BSM exercises

1. **Discussion**

   a) What is the difference between a renormalizable and a non-renormalizable model ? How do UV divergences appear in these type of models ?

   b) What does it mean that a symmetry is anomalous ? Discuss how anomalies can arise in QFT.

2. **Anomalies in the SM**

   Consider the theory of nature which is the standard model (SM) of particle physics. The gauge group of the SM consist of $SU(3)_C \times SU(2)_L \times U(1)_Y$ where C, L and Y correspond to color charge, left handed and hyper charge, respectively. We later extend the SM by a global $U(1)_{B+L}$ gauge symmetry where B is the baryon number and L is the lepton number. The representations and charge assignments under the different gauge group are given in Table **??**.

Gauge	Baryon Fields			Lepton Fields			Scalar Field
Group	$Q_L^i = (u_L^i, d_L^i)^T$	u_R^i	d_R^i	$L_L^j = (\nu_L^j, e_L^j)^T$	e_R^j	ν_R^j	ϕ_h
$SU(3)_C$	3	3	3	1	1	1	0
$SU(2)_L$	2	1	1	2	1	1	2
$U(1)_Y$	1/6	2/3	$-1/3$	$-1/2$	-1	0	1/2
$U(1)_{B+L}$	1/3	1/3	1/3	1	1	1	0

Table 1: SM particle content, together with the RH neutrinos, and their corresponding representations and charges under various symmetry groups.

In the Table **??**, $u_{L,R}^i = u_{L,R}, c_{L,R}, t_{L,R}$ and $d_{L,R}^i = d_{L,R}, s_{L,R}, b_{L,R}$ correspond to up, charm, top, down, strange, bottom quarks whereas for lepton fields $\nu_L^j = \nu_L^e, \nu_L^\mu, \nu_L^\tau$ and $e_{L,R}^j = e_{L,R}, \mu_{L,R}, \tau_{L,R}$ are electron neutrino, mu neutrino and tauon neutrino, electron, muon and tau leptons, respectively. Note that the RH neutrinos are not usually considered as part of the SM.

(a) Show that the SM is gauge anomaly free by considering different gauge combinations for A_1, A_2 and A_3 as given in Fig. **??**.

(b) Show that the global $U(1)_{B+L}$ symmetry is anomalous and compute the relevant anomaly term.

Hint: Recall that the triangle diagram gives the trace of the three generators at the vertices, which You can determine depending on the

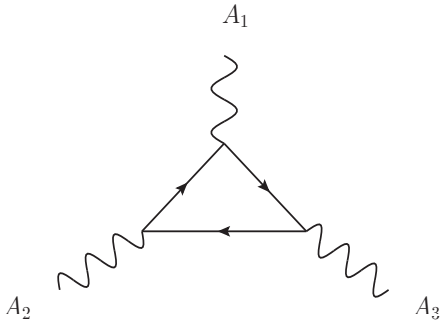

Figure 1: Gauge anomaly diagram.

fermionic representation or on the charge q for the $U(1)$ group, i.e. for a $U(1)$ local symmetry the generators can be written simply as qI, where I is the identity matrix in the appropriate dimension.

3. **Running of the gauge couplings**

(a) Compute the one-loop coefficients of the β functions for the gauge couplings in the Standard Model $U(1)_Y \times SU(2)_L \times SU(3)_c$ knowing that

$$\beta_i \equiv \mu \frac{dg_i}{d\mu} = \frac{b_i}{16\pi^2} g_i^3 \qquad \text{for} \qquad i = 1, 2, 3$$

where

$$b_i = -\frac{11}{3} C_2(G_i)|_{gaugebosons} + \frac{2}{3} T(R_i)|_{Weylfermions} + \frac{1}{3} T(R_i)|_{Complexscalar}$$

where $C_2(SU(N)) = N, C_2(U(1)) = 0, T(R_i = F) = \frac{1}{2}, T(R_i = A) = C_2(G_i)$, for the fundamental F and adjoint A representations of $SU(N)$ respectively, and $T(R_1) = Y^2$ for $U(1)_Y$.

(b) Compute the running of the gauge couplings of $U(1)_Y$ and $SU(2)_L$ from the EW scale at M_Z and find where the couplings meet (α_{GUT}) and at what scale M_{GUT}.

Note: Normalize the generator of the $U(1)_Y$ gauge group by a factor $C = \sqrt{5/3}$, as $g_1 = C g_Y$. The running is linear for $\alpha_i^{-1} = \frac{4\pi}{g_i^2}$ as a function of $\log \mu$.

(c) Compute the QCD coupling α_3 at the electroweak scale M_Z from the GUT value α_{GUT} found in (b) and compare it with the measured value given in the PDG, $\alpha_{QCD}(M_Z) = 0.1179 \pm 0.0009$. Do they agree ?

(d) Consider moreover a supersymmetric model with two Higgs doublets, i.e. a model where every Weyl fermion or complex/real scalar in a particular representation has a corresponding scalar/fermionic partner in the same representation. Show that then the coefficients in the β function change to

$$b_i = -3C_2(G_i)|_{vectormultiplet} + T(R_i)|_{chiralmultiplet} \ .$$

Check if the value of α_3 improves compared to the SM.