BSM exercises

0. **Discussion**

   a) Discuss what are possible "portals" between the SM and DM for the case of a scalar DM candidate, e.g. possible interactions of dimension less than 6 between DM and the SM fields.

   b) Add to the SM three RH neutrinos and the Yukawa coupling

   $$Y_{ij}\bar{N}LH$$

   and discuss the interactions that arise from integrating out the heavy RH neutrinos.

1. **Two Higgs Doublets Model**

   Let us consider an $SU(2)_L \times U(1)_Y$ gauge theory with two complex Higgs doublets $H_{1,2}$ in the same representation under the gauge group.

   a) Write down the most general renormalizable potential $V(H_1, H_2)$ for the two doublets and discuss the conditions arising from CP conservation and from additional symmetries in the scalar sector as Z_2-Parity defined as $P_{Z_2}(H_i) = (-1)^{i-1}H_i$, or an $O(2)$-symmetry between the doublets (i.e. (H_1, H_2) transform as a fund representation of O(2)).

   b) Give the kinetic terms of the two Higgs doublets and compute the mass terms of the gauge bosons, assuming aligned real vacuum expectation values v_i's in the neutral components.

   c) Consider the simplest O(2)-symmetric potential and compute the mass matrix for the real scalar components of the neutral Higgs. Compare the mass eigenstates with the v.e.v directions and discuss the difference with the general case.

2. **Wess-Zumino model and non-renormalization theorem**

   Consider the simplest supersymmetric model with only one chiral multiplet, the Wess-Zumino model, given by the superpotential

   $$W = \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{3!}\Phi^3$$

   and canonical kinetic term with Kähler potential $K(\Phi, \bar{\Phi}) = \bar{\Phi}\Phi$. Here we choose $m > 0$ and $\lambda > 0$ and both real.

a) Write down the lagrangian for the scalar and fermionic components of the chiral multiplets.

b) Derive the Feynman rules for the interactions in the model and give the expressions for all vertices.

c) Compute the one-loop corrections to the mass term of the scalar field in the limit $m = 0$. Verify that the sum of all terms does not contain any quadratic divergence in the UV and that it exactly vanishes.

Hint: Note that to single out the mass correction You can take the external momentum to vanish.

d) Consider now that the mass of the scalar field is up-lifted by a supersymmetry breaking term $m^2_{soft}\varphi^2$ and show that this leads to a non-vanishing logarithmically divergent mass correction proportional to the soft mass itself (or generically to the mass difference between the scalar and fermionic component).