Eigen-Microstate Signatures of Criticality in Relativistic Heavy-Ion Collisions

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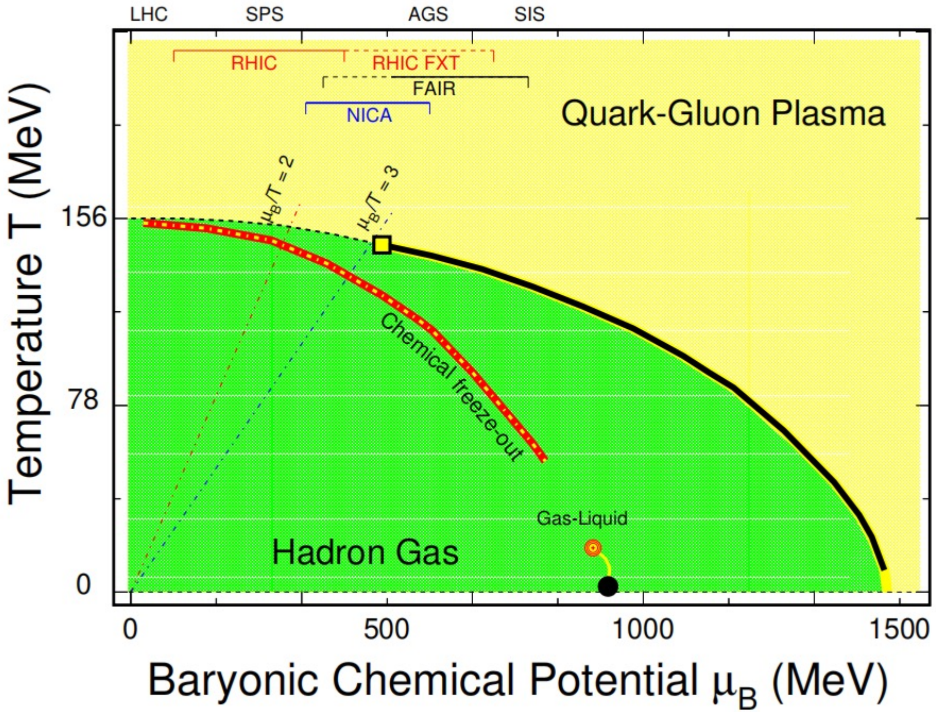
[1] Ranran Guo, Jin Wu, Mingmei Xu, Xiaosong Chen, Zhiming Li, Zhengning Yin, Yuanfang Wu arXiv:2510.20336.

[2] Ranran Guo, Jin Wu, Mingmei Xu, Zhiming Li, Zhengning Yin, Yufu Lin, Lizhu Chen, Yanhua Zhang, Jinghua Fu Xiaosong Chen, Yuanfang Wu, arXiv:2602.00537.

OUTLINE

- Motivation
- Eigen-Microstate Approach (EMA) in HIC
- Models and corresponding EM patterns & eigenvalues
- Finite size scaling (FSS) behavior of signal pattern & eigenvalue
- Summary

1. Motivation



- **One main goals of HIC:**

  Mapping QCD phase diagram, in particular, critical point (CP)

Features of CP:

Diverge of corr. length, or present of all lengths of corr., in particular, long range corr.

Large intermittent fluc., or self-similar fractal structure!

➢ Critical sensitive observables in HIC

Difficulty:

Order parameter of QCD deconfiment phase transition

→ **Polyakov loop**

→ unknown corresponding observable?

(1) Higher order moments of conserved charges

Based on: **equilibrium statistics, or thermal equilibrium models**

$S\sigma \sim \xi^{4.5}$,

$\kappa\sigma^2 \sim \xi^7$

M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); Phys. Rev. Lett. 107, 052301 (2011)

S. Ejiri, F. Karsch, and K. Redlich, Phys. Lett. B 633,275 (2006).

Problems in experimental measurements:

- Baryon ~ net-proton, unknow neutron
- Statistical fluc., need to subtract the Poison baseline
- Non-equilibrium relaxation
- High statistics for precision measurements
- Correlations from conventional mechanisms

Expected critical related non-monotonic energy dependence has not yet been observed!

(2) Factorial moments of multiplicity distributions

$$F_q(M) = \frac{\langle \frac{1}{M^D} \sum_{i=1}^{M^D} n_i(n_i - 1)\cdots(n_i - q + 1)\rangle}{\langle \frac{1}{M^D} \sum_{i=1}^{M^D} n_i \rangle^q},$$

A. Bialas and R. Peschanski, Nucl. Phys. B 273, 703(1986).

Based on: fractal like large fluctuations, i.e., intermittency

Factorial moments is expected to have power law behavior :

$$F_q(M) \sim (M^D)^{\phi_q}, M \gg 1.$$

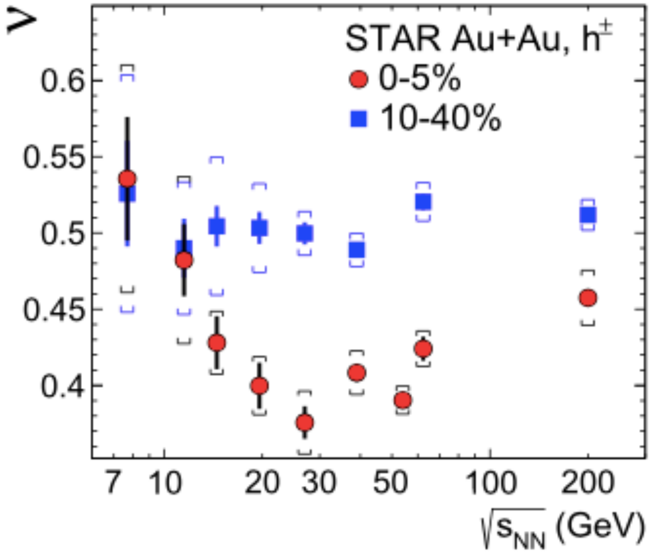
However, saturate in 1D, no exact power law in 2D,

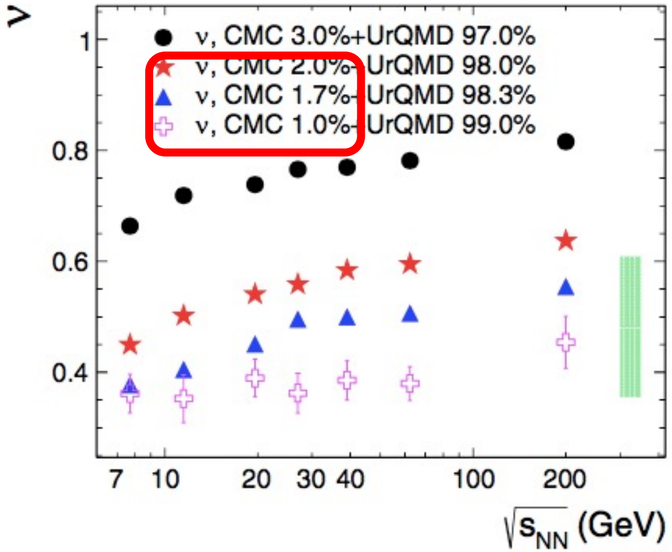
Maybe:

- $F_q(M) \propto F_2(M)^{\beta_q}, M \gg 1.$ $\beta_q \propto (q-1)^\nu$
- background subtraction: $\Delta F_q(M) = F_q(M)^{\text{data}} - F_q(M)^{\text{mix}}.$

Experimental results of factorial moments measurements

$F_q(M) \propto F_2(M)^{\beta_q}$ $\beta_q \propto (q-1)^{\nu}$





Only a few 1-2% critical signal!

STAR, Phys. Lett. B 845, 138165 (2023).

J. Wu, Y. Lin, Z. Li, X. Luo, and Y. Wu, Phys. Rev. C 104, 034902 (2021).

J.Wu, Z. Li, X. Luo, M. Xu,and Y. Wu, Phys. Rev. C 106, 054905 (2022).

R. Wang, C. Qiu, C. Hu, Z. Li, and Y. Wu, Phys. Lett. B 864, 139405 (2025).

Search for the CP is challenging !

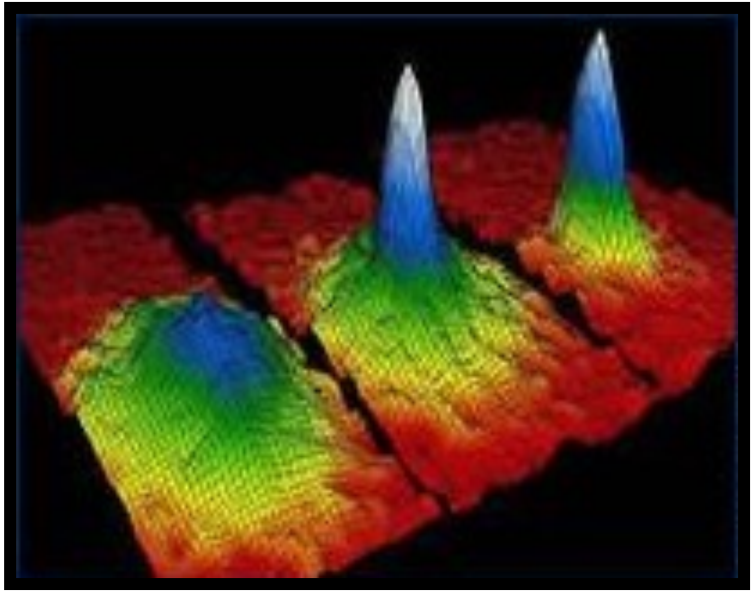
More sensitive, or effective, observables are called for!

➢ Current development on Eigen-Microstate Approach (EMA)

Xiaosong Chen et al., Sci. China Phys. Mech. Astron. 62, 990511 (2019).

Based on: generalized the microstate description from Gibbs-ensemble theory to arbitrary event ensembles, by constructing the temporal-spatial correlated matrix, EMA as well as principal component analysis (PCA), can extract the dominant collective mode and reveal its condensation, analogous to Bose–Einstein condensation!

L. Wang, Phys. Rev. B 94, 195105 (2016).

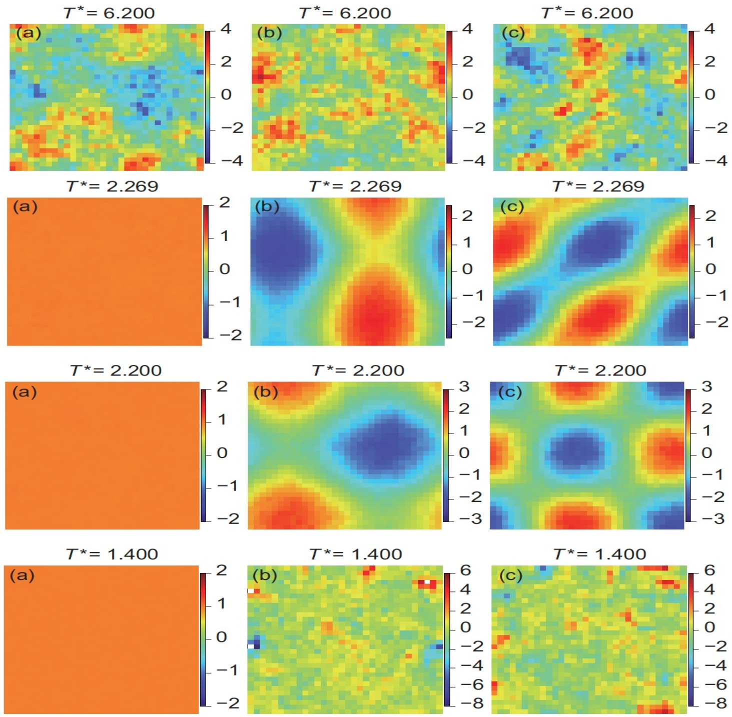


EMA is able to reveal critical patterns for both equilibrium, i.e., 1D, 2D and 3D Ising models, and non-equilibrium systems, i.e., Vicsek model, Karman vortex street, Atmospheric, Social systems, …, and so on,

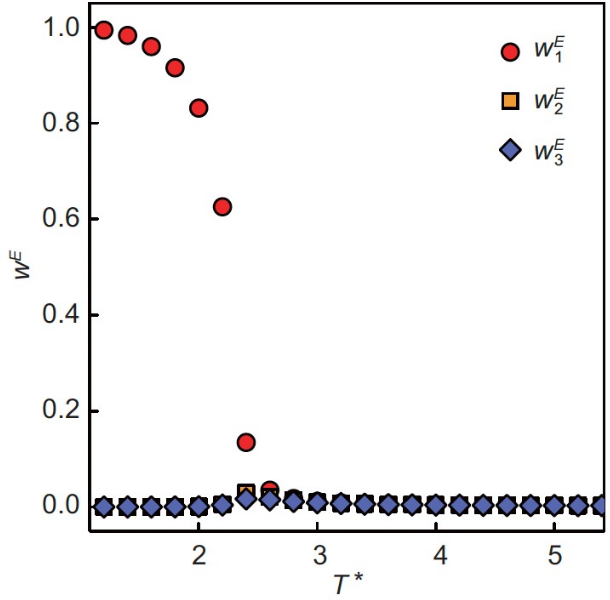
and provide accessible to order parameter!

Xiaosong Chen et al., Commun. Theor. Phys. 73, 065603 (2021);
Chin. Phys. B 30, 128703 (2021); Sci. China-Phys. Mech. Astron. 67, 110511 (2024);
Chinese Phys. Lett. 39, 080503 (2022).

EM patterns for first three largest eigenvalues in 2D Ising near Tc



First three largest
eigen microstate patterns.



First three largest
eigen values
near critical temperature

2. Eigen-Microstate Approach (EMA) in HIC

Ranran Guo, Jin Wu, Mingmei Xu, Xiaosong Chen, Zhiming Li, Zhengning Yin, Yuanfang Wu arXiv:2510.20336.

➢ Formulation of Original Microstates (OM)

Generalizing the concept of microstate in Gibbs ensemble!

An event → An original microstate (OM)

M event sample → Generalized ensemble with M microstates

Temporal:

Under identical macro conditions, such as,

collider nuclear, collision energy, impact parameter （centrality)

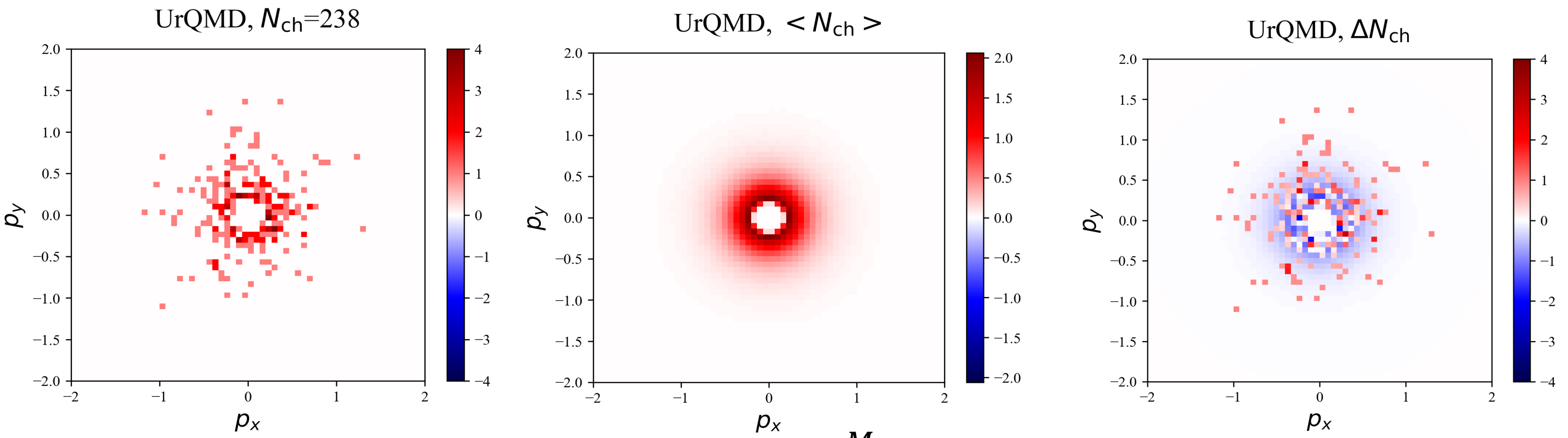
A given event → a specified evolution time

Spatial:

Transverse momentum space

Binning the space, each bin is specified by $\triangle Nch$

➢ Original microstate --- An event in HIC



An event: $N_{\rm ch}$ in p_x-p_y

$$\langle N_{{\rm ch},i}\rangle = \frac{1}{M}\sum_{I=1}^{M} N^{I}_{{\rm ch},i}$$

$$\Delta N^{I}_{{\rm ch},i} = N^{I}_{{\rm ch},i} - \langle N_{{\rm ch},i}\rangle$$

An original microstate (OM) →

$$\boldsymbol{A}^{I} = \frac{1}{\mathcal{N}}\begin{bmatrix}\Delta N^{I}_{{\rm ch},1}\\ \Delta N^{I}_{{\rm ch},2}\\ \vdots \\ \Delta N^{I}_{{\rm ch},N}\end{bmatrix}$$

$N = L^2$ bins

$I = 1, 2, \ldots M$ events

➤ Temporal-spatial correlation matrix of M microstates

$$C_{IJ} = [\boldsymbol{A}^I]^T \cdot \boldsymbol{A}^J$$

Its eigenvalue (λ_I) equation,

$$C\boldsymbol{b}_I = \lambda_I \boldsymbol{b}_I$$

Define **Eigen-Microstate (EM_I)**

$$\boldsymbol{E}^I = \sum_{l=1}^{M} b_{lI} \boldsymbol{A}^l$$

They are orthogonal and normalized to 1 !

$$[\boldsymbol{E}^I]^T \cdot \boldsymbol{E}^J = 0$$

$$|\boldsymbol{E}^I|^2 = |\boldsymbol{E}^I|^T \cdot \boldsymbol{E}^I = \lambda_I$$

Weight of each EM $w_I = \lambda_I$

Satisfying, $\sum_{I=1}^{M} w_I = 1$

Order EM_1, EM_2, EM_3 ..., by

$$w_1 > w_2 > w_3 \ \dots$$

- For a disordered system

$$w_I = \frac{1}{M}, \quad \lim_{M\to\infty} w_I = 0$$

- If $\lim_{M\to\infty} w_I =$ finite,

→ **a condensation !**

- Weight cumulant is defined as

$$C(m) = \sum_{n=1}^{m} w_n$$

The speed of EM goes to saturation.

The faster the speed, the more dominated leading EM's contributions!

3. Models and corresponding EM patterns and eigenvalues

(A) Non-critical models

Ranran Guo, et,al, arXiv:2602.00537.

- **UrQMD**

  Including almost all conventional mechanisms,

  i.e., energy-momentum, charge conservations, resonance decay, elliptic flow, and so on.

  Aim: see the contribution of background to its EM.

- **Stochastic model (Stoch. I))**

  same Nch dis. as UrQMD, p_{Tx}, p_{Ty}, uniformly & randomly dis. in (-1, 1) GeV

  No specified mechanism at all!

  Aim: compare with UrQMD's EM

- **Restricted stochastic model (Stoch. II)**

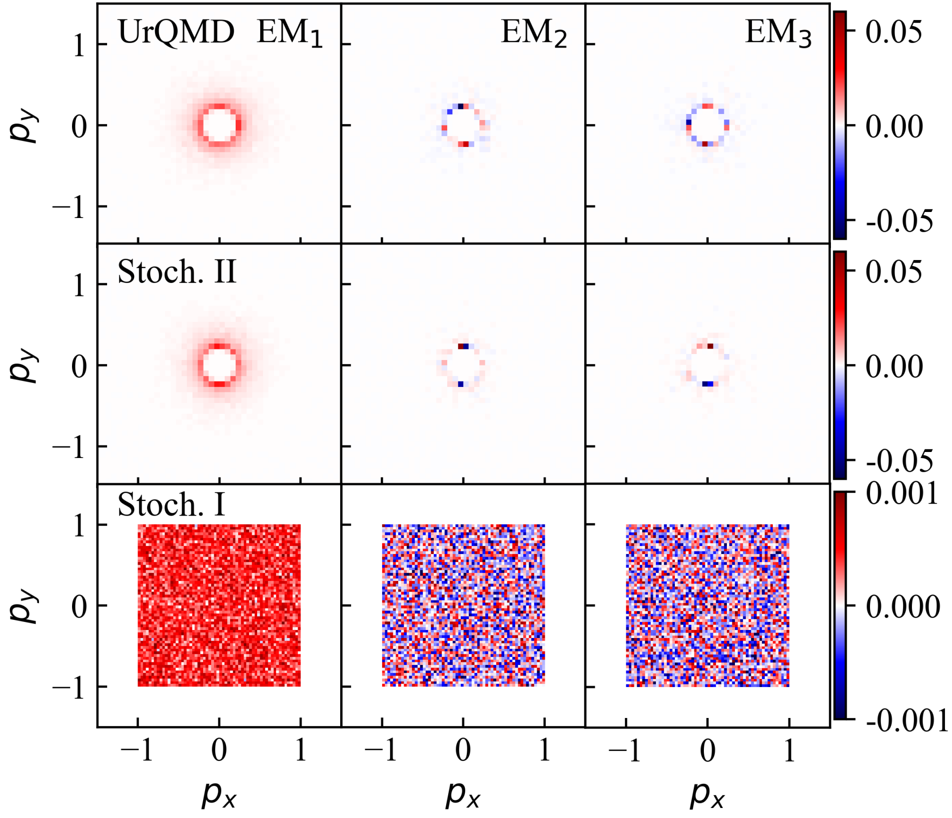
  same Nch dis. as UrQMD

  $p(p_t) = \frac{1}{\Gamma(k)\,\beta^k}\, p_t^{k-1}\, e^{-p_t/\beta} \quad p(\phi) = \frac{1}{2\pi}(1 + 0.08\cos 2\phi)$

  basic energy-momentum conservations.

  Aim: see the contributions of energy-momentum conservation and elliptic flow

➢ EM patterns of three largest eigenvalues for three non-critical models



Stable pattern for M = 20,000 !

UrQMD

EM1: similar to <Nch>

uniform & rotational symmetry

EM2: random red–blue fluctuations

EM3: random red–blue fluctuations

Stoch. II

EM1: same as that of UrQMD

EM2: random red–blue fluctuations

EM3: random red–blue fluctuations

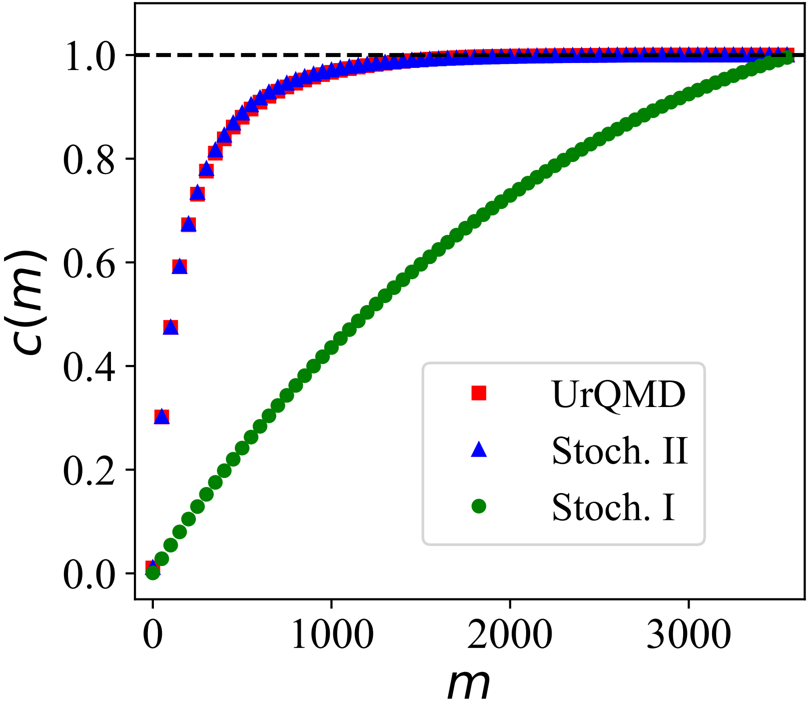
Stoch. I

EM1: uniform distribution

EM2: random red–blue fluctuations

EM3: random red–blue fluctuations

➤ Weight cumulants for three non-critical models



Stoch. I

C(m) increases almost linearly with m !

No preferred mode !

UrQMD & Stoch. II

A rapid initial rise followed by early saturation !

→ A common global energy-momentum cons. enhances leading noncritical modes!

The overlap of UrQMD and Stoch. II

→ EMA is sensitive to the global pt and ϕ dis. rather than detail mechanisms of UrQMD.

(B) Critical Monte Carlo (CMC) model

N.G. Antoniou, F.K. Diakonos and A.S. Kapoyannis, Phys. Rev. Lett. 97, 032002 (2006).

- Transverse momentum follows Levy random walks, i.e.,

$$\tilde{\rho}(p_i) = \frac{\nu p_{\min}^{\nu}}{1 - \left(\frac{p_{\min}}{p_{\max}}\right)^{\nu}} p_i^{-1-\nu}, \qquad i = x, y,$$

ν : fractal exponent, belonging to the 3D-Ising universality class!

→ **particles have fractal fluctuations in momentum space!**

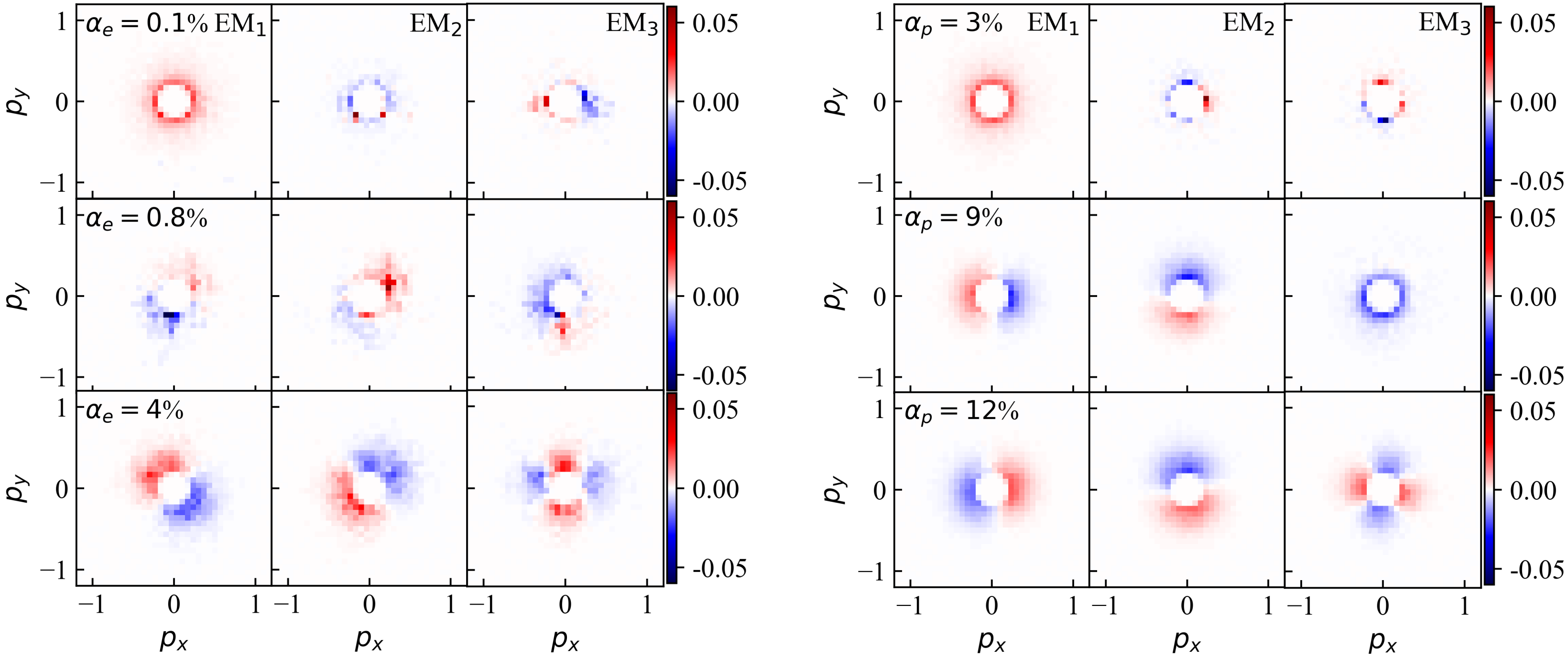
Two ways to add critical components:

(1) Replace a fraction α_p of particles in each of UrQMD events by CMC particles!

(2) Replace a fraction α_e of UrQMD events by CMC events!

critical signal strength!

➢ First three EM patterns for two ways of adding critical signal



With increase of critical component, 2- & 4-patch pattern appear, which indicate the appearance of **nascent critical mode** (**new phase**)!

- Why 2-, 4-, 6-patch, or cluster like pattern?

At critical point → correlation length diverges

& all length correlations appear,

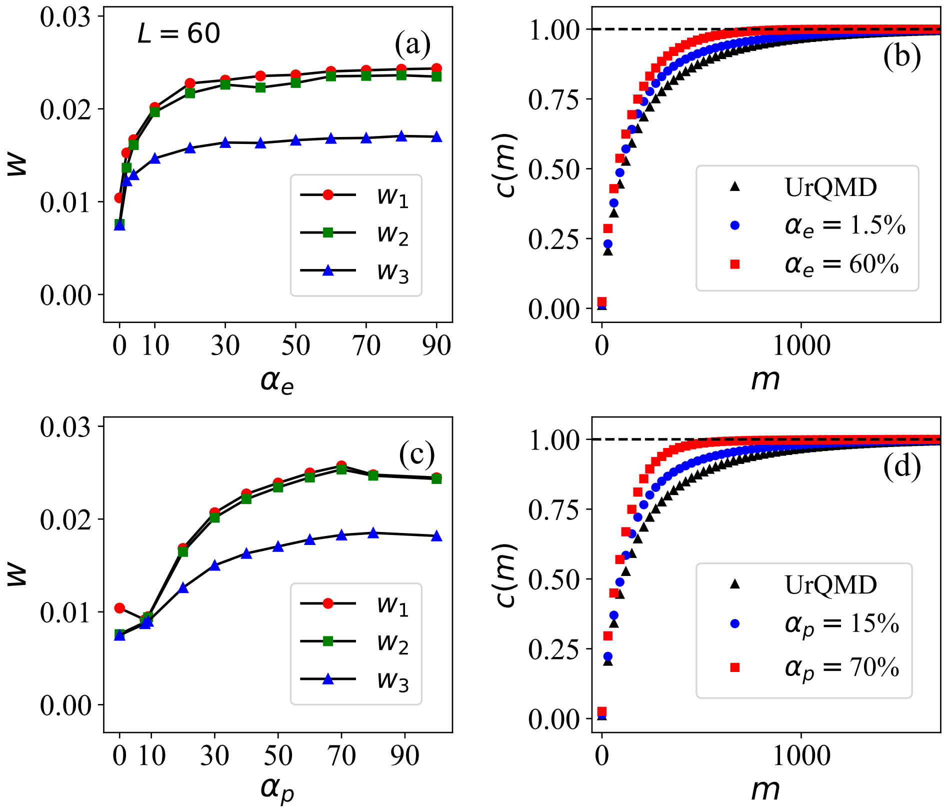
in particular, long range correlations,

in contrast to short range corr. in UrQMD

Different number of clusters in fact correspond to various sizes or corr. lengths. The largest size of cluster corresponds to the longest correlation length, similar to those in 2D Ising model.

Critical EM pattern is independent of specified critical model!

➢ First three eigenvalues and weight cumulant



➢ **w_1 increases with α_e, or α_p, then saturates.**

Acts as an order parameter!

➢ Larger critical fraction,

Fewer leading EMs dominate,

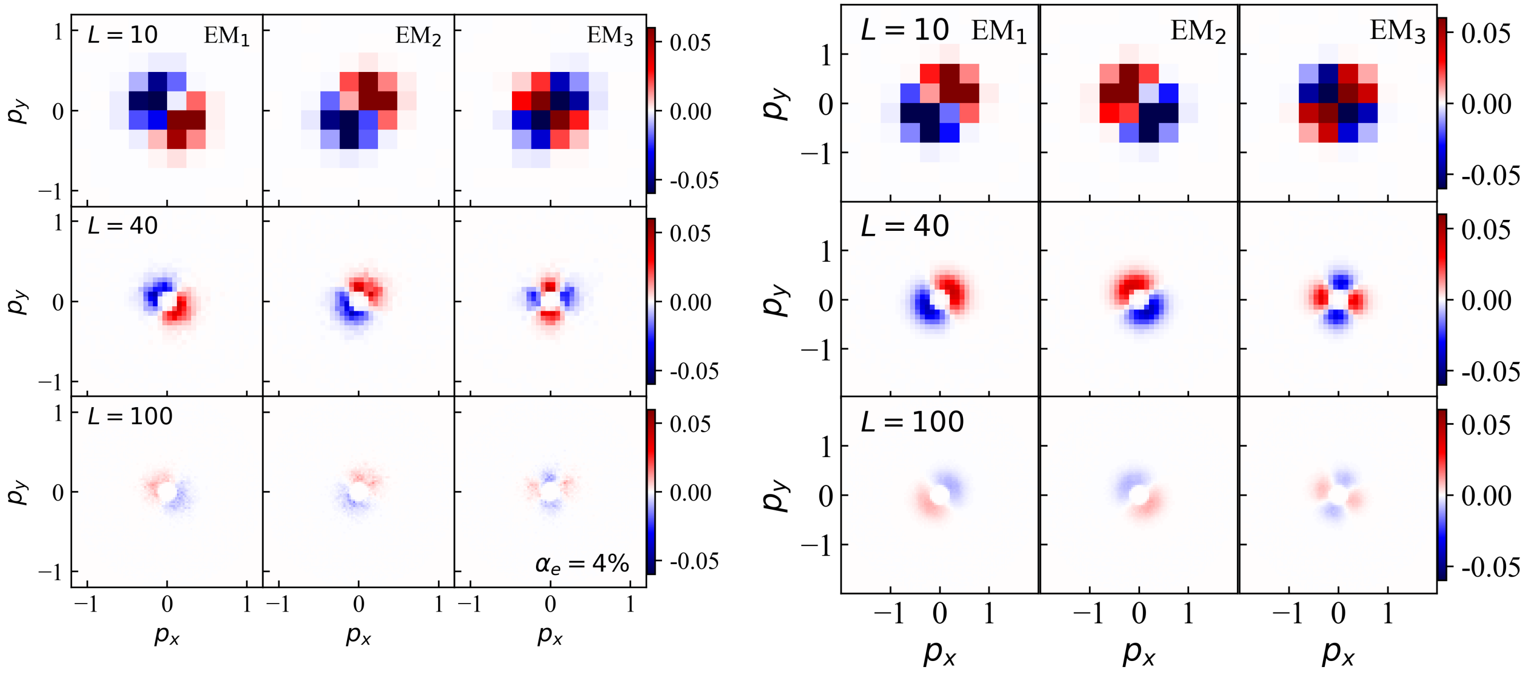
More concentrated,

& enhanced condensation!

EM pattern & eigenvalue are sensitive to critical fluctuations!

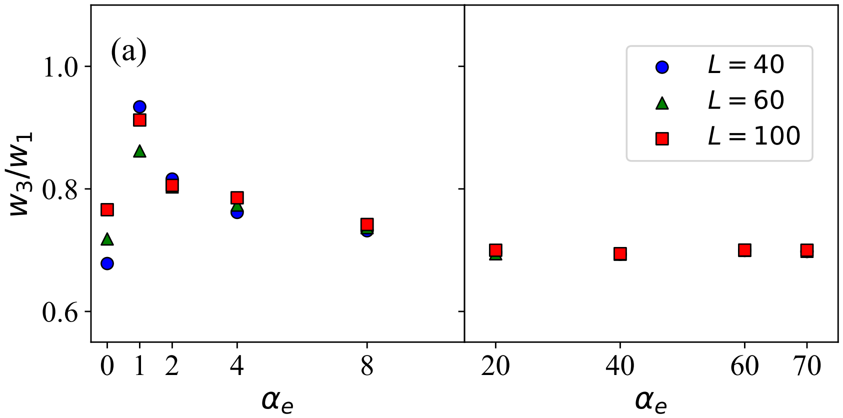
4. Finite size scaling behavior of EM pattern and eigenvalue

(1). Scale Invariance of Critical Patterns



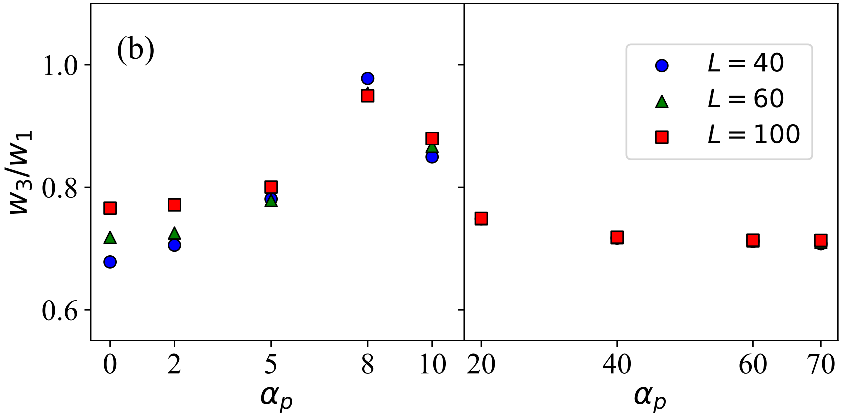
Self-similar across different scales *L*, i. e., fractality!

(2) Fixed-point behavior



$\alpha_e > 2\%$,

all size points overlap to a point!



$\alpha_p > 9\%$

all size points overlap to a point!

Signal EM pattern & eigenvalue

have well defined finite size scaling behavior!

Summary

- EMA successfully extracts critical patterns from event samples.
- Eigenvalue provides robust order-parameter–like indicator.
- Advantages:

-- Works without equilibrium assumptions, ideal for HIC
-- Insensitive to backgrounds
-- Much less events than high-order cumulants (~20k sufficient)

Thank you for your attention!

Replacements:

$$\left|p_T^{\mathrm{CMC}} - p_T^{\mathrm{UrQMD}}\right| < 0.2\ \mathrm{GeV}/c$$

Applying STAR kinematic cuts:

- pseudorapidity $|\eta| < 0.5$,
- transverse momentum windows

$$0.2 < p_{\mathrm{t}} < 1.6\ \mathrm{GeV}/c \text{ for } \pi^{\pm} \text{ and } K^{\pm},$$
$$0.4 < p_{\mathrm{t}} < 2.0\ \mathrm{GeV}/c \text{ for } p \text{ and } \bar{p}.$$