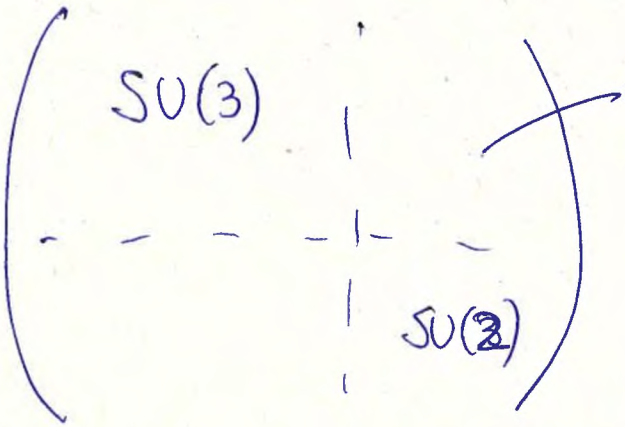
GUT symmetries

$$G_{GUT} \xrightarrow{M_{GUT}} G_{SM} \xrightarrow{M_Z} SU(3)_c \times U(1)_{em}$$

since $\text{rank}(G_{SM}) = 4$ need $\text{rank}(G_{GUT}) \geq 4$

Minimal case $G_{GUT} = SU(5)$. Adjoint rep is 5×5 matrix, traceless such that



mixed $SU(3)/SU(2)$ → "leptoquarks"

$$U(1)_Y \Rightarrow T^{12} = \frac{1}{\sqrt{15}} \text{diag}\left(1, -1, -1, \frac{3}{2}, \frac{3}{2}\right)$$

Matter fields: $\bar{5} = \begin{pmatrix} \bar{3} \\ \bar{2} \end{pmatrix} \rightarrow (\bar{3}, 1, \bar{q}_3)$ $\rightarrow (0, \bar{2}, \bar{q}_2)$

$U(1)_Y$ relations: $-\frac{1}{\sqrt{15}} g_{GUT} = \bar{q}_3 g_1$ $\quad \frac{3}{2\sqrt{15}} g_{GUT} = \bar{q}_2 g_1$

giving for $\bar{q}_2 = 1/2$

$$q_1 = \sqrt{\frac{3}{5}}\, q_{GUT} \quad \& \quad q_3 = -1/3$$

i.e. $\bar{5} = \begin{pmatrix} d_R \\ \bar{\ell}_L \end{pmatrix}$ i.e. $5 = \begin{pmatrix} d_R^c \\ \ell_L \end{pmatrix}$

10 reps is the 5×5 antisymmetric

$$\begin{pmatrix} 0 & u_R^c & | & q_L \\ -u_R^c & 0 & | & \\ - & - & - & e_R^c \\ -q_L & & 0 & \\ & & -e_R^c & 0 \end{pmatrix}$$

Need to break GUT symmetry

→ Adjoint of $SU(5)$ Σ

BUSSTEPP 2025 - Nottingham U

Nottingham, 26 August - 6 September 2025

Beyond the Standard Model II

Laura Covi

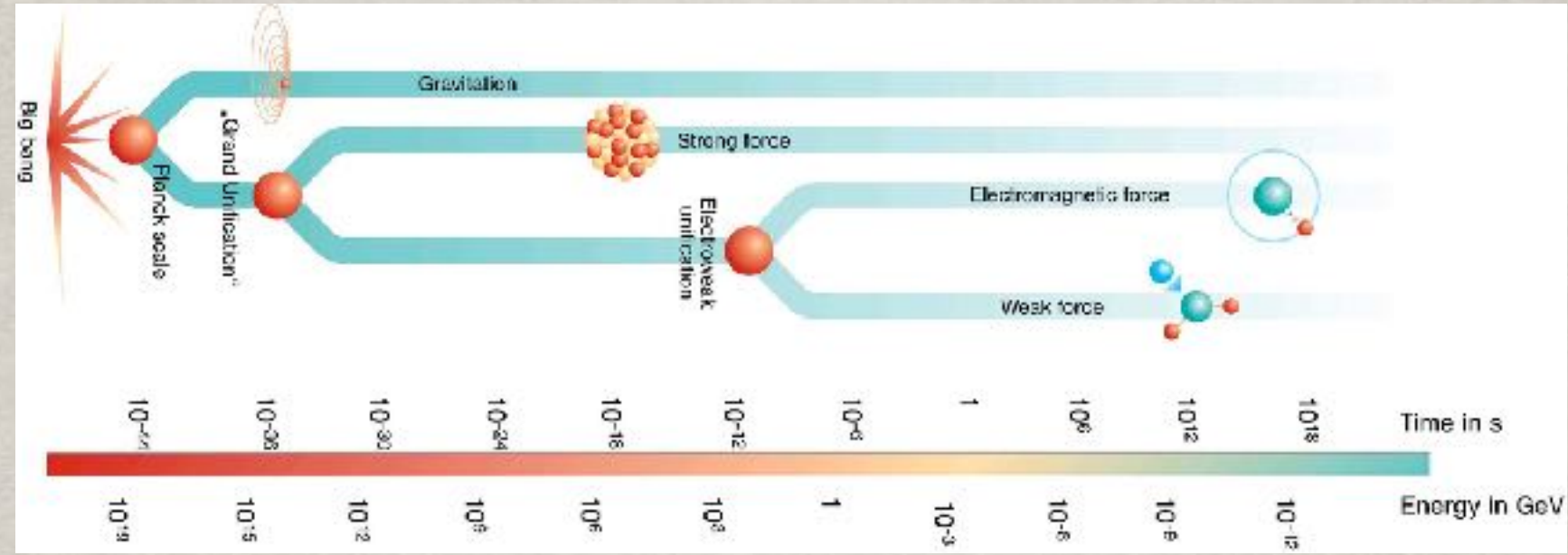
Institute for Theoretical Physics
Georg-August-University Göttingen

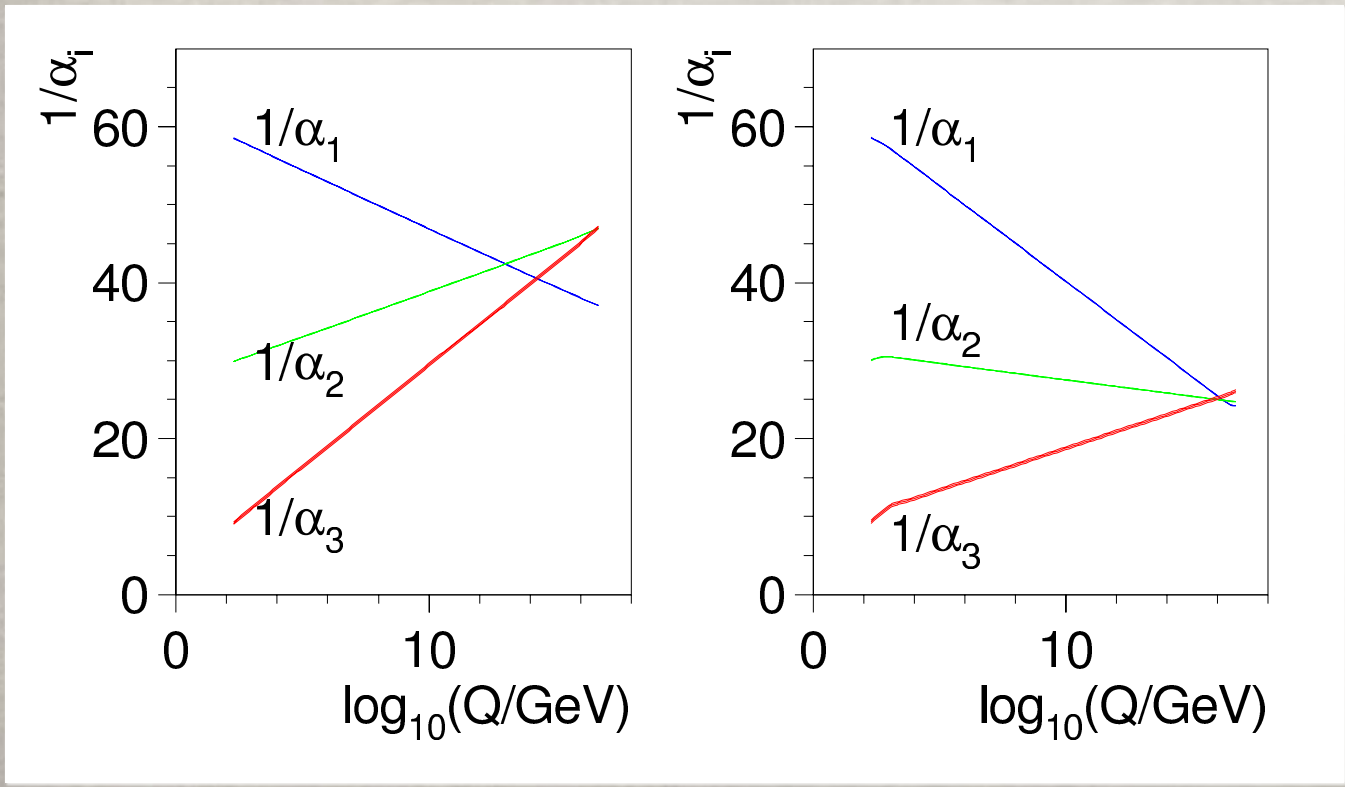
Lecture 4: Outline

- GUT's and proton decay

GUTs and Proton Decay

Long extrapolation...





SMALLEST GUT MODEL

SU(3) x SU(2)xU(1) ⇒ SU(5)

$$\begin{pmatrix} d_r^c \\ d_b^c \\ d_g^c \\ e^- \\ \nu_e \end{pmatrix}$$

RH down quarks and Lepton doublets are arranged in a "Fiveplet" rep, while the LH quark doublets, RH up and electron in a "Tenplet" rep.

The 3x3, 2x2 Matrices of the gauge groups combine in a 5x5 Matrix in Adjoint of SU(5):

$$\begin{pmatrix} g - \sqrt{\frac{2}{15}}B & \mathcal{X} \\ \mathcal{X} & W + \sqrt{\frac{3}{10}}B \end{pmatrix}$$

So there are additional transitions driven by the heavy X gauge bosons -> Proton decay !!!

Proton decay

Like for the Neutron decay we have $\tau_p \propto (G_p^2 m_p^5)^{-1}$

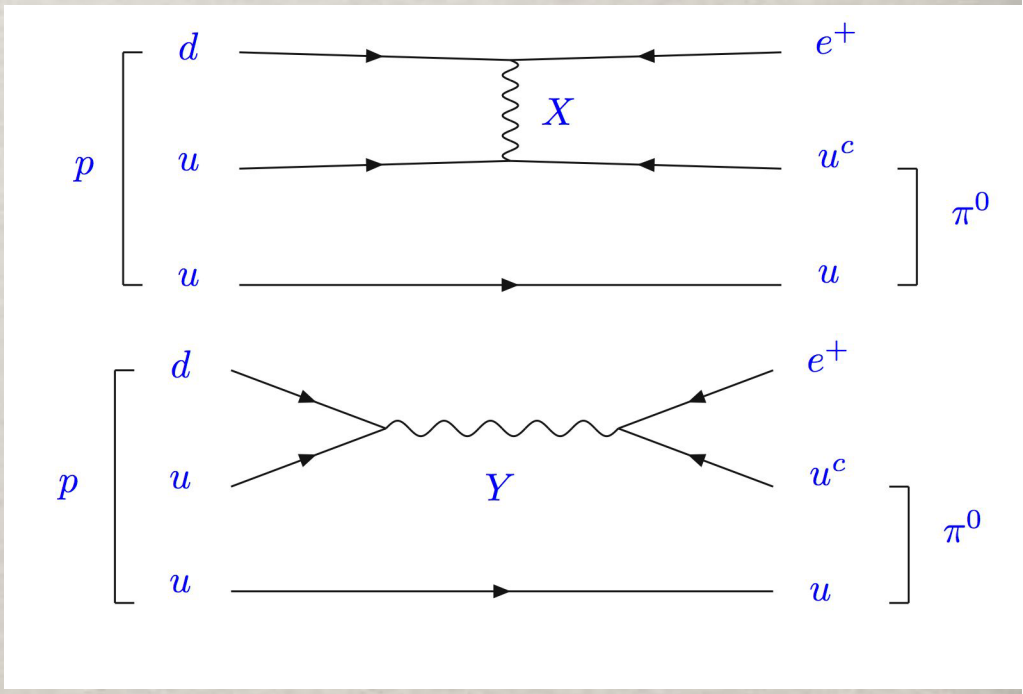
but the effective coupling ist here not G_F …

$$G_p = \frac{g_{GUT}^2}{4\sqrt{2}M_X^2} \sim \frac{\alpha_{GUT}}{M_{GUT}^2}$$

$$\sim 10^{-34} GeV^{-2}$$

So this gives

$$\tau_p \sim 10^{34} y$$



VERY LONG LIFETIME !!!

Measuring Proton decay

The probability that one proton decays in a year is $1 - e^{-10^{-34}} \sim 10^{-34}$

But for N protons, the probabily grows with N !

I need a lot of protons...

How many are there in a litre of water ?

$$\frac{10}{18} \times 10^3 \; N_A \sim 3 \times 10^{26}$$

So we need at least $10^8 L \sim 100 \; kton$ water

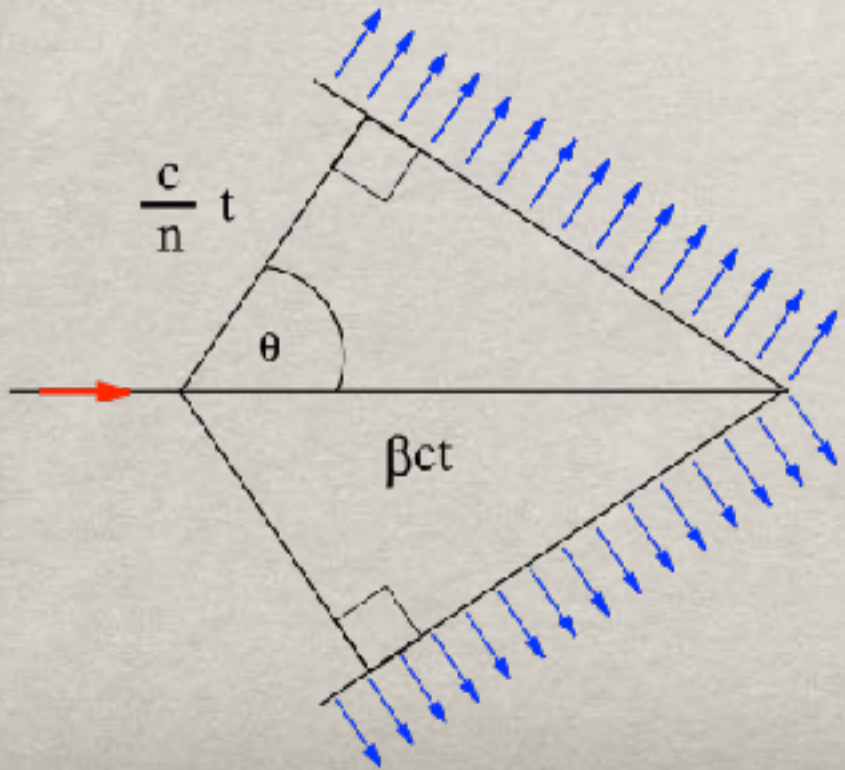
What can we see? The lepton and perhaps also the produced meson (Pion, Kaon) via its decay…

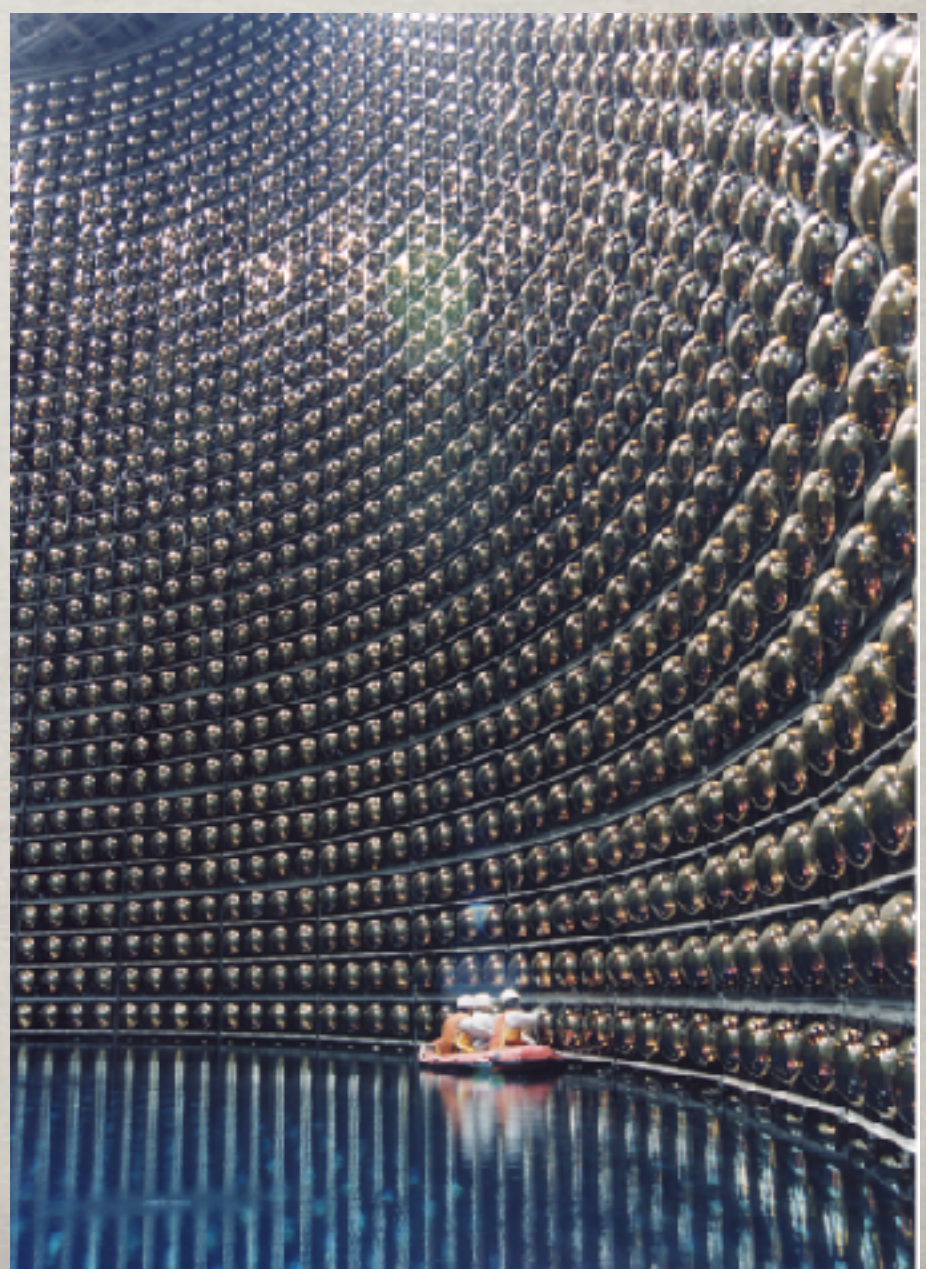
SUPERKAMIOKANDE

SUPERKAMIOKANDE

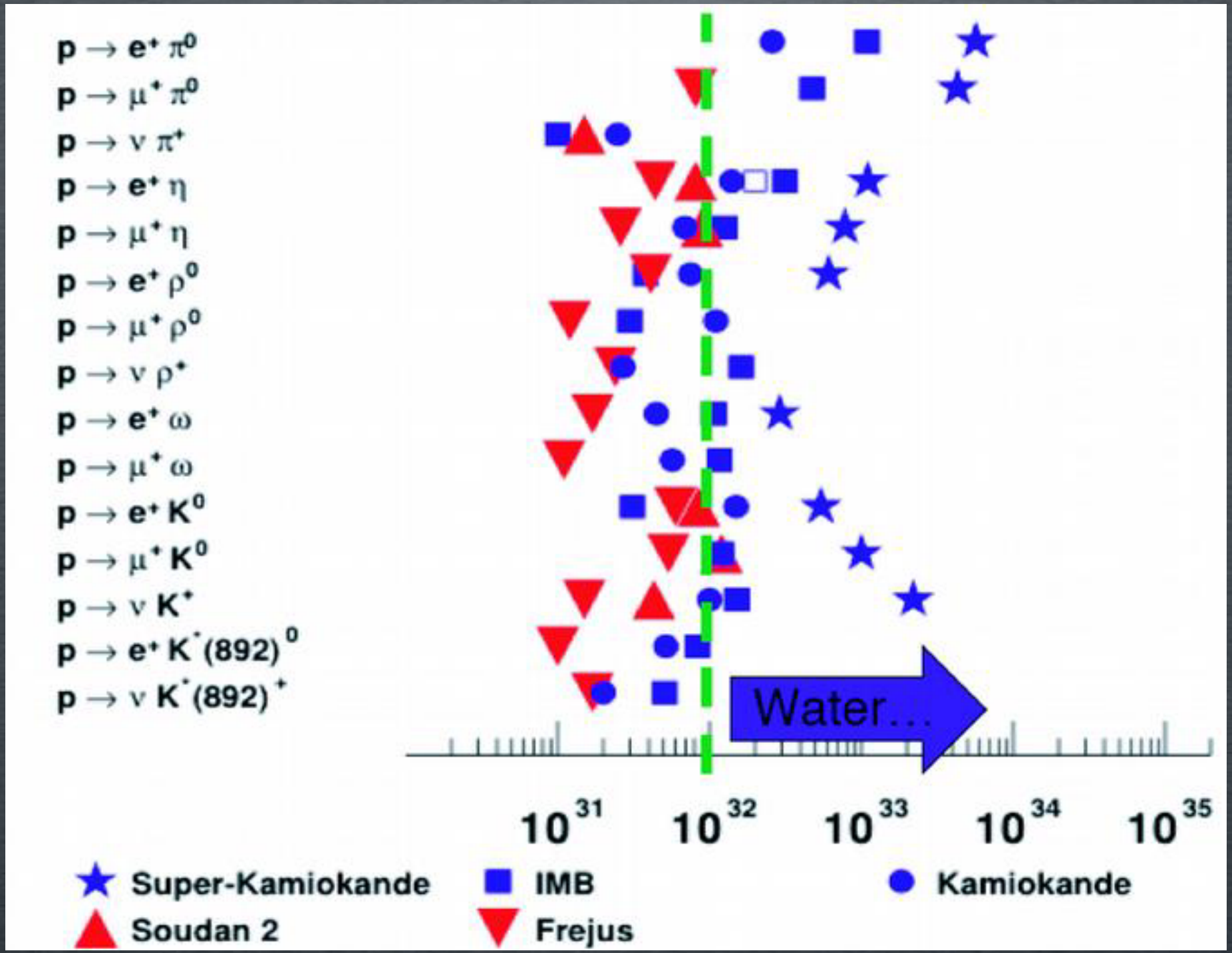
Wasser Cherenkov Detektor with 50 (22.5) kton water

Phototubes on the surface measure the Cherenkov Light of charged particles from the decay





(OLD)Proton decay limits



Conclusions & Outlook

- We have strong evidence for BSM from cosmology and from some SM puzzles, but the right direction is still unclear… It requires to go Beyond the Standard Model, to have DM, inflation and baryogenesis!
- Extending the SM symmetries leads to very different models SUSY, GUTs, etc… and many new phenomenological signatures at colliders but also proton decay…
- Keep looking and developing interesting BSM models !

EFTs & Wilson action

Path integral formulation for 1PI generating functional for light fields l:

$$e^{i\gamma[e]} = \int \mathcal{D}l\,\mathcal{D}H\; e^{i\int d^4x\left[\mathcal{L}(l+1, H) + jl\right]}$$

with external current $j[e] = -\dfrac{\delta\gamma[e]}{\delta e}$ only for the light field & not for heavy fields H

Now divide the functional integral into low/high energy $\mathcal{D}l\,\mathcal{D}H = [\mathcal{D}l]_{l.e.}[\mathcal{D}l\,\mathcal{D}H]_h$ for some intermediate scale & perform the integration in the high energy modes (integrate them out):

$$e^{i\gamma[e]} = \int [\mathcal{D}l]_{l.e.}\; e^{i\int d^4x\left[\mathcal{L}_W(l+l_{l.e.}) + j l_{l.e}\right]}$$

where $S_W = \int d^4x\,\mathcal{L}_W$ is the Wilson action

$$e^{iS_W[\ell+l_{e.e}]} = \int [\mathcal{D}l\,\mathcal{D}H]_{h.e.}\; e^{iS[\ell+l, H]}$$

⟶ action only for low energy modes

⟶ local action if expanded in inverse powers of heavy scales

$$\frac{1}{-\Box + M^2} = \frac{1}{M^2} + \frac{\Box}{M^4} + \dots$$

local contact interactions

S_W contains all possible interactions for the light fields consistent with the low energy symmetries

Ref.: hep-th/0701053

C.P. Burgess - Introduction to EFT