

Hawking Radiation from Primordial Black Holes

Christopher Hirata
Baltimore • July 2025

conference in celebration of
Marc Kamionkowski and Rob Caldwell

A bit of reflection on my trajectory through cosmology (since I'm at Hopkins)

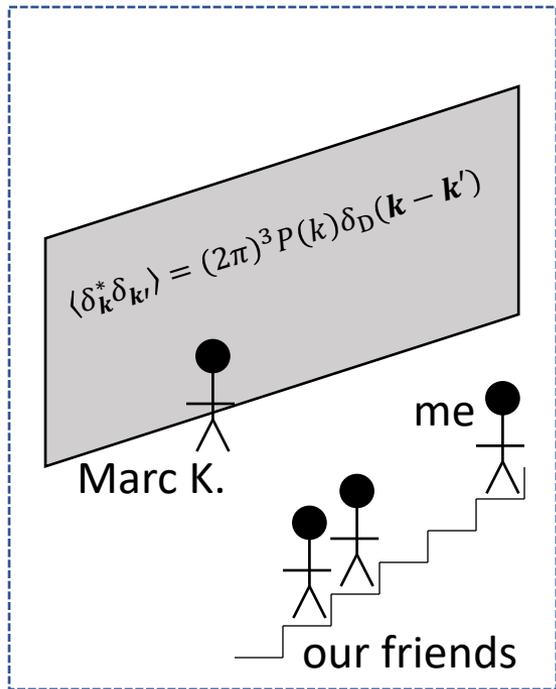


Figure [0]
Lauritsen Room 269, Caltech
(September 2000)



My first time in Bloomberg Hall:
*Planning meeting for a large scale
structure survey mission that
would eventually get merged into
the Roman Space Telescope*

(August 2005)



Figure [-1]
Building #29, Goddard SFC
(July 2025)

... but today I will put my other hat on and discuss my group's theory work ...

I'd like to thank the Hawking radiation team ...



Makana Silva



Gabriel Vasquez



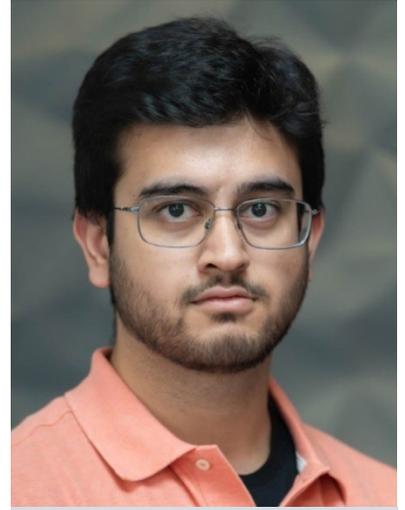
Emily Koivu



Cara Nel

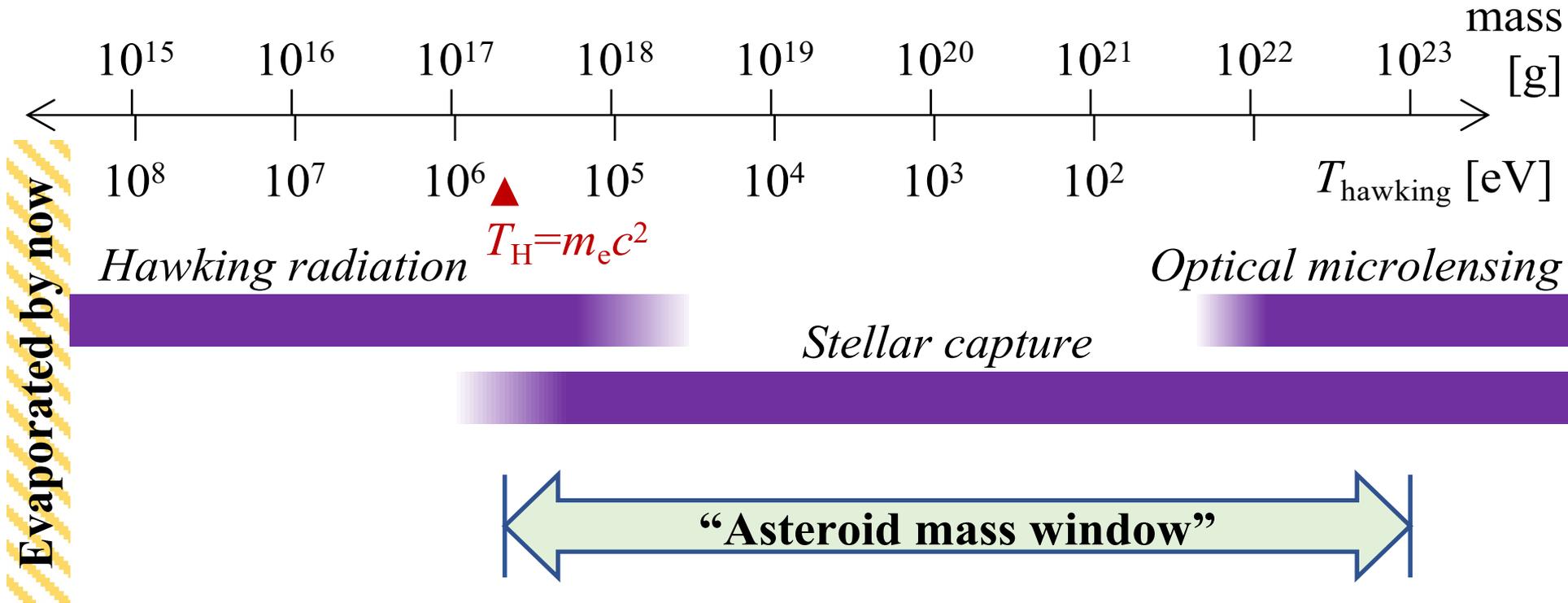


Bowen Chen



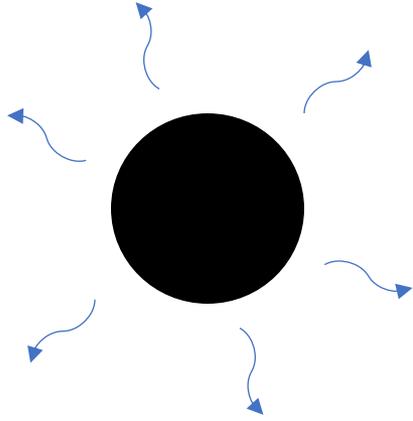
Arijit Das

Primordial Black Holes as Dark Matter Candidates



- ✓ Cold dark matter (from the perspective of structure formation)
- ✓ Not a new stable particle (but requires a formation mechanism in the early Universe)
- ✓ If massive enough, detectable through gravitational effects (dynamics, microlensing)
- ✓ If light enough, detectable via Hawking radiation
 - gamma ray background
 - positrons (+ 511 keV line)
- ✓ Various ideas for the intermediate masses (not this talk)

Hawking Radiation



- Horizon has a temperature related to surface gravity κ :

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{8\pi M} \quad (\text{for Schwarzschild spacetime — I'll focus on this case})$$

- A black hole will emit a spectrum of particles, most easily evaluated via partial wave expansion:

$$\frac{dN}{dE dt} = \sum_{j=s}^{\infty} \sum_p \frac{2j+1}{2\pi} \frac{1}{e^{E/T_H} \pm 1} |T_{j,p}(E)|^2$$

energy E

angular momentum j
(starts at particle spin)

polarization state p
(if nontrivial)

mode counting

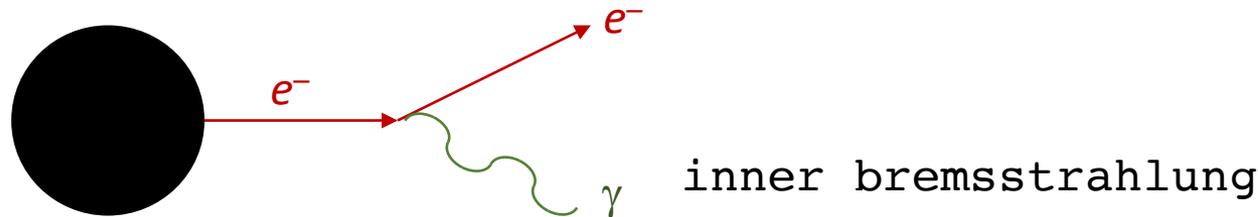
blackbody spectrum
(\pm for fermion/boson)

partial wave transmission coefficient (through angular momentum barrier)

- Numerically evaluated for the common particles (Page 1977) — now in common tools such as BlackHawk (Arbey & Auffinger 2019, 2021).

... but what happens when particles interact?

- At least one effect is relevant at \sim MeV energies and qualitatively changes the spectrum (Page et al. 2008, Coogan et al. 2021), even though it is formally $O(\alpha)$.



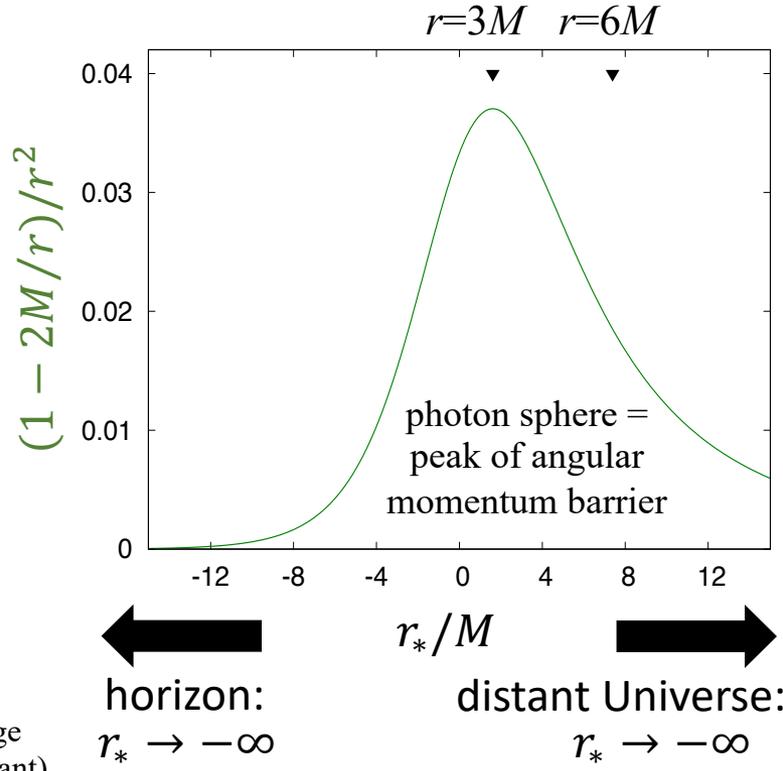
$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

- Given that current constraints explore temperatures where e^\pm can be produced, we wanted to explore what happens in the full theory of quantum electrodynamics (QED) on a Schwarzschild background:
 1. [astrophysics question] Do the corrections to the emitted spectrum affect PBH observables and constraints?
 2. [high energy physics question] What issues arise in numerically computing the corrections to $O(\alpha)$, and how can divergences be cured (if at all)?

Electrodynamics in the Schwarzschild spacetime

$$ds^2 = \underbrace{\left(1 - \frac{2M}{r}\right)}_{\text{conformal factor}} \left[\underbrace{-dt^2 + dr_*^2}_{\text{causal structure}} + \underbrace{\frac{d\theta^2 + \sin^2 \theta d\phi^2}{(1 - 2M/r)/r^2}}_{\text{angular metric}} \right]$$

angular momentum barrier



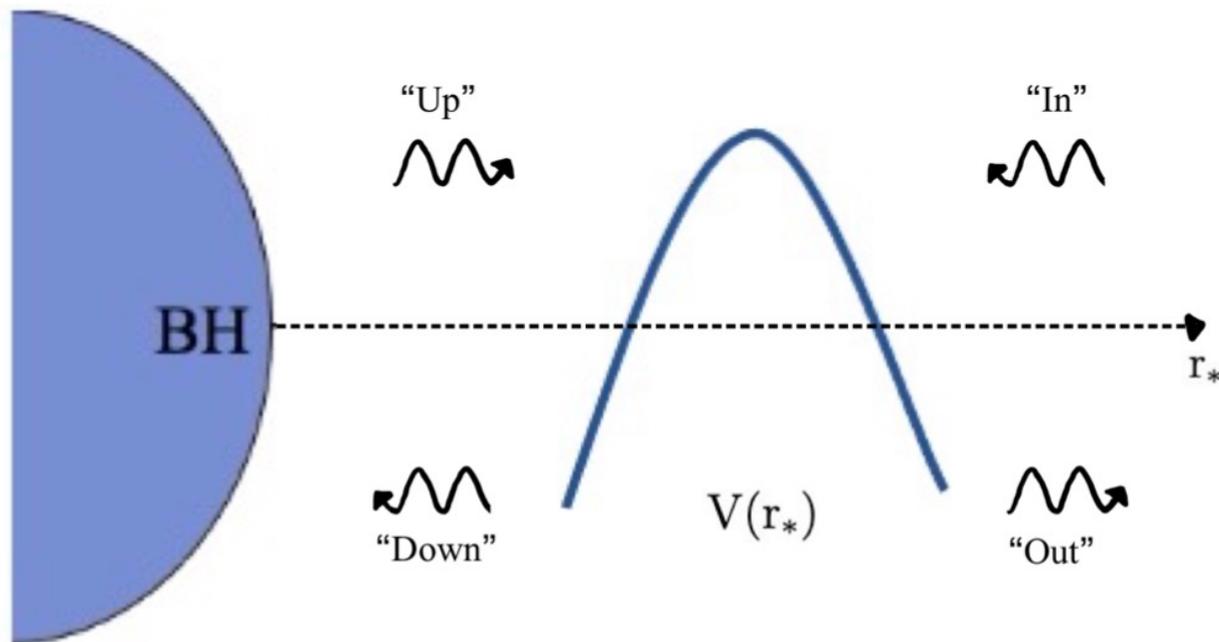
Radial coordinate: $r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$

$$S_{\text{QED}} = \int \left[\overset{\text{vierbein}}{\downarrow} \bar{\psi} \left(\overset{\text{spinor connection}}{\downarrow} a^\sigma_{\hat{\beta}} \overset{\text{electron charge (coupling constant)}}{\downarrow} \tilde{\gamma}^{\hat{\beta}} (\partial_\sigma - \Gamma_\sigma - ieA_\sigma) + \mu \right) \psi - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} \right] \sqrt{-g} d^4x$$

- Canonical quantization in Coulomb gauge works.
 - ✓ Scalar potential A_t is non-dynamical, all electromagnetic waves are in spatial components.
- Expand around Boulware vacuum (ground state with respect to Killing field ∂_t).
- But use spherical waves instead of plane waves ...

Electrodynamics in the Schwarzschild spacetime

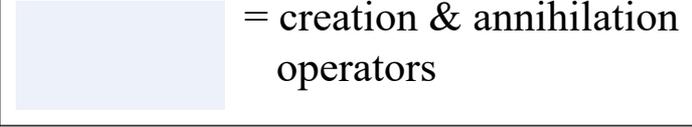
- Photon & electron mode functions have quantum numbers:
 - ✓ Angular momentum is discrete (l, m, p or k, m)
 - ✓ Energy is continuous (ω or h)
- There is also channel index X (“in” or “up”).
 - ✓ Alternative basis: “out” and “down”.
 - ✓ Initial conditions (Hawking radiation) are simple in in/up, observable is simple in out/down.



It's like being in QFT class again!

[Silva et al. 2023]



 = radial mode functions
 = angular mode functions
 = creation & annihilation operators

Vector potential operator:

$$\begin{aligned}
 A_{r_*} &= \frac{1 - 2M/r}{r^2} \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \sqrt{\ell(\ell+1)} \int_0^{\infty} \frac{d\omega}{2\pi} \frac{1}{\sqrt{2\omega^3}} \\
 &\times \sum_{X \in \{\text{in, up}\}} \left[\Psi_{X, \ell, \omega}(r_*) \hat{a}_{X, \ell, m, \omega, (e)} + (-1)^m \Psi_{X, \ell, \omega}^*(r_*) \hat{a}_{X, \ell, -m, \omega, (e)}^\dagger \right] Y_{\ell, m}(\theta, \phi) \quad \text{and} \\
 A_\theta \pm \frac{i}{\sin\theta} A_\phi &= \mp \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} \frac{d\omega}{2\pi} \frac{1}{\sqrt{2\omega}} \sum_{X \in \{\text{in, up}\}} \left\{ \frac{1}{\omega} \Psi'_{X, \ell, \omega}(r_*) \hat{a}_{X, \ell, m, \omega, (e)} + (-1)^m \frac{1}{\omega} \Psi'_{X, \ell, \omega}{}^*(r_*) \hat{a}_{X, \ell, -m, \omega, (e)}^\dagger \right. \\
 &\left. \pm i \left[\Psi_{X, \ell, \omega}(r_*) \hat{a}_{X, \ell, m, \omega, (o)} + (-1)^m \Psi_{X, \ell, \omega}^*(r_*) \hat{a}_{X, \ell, -m, \omega, (o)}^\dagger \right] \right\} \pm Y_{\ell, m}(\theta, \phi)
 \end{aligned}$$

Electron field operator:

$$\begin{aligned}
 \psi(r, \theta, \phi) &= \int_0^{\infty} \frac{dh}{2\pi} \frac{1}{\sqrt{2h}} \sum_{Xkm} \frac{1}{r(1 - 2M/r)^{1/4} \sqrt{\sin\theta}} \left\{ [F_{X, k, h}(r) \Theta_{k, m}^{(F)}(\theta, \phi) + G_{X, k, h}(r) \Theta_{k, m}^{(G)}(\theta, \phi)] \hat{b}_{X, k, m, h} \right. \\
 &\left. + [G_{X, -k, h}^*(r) \Theta_{k, m}^{(F)}(\theta, \phi) + F_{X, -k, h}^*(r) \Theta_{k, m}^{(G)}(\theta, \phi)] \hat{d}_{X, k, m, h}^\dagger \right\}
 \end{aligned}$$

Radial equations described by Brill & Wheeler (1957).

The $ee\gamma$ “vertex” now depends on 3-mode angular integrals (Clebsch-Gordan coefficients) and 3-mode radial integrals.

Hawking radiation spectrum in QED

We find that there are two types of corrections to the particle number in the “out” states. Both types contain UV or IR divergences that need to be addressed.

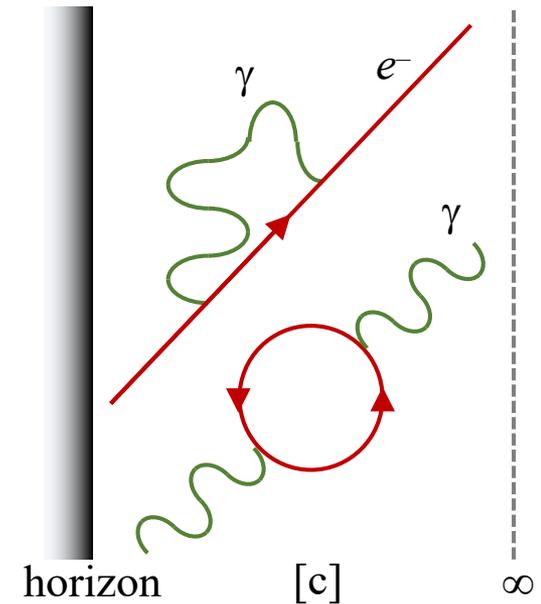
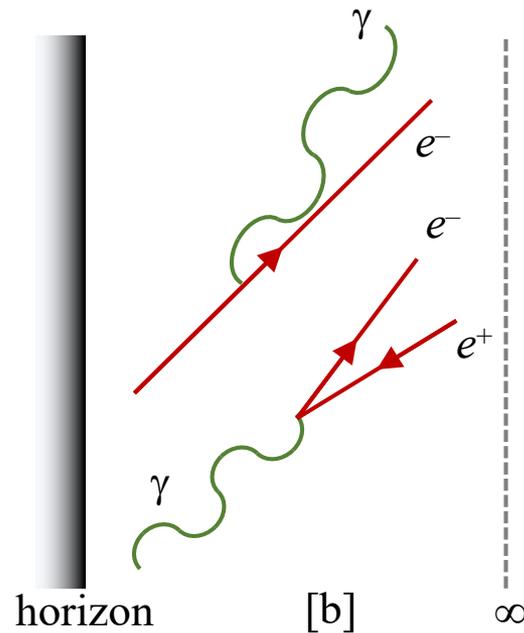
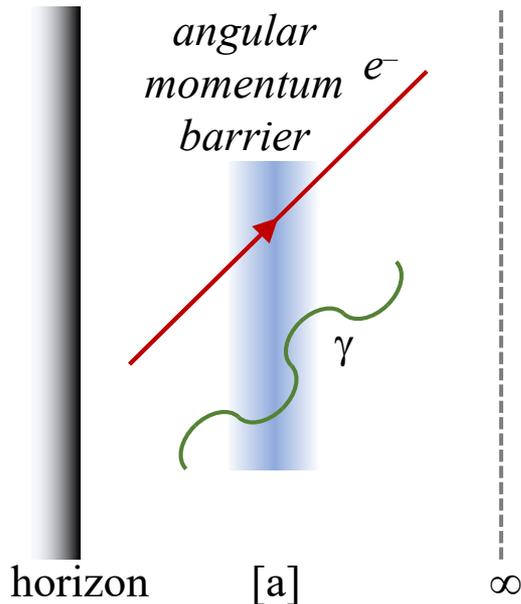
Conventional calculation
(e.g., Page 1977)

$O(\alpha)$ corrections
(ongoing work)

Free fields
(spectrum given by barrier transmission probability)

Dissipative terms
(change number of particles)

Conservative terms
(change transmission probability)



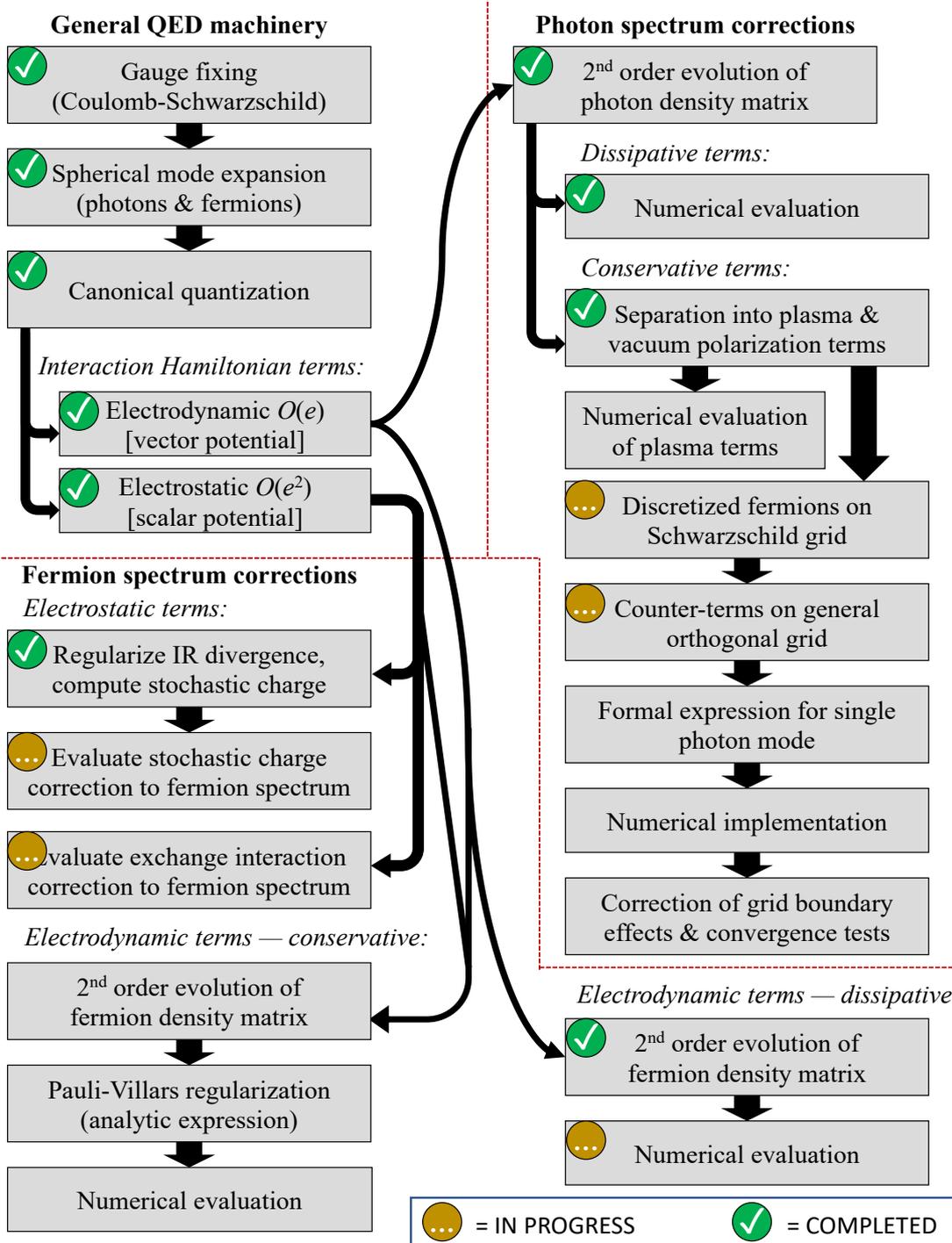
Progress toward the complete $O(\alpha)$ spectra

Silva et al.
PRD 107:045004
(2023)

Vasquez et al.
PRD (accepted)

Vaquez et al.
(in prep)

Nel et al.
(in prep)



Koivu et al.
PRD 111:045011
(2025)

Koivu et al.
(in prep)

Chen et al.
(in prep)

Corrections to photon spectrum

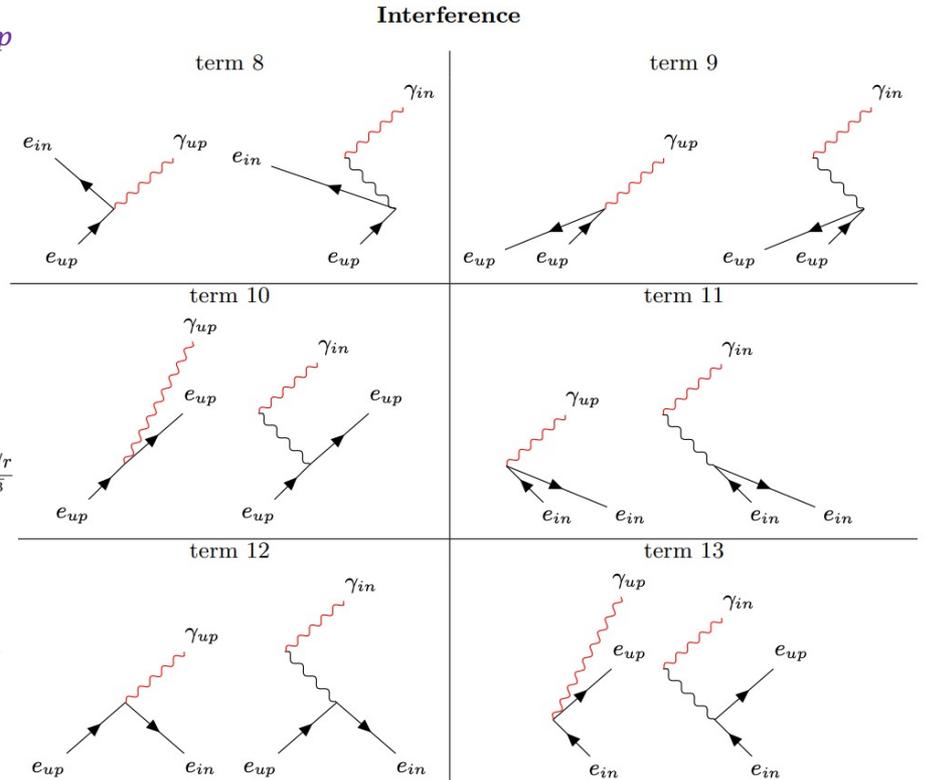
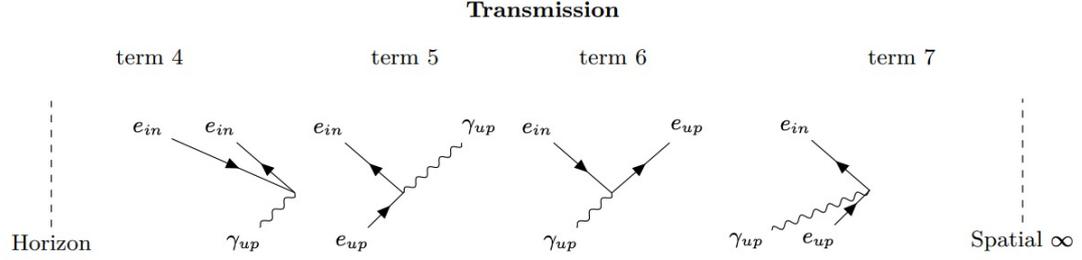
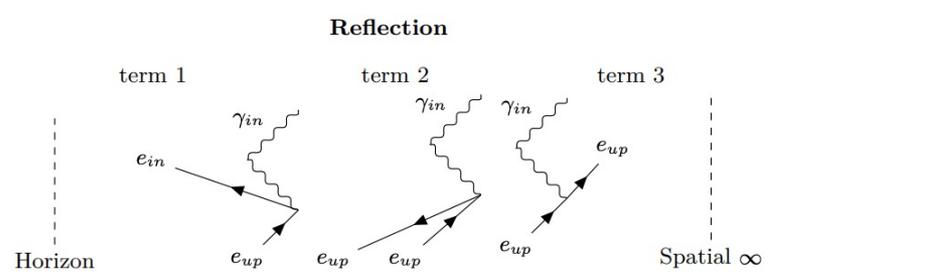
- Reduces to 13 terms. Example:

$$\left. \frac{dN}{d\omega dt} \right|_{\text{term 1}} = \frac{e^2}{\pi} \sum_{\ell p k k'} \int \frac{dh}{2\pi} \Delta(j, j', \ell) \delta_{ss', (-1)^{j-j'+\ell+p}}$$

mode sum → $\sum_{\ell p k k'}$
angular momentum & parity conservation → $\Delta(j, j', \ell) \delta_{ss', (-1)^{j-j'+\ell+p}}$
reflection coefficient → $\frac{|R_{1,\ell,\omega}|^2}{e^{8\pi M(h+\omega)} + 1}$
thermal phase space density → $\left[\left[I_{\text{in},k,\text{up},k',\text{in},\ell,(p)}^-(h, h+\omega, \omega) \right] \right]^2$
3-mode integral → $I_{\text{in},k,\text{up},k',\text{in},\ell,(p)}^-(h, h+\omega, \omega)$

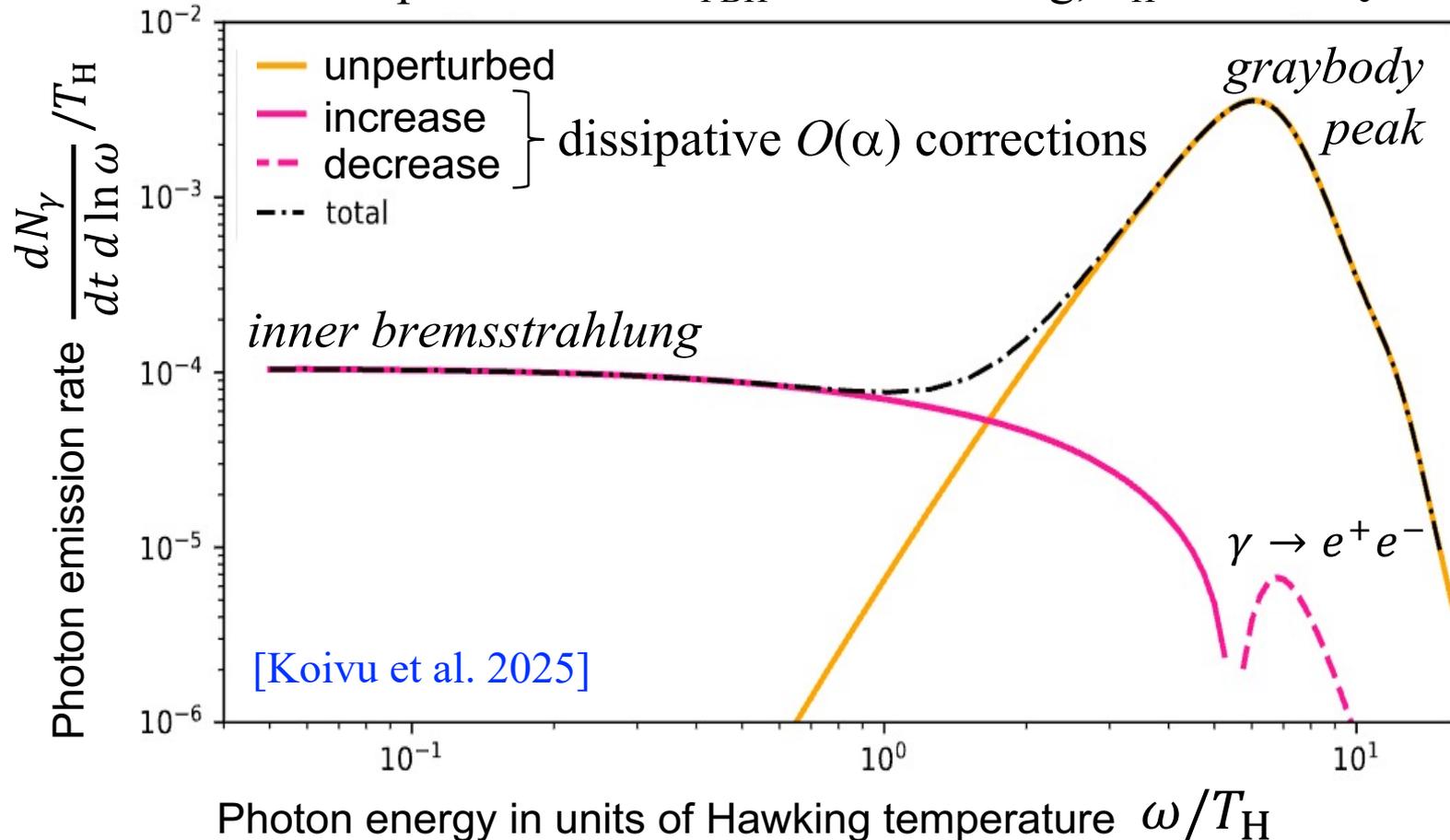
$$I_{Xkm, X'k'm', X, \ell m_\gamma(e)}^-(h, h', \omega) = \frac{-i}{\sqrt{4hh'}} \Delta_{mm'm_\gamma}^{kk'l} \int_{-\infty}^{\infty} \left[(G_{X-kh} F_{X'-k'h'}^* - F_{X-kh} G_{X'-k'h'}^*) \Psi_{X,\ell\omega} \sqrt{\ell(\ell+1)} \frac{1-2M/r}{r^2 \sqrt{2\omega^3}} \right. \\ \left. + (G_{X-kh} F_{X'-k'h'}^* + F_{X-kh} G_{X'-k'h'}^*) \frac{k-k'}{\sqrt{\ell(\ell+1)}} \frac{1}{\omega} \Psi_{X,\ell\omega} \frac{\sqrt{1-2M/r}}{r\sqrt{2\omega}} \right] dr_*$$

$$\Delta_{mm'm_\gamma}^{kk'l} \equiv \frac{(-1)^{m+1/2}}{2} \sqrt{\frac{(2j+1)(2j'+1)(2\ell+1)}{4\pi}} \begin{pmatrix} j & j' & \ell \\ -m & m' & m_\gamma \end{pmatrix} \begin{pmatrix} j & j' & \ell \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} (1 + ss'(-1)^{j-j'+\ell})$$



Results: dissipative terms (for photon spectrum)

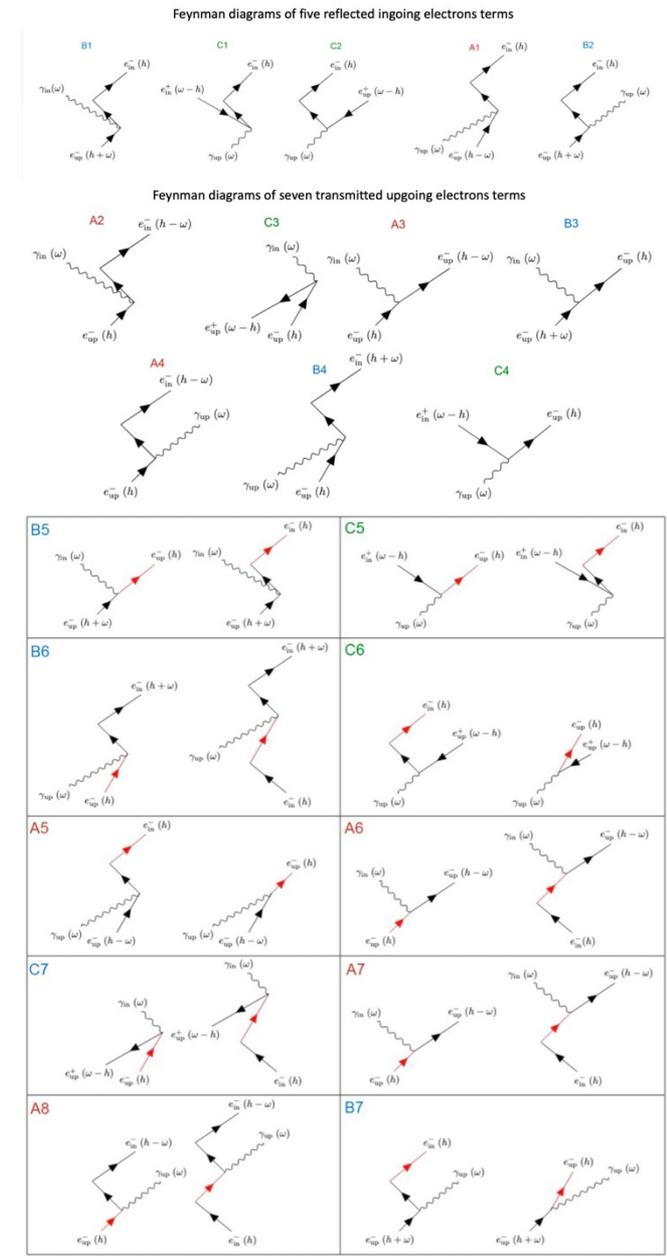
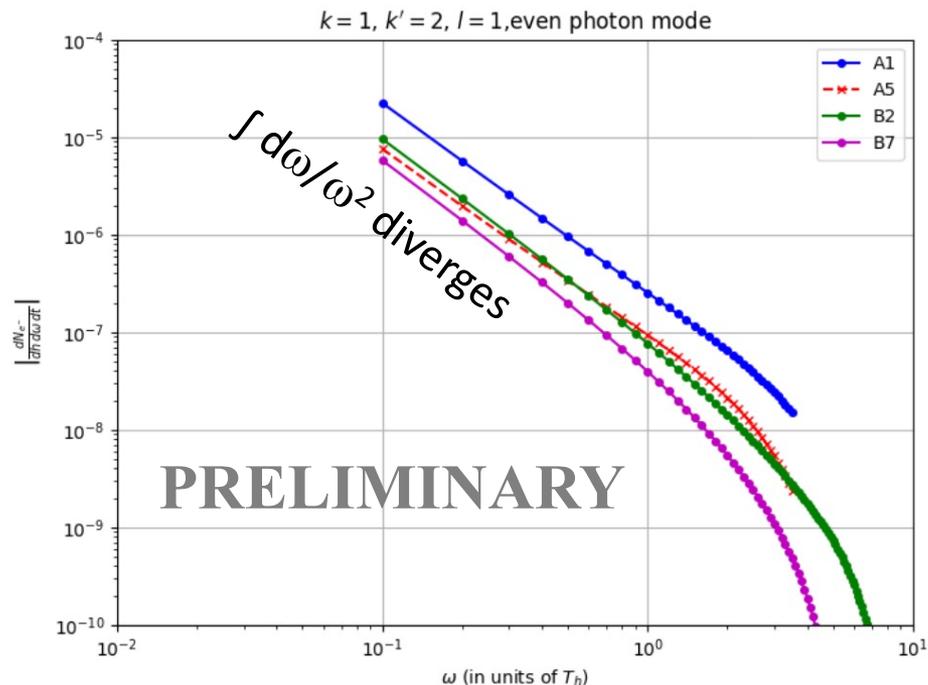
Emission Spectrum for $M_{\text{PBH}} = 2.17 \times 10^{16}$ g, $T_{\text{H}} = 0.97 m_e c^2$



- ✓ This was a major numerical effort, but no infinities / worrisome issues arise.
- ✓ Inner bremsstrahlung emerges in the full QED/Schwarzschild treatment and approaches the classical flat spacetime result in the limit of low ω .

Dissipative terms in electron spectrum [Chen et al., in prep]

- There are 22 terms in the correction to the electron spectrum.
- 4 of these terms are IR-divergent, picking up an *infinite contribution* from interaction with the low-energy (Rayleigh-Jeans) radiation from the black hole.
- We believe this is due to an infinite phase shift picked up from the horizon to the angular momentum barrier — working on the best way to handle this in our calculation.



Stochastic charge effect

- Semi-classical statement: as a black hole emits e^\pm , its charge (analogous to atomic number Z) undergoes a random walk (IR divergence).
- Page (1979): this random walk is biased toward zero
 - ✓ Positive [**negative**] black holes more likely to emit e^+ [e^-]
 - ✓ Results in a finite variance $\langle Z^2 \rangle$ of order $O(1/\alpha)$.
- Quantum statement: define an “effective charge” operator \hat{Z} .

$$-e\hat{Z} = \hat{Q}_- + \int \frac{2M}{r} : \psi^\dagger \psi : \sqrt{1 - \frac{2M}{r}} r^2 dr_* \sin \theta d\theta d\phi$$

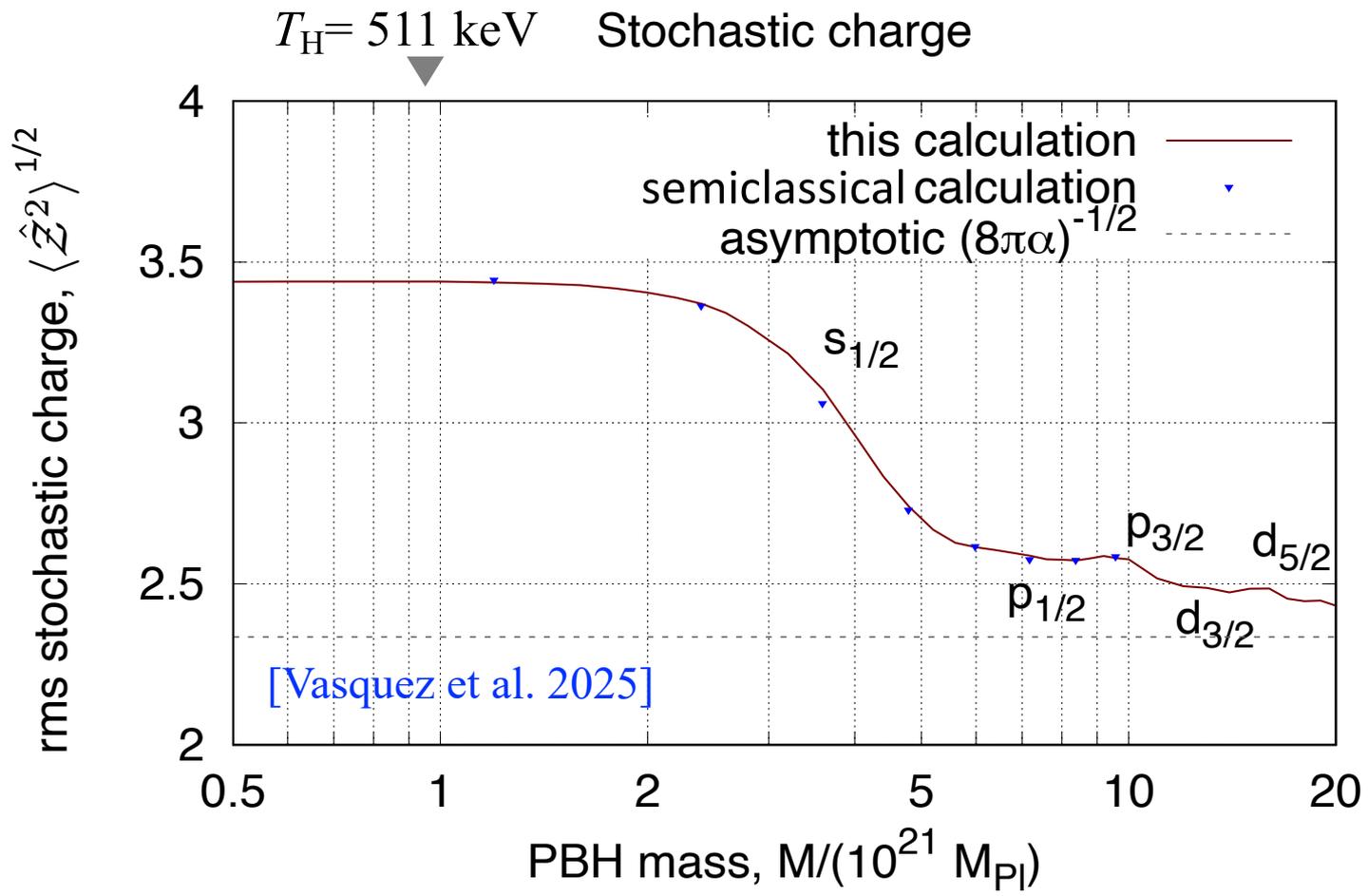
“charge on horizon”
(boundary term added to
Lagrangian)

charge of fermions weighted by $2M/r$

- The fermion contribution to \hat{Z} diverges (infinitely many particles), but the total is well-behaved.

Stochastic charge effect

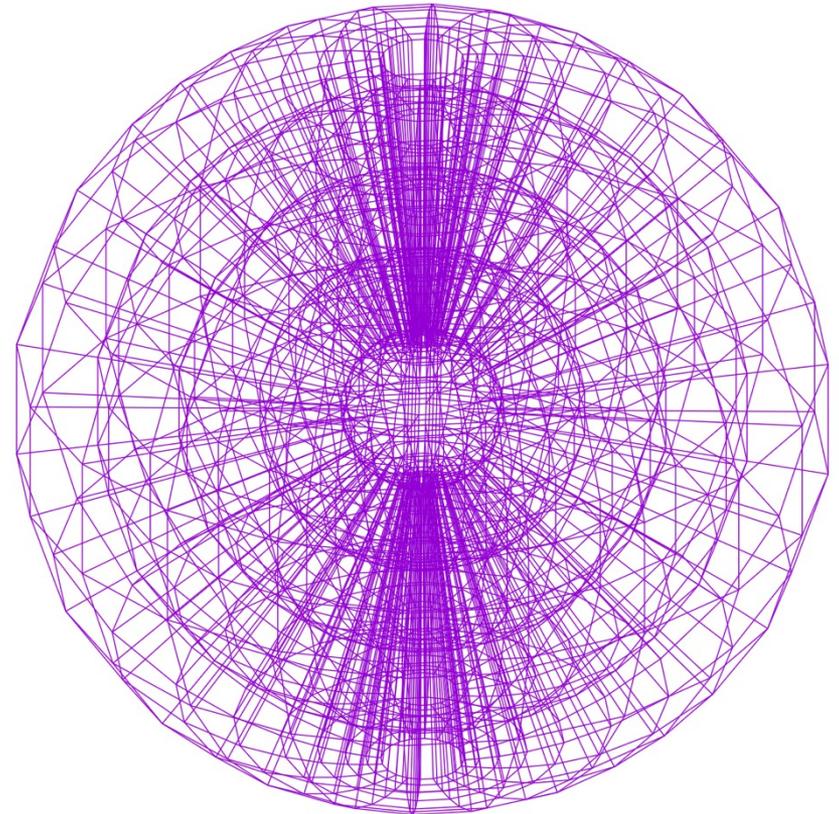
QED produces a solution similar to Page (1979), but with the charge being a quantum operator.



Effect on e^+e^- spectrum coming soon!

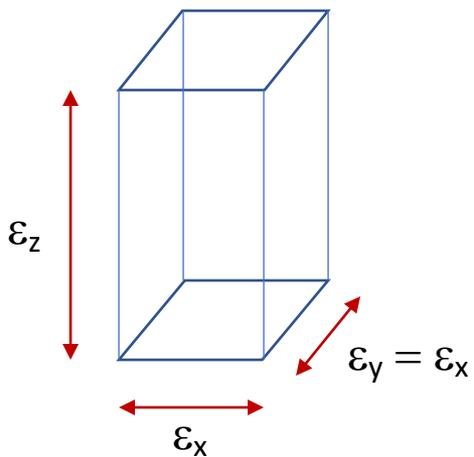
Conservative effects on photon spectrum [Koivu et al., in prep]

- Conservative terms (e.g., vacuum polarization) must be renormalized.
 - ✓ Solve by discretizing the spatial directions — preserves gauge invariance.
- Leave time un-rotated and continuous.
 - ✓ This became more similar to QFT applications in condensed matter than, e.g., lattice QCD.
- Challenges
 - Need 4 spatially varying “bare” susceptibilities (2 electric, 2 magnetic) given symmetries of the grid.
 - But these run only logarithmically.
 - Numerics — maintain discrete symmetry in longitude ϕ ; can use FFT when computing the vacuum polarization diagram of the $m=0$ photon mode.
 - Boundary conditions to avoid spurious reflection in r .

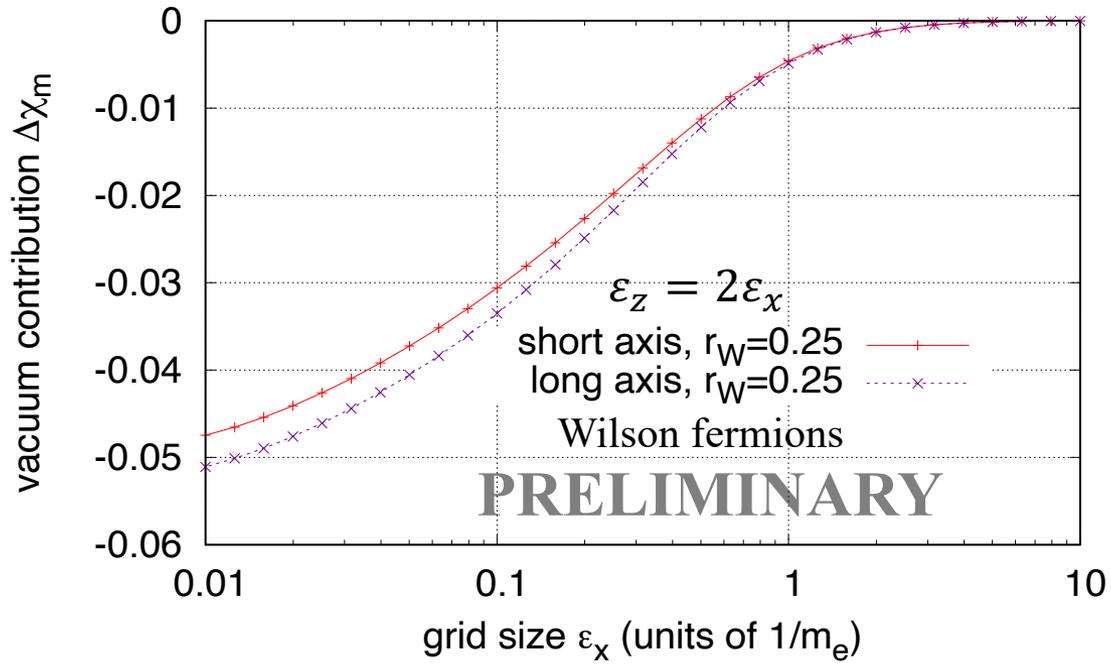


Conservative effects on photon spectrum

- Schwarzschild spacetime can be covered with a (locally) tetragonal grid.
 - ✓ Essentially: diagonal metric, $g_{11}=g_{22}$
 - ✓ “Bare” theory is not Lorentz invariant (speed of light $\neq 1$, birefringent ...)
but renormalized theory can be.
 - ✓ Principles should be OK for other choices, but numerics are easiest with the most symmetric possible cell.
- “Susceptibility of the vacuum” is an integral over 1st Brillouin zone.



Magnetic susceptibility: N=512 box



- Stay tuned!

Summary

- Hawking radiation was solved in the 1970s for non-interacting particles, but real-world particles interact.
- Current PBH constraints are in a temperature range where QED interactions may be expected ($T_H \sim m_e$). Even effects that are formally $O(\alpha)$ can have a qualitative impact on the Hawking spectrum.
- We are partway through the full $O(\alpha)$ computation. This is a technically difficult calculation:
 - ✓ Both UV and IR divergences
 - ✓ Both analytical and numerical challenges due to the reduced symmetry of Schwarzschild vs. Minkowski
- But half a century after the pioneering work of Hawking, Page, ... we think we have the computational methods to do this.