BUSSTEPP 2025

New directions for BSM: larger symmetries!

SM symmetry groups are

$$\mathbb{R}^{1,3} \rtimes SO(1,3) \times SU(3)_c \times SU(2)_L \times U(1)_Y$$

global, non-compact Poincaré group — space-time symmetry

local internal gauge symmetries

→ Supersymmetry (SUSY)

→ Supergravity (SUGRA)

→ Grand Unification $SU(5)$ / $SO(10)$ ....

Let us consider first SUSY & let us have a look at the Poincaré group:

$\mathbb{R}^{1,3}$ → translations in 4D

$SO(1,3)$ → Lorentz symmetry { 3D rotations; 3D Lorentz boosts }

⇒ 10 dimensional group

Generators:

$$\hat{M}_{\mu\nu} = i\left(x_\mu \partial_\nu - x_\nu \partial_\mu\right) + \hat{S}_{\mu\nu}$$

"angular momentum" Spin

$$\hat{P}_\mu = -i\partial_\mu$$

General transformation: $\hat{U}(\theta, a) = e^{-\frac{i}{2}\theta_{\mu\nu}\hat{M}^{\mu\nu} - i a_\mu P^\mu}$

with $\theta_{\mu\nu} = \begin{cases} \theta_{ij} = \varepsilon_{ijk}\theta_k & \text{Euler angles} \\ \theta_{0i} = \beta_i = v_i/c & \text{boost velocities} \end{cases}$

$a_\mu = (a_0, \vec{a})$ shift vector

The Poincare algebra closes:

$$\left[\hat{P}_\mu, \hat{P}_\nu\right] = 0$$

$$\left[\hat{M}_{\mu\nu}, \hat{P}_\rho\right] = -i\eta_{\mu\rho}\hat{P}_\nu + i\eta_{\nu\rho}\hat{P}_\mu$$

$$\left[\hat{M}_{\mu\nu}, \hat{M}_{\rho\sigma}\right] = i\eta_{\nu\rho}\hat{M}_{\mu\sigma} - i\eta_{\mu\rho}\hat{M}_{\nu\sigma} - i\eta_{\nu\sigma}\hat{M}_{\mu\rho} + i\eta_{\mu\sigma}\hat{M}_{\nu\rho}$$

Casimir operators are $\hat{P}_\mu \hat{P}^\mu$ & $\hat{W}_\mu \hat{W}^\mu$

with $\hat{W}^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \hat{P}_\nu \hat{M}_{\rho\sigma}$ Pauli-Lubanski pseudovector

⇒ the representations are labelled by the eigenvalues of the two operators $\hat{P}^2$ & $\hat{W}^2$

Physically relevant reps:

1. Massive states with definite spin

$\hat{P}_\mu \hat{P}^\mu \longrightarrow m^2 > 0$ mass

$\hat{W}_\mu \hat{W}^\mu \longrightarrow -m^2 s(s+1)$ with $s = 0, \frac{1}{2}, 1, \ldots$

2. Massless states defined via helicity since they have $\hat{W}_\mu = h \hat{P}_\mu$ with $h = 0, \pm\frac{1}{2}, \pm 1, \ldots$ & both eigenvalues vanish.

Lorentz group reps: $SO(1,3) \sim SO(4) \sim SU(2) \times SU(2)$ with two copies of the $SU(2)$ group. Indeed we can identify two sets of $SU(2)$ generators

$\vec{J} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12})$ angular momentum

$\vec{K} = (\hat{M}^{10}, \hat{M}^{20}, \hat{M}^{30})$ boosts

Define: $\hat{\vec{A}} = \frac{1}{2}\left(\hat{\vec{J}} + i\hat{\vec{K}}\right)$ both satisfy the SU(2) algebra

$\hat{\vec{B}} = \frac{1}{2}\left(\hat{\vec{J}} - i\hat{\vec{K}}\right)$

$$\left[\begin{matrix}\hat{A}^i, \hat{A}^j \\ \hat{B}^i, \hat{B}^j\end{matrix}\right] = i\,\varepsilon^{ijk}\begin{matrix}\hat{A}^k \\ \hat{B}^k\end{matrix} \qquad \left[\hat{A}^i, \hat{B}^j\right] = 0$$

So we have two different spin $\frac{1}{2}$ reps:

$\left(\frac{1}{2}, 0\right) \longrightarrow$ LH spinors ψ (Weyl spinors)

$\left(0, \frac{1}{2}\right) \longrightarrow$ RH spinors $\bar{\psi}$

In SUSY notation $\psi_\alpha, \bar{\psi}^{\dot{\alpha}}$ <u>different indices</u>

Note that $\psi^c = -i\sigma^2\psi^* = \bar{\psi}$

Can we extend the Poincaré algebra?

→ Coleman-Mandula theorem: no extension with bosonic symmetries (apart internal symmetries) only direct products are allowed

Loophole: Haag-Łopuszański-Sohnius theorem: introduce fermionic generators in a non-trivial reps of SO(1,3)

Simplest choice add two fermionic generators

$$\hat{Q}^{\alpha} \in \left(\tfrac{1}{2},0\right) \quad \& \quad \hat{\bar{Q}}_{\dot{\alpha}} \in \left(0,\tfrac{1}{2}\right)$$

with commutators:

$$\left[\hat{Q}^{\alpha}, \hat{P}^{\mu}\right] = 0 = \left[\hat{\bar{Q}}_{\dot{\alpha}}, \hat{P}^{\mu}\right]$$

trivial transf under translation

$$\left[\hat{Q}^{\alpha}, \hat{M}^{\mu\nu}\right] = (\sigma^{\mu\nu})^{\alpha}{}_{\beta}\hat{Q}^{\beta}$$

$$\left[\hat{\bar{Q}}_{\dot{\alpha}}, \hat{M}^{\mu\nu}\right] = (\bar{\sigma}^{\mu\nu})_{\dot{\alpha}}{}^{\dot{\beta}}\hat{\bar{Q}}_{\dot{\beta}}$$

where

$$\sigma^{\mu\nu} = \frac{i}{4}\left(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}\right)$$

$$\bar{\sigma}^{\mu\nu} = \frac{i}{4}\left(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}\right)$$

for $\sigma^{\mu} = (\mathbb{I}, \vec{\sigma})$ & $\bar{\sigma}^{\mu} = (\mathbb{I}, -\vec{\sigma})$

$$\left\{\hat{Q}_{\alpha}, \hat{\bar{Q}}^{\dot{\beta}}\right\} = 2(\sigma^{\mu})_{\alpha}{}^{\dot{\beta}}\hat{P}_{\mu}$$

since $\left(\tfrac{1}{2},0\right)\otimes\left(0,\tfrac{1}{2}\right) = \left(\tfrac{1}{2},\tfrac{1}{2}\right)$ vector

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = 0 = \{\hat{\bar{Q}}^{\dot\alpha}, \hat{\bar{Q}}^{\dot\beta}\}$$

[For more charges $\hat{Q}_\alpha^i$ & $\hat{\bar{Q}}_i^{\dot\beta}$ $i = 1, \ldots N$

instead $\{\hat{Q}_\alpha^i, \hat{Q}_\beta^j\} = \varepsilon_{\alpha\beta} Z_{ij}$ antisymmetric

$$\{\hat{\bar{Q}}_i^{\dot\alpha}, \hat{\bar{Q}}_j^{\dot\beta}\} = -\varepsilon^{\dot\alpha\dot\beta} (Z^+)_{ij}$$

]

Define the SUSY ground state as

$$\hat{Q}_\alpha |0\rangle = 0 = \hat{\bar{Q}}^{\dot\beta} |0\rangle$$

$$\Rightarrow \langle 0 | \{\hat{Q}_\alpha, \hat{\bar{Q}}^{\dot\beta}\} | 0\rangle = 0 =$$

$$= 2 (\sigma^\mu)_\alpha{}^{\dot\beta} \langle 0 | \hat{P}_\mu | 0\rangle$$

Consider the trace $\mathrm{tr}\,\sigma^0 = 2$ $\mathrm{tr}\,\sigma^i = 0$

$$\Rightarrow 4 \langle 0 | \hat{P}_0 | 0\rangle = 4 \langle 0 | \hat{H} | 0\rangle = 0$$

SUSY vacuum has zero energy

no Λ for unbroken SUSY

SUSY generators are like ladder operators in QM & allow to construct the states of a full SUSY multiplet: start with ℂ scalar φ

$$\hat{Q}_\alpha[\varphi] = \psi_\alpha \quad \text{LH fermion}$$

$$\hat{Q}_\alpha \hat{Q}_\beta[\varphi] = \hat{Q}_\alpha[\psi_\beta] = \varepsilon_{\alpha\beta} F + \text{deriv.}$$

SUSY multiplets contain particles with different spin, e.g.

$$SU(2)\begin{pmatrix}\nu_L\\ e_L\end{pmatrix} \to \hat{W}^\pm \qquad SUSY\begin{pmatrix}\varphi\\ \psi_\alpha\\ \overline{F}\end{pmatrix} \to \hat{Q}_\alpha,\ \hat{\bar{Q}}^{\dot\alpha}$$

Simplest multiplet: chiral multiplet

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta_\alpha \psi^\alpha(y) + \theta^2 F(y)$$

ℂ scalar — Weyl fermion — auxiliary field

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ for $\theta, \bar{\theta}$ Grassman variables → superfield formalism

Then a model is defined by

$K(\bar{\Phi}, \Phi)$: Kähler potential, $\mathbb{R}$ function of Φ

$W(\Phi)$: superpotential, holomorphic function of Φ

$K(\bar{\Phi}, \Phi) = \bar{\Phi}\Phi \longrightarrow$ Kähler metric $K_{\bar{i}j} = \frac{\partial K}{\partial \bar{\phi}_j \partial \phi_i}$

determines the kinetic terms

$$\Rightarrow K_{\bar{i}j}\left[\bar{F}^i F^j + \frac{i}{2}\bar{\psi}^i \bar{\sigma}^\mu \partial_\mu \psi + \right.$$

$$\left. + \frac{i}{2}\psi^i \sigma^\mu \partial_\mu \bar{\psi}^j + \partial_\mu \varphi_i^* \partial^\mu \varphi_j \right]$$

$W = \frac{m}{2}\phi^2 + \frac{\lambda}{3!}\phi^3$ determines the potential & Yukawa terms

$$V(\varphi) = \left|\frac{\partial W}{\partial \phi}\right|^2_{\phi=\varphi}$$

Fermion couplings: $\frac{\partial W}{\partial \phi_i \partial \phi_j}\psi_i \psi_j + \text{h.c.}$

More complex construction for the gauge fields

$(A_\mu, \lambda^\alpha, D)$ → vector multiplet

- A_μ: gauge field
- λ^α: Majorana fermion
- D: auxiliary field

⇒ MSSM: Matter fields / Higgs → chiral multiplets

Gauge fields → vector multiplets

Moreover need two Higgs doublet of opposite charges / reps due to holomorphicity of W & anomaly cancellation for fermionic partners

⇒ Type II 2HDM

$\tilde{h} \longrightarrow H_u\,(h_u, \tilde{h}_u)$

$h \longrightarrow H_d\,(h_d, \tilde{h}_d)$

$$W = \mu H_u H_d + Y_u Q H_u U + Y_d Q H_d D + Y_e L H_d E$$

$$(+ \mu_i H_u L_i + \lambda' QLD + \lambda LLE + \lambda'' UDD)$$

Ref.: Stephen P. Martin – A Supersymmetry primer
hep-ph/9709356

M.F. Sohnius – Introducing Supersymmetry
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