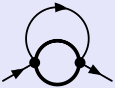

$+ \quad \mathcal{O}(G_F^3)$

$$H_{\mathrm{int}}(t) = \int_{\boldsymbol{x}} \frac{G_F}{8\sqrt{2}}\Big\{2\,\bar{\nu}_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha\,\bar{\ell}_e\gamma^\mu\left[2\delta_{\alpha,e}-1+4s_W^2+(1-2\delta_{\alpha,e})\gamma_5\right]\ell_e$$
$$+\,\bar{\nu}_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha\,\bar{\nu}_\beta\gamma^\mu(1-\gamma_5)\nu_\beta\Big\}$$

sum over $\alpha, \beta = \{e, \mu, \tau\}$

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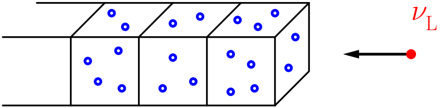

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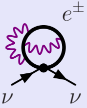
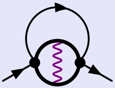
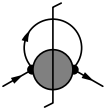
$\Sigma(\mathcal{K}) \;=$  $+$  $+\; \mathcal{O}(G_{\rm F}^3)$

QED corrections?

$$\alpha_{\rm em} = \frac{e^2}{4\pi}$$

$$H_{\sf int}(t) = \int_{\boldsymbol{x}} \frac{G_{\rm F}}{8\sqrt{2}} \Big\{ 2\,\bar{\nu}_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha\,\bar{\ell}_e\gamma^\mu\left[2\delta_{\alpha,e} - 1 + 4s_W^2 + (1-2\delta_{\alpha,e})\gamma_5\right]\ell_e$$
$$+\,\bar{\nu}_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha\,\bar{\nu}_\beta\gamma^\mu(1-\gamma_5)\nu_\beta + ieC_\alpha\bar{\nu}_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha\,\partial_\nu F^{\nu\mu} \Big\}$$

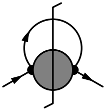
sum over $\alpha, \beta = \{e, \mu, \tau\}$ 1-loop level operator!



$$= \frac{G_F^2 k}{8\pi^2} \int \mathrm{d}p^0\, \mathrm{d}p\, \mathcal{P}^2 \left[\, 1 - n_F(k-p^0) + n_B(p^0)\, \right]$$

$$\times \underbrace{L_{\mu\nu}\big(\mathcal{K},\mathcal{K}-\mathcal{P}\big)}_{\text{leptonic tensor}}\ \mathrm{Im}\, \underbrace{\Pi^{\mu\nu}(p^0,p)}_{\text{self-energy}}$$

GJ, Laine [2312.07015]



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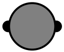
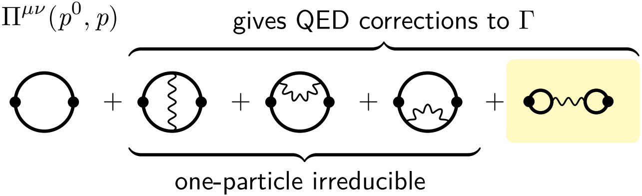
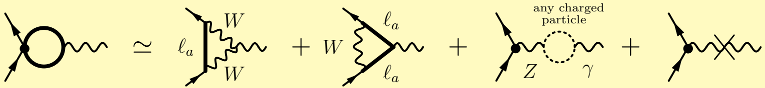


$$\equiv \; \Pi^{\mu\nu}(p^0, p)$$

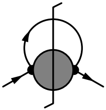
$$=$$





$$e^2 \, C_\alpha^{(\text{bare})} \; = \; 8(2\delta_{\alpha,e} - 1 + 4 s_W^2) \frac{e^2}{3} \frac{\mu^{-2\epsilon}}{(4\pi)^2} \left(\frac{1}{\epsilon} + \ln \frac{\bar{\mu}^2}{m_e^2} \right) \; \pm \; 0.1$$

Hill, Tomalak [1911.01493]

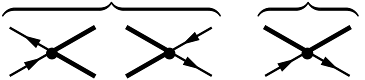


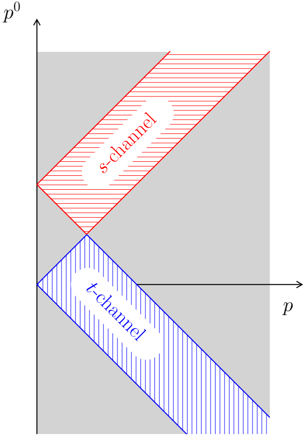
$$= \quad \frac{G_{\rm F}^2 k}{8\pi^2} \int \mathrm{d}p^0\, \mathrm{d}p \; \mathcal{P}^2 \left[\, 1 - n_{\rm F}(k - p^0) + n_{\rm B}(p^0) \,\right]$$

$$\times \quad L_{\mu\nu}\big(\mathcal{K}, \mathcal{K} - \mathcal{P}\big) \; \mathrm{Im}\; \Pi^{\mu\nu}(p^0, p)$$

GJ, Laine [2312.07015]

s-channel (inel) t-channel (el)

LO: 



phase space:

$$\int \mathrm{d}p^0\, \mathrm{d}p \;\to\; \int \mathrm{d}\Omega^{(s)} + \int \mathrm{d}\Omega^{(t)}$$

where measures are defined by

$$\int \mathrm{d}\Omega^{(s)} \quad \equiv \quad -\frac{1}{k^3} \int_0^k \mathrm{d}p_- \int_k^\infty \mathrm{d}p_+ \; p$$

$$\int \mathrm{d}\Omega^{(t)} \quad \equiv \quad \frac{1}{k^3} \int_{-\infty}^0 \mathrm{d}p_- \int_0^k \mathrm{d}p_+ \; p$$

full non-linear evolution equations

time dep. of classical phase space distr. $f_{\boldsymbol{k}_\nu} = \left\langle \hat{a}^\dagger_{\boldsymbol{k}_\nu} \hat{a}_{\boldsymbol{k}_\nu} \right\rangle$,

$$\begin{aligned} \dot{f}_{\boldsymbol{k}_\nu} &\simeq \int_{\boldsymbol{q}_{\bar{\nu}}} \Big[\Psi(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}})(1 - f_{\boldsymbol{k}_\nu})(1 - f_{\boldsymbol{q}_{\bar{\nu}}}) - f_{\boldsymbol{k}_\nu} f_{\boldsymbol{q}_{\bar{\nu}}} \widetilde{\Psi}(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}}) \Big] \\ &+ \int_{\boldsymbol{q}_\nu} \Big[\Theta(\boldsymbol{q}_\nu \to \boldsymbol{k}_\nu) f_{\boldsymbol{q}_\nu} (1 - f_{\boldsymbol{k}_\nu}) - \Theta(\boldsymbol{k}_\nu \to \boldsymbol{q}_\nu) f_{\boldsymbol{k}_\nu} (1 - f_{\boldsymbol{q}_\nu}) \Big] \end{aligned}$$

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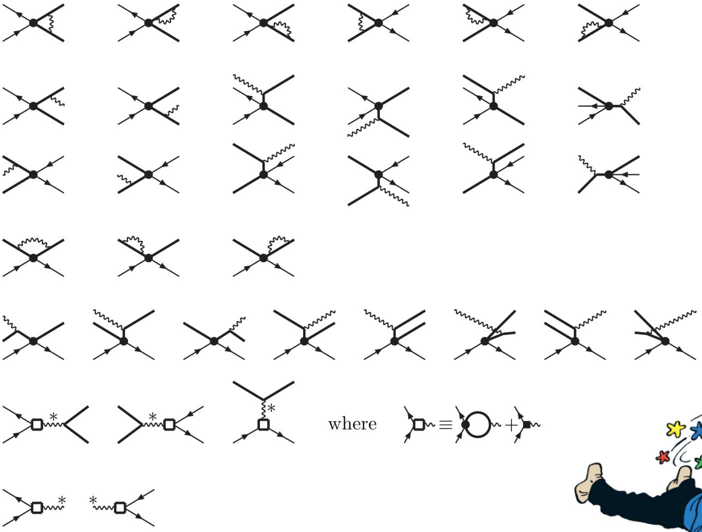
double-differential rates i.t.o. $\operatorname{Im}\Pi(p^0, p)$ evaluated[4] at...

$$\left.\begin{array}{rl} \textbf{production:} & \Psi(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar\nu}) \\[1em] \textbf{annihilation:} & \widetilde{\Psi}(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar\nu}) \end{array}\right\} \quad p^0 = k + q\,, \;\; p = |\boldsymbol{k}_\nu + \boldsymbol{q}_{\bar\nu}|$$

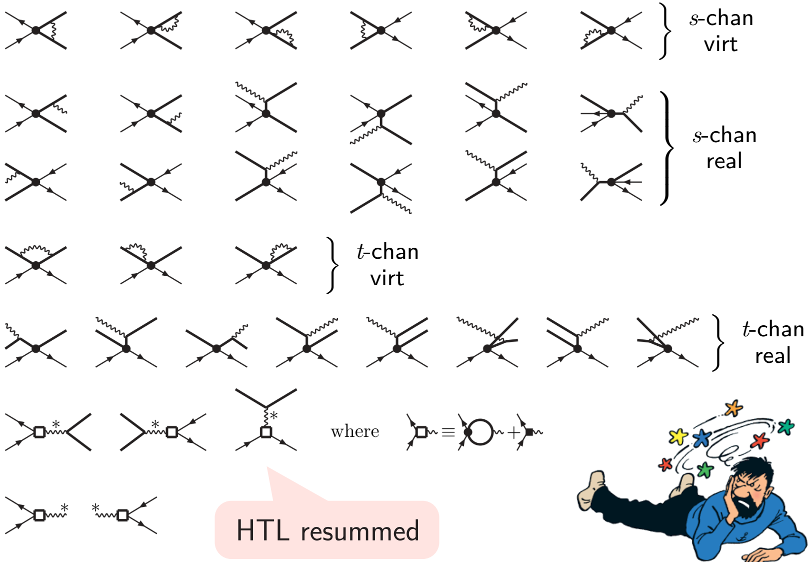
$$\textbf{scattering:} \;\; \Theta(\boldsymbol{q}_\nu \to \boldsymbol{k}_\nu) \qquad p^0 = k - q\,, \;\; p = |\boldsymbol{k}_\nu - \boldsymbol{q}_\nu|$$

[4] tabulation & interpolation code for the e^+e^- spectral function made available in relevant kinematic domains: zenodo.14217713

the spectral function $\mathrm{Im}\,\Pi_{\mu\nu}$ neatly encodes many scatterings:



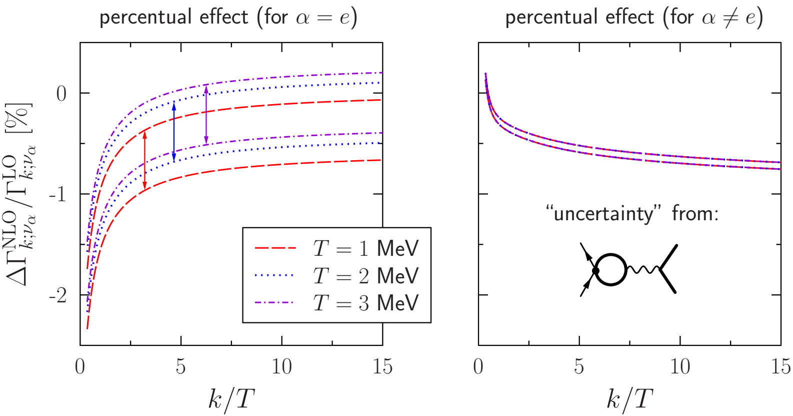
the spectral function $\mathrm{Im}\,\Pi_{\mu\nu}$ neatly encodes many scatterings:



equilibration rate: $\dot{f}_{\boldsymbol{k}} \simeq -\Gamma_{\boldsymbol{k}}\, \delta f_{\boldsymbol{k}} - \int_{\boldsymbol{q}} \Upsilon_{\boldsymbol{k},\boldsymbol{q}}\, \delta f_{\boldsymbol{q}} + \mathcal{O}(\delta^2)$

$\Rightarrow$ tiny QED corrections to Γ, relatively less than ~ 1 %

(neglecting m_e) GJ, Laine [2312.07015]



3. $\nu, \bar{\nu}$ decoupling in the SM

Why is $N_{\rm eff}$ in the SM not 3?

several effects well-studied: $N_{\rm eff}^{\sf SM} - 3 =$

- some $e^+e^- \to \nu\bar{\nu}$ heating since $T_{\sf dec} \approx m_e$ $+0.03$
  [Dicus, *et al.* (1982)] [Dolgov, *et al.* (1997)]
- corrections to $e^\pm, \gamma$ equation of state $P_{\sf em}(T)$ $+0.01$
  [Heckler (1994)]

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- some $e^+e^- \to \nu\bar{\nu}$ heating since $T_{\sf dec} \approx m_e$ $+0.034$
  [Dicus, *et al.* (1982)] [Dolgov, *et al.* (1997)]
- corrections to $e^\pm, \gamma$ equation of state $P_{\sf em}(T)$ $+0.009$
  [Heckler (1994)] Bennet, *et al.* [1911.04504]
  Akita, Yamaguchi [2005.07047]
- neutrino oscillations $+0.001$
  de Salas, Pastor [1606.06986] Froustey, *et al.* [2008.01074]
  Bennet, *et al.* [2012.02726]
- QED corrections to interaction rates -0.001
  Cielo, *et al.* [2306.05460]

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- neutrino oscillations $+0.001$
  de Salas, Pastor [1606.06986] Froustey, *et al.* [2008.01074]
  Bennet, *et al.* [2012.02726]

- QED corrections to interaction rates $(-0.001?)$ $\lesssim 10^{-4}$
  Cielo, *et al.* [2306.05460]
  Drewes, *et al.* [2402.18481]
  GJ, Laine [2412.03958]

frameworks for describing $\nu, \bar{\nu}$ decoupling:

instantaneous decoupling (zeroth order approx. $N_{\rm eff}^{\rm SM} \simeq 3$)
→ trivial to implement, but misses dynamics

solve quantum kinetic eqs (e.g. `FortEPiaNO` code [1905.11290])
→ gold standard, very few approximations
→ computationally expensive (stiff integrodifferential equations)

momentum-averaged approx (⇒ focus of this talk)
→ flast & flexible (easy to include BSM cases)
→ less accurate when spectral distortions are large

monte carlo sim (Ovchynnikov, Syvolap [2409.15129] [2409.07378])
→ numerical, good for far from equilibrium

momentum-ave. approach: Escudero [1812.05605] , [2001.04466]

$$\rho(t, \boldsymbol{q}) = \begin{pmatrix} f_{\nu_e}(t,q) & & \\ & f_{\nu_\mu}(t,q) & \\ & & f_{\nu_\tau}(t,q) \end{pmatrix}$$

$f_{\nu_e} \neq f_{\nu_\mu,\nu_\tau}$ ["no osc"]

$f_{\nu_e} = f_{\nu_\mu,\nu_\tau}$ ["fast osc"]

with thermal phase space distributions[5]

$$f_{\nu_\alpha} \equiv f_{\bar{\nu}_\alpha} \equiv \left[e^{(q-\mu_{\nu_\alpha})/T_{\nu_\alpha}} + 1\right]^{-1} \quad \left(T_{\nu_\alpha},\, \mu_{\nu_\alpha} \text{ are \underline{effective} params!}\right)$$

[5] nota bene: $\mu_\nu = \mu_{\bar{\nu}}$ ($\Rightarrow$ not lepton asymmetries!)

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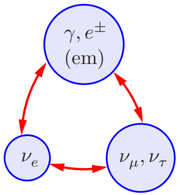
$$f_{\nu_\alpha} \;\equiv\; f_{\bar\nu_\alpha} \;\equiv\; \left[e^{(q-\mu_{\nu_\alpha})/T_{\nu_\alpha}}+1\right]^{-1}$$ (T_{ν_α}, μ_{ν_α} are <u>effective</u> params!)

$$\frac{\mathrm{d}T_i}{\mathrm{d}t} \;=\; \frac{-3H\left[(\varepsilon_i+P_i)\frac{\partial n_i}{\partial\mu_i}-n_i\frac{\partial\varepsilon_i}{\partial\mu_i}\right]+Q_i\,\frac{\partial n_i}{\partial\mu_i}-J_i\,\frac{\partial\varepsilon_i}{\partial\mu_i}}{\frac{\partial n_i}{\partial\mu_i}\frac{\partial\varepsilon_i}{\partial T_i}-\frac{\partial n_i}{\partial T_i}\frac{\partial\varepsilon_i}{\partial\mu_i}}$$

$$\frac{\mathrm{d}\mu_i}{\mathrm{d}t} \;=\; \frac{3H\left[(\varepsilon_i+P_i)\frac{\partial n_i}{\partial T_i}-n_i\frac{\partial\varepsilon_i}{\partial T_i}\right]-Q_i\,\frac{\partial n_i}{\partial T_i}+J_i\,\frac{\partial\varepsilon_i}{\partial T_i}}{\frac{\partial n_i}{\partial\mu_i}\frac{\partial\varepsilon_i}{\partial T_i}-\frac{\partial n_i}{\partial T_i}\frac{\partial\varepsilon_i}{\partial\mu_i}}$$



$\Rightarrow$ transfer rates[6] ($Q = \dot\varepsilon_{\nu+\bar\nu}$ and $J = \dot n_{\nu+\bar\nu}$) enter as "coefficients"

[6] the Q and J are moments of Ψ, $\widetilde\Psi$ and Θ introduced on slide 13

Updated framework (fast & flexible):

Escudero, GJ, Laine, Sandner [2511.04747]

Part I, Standard Model[7]

- improved interaction rates
- factor ~ 10 speedup
- QED eq. of state, $\mathcal{O}(e^4)$ & $\mathcal{O}(e^5)$
- also: g_*, h_{eff}, $\sum m_\nu/[\Omega_\nu h^2]$

[7] available in Python or Mathematica: github.com/MiguelEA/nudec_BSM

Updated framework (fast & flexible): Part I, Standard Model[7]

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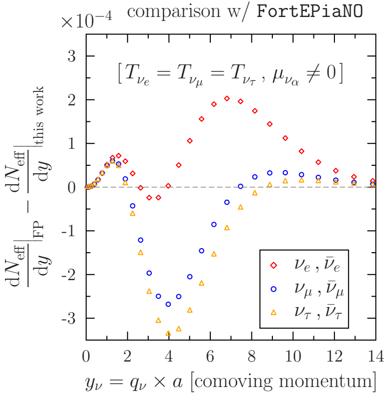

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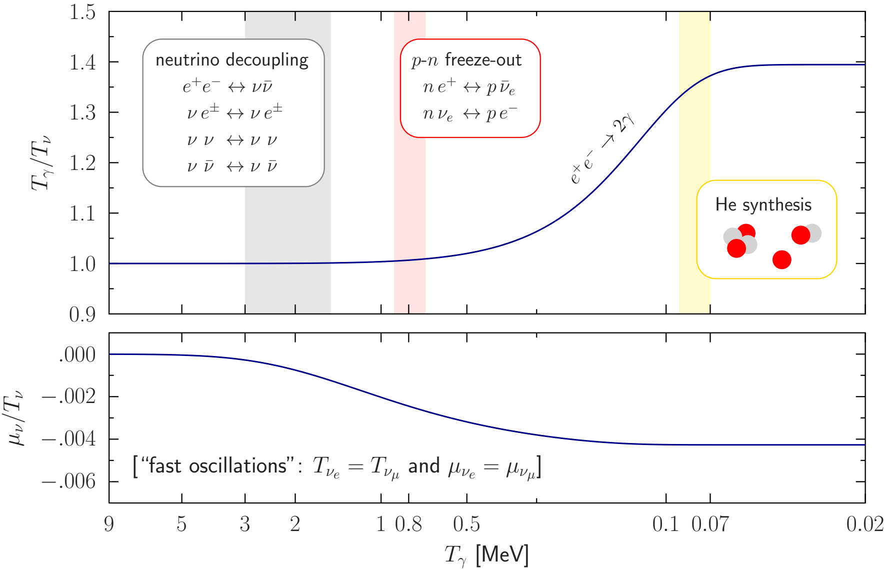
- QED eq. of state, $\mathcal{O}(e^4)$ & $\mathcal{O}(e^5)$
- also: g_*, $h_{\rm eff}$, $\sum m_\nu/[\Omega_\nu h^2]$

case/setup	$N_{\rm eff}$	
FortEPiaNO	3.0439	
this work $\mu_\nu = 0$	3.0453	[0.044 %]
this work $\mu_\nu \neq 0$	3.0446	[0.020 %]
this work (w/ osc)	3.0443	[0.012 %]

$\Rightarrow$ Part II will consider various BSM implementations

[7] available in Python or Mathematica: github.com/MiguelEA/nudec_BSM

evolution in the SM:



thermodynamic functions

$$P_{\mathsf{em}} = \frac{\pi^2 T_\gamma^4}{45}\left[c_0 + e^2 c_2 + e^3 c_3 + e^4\left(\tilde{c}_4 \ln\frac{\bar{\mu}}{T_\gamma} + c_4\right) + e^5\left(\tilde{c}_5 \ln\frac{\bar{\mu}}{T_\gamma} + c_5\right) + \ldots\right]$$

Arnold, Zhai [hep-ph/9410360] Zhai, Kastening [hep-ph/9507380]

... where $\bar{\mu} \to m_e$ is the $\overline{\mathrm{MS}}$ renorm scale, the coefficients $c_i = c_i\left(\frac{m_e}{T_\gamma}\right)$

	$P_{\mathsf{em}}^{(0)}$	$P_{\mathsf{em}}^{(2)}$	$P_{\mathsf{em}}^{(3)}$	$P_{\mathsf{em}}^{(4,5)}$
δN_{eff}		$\approx 10^{-2}$	$\approx 10^{-3}$	$\lesssim 2 \times 10^{-5}$

[c_4 and c_5 only inown for $m_e = 0$]

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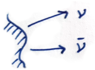
δN_{eff}		$\approx 10^{-2}$	$\approx 10^{-3}$	$\lesssim 2 \times 10^{-5}$

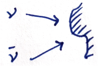
[c_4 and c_5 only inown for $m_e = 0$]

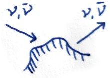
$$\left(\begin{array}{c} \text{for neutrinos: } \varepsilon_\nu = -2\frac{3T_\nu^4}{\pi^2}\mathrm{Li}_4\left(-e^{\mu_\nu/T_\nu}\right),\ n_\nu = -2\frac{T_\nu^3}{\pi^2}\mathrm{Li}_3\left(-e^{\mu_\nu/T_\nu}\right) \\ \frac{\partial n}{\partial T} = \ldots\ , \qquad \text{etc.} \end{array}\right)$$

use double-differential rates (slide 13), e.g.

$$
\begin{aligned}
Q &\equiv \dot{\varepsilon}_{\nu+\bar{\nu}} \\
&= \int_{\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}}} (k_\nu + q_{\bar{\nu}})\, \Psi(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}})\, (1 + f_{\boldsymbol{k}_\nu})(1 + f_{\boldsymbol{q}_{\bar{\nu}}}) \\
&- \int_{\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}}} (k_\nu + q_{\bar{\nu}})\, \widetilde{\Psi}(\boldsymbol{k}_\nu, \boldsymbol{q}_{\bar{\nu}})\, f_{\boldsymbol{k}_\nu} f_{\boldsymbol{q}_{\bar{\nu}}} \\
&+ \int_{\boldsymbol{k}_\nu, \boldsymbol{q}_\nu} (k_\nu - q_\nu)\, \Theta(\boldsymbol{q}_\nu \to \boldsymbol{k}_\nu)\, f_{\boldsymbol{q}_\nu} (1 + f_{\boldsymbol{k}_\nu})
\end{aligned}
$$





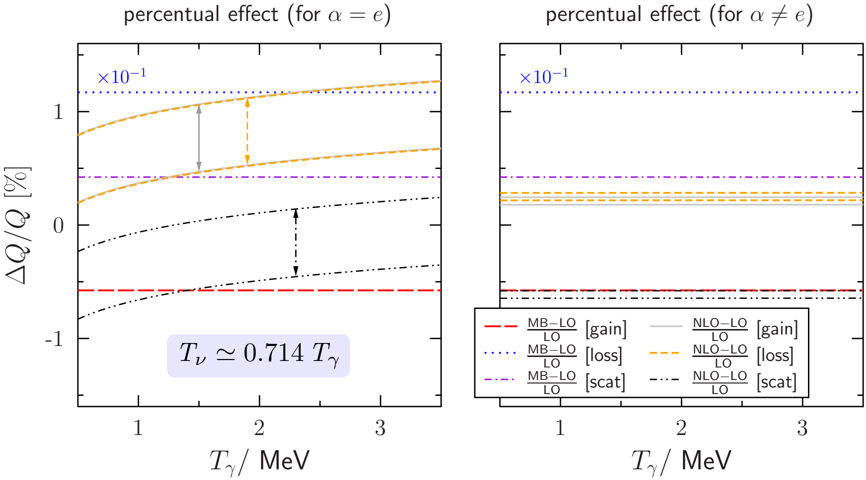


$$J \equiv \dot{n}_{\nu+\bar{\nu}} = \ldots$$ (exercise for the reader)

$\Rightarrow$ tiny QED corrections ($\delta N_{\text{eff}} \approx 10^{-4}$) [2312.07015] [2412.03958]

(numerically much smaller than estimated in Cielo, *et al.* [2306.05460])

energy & number exchange rates



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(numerically much smaller than estimated in Cielo, *et al.* [2306.05460])

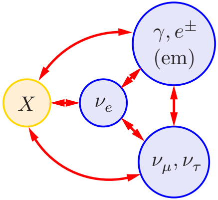
4. a few BSM examples

easy to include additional subsystems:

$$\dot{\varepsilon}_X + 3H(\varepsilon_X + P_X) \;=\; Q_X \,, \quad \dot{n}_X + 3Hn_X \;=\; J_X$$

with their own transfer coefficients, e.g.

$$Q_X = \sum_i Q_{X\leftarrow i}\,;\; i \in \{\mathrm{em}, \nu_e, \nu_\mu, \nu_\tau\}$$



$\Rightarrow$ need thermal production rates!

K. Bouzoud, (Wed, 15:15)

AUTOTHERM

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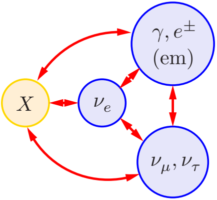

$\Rightarrow$ need thermal production rates!

K. Bouzoud, (Wed, 15:15)

AUTOTHERM

Boltzmann equation for BSM sector: $\dot{f}_X \;=\; \sum_i C_{X\leftarrow i}\,[f_X]$

$$Q_{X\leftarrow i} \;=\; g_X \int_{\boldsymbol{k}} \sqrt{k^2 + m_X^2}\;\; \underbrace{C_{X\leftarrow i}\,[f_X]}_{=\text{ collision operator}}$$

$f_X(\boldsymbol{k})$ w/ Gauss-Laguerre weights, or $T_X(t)$ and $\mu_X(t)$

trivial case: $Q_X = J_X = 0$ during $\nu, \bar{\nu}$ decoupling...

- $P_X \ll \varepsilon_X$ [cold dark matter] $\Rightarrow \varepsilon_{X,0} = \varepsilon_{X,\text{ini}} \left(a_{\text{ini}} / a_0\right)^3$
- $P_X = \dfrac{\varepsilon_X}{3}$ [extra radiation] $\Rightarrow \varepsilon_{X,0} = \varepsilon_{X,\text{ini}} \left(a_{\text{ini}} / a_0\right)^4$

  (e.g. light axions: G. Moore, (Mon, 18:00) , E. Weitz, (Wed, 15:30))

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(e.g. light axions: G. Moore, (Mon, 18:00) , E. Weitz, (Wed, 15:30))

coupled MeV-scale particles: $(Q_X, J_X \neq 0)$

$\Rightarrow$ modified dynamics and observables, N_{eff} etc.

- inflaton, scalar populates the SM bath via Yukawa int.

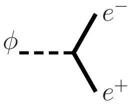
  (cf. next slide, "low reheating" Barbieri, *et al.* [2501.01369])

- majoron, pesudo-Goldstone interacting with Majorana ν's

  (backup slides, "neutrophilic" int. Sandner, *et al.* [2305.01692])

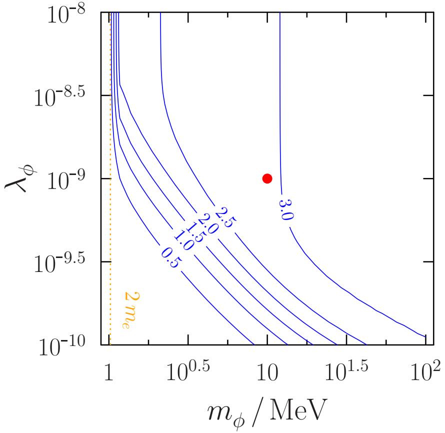
$X = \phi$ [inflaton]

$$\mathcal{L} \;=\; \mathcal{L}_{\mathsf{SM}} \;+\; \frac{1}{2}\Big(\partial^\mu\phi\,\partial_\mu\phi - m_\phi^2\,\phi^2\Big) - i\,\lambda_\phi\,\bar{e}\,\gamma_5\,e\,\phi$$

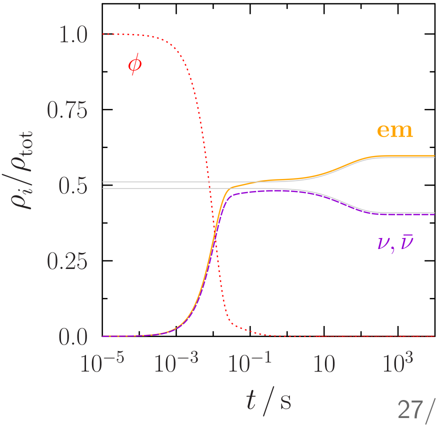


reheating: $\rho_\phi(t_{\text{ini}})$ generates SM bath at $T_{\text{RH}} \sim$ MeV

contour plot of N_{eff}



$\lambda_\phi = 10^{-9}, m_\phi = 10$ MeV (●)



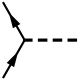
5. summary

- active neutrinos play an important role in cosmology
  $\Rightarrow$ test of SM physics around $T \sim$ MeV
- measurements will continue to increase in precision
  $\Rightarrow$ from CMB (Simons, ...) and BBN (LBT,...)
- rich theory challenge
  $\Rightarrow$ devil is in the details

[péringuey's adder]

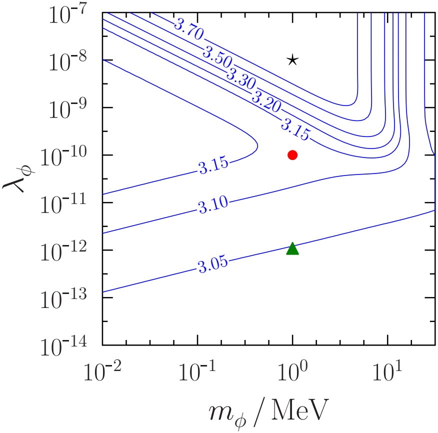
$X = \phi$ [majoron]

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{2}\Big(\partial^\mu\phi\,\partial_\mu\phi - m_\phi^2\,\phi^2\Big) - i\sum_{i=1,2,3}\frac{\lambda_{\nu_i}}{2}\,\bar{\nu}_i\gamma_5\,\nu_i\,\phi$$

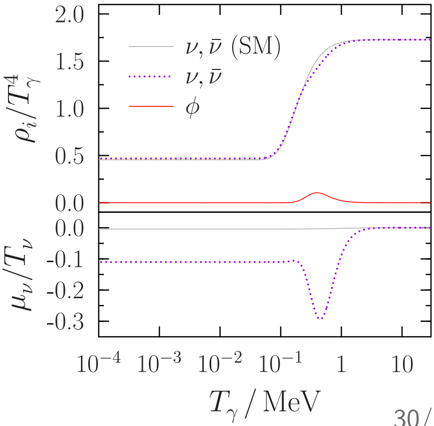


$f_{\phi,\boldsymbol{k}}$ w/ Gauss-Laguerre weights

contour plot of $N_{\rm eff}$

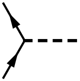


$\lambda_\phi = 10^{-10}, m_\phi = 1$ MeV (●)



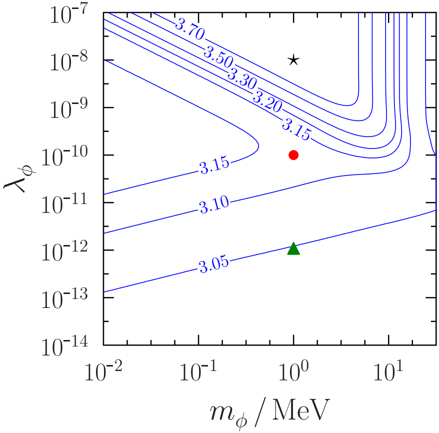
$X = \phi$ [majoron]

$$\mathcal{L} \;=\; \mathcal{L}_{\mathsf{SM}} \;+\; \frac{1}{2}\Big(\partial^\mu\phi\,\partial_\mu\phi - m_\phi^2\,\phi^2\Big) - i\sum_{i=1,2,3}\frac{\lambda_{\nu_i}}{2}\,\bar{\nu}_i\gamma_5\,\nu_i\,\phi$$

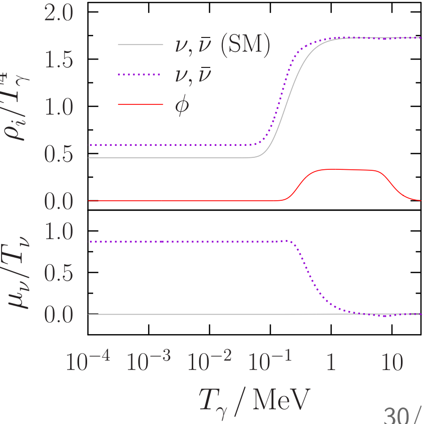


$f_{\phi,\boldsymbol{k}}$ w/ Gauss-Laguerre weights

contour plot of $N_{\rm eff}$

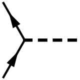


$\lambda_\phi = 10^{-8}, m_\phi = 1$ MeV ($\star$)



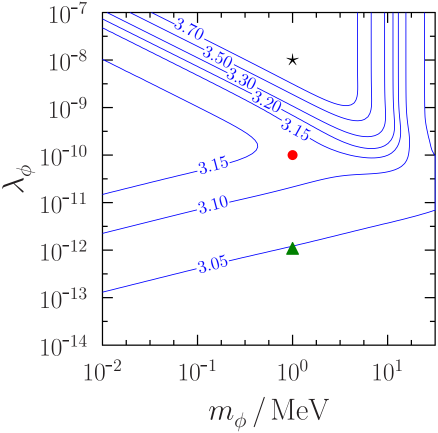
$X = \phi$ [majoron]

$$\mathcal{L} \;=\; \mathcal{L}_{\mathsf{SM}} \;+\; \frac{1}{2}\Big(\partial^\mu\phi\,\partial_\mu\phi - m_\phi^2\,\phi^2\Big) - i\sum_{i=1,2,3}\frac{\lambda_{\nu_i}}{2}\,\bar\nu_i\gamma_5\,\nu_i\,\phi$$



$f_{\phi,\boldsymbol{k}}$ w/ Gauss-Laguerre weights

contour plot of $N_{\rm eff}$



$\lambda_\phi = 10^{-12}, m_\phi = 1$ MeV (▲)