Open Quantum Systems approaches for Heavy-ion Collisions

Alexander Rothkopf
Department of Physics
Korea University
South Korea

The 22nd International Conference on
Strangeness in Quark Matter
22-27 March, 2026, Los Angeles, CA

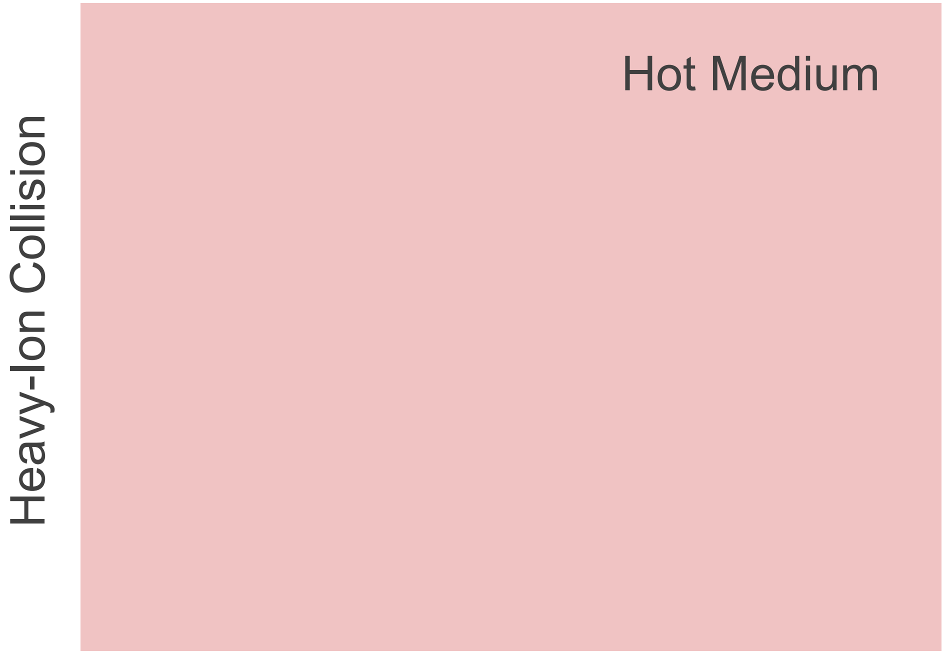
Outline

- Motivation: exploring the properties of hot nuclear matter
- The Quarkonium OQS ecosystem in heavy-ion collisions
- OQS for discovery in heavy-ion collisions – optimal observables
- Conclusion & Outlook

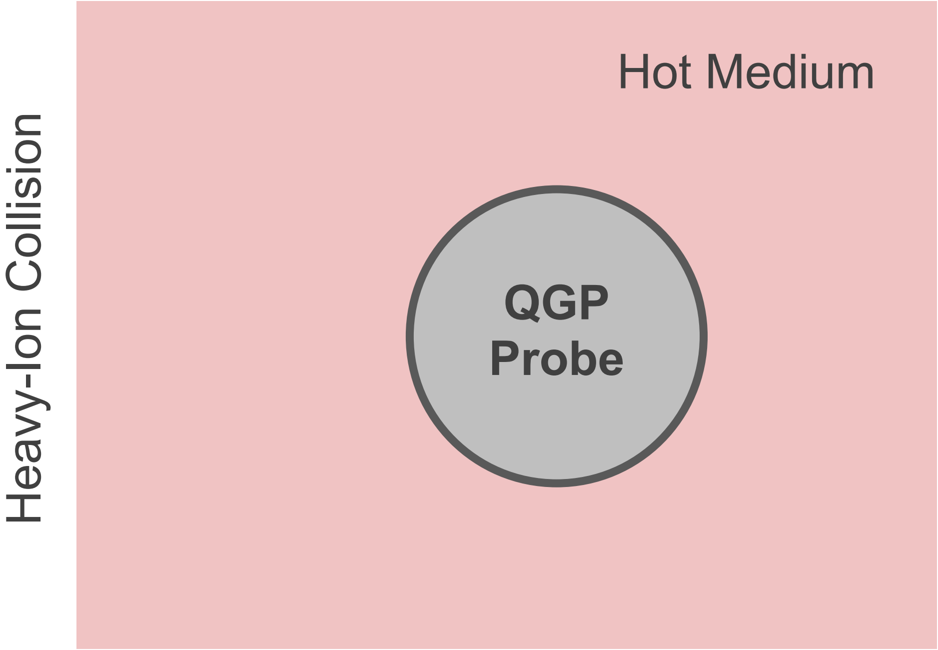
Outline

- Motivation: exploring the properties of hot nuclear matter
- The Quarkonium OQS ecosystem in heavy-ion collisions
- OQS for discovery in heavy-ion collisions – optimal observables
- Conclusion & Outlook

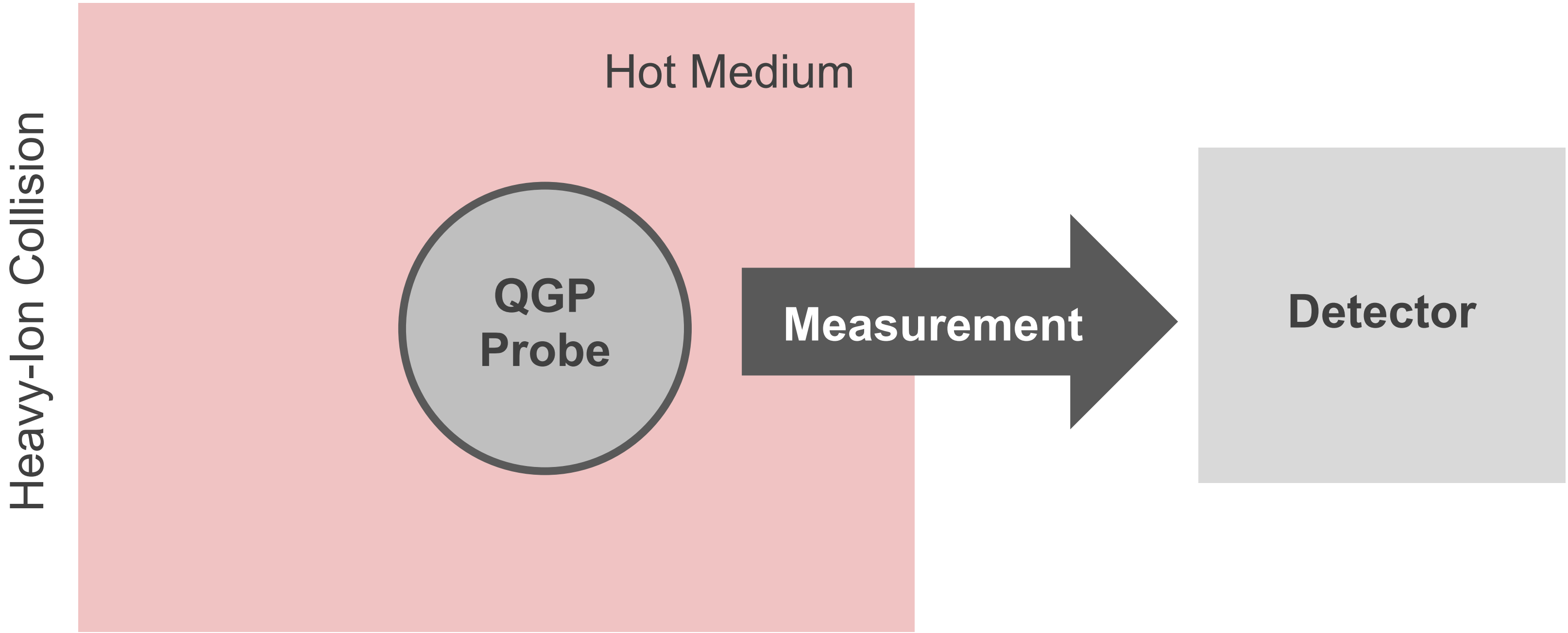
A tale of two measurements



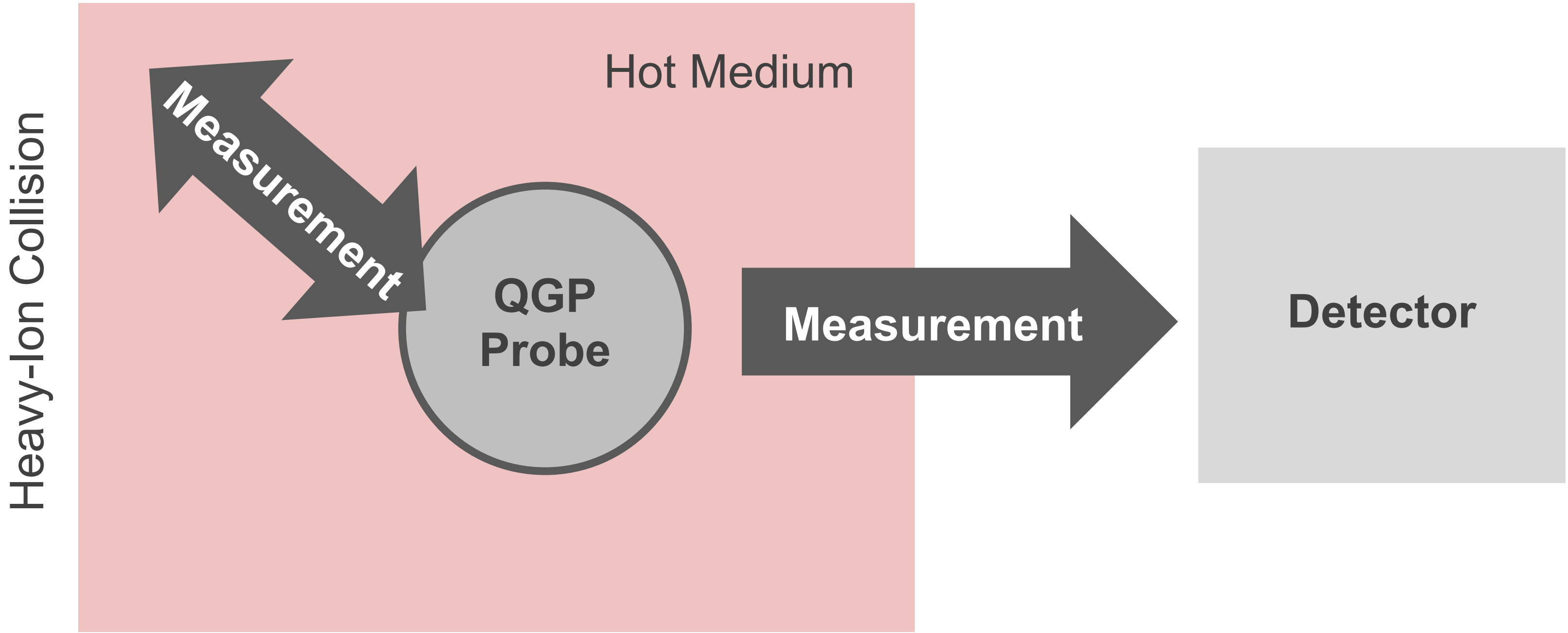
A tale of two measurements



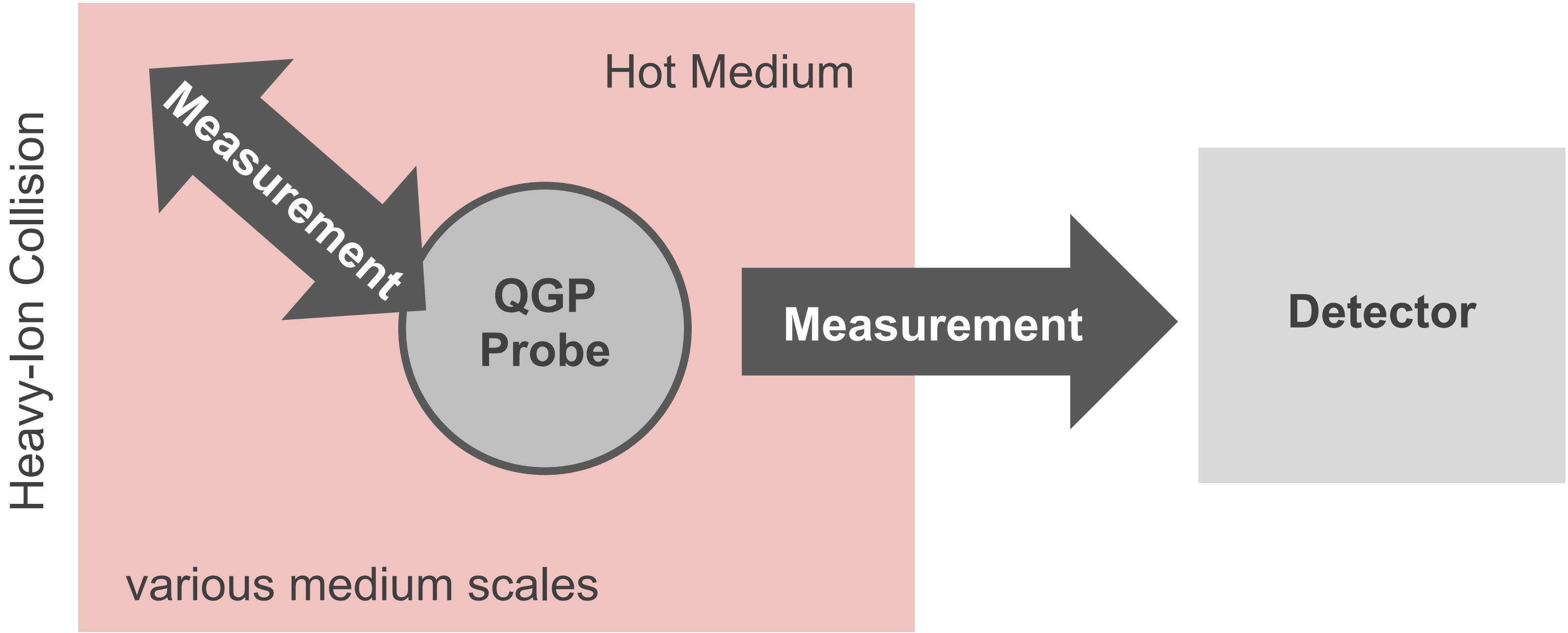
A tale of two measurements



A tale of two measurements

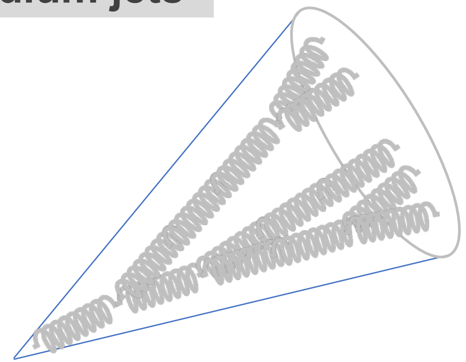


A tale of two measurements



Probing & being probed by the medium

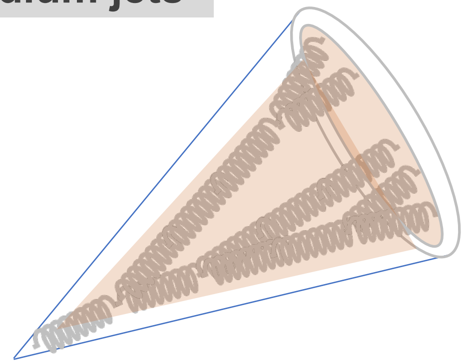
in-medium jets



for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

Probing & being probed by the medium

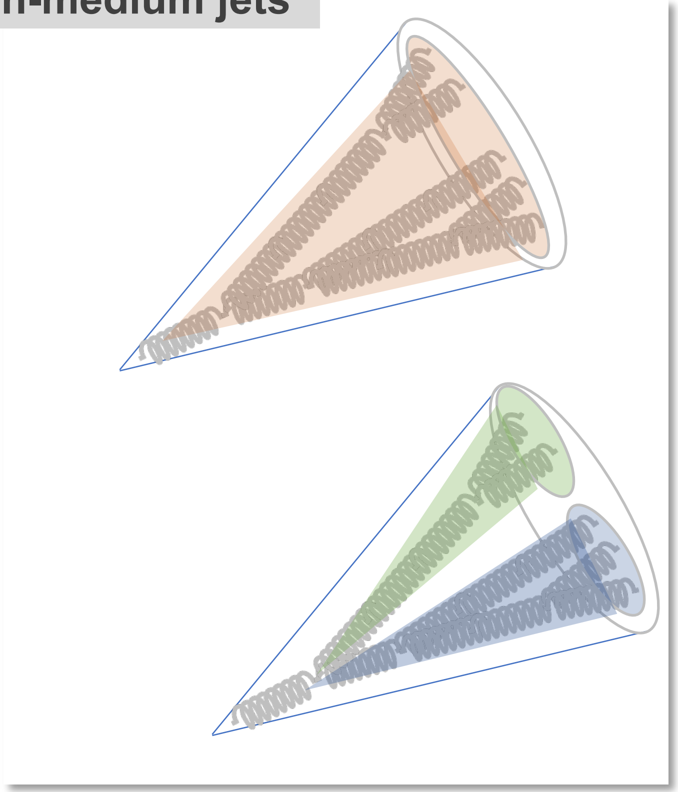
in-medium jets



for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

Probing & being probed by the medium

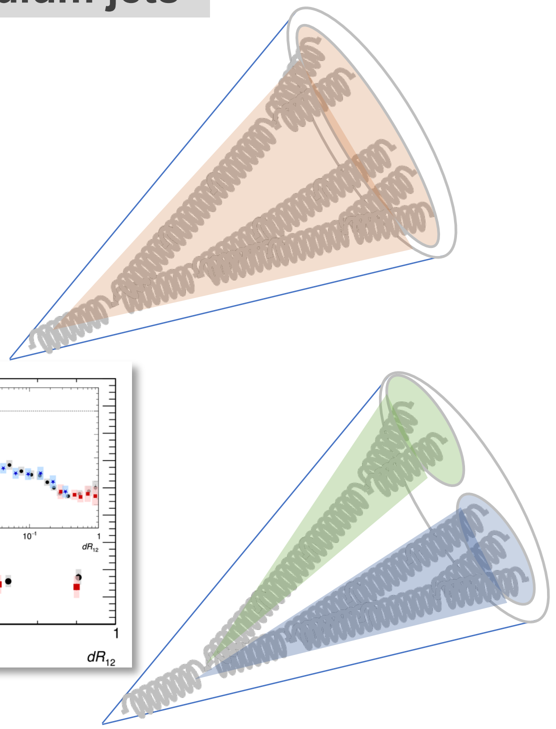
in-medium jets

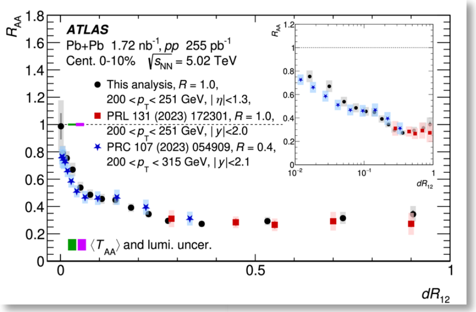


for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

Probing & being probed by the medium

in-medium jets



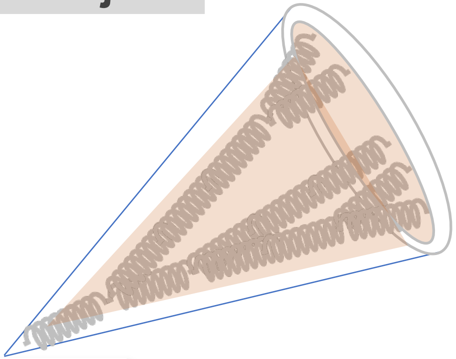


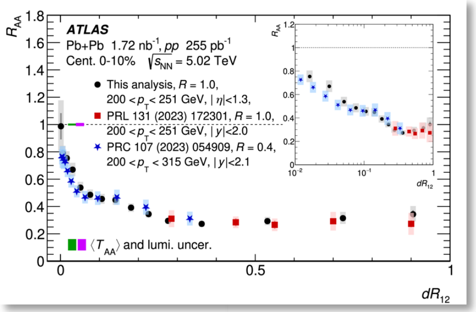
[ATLAS collaboration]
PLB 871 (2025) 139929

for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

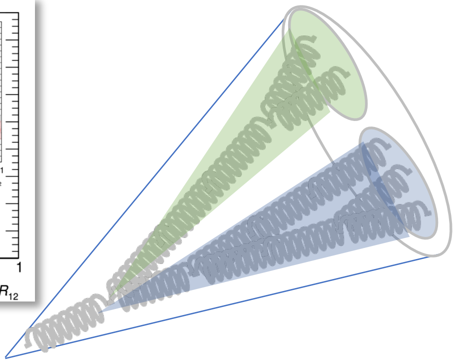
Probing & being probed by the medium

in-medium jets



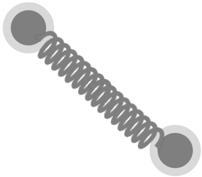


[ATLAS collaboration]
PLB 871 (2025) 139929



for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

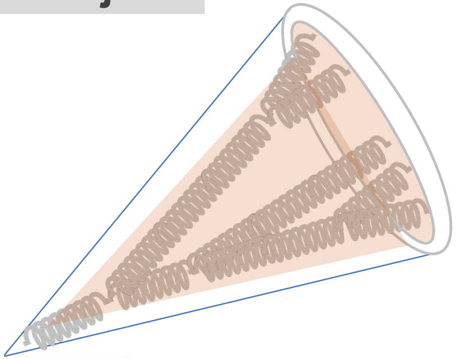
in-medium QQ

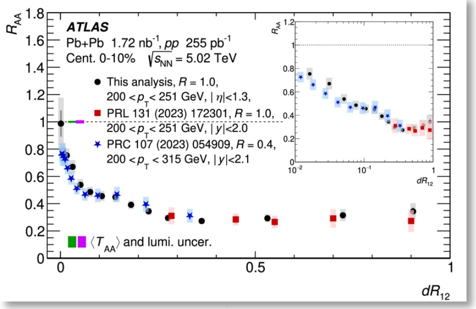


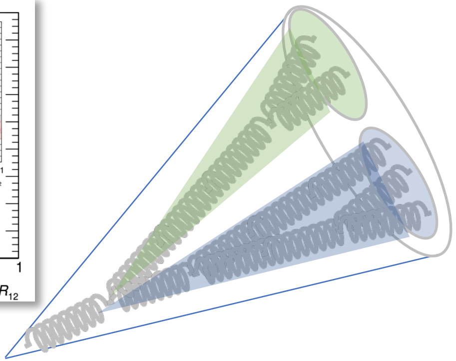
For reviews see: A.R. Phys. Rept. 858, 1 (2020), R. Sharma EPJ. ST 230, 3, 697 (2021), X. Yao IJMP A 36, 20, 2130010 (2021), Y. Akamatsu PPNP 123, 103932 (2022), for recent lattice results see [HotQCD] PRD 109 (2024) 7, 074504

Probing & being probed by the medium

in-medium jets



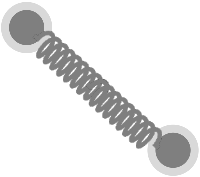


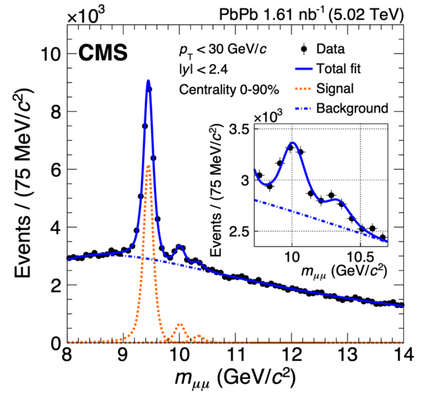


[ATLAS collaboration]
PLB 871 (2025) 139929

for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

in-medium QQ



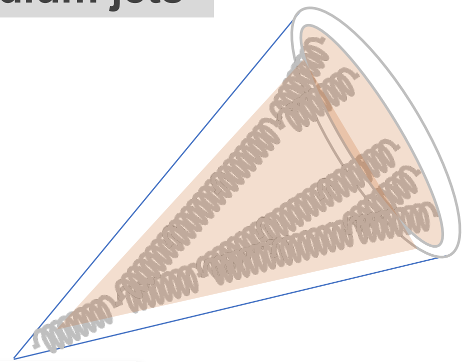


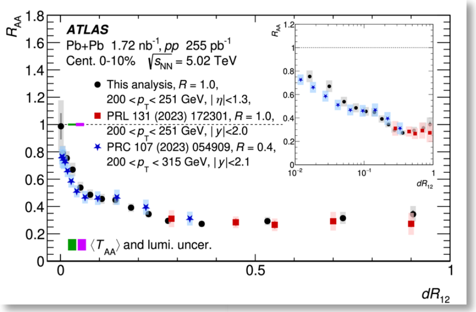
[CMS collaboration]
PRL 133 (2024) 022302

For reviews see: A.R. Phys. Rept. 858, 1 (2020), R. Sharma EPJ. ST 230, 3, 697 (2021), X. Yao IJMP A 36, 20, 2130010 (2021), Y. Akamatsu PPNP 123, 103932 (2022), for recent lattice results see [HotQCD] PRD 109 (2024) 7, 074504

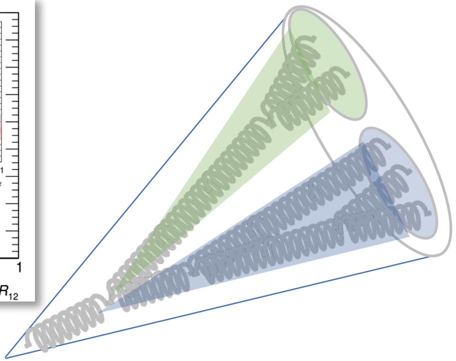
Probing & being probed by the medium

in-medium jets



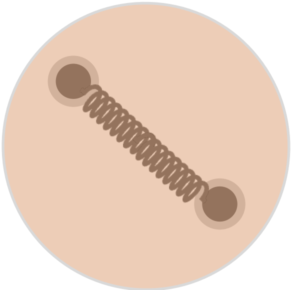


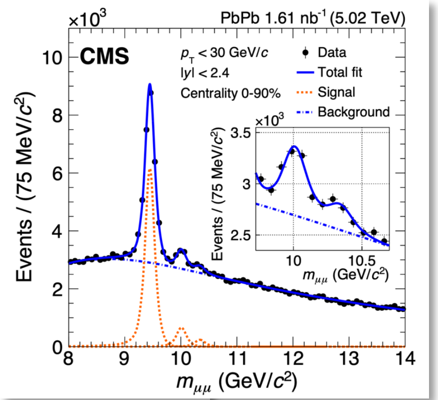
[ATLAS collaboration]
PLB 871 (2025) 139929



for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

in-medium QQ



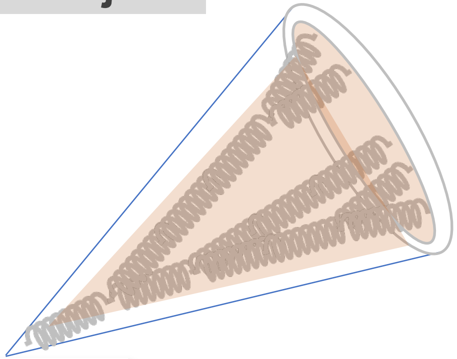


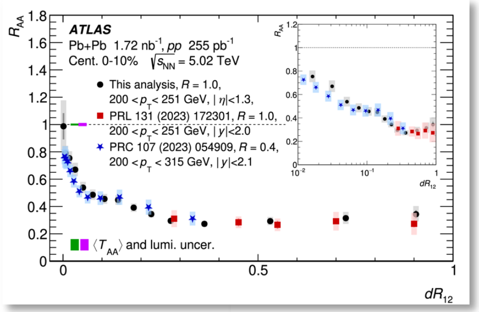
[CMS collaboration]
PRL 133 (2024) 022302

For reviews see: A.R. Phys. Rept. 858, 1 (2020), R. Sharma EPJ. ST 230, 3, 697 (2021), X. Yao IJMP A 36, 20, 2130010 (2021), Y. Akamatsu PPNP 123, 103932 (2022), for recent lattice results see [HotQCD] PRD 109 (2024) 7, 074504

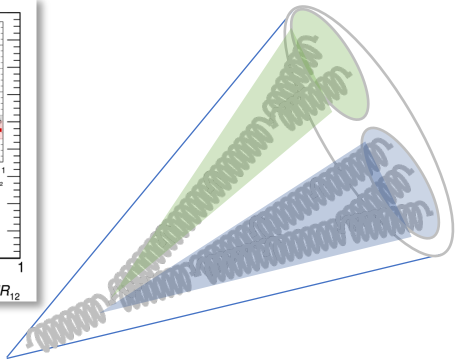
Probing & being probed by the medium

in-medium jets



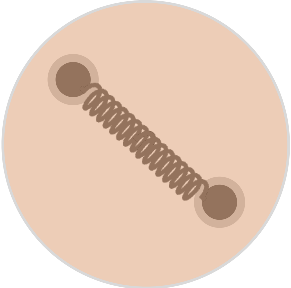


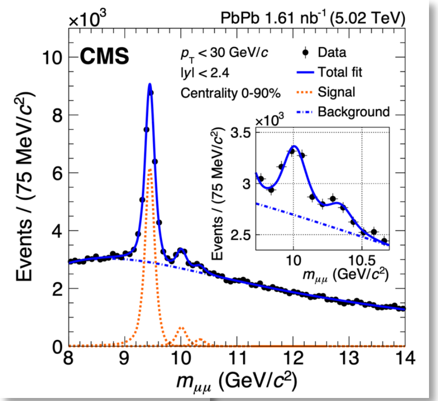
[ATLAS collaboration]
PLB 871 (2025) 139929



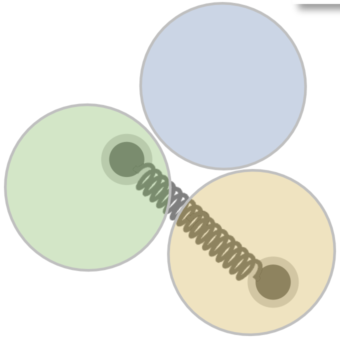
for recent theory results see e.g. Mehtar-Tani et.al. PLB 869 (2025) 139827, JHEP 02 (2026) 048 and Vaidya arXiv:2603.00238

in-medium QQ





[CMS collaboration]
PRL 133 (2024) 022302

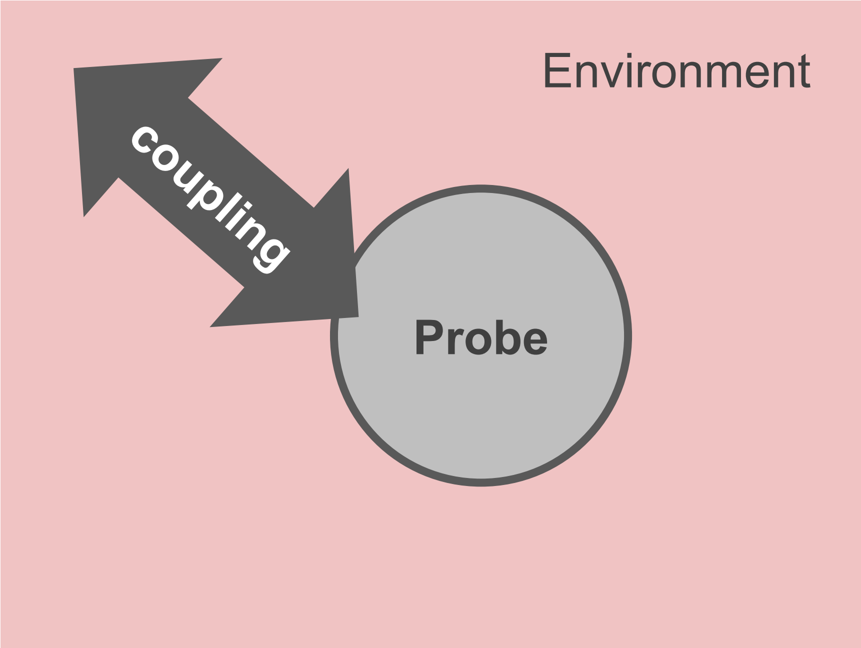


For reviews see: A.R. Phys. Rept. 858, 1 (2020), R. Sharma EPJ. ST 230, 3, 697 (2021), X. Yao IJMP A 36, 20, 2130010 (2021), Y. Akamatsu PPNP 123, 103932 (2022), for recent lattice results see [HotQCD] PRD 109 (2024) 7, 074504

Open Quantum Systems

- Demystifying the measurement process: coupling to environment

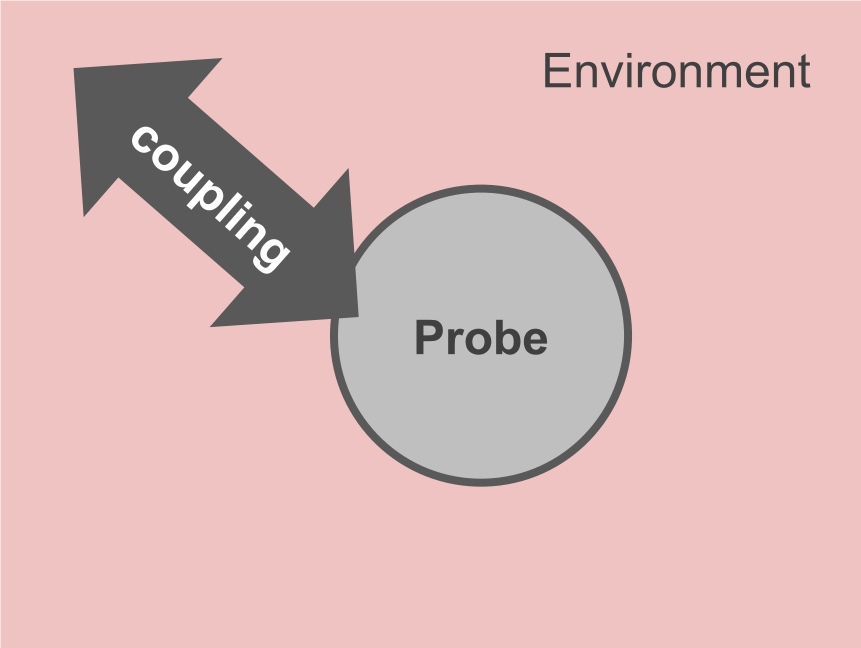
for a textbook see Breuer, Pettrucione The Theory of Open Quantum Systems



Open Quantum Systems

- Demystifying the measurement process: coupling to environment

for a textbook see Breuer, Pettrucione The Theory of Open Quantum Systems

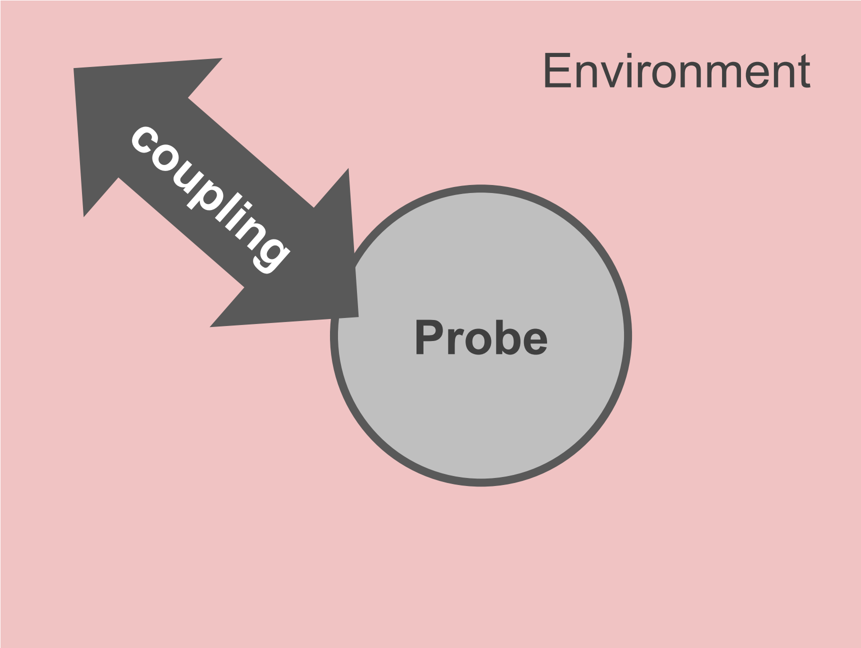


- Measurement is a **dynamical process**: focus on real-time evolution

Open Quantum Systems

- Demystifying the measurement process: coupling to environment

for a textbook see Breuer, Pettrucione The Theory of Open Quantum Systems



- Measurement is a **dynamical process**: focus on real-time evolution

- In presence of **separation of scales**: simplification (close relation to EFTs)

The Open Quantum System framework

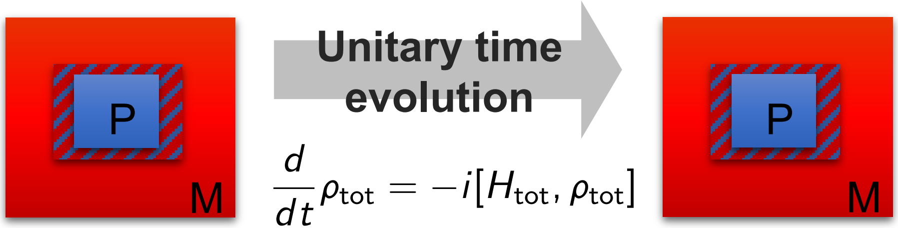
$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^{\dagger}$$

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H^{\dagger}_{\rm tot}$$

The Open Quantum System framework

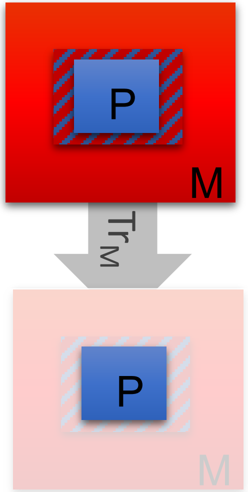
$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^{\dagger}$$



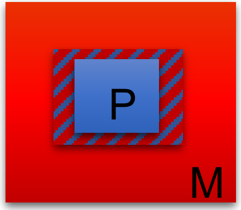
$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^{\dagger}$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

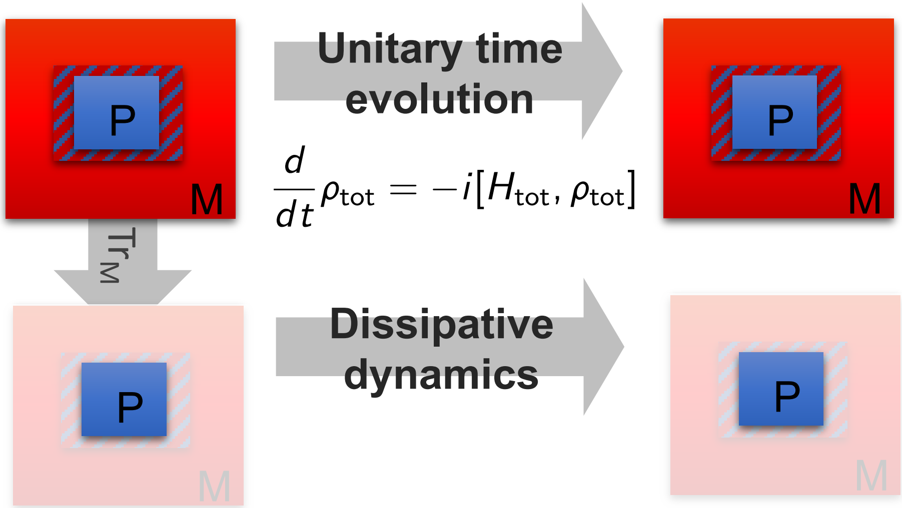


$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



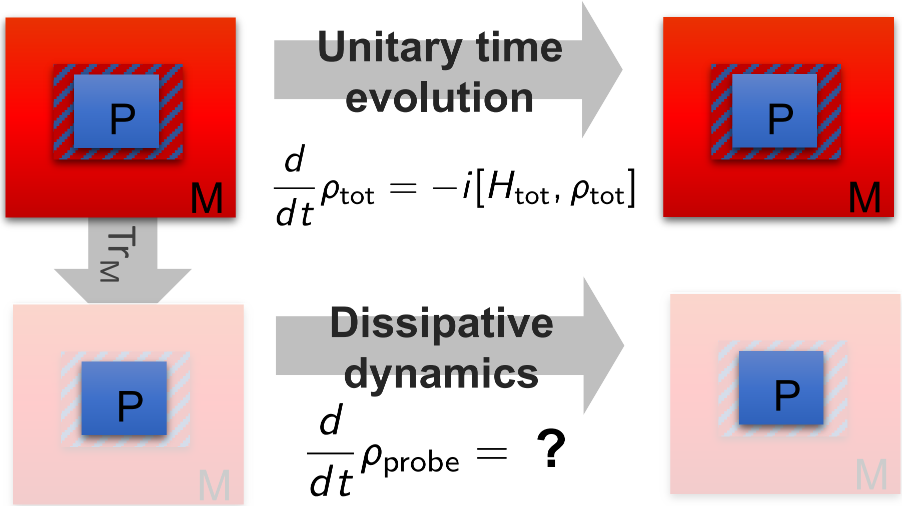
$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\rho_{\rm probe} = \mathrm{Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^{\dagger}$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

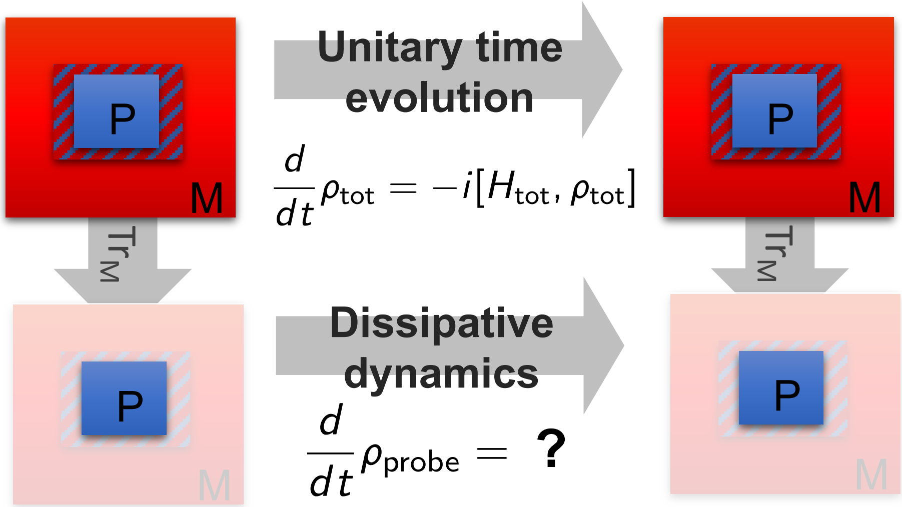
$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

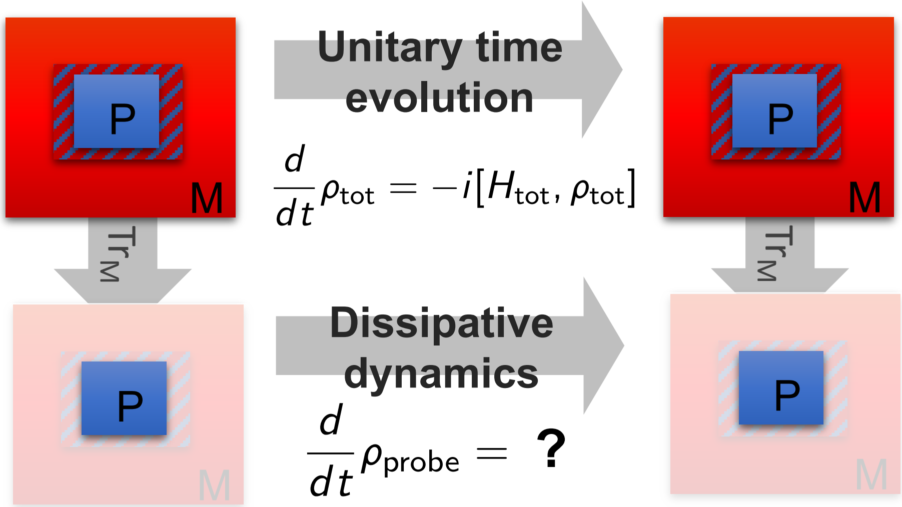
$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe}\otimes I_{\rm M} + I_{\rm probe}\otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

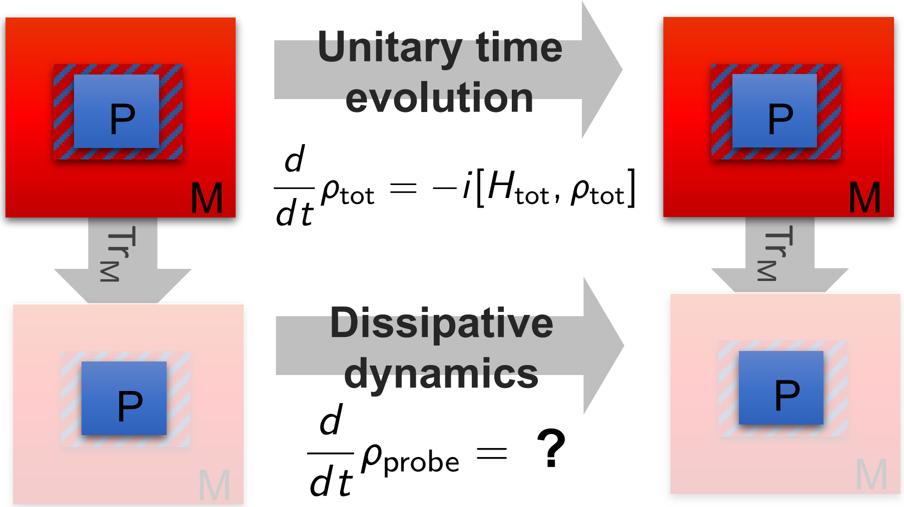
$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

Separation of time-scales
determines nature of e.o.m. :

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr_M}[\rho_{\rm tot}]$$

reduced density matrix

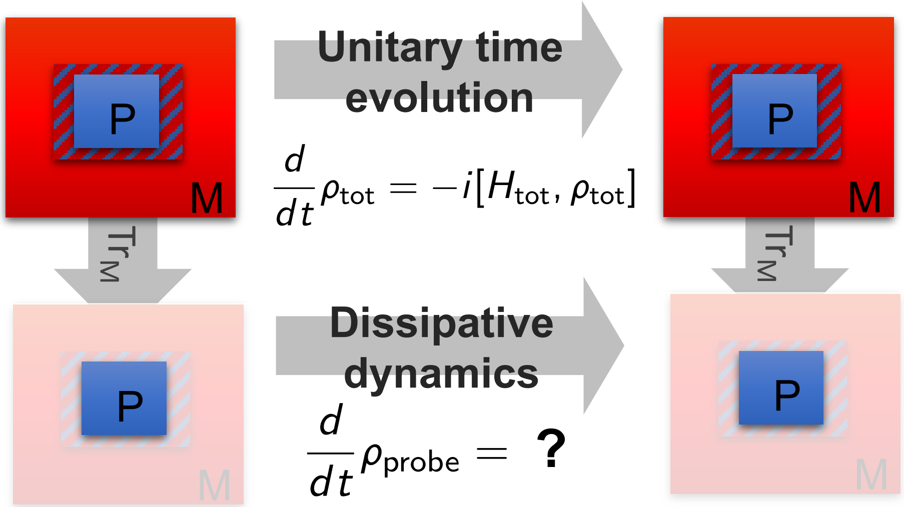
Separation of time-scales determines nature of e.o.m. :

Environment relaxation scale τ_E :

$$\langle \Xi_m(t)\Xi_m(0)\rangle \sim e^{-t/\tau_E}$$

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H^\dagger_{\rm tot}$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

Separation of time-scales determines nature of e.o.m. :

Environment relaxation scale τ_E :

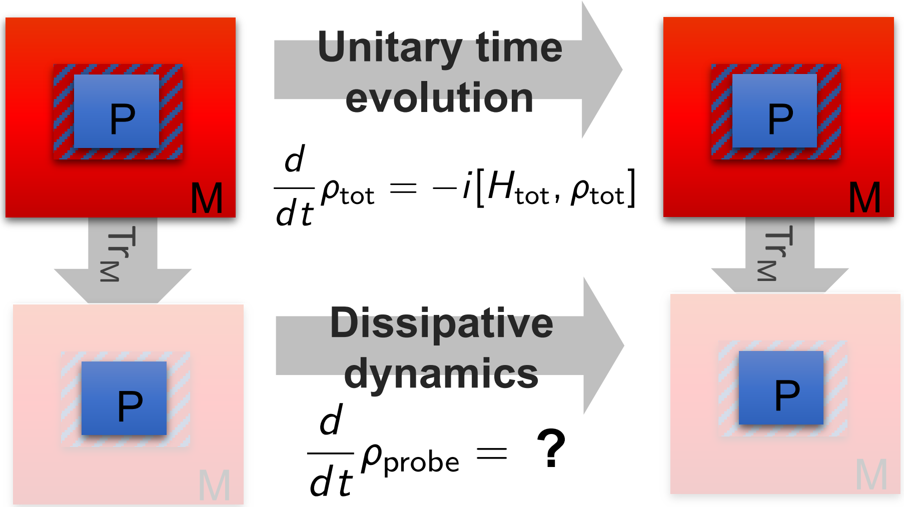
$$\langle \Xi_m(t)\Xi_m(0)\rangle \sim e^{-t/\tau_E}$$

probe system scale τ_S :

$$\tau_S \sim 1/|\omega - \omega'|$$

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm M} + I_{\rm probe} \otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = \mathrm{Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

Separation of time-scales determines nature of e.o.m. :

Environment relaxation scale τ_E :

$$\langle \Xi_m(t)\Xi_m(0)\rangle \sim e^{-t/\tau_E}$$

probe system scale τ_S :

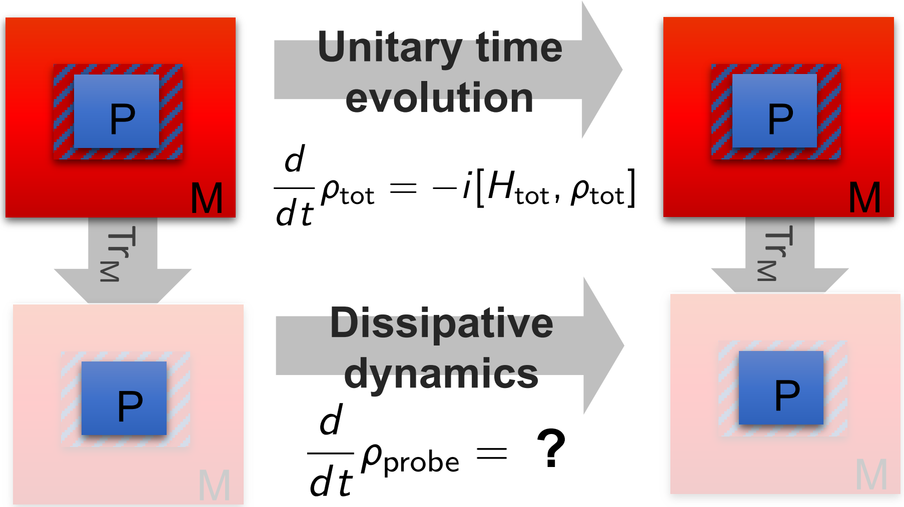
$$\tau_S \sim 1/|\omega - \omega'|$$

probe relaxation scale $\tau_{\rm rel}$:

$$\langle p(t)\rangle \propto e^{-t/\tau_{\rm rel}}$$

The Open Quantum System framework

$$H_{\rm tot} = H_{\rm probe}\otimes I_{\rm M} + I_{\rm probe}\otimes H_{\rm M} + H_{\rm int} = H_{\rm tot}^\dagger$$



$$\frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

$$\frac{d}{dt}\rho_{\rm probe} = \textbf{?}$$

$$\rho_{\rm probe} = {\rm Tr}_{\rm M}[\rho_{\rm tot}]$$

reduced density matrix

Separation of time-scales determines nature of e.o.m. :

Environment relaxation scale τ_E :

$$\langle \Xi_m(t)\Xi_m(0)\rangle \sim e^{-t/\tau_E}$$

probe system scale τ_S :

$$\tau_S \sim 1/|\omega - \omega'|$$

probe relaxation scale $\tau_{\rm rel}$:

$$\langle p(t)\rangle \propto e^{-t/\tau_{\rm rel}}$$

- In case of Markovian time evolution ($\tau_E \ll \tau_{\rm rel}$) leads to a **Lindblad equation:**

$$\frac{d}{dt}\rho_{Q\bar Q} = -i[\tilde H_{Q\bar Q}, \rho_{Q\bar Q}] + \sum_k \gamma_k\left(L_k\rho_{Q\bar Q}L_k^\dagger - \frac{1}{2}L_k^\dagger L_k\rho_{Q\bar Q} - \frac{1}{2}\rho_{Q\bar Q}L_k^\dagger L_k\right)$$

$$\langle n|\rho_{Q\bar Q}|n\rangle > 0, \forall n,$$
$$\rho_{Q\bar Q}^\dagger = \rho_{Q\bar Q}, \quad {\rm Tr}[\rho_{Q\bar Q}] = 1$$

Outline

- Motivation: exploring the properties of hot nuclear matter
- The Quarkonium OQS ecosystem in heavy-ion collisions
- OQS for discovery in heavy-ion collisions – optimal observables
- Conclusion & Outlook

The EFT approach to heavy quarkonium

- Separation of inherent energy scales (EFT) mirrored in timescales (OQS)
- Exploit $\frac{T}{m_Q} \ll 1, \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

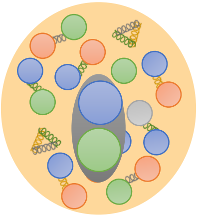
see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

The EFT approach to heavy quarkonium

- Separation of inherent energy scales (EFT) mirrored in timescales (OQS)
- Exploit $\frac{T}{m_Q} \ll 1, \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

Relativistic T>0 field theory

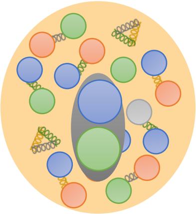


The EFT approach to heavy quarkonium

- Separation of inherent energy scales (EFT) mirrored in timescales (OQS)
- Exploit $\frac{T}{m_Q} \ll 1, \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

Relativistic T>0 field theory



$E \sim m_Q$

QCD

Dirac fields

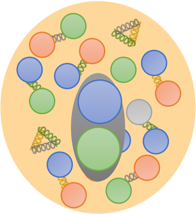
$Q(x), \bar{Q}(x)$

The EFT approach to heavy quarkonium

- Separation of inherent energy scales (EFT) mirrored in timescales (OQS)
- Exploit $\frac{T}{m_Q} \ll 1, \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

Relativistic T>0 field theory



$E \sim m_Q$ → **QCD** → $E \sim m_Q v$ → **NRQCD**

Dirac fields $Q(x), \bar{Q}(x)$

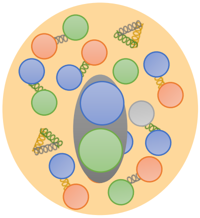
Pauli fields $\psi, \psi^\dagger, \chi, \chi^\dagger$

The EFT approach to heavy quarkonium

- Separation of inherent energy scales (EFT) mirrored in timescales (OQS)
- Exploit $\frac{T}{m_Q} \ll 1, \; \frac{\Lambda_{QCD}}{m_Q} \ll 1$ to treat heavy quarks non-relativistically

see Brambilla et. al. Rev.Mod.Phys. 77 (2005) 142

Relativistic T>0 field theory



$E \sim m_Q$	$E \sim m_Q v$	$E \sim m_Q v^2$
QCD	**NRQCD**	**pNRQCD**
Dirac fields	**Pauli fields**	**Color singlet and octet wavefunctions**
$Q(x), \bar{Q}(x)$	$\psi, \psi^\dagger, \chi, \chi^\dagger$	$\psi_S(R,t), \psi_O(R,t)$

The Quarkonium OQS ecosystem

QCD

The Quarkonium OQS ecosystem

QCD

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N_\psi^{eq}(T)\Big]$$

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD → multipole expansion → pNRQCD

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N_\psi^{eq}(T)\right]$$

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

weak coupling

Lindblad equation

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N_\psi^{eq}(T)\Big]$$

Deterministic Schrödinger

$$i\partial_t\psi = \Big[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\Big]\psi$$

The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

weak coupling

Lindblad equation

"quantum optical limit"

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N_\psi^{eq}(T)\Big]$$

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

weak coupling

Lindblad equation

"quantum optical limit"

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

time scale separation
close to equilibrium

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N_\psi^{eq}(T)\Big]$$

Deterministic Schrödinger

$$i\partial_t\psi = \Big[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\Big]\psi$$

The Quarkonium OQS ecosystem

QCD → multipole expansion → pNRQCD → small dipole approximation → Lindblad equation

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

pNRQCD → weak coupling → Lindblad equation

"quantum optical limit"

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

time scale separation
close to equilibrium

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N^{eq}_\psi(T)\Big]$$

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

small dipole approximation

Lindblad equation

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

non-relativistic quarks

NRQCD

weak coupling

Lindblad equation

"quantum optical limit"

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

time scale separation
close to equilibrium

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N_\psi^{eq}(T)\Big]$$

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD → *multipole expansion* → **pNRQCD** → *small dipole approximation* → **Lindblad equation**

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

QCD → *non-relativistic quarks* → **NRQCD**

NRQCD → *weak coupling "quantum Brownian motion limit"* → **Lindblad equation**

Y. Akamatsu PRD91 (2015) 056002
T. Miura et.al. PRD 106 (2022) 7, 074001

pNRQCD → *weak coupling* → **Lindblad equation**

Lindblad equation → *"quantum optical limit"* → **Boltzmann equation**

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

Boltzmann equation → time scale separation **close to equilibrium** → **Rate equation**

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\Big[N_\psi - N^{eq}_\psi(T)\Big]$$

Lindblad equation (quantum Brownian motion limit) → **Deterministic Schrödinger**

$$i\partial_t\psi = \Big[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\Big]\psi$$

The Quarkonium OQS ecosystem

QCD

multipole expansion → pNRQCD

small dipole approximation → Lindblad equation

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

non-relativistic quarks → NRQCD

weak coupling → Lindblad equation

"quantum optical limit" → Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

time scale separation
close to equilibrium → Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N^{eq}_\psi(T)\right]$$

weak coupling
"quantum Brownian motion limit" → Lindblad equation

Y. Akamatsu PRD91 (2015) 056002
T. Miura et.al. PRD 106 (2022) 7, 074001

color singlet projection
quantum state diffusion → Non-linear stochastic Schrödinger

T. Miura et.al.
PRD101 (2020) 034011

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

small dipole approximation

Lindblad equation

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

non-relativistic quarks

NRQCD

weak coupling

Lindblad equation

weak coupling
“quantum Brownian motion limit”

Y. Akamatsu PRD91 (2015) 056002
T. Miura et.al. PRD 106 (2022) 7, 074001

Lindblad equation

”quantum optical limit”

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}_{nl}^{(rec)}(\mathbf{X},\mathbf{K},t) - \mathcal{C}_{nl}^{(diss)}(\mathbf{X},\mathbf{K},t)$$

color singlet projection
quantum state diffusion

T. Miura et.al. PRD101 (2020) 034011

Non-linear stochastic Schrödinger

time scale separation
close to equilibrium

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N_\psi^{eq}(T)\right]$$

dissipationless limit

Y. Akamatsu, A.R. PRD85 (2012) 105011

Stochastic Schrödinger

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

The Quarkonium OQS ecosystem

QCD → multipole expansion → **pNRQCD** → small dipole approximation → **Lindblad equation**

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

QCD → non-relativistic quarks → **NRQCD**

NRQCD → weak coupling “quantum Brownian motion limit” → **Lindblad equation**

Y. Akamatsu PRD91 (2015) 056002
T. Miura et.al. PRD 106 (2022) 7, 074001

Lindblad equation → color singlet projection **quantum state diffusion** → **Non-linear stochastic Schrödinger**

T. Miura et.al. PRD101 (2020) 034011

Non-linear stochastic Schrödinger → dissipationless limit → **Stochastic Schrödinger**

Y. Akamatsu, A.R. PRD85 (2012) 105011

Stochastic Schrödinger → adiabatic approximation → **Deterministic Schrödinger**

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

pNRQCD → weak coupling → **Lindblad equation** → ”quantum optical limit” → **Boltzmann equation**

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}^{(rec)}_{nl}(\mathbf{X},\mathbf{K},t) - \mathcal{C}^{(diss)}_{nl}(\mathbf{X},\mathbf{K},t)$$

Boltzmann equation → time scale separation **close to equilibrium** → **Rate equation**

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N^{eq}_\psi(T)\right]$$

The Quarkonium OQS ecosystem

QCD

multipole expansion

pNRQCD

small dipole approximation

Lindblad equation

Dynamics in terms of non-perturbative **transport coefficients**

N. Brambilla et.al. PRD99 (2019) 096028, PRD 104 (2021) 9, 094049, PRD 108 (2023) 1, L011502, arXiv: 2508.11743

non-relativistic quarks

NRQCD

weak coupling

Lindblad equation

weak coupling

"quantum Brownian motion limit"

Y. Akamatsu PRD91 (2015) 056002
T. Miura et.al. PRD 106 (2022) 7, 074001

"quantum optical limit"

Lindblad equation

Boltzmann equation

X. Yao, T. Mehen PRD99 (2019) 096028
X. Yao et.al. JHEP 01 (2021) 046

$$\frac{\partial}{\partial t} f_{nl}(\mathbf{X},\mathbf{K},t) + \mathbf{v}\cdot\nabla_{\mathbf{X}} f_{nl}(\mathbf{X},\mathbf{K},t) = \mathcal{C}_{nl}^{(rec)}(\mathbf{X},\mathbf{K},t) - \mathcal{C}_{nl}^{(diss)}(\mathbf{X},\mathbf{K},t)$$

color singlet projection
quantum state diffusion

T. Miura et.al. PRD101 (2020) 034011

Non-linear stochastic Schrödinger

fast decoherence

time scale separation
close to equilibrium

Rate equation

$$\frac{dN_\psi}{dt} = -\Gamma_\psi(T)\left[N_\psi - N_\psi^{eq}(T)\right]$$

dissipationless limit

Y. Akamatsu, A.R. PRD85 (2012) 105011

Stochastic Schrödinger

Semi-classical Langevin

J.P. Blaizot, M. Escobedo JHEP 1806 (2018) 034
A. Daddi-Hammou et.al. PRD 113 (2026) 1, 014017

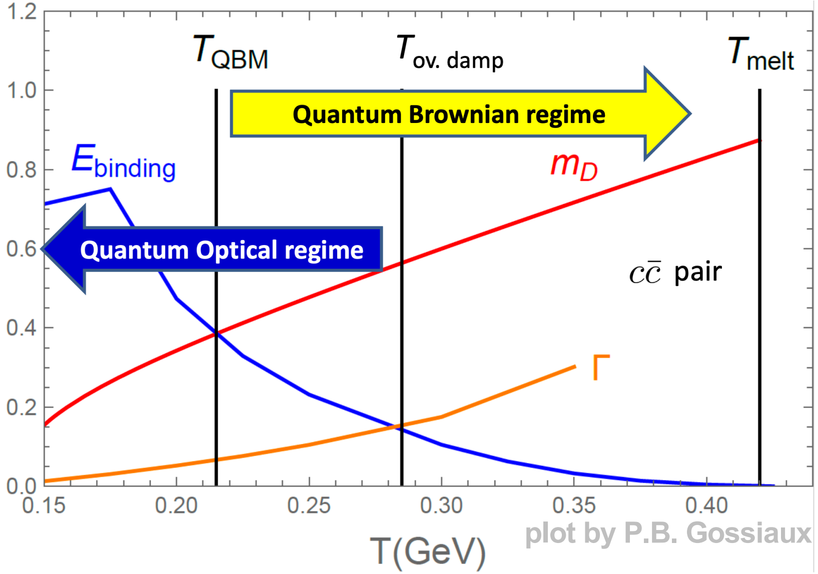
adiabatic approximation

Deterministic Schrödinger

$$i\partial_t\psi = \left[\frac{-\nabla^2}{2m_Q} + Re[V](R,T) - iIm[V](R,T)\right]\psi$$

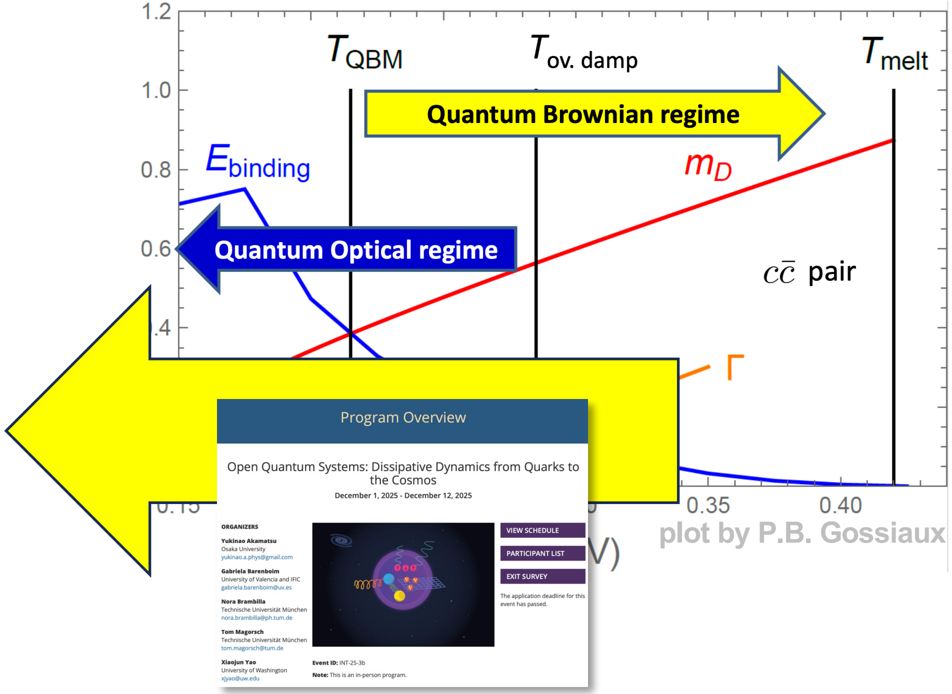
Recent progress I: range of validity

- Traditional OQS approaches cover only a **limited range** of relevant scales



Recent progress I: range of validity

- Traditional OQS approaches cover only a **limited range** of relevant scales



Program Overview

Open Quantum Systems: Dissipative Dynamics from Quarks to the Cosmos

December 1, 2025 - December 12, 2025

ORGANIZERS

Yukinao Akamatsu
Osaka University
yukinao.a.phys@gmail.com

Gabriela Barenboim
University of Valencia and IFIC
gabriela.barenboim@uv.es

Nora Brambilla
Technische Universität München
nora.brambilla@ph.tum.de

Tom Magorsch
Technische Universität München
tom.magorsch@tum.de

Xiaojun Yao
University of Washington
xjyao@uw.edu

VIEW SCHEDULE

PARTICIPANT LIST

EXIT SURVEY

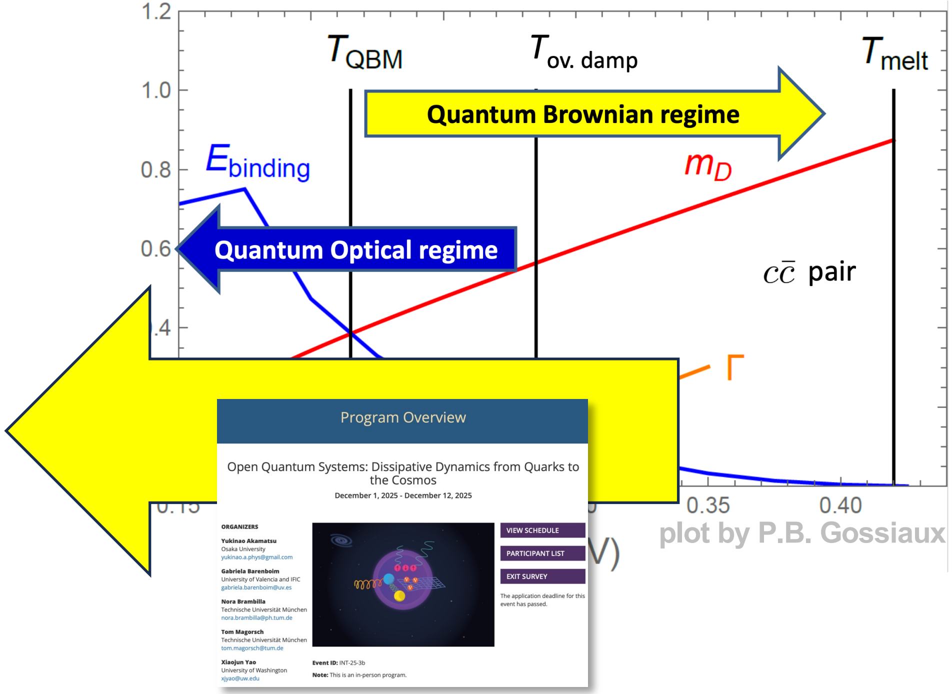
The application deadline for this event has passed.

Event ID: INT-25-3b

Note: This is an in-person program.

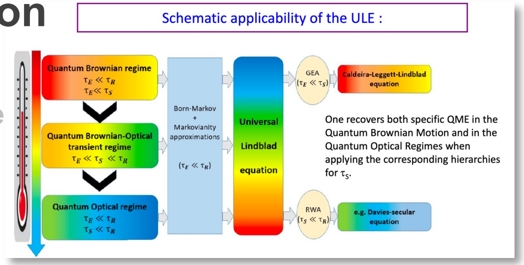
Recent progress I: range of validity

- Traditional OQS approaches cover only a **limited range** of relevant scales



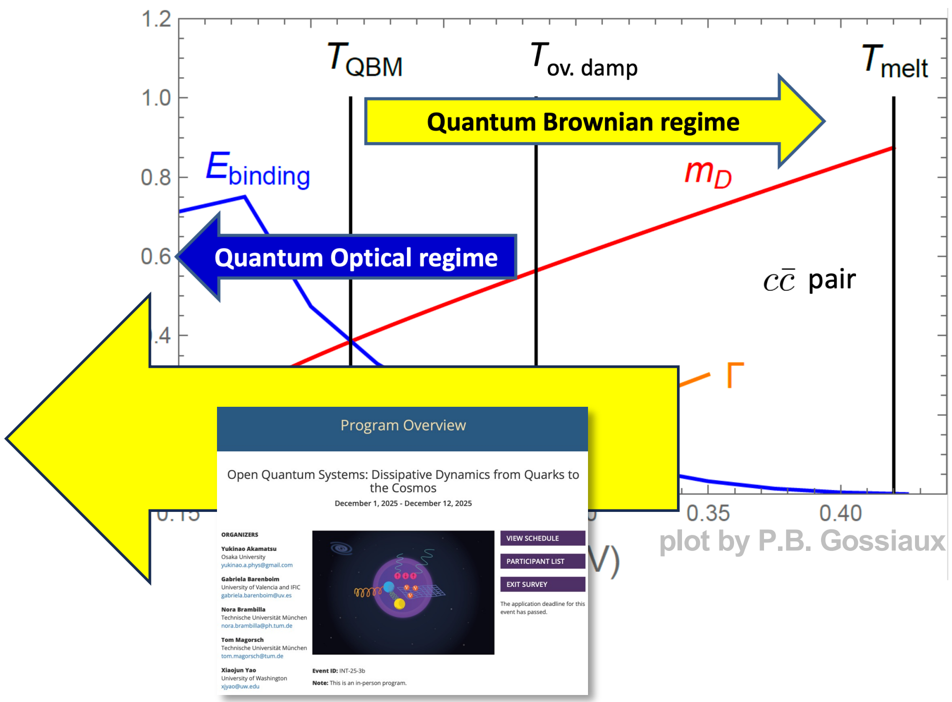
SUBATECH group: Towards a Universal Lindblad equation

(in progress, for update see PoS EPS-HEP2025 (2026) 213)



Recent progress I: range of validity

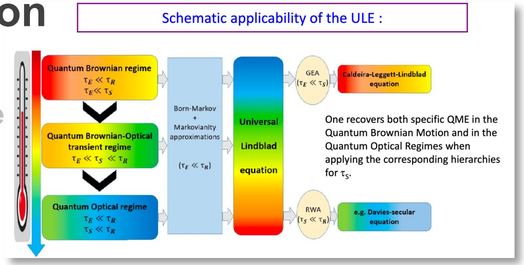
- Traditional OQS approaches cover only a **limited range** of relevant scales



plot by P.B. Gossiaux

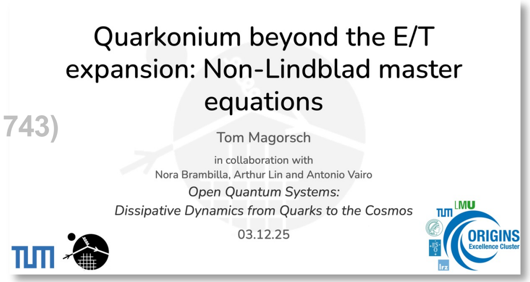
SUBATECH group: Towards a Universal Lindblad equation

(in progress, for update see PoS EPS-HEP2025 (2026) 213)



TUM group: improving the E_{bind}/T expansion

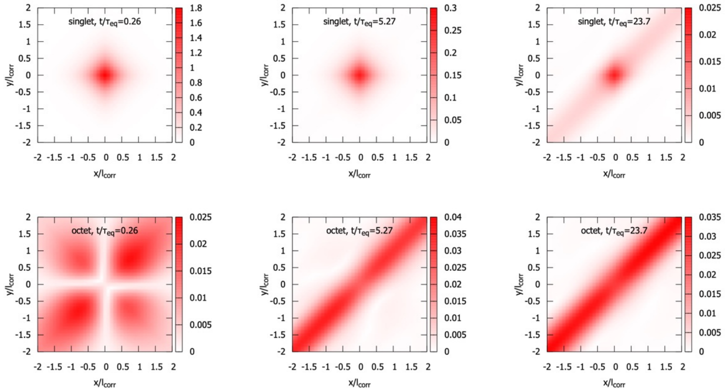
(see also arXiv:2508.11743)



(now full thermalization possible)

Recent progress II: semiclassical limit

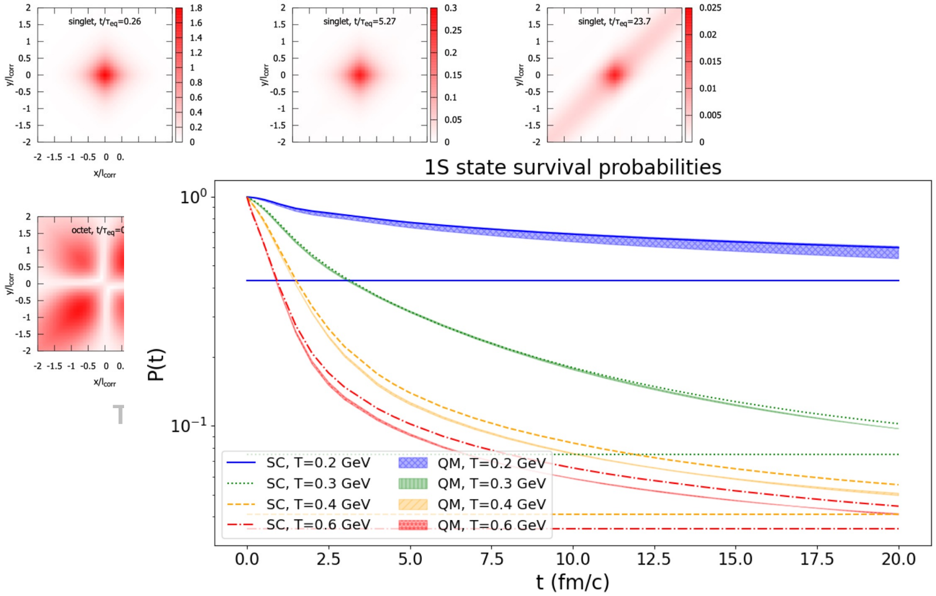
- Fully quantum approaches suffer from curse of dimensionality for multiple QQ



T. Miura et.al. PRD 106 (2022) 7, 074001

Recent progress II: semiclassical limit

- Fully quantum approaches suffer from curse of dimensionality for multiple QQ



A. Daddi-Hammou et.al. PRD 113 (2026) 1, 014017

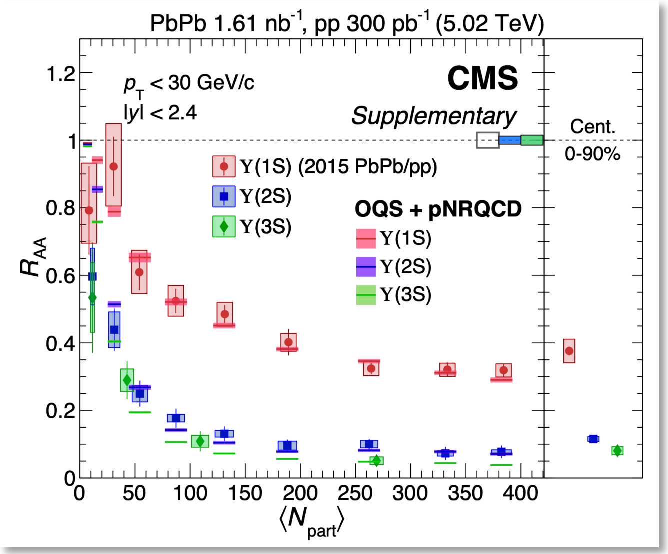
Promising results providing uncertainty quantification for the semiclassical approx.

Outline

- Motivation: exploring the properties of hot nuclear matter
- The Quarkonium OQS ecosystem in heavy-ion collisions
- OQS for discovery in heavy-ion collisions – optimal observables
- Conclusion & Outlook

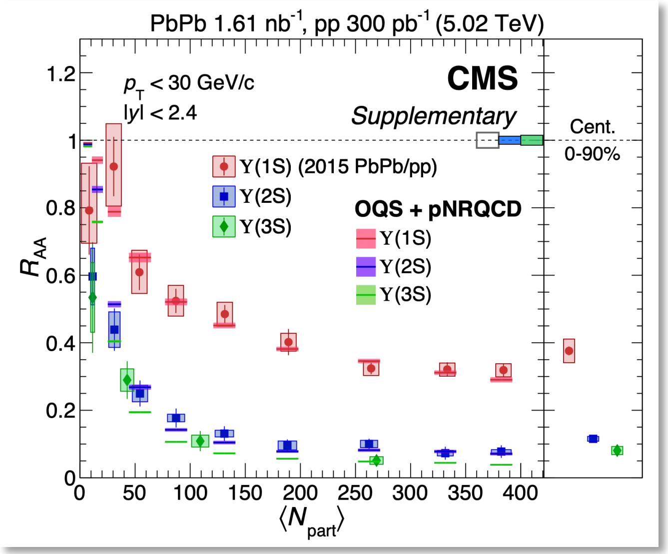
OQS for discovery in heavy-ion collisions

- Need to translate **methods development** into **physics opportunities**



OQS for discovery in heavy-ion collisions

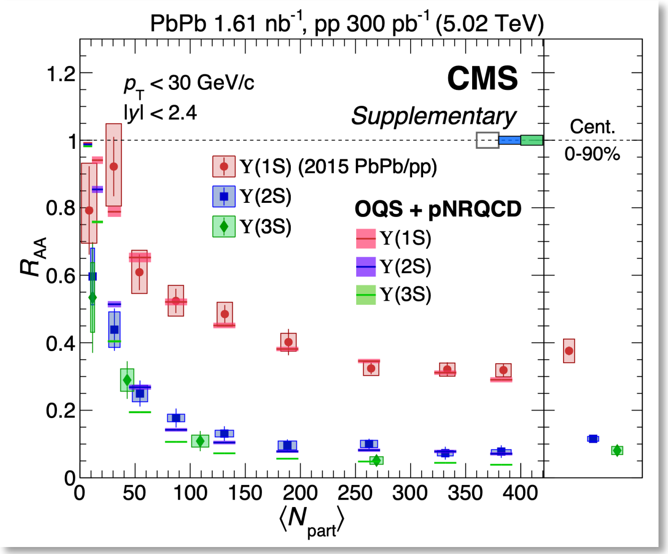
- Need to translate **methods development** into **physics opportunities**



- Key challenge I: **scarcity** of existing observables
  - How to exploit optimally existing data

OQS for discovery in heavy-ion collisions

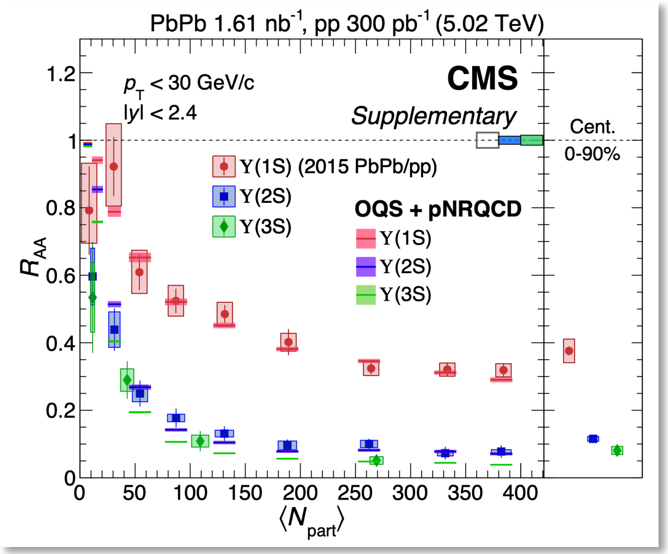
- Need to translate **methods development** into **physics opportunities**



- Key challenge I: **scarcity** of existing observables
  - How to exploit optimally existing data
- Key challenge II: QGP properties are **not direct observables**
  - How to exploit OQS approach to achieve a (thermo-), (viscosi-), (diffusi-) meter?

OQS for discovery in heavy-ion collisions

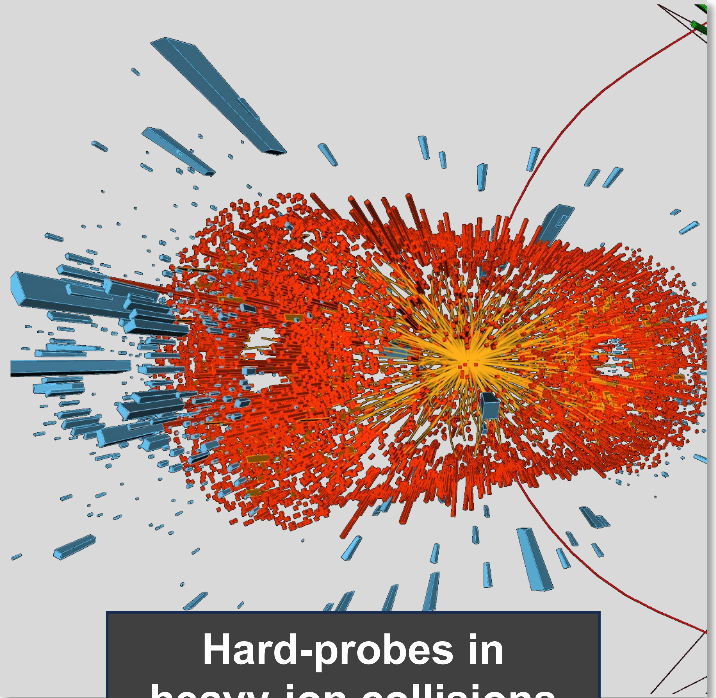
- Need to translate **methods development** into **physics opportunities**



- Key challenge I: **scarcity** of existing observables
  - How to exploit optimally existing data
- Key challenge II: QGP properties are **not direct observables**
  - How to exploit OQS approach to achieve a (thermo-), (viscosi-), (diffusi-) meter?
- Key challenge III: **limited** resources
  - How to decide which observable provides best ROI?

A fruitful analog: impurity physics

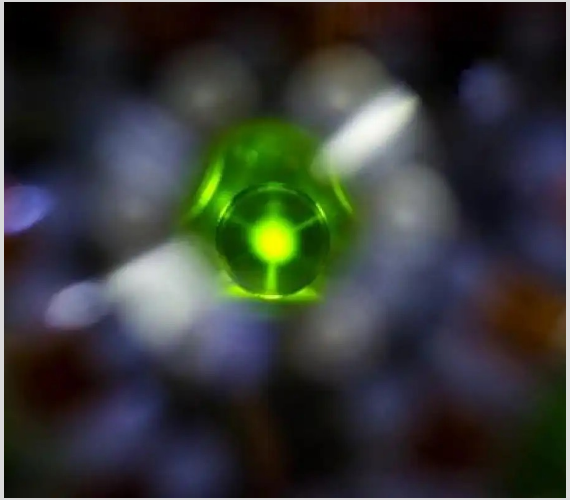
Heavy-ion Collisions



[CMS Collaboration]

Hard-probes in heavy-ion collisions

Ultracold atoms

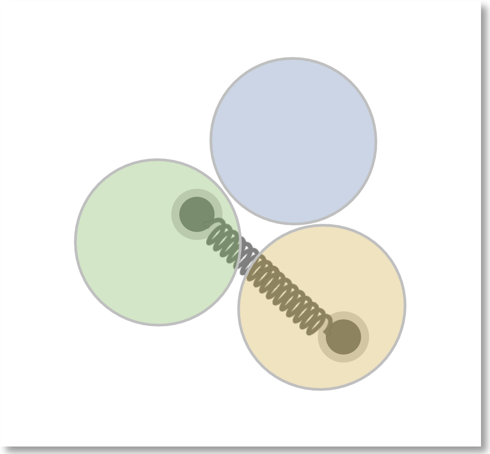


Joint Quantum Institute, UMD

Bosonic or Fermionic Polarons

Impurity physics: metrology at extremes

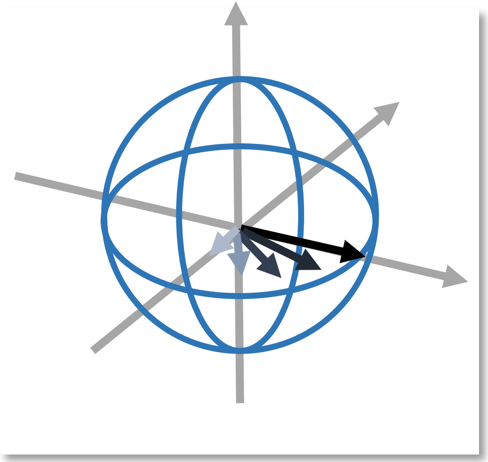
T=10^{12}K



color decoherence of heavy quarkonium in HIC

see e.g. S. Kajimoto, Y.Akamatsu, M. Asakawa, A.R., PRD97 (2018), 014003

T=10^{-9}K



decoherence of a qubit in a quantum gas

M. T. Mitchison et.al. PRL 125, 080402 (2020)

Quantum Metrology

- The study of optimizing the measurement process exploiting quantum features

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

Quantum Metrology

- The study of optimizing the measurement process exploiting quantum features

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

error in T measurement — spread in impurity property

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2\rangle - \langle \hat{O}\rangle^2}{\sqrt{N\chi_T^2[\hat{O}]}}$$

# of measurements — how sensitive is O to T

Quantum Metrology

- The study of optimizing the measurement process exploiting quantum features

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

error in T measurement — spread in impurity property

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2}{\sqrt{N \chi_T^2[\hat{O}]}}$$

# of measurements — how sensitive is O to T

Optimum sensitivity $\chi_T[\hat{O}] = \partial_\xi \mathrm{Tr}[\rho_{\mathrm{probe}}(\xi)\hat{O}]_{\xi=T}$ reached via unique quantity $\hat{\Lambda}_T$:

$$\hat{\Lambda}_T \hat{\rho}_{\mathrm{probe}} + \hat{\rho}_{\mathrm{probe}} \hat{\Lambda}_T = 2\partial_T \hat{\rho}_{\mathrm{probe}}$$

“symmetric logarithmic derivative” (SLD)

Quantum Metrology

- The study of optimizing the measurement process exploiting quantum features

see e.g. M. Mehboudi et.al. J. Phys. A52 (2019) 303001

error in T measurement — spread in impurity property

$$\delta T[\hat{O}] = \frac{\langle \hat{O}^2\rangle - \langle \hat{O}\rangle^2}{\sqrt{N\chi_T^2[\hat{O}]}}$$

\# of measurements — how sensitive is O to T

Optimum sensitivity $\chi_T[\hat{O}] = \partial_\xi \mathrm{Tr}[\rho_{\rm probe}(\xi)\hat{O}]_{\xi=T}$ reached via unique quantity $\hat{\Lambda}_T$:

$$\hat{\Lambda}_T\hat{\rho}_{\rm probe} + \hat{\rho}_{\rm probe}\hat{\Lambda}_T = 2\partial_T\hat{\rho}_{\rm probe}$$

“symmetric logarithmic derivative” (SLD)

- Fisher information $\mathcal{F}_\theta = \mathrm{Tr}[\hat{\rho}(\theta)\hat{\Lambda}_\theta^2]$ (“how much can we learn about θ”)

Explicit construction of SLD now possible

- SLD for **temperature T & relaxation rate γ** from Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

Explicit construction of SLD now possible

- SLD for **temperature T** & **relaxation rate γ** from Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe},\hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x},[\hat{x},\hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x},\{\hat{p},\hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics

Explicit construction of SLD now possible

- SLD for **temperature T** & **relaxation rate γ** from Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe},\hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x},[\hat{x},\hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x},\{\hat{p},\hat{\rho}_{\rm probe}\}]$$

coherent dynamics | fluctuations | dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

Explicit construction of SLD now possible

- SLD for **temperature T** & **relaxation rate γ** from Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\text{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\text{probe}}\}]$$

coherent dynamics   fluctuations   dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

V. López-Pardo, A.R. arXiv:2506.23600

Explicit construction of SLD now possible

- SLD for **temperature T & relaxation rate γ** from Caldeira-Leggett master equation

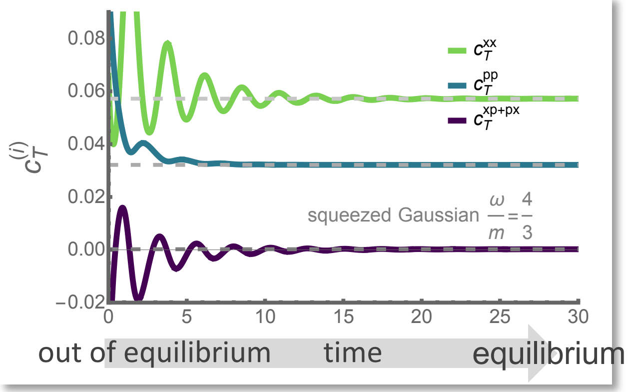
$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\text{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\text{probe}}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics — fluctuations — dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

V. López-Pardo, A.R. arXiv:2506.23600



Explicit construction of SLD now possible

- SLD for **temperature T & relaxation rate γ** from Caldeira-Leggett master equation

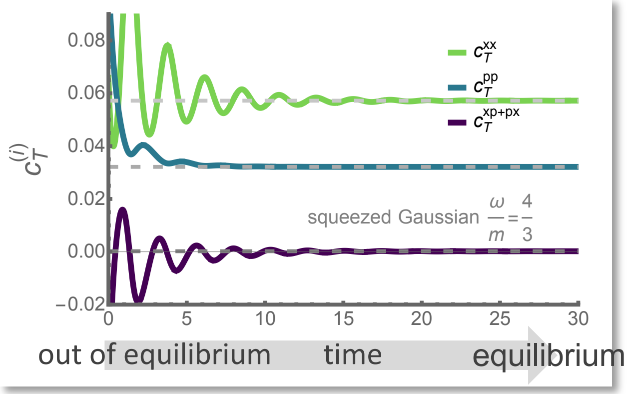
$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x},[\hat{x},\hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x},\{\hat{p},\hat{\rho}_{\rm probe}\}]$$

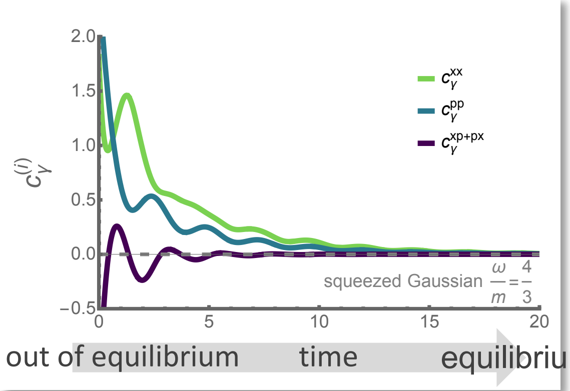
coherent dynamics — fluctuations — dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

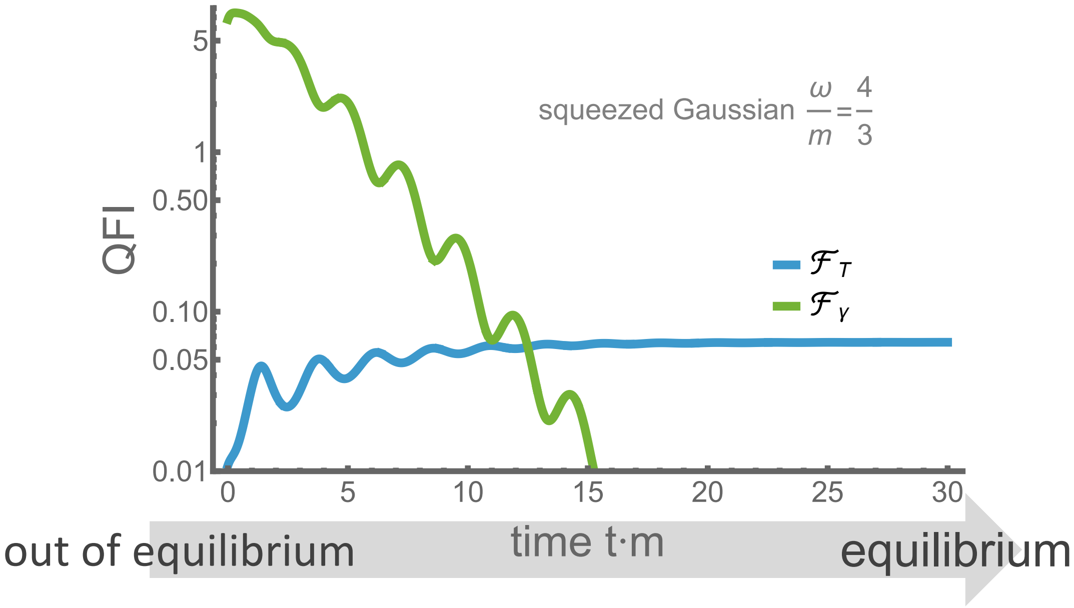
V. López-Pardo, A.R. arXiv:2506.23600





A new tool: Quantum Fisher Information

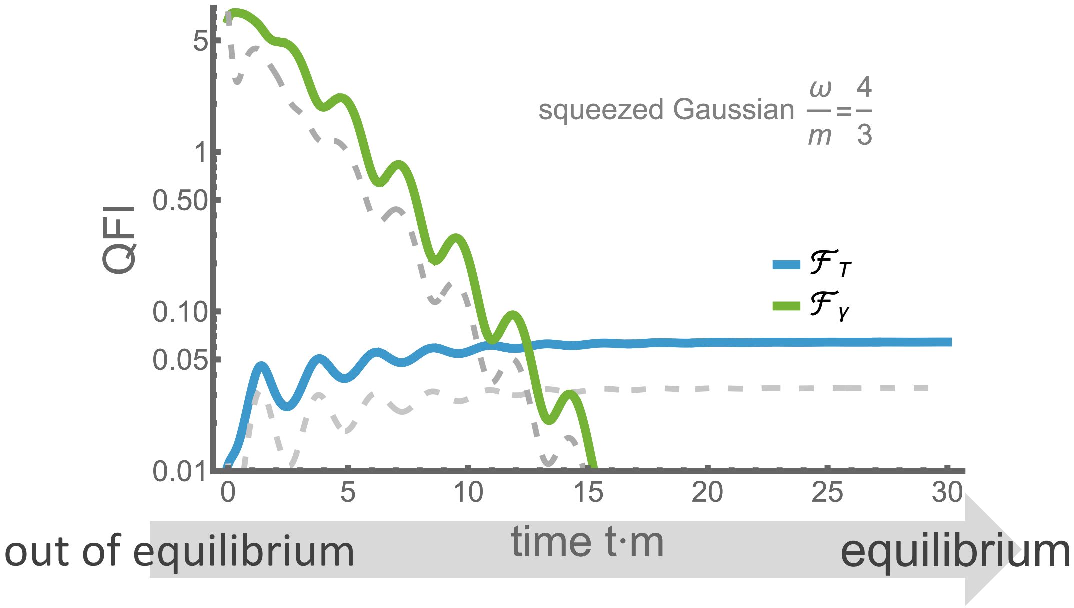
- Tells us how sensitive we are to medium properties during evolution



$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

A new tool: Quantum Fisher Information

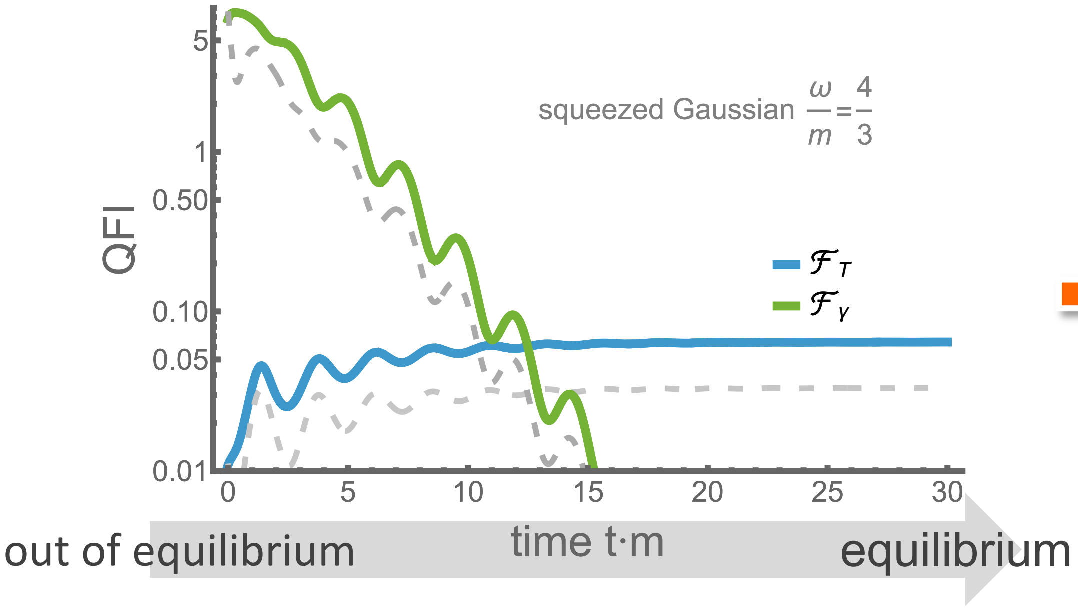
- Tells us how sensitive we are to medium properties during evolution



$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

A new tool: Quantum Fisher Information

- Tells us how sensitive we are to medium properties during evolution



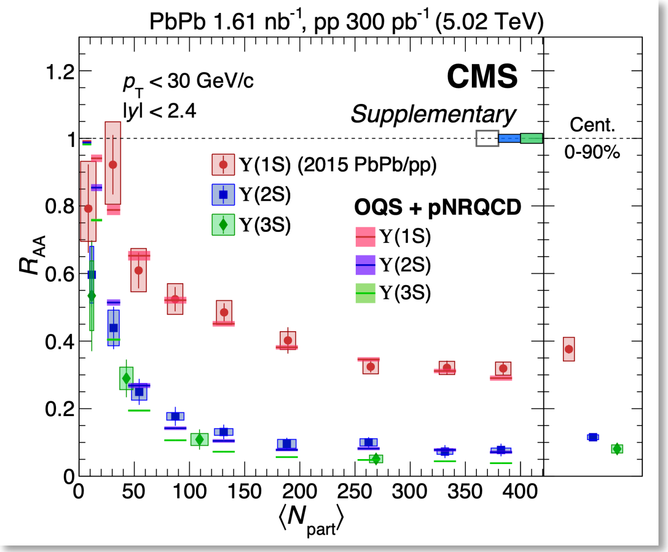
$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

- Important insight:
  by adding / removing observables change in QFI indicates gain/loss in information about medium properties.

Towards OQS insight in quarkonium

- Constructing SLD for pNRQCD Lindblad equation with transport coefficients κ & γ

(**PRELIMINARY** results – work in progress)

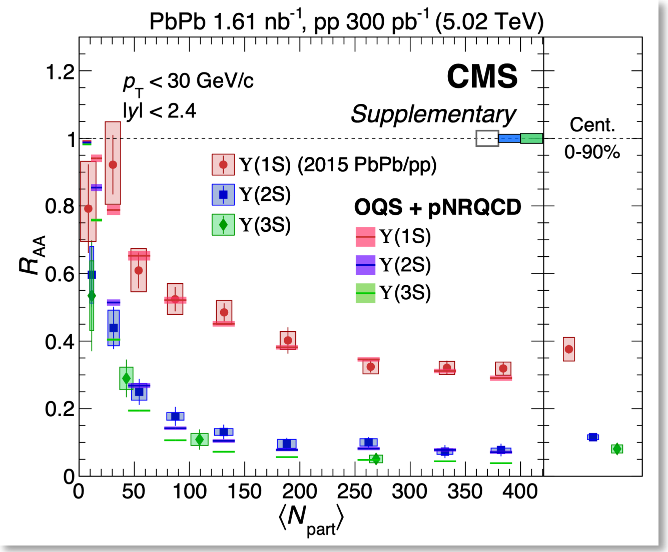


- Restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P) (so far only qualitative assessment possible)

Towards OQS insight in quarkonium

- Constructing SLD for pNRQCD Lindblad equation with transport coefficients κ & γ

(**PRELIMINARY** results – work in progress)



- Restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P) (so far only qualitative assessment possible)

- Υ(1S) survival most sensitive to quarkonium diffusion (κ) while potential modification (γ) requires excites state information.

Conclusion & Outlook

Conclusion & Outlook

- Open Quantum Systems: versatile framework for a probe coupled to environment

Conclusion & Outlook

- Open Quantum Systems: versatile framework for a probe coupled to environment
- OQS ecosystem for heavy quarkonium mature with steady progress

Conclusion & Outlook

- Open Quantum Systems: versatile framework for a probe coupled to environment
- OQS ecosystem for heavy quarkonium mature with steady progress
- Optimal observables defined in OQS metrology: Symmetric Logarithmic Derivative

Conclusion & Outlook

- Open Quantum Systems: versatile framework for a probe coupled to environment
- OQS ecosystem for heavy quarkonium mature with steady progress
- Optimal observables defined in OQS metrology: Symmetric Logarithmic Derivative
- Explicit construction of SLD gives access to QFI: key insights into sensitivity

Conclusion & Outlook

- Open Quantum Systems: versatile framework for a probe coupled to environment
- OQS ecosystem for heavy quarkonium mature with steady progress
- Optimal observables defined in OQS metrology: Symmetric Logarithmic Derivative
- Explicit construction of SLD gives access to QFI: key insights into sensitivity
- Work in progress: application to quarkonium master equations

Conclusion & Outlook

- Open Quantum Systems: versatile framework for a probe coupled to environment
- OQS ecosystem for heavy quarkonium mature with steady progress
- Optimal observables defined in OQS metrology: Symmetric Logarithmic Derivative
- Explicit construction of SLD gives access to QFI: key insights into sensitivity
- Work in progress: application to quarkonium master equations

Thank you for your attention

SLD from the master equation

master equation

$$\partial_t \hat{\rho}_S(t,\theta) = \hat{\mathcal{L}}[\theta]\hat{\rho}_S(t,\theta)$$

definition of SLD

$$\partial_\theta \hat{\rho}(\theta) = \frac{1}{2}\left(\hat{\Lambda}_\theta \hat{\rho}(\theta) + \hat{\rho}(\theta)\hat{\Lambda}_\theta\right) - \hat{\rho}(\theta)\langle\hat{\Lambda}_\theta\rangle$$

SLD from the master equation

master equation

$$\partial_t \hat{\rho}_S(t,\theta) = \hat{\mathcal{L}}[\theta]\hat{\rho}_S(t,\theta)$$

definition of SLD

$$\partial_\theta \hat{\rho}(\theta) = \frac{1}{2}\left(\hat{\Lambda}_\theta \hat{\rho}(\theta) + \hat{\rho}(\theta)\hat{\Lambda}_\theta\right) - \hat{\rho}(\theta)\langle\hat{\Lambda}_\theta\rangle$$

$$\partial_\theta\left(\partial_t \hat{\rho}(t,\theta)\right) = \partial_t\left(\partial_\theta \hat{\rho}(t,\theta)\right)$$

Schwarz' theorem: symmetry of second partial derivatives

V. López-Pardo, A.R. arXiv:2506.23600

SLD from the master equation

master equation

$$\partial_t \hat{\rho}_S(t,\theta) = \hat{\mathcal{L}}[\theta]\hat{\rho}_S(t,\theta)$$

definition of SLD

$$\partial_\theta \hat{\rho}(\theta) = \frac{1}{2}\left(\hat{\Lambda}_\theta \hat{\rho}(\theta) + \hat{\rho}(\theta)\hat{\Lambda}_\theta\right) - \hat{\rho}(\theta)\langle \hat{\Lambda}_\theta \rangle$$

$$\partial_\theta \left(\partial_t \hat{\rho}(t,\theta)\right) = \partial_t \left(\partial_\theta \hat{\rho}(t,\theta)\right)$$

Schwarz' theorem: symmetry of second partial derivatives

V. López-Pardo, A.R. arXiv:2506.23600

choice of experimentally accessible operators

$$\hat{\Lambda}_\theta = \sum_i c_\theta^{(i)} \hat{A}_i$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat{\rho}(t,\theta)\, \hat{A}_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat{\rho}(t,\theta)\, \hat{A}_j\right]$$

SLD from the master equation

master equation

$$\partial_t \hat{\rho}_S(t,\theta) = \hat{\mathcal{L}}[\theta]\hat{\rho}_S(t,\theta)$$

definition of SLD

$$\partial_\theta \hat{\rho}(\theta) = \frac{1}{2}\left(\hat{\Lambda}_\theta \hat{\rho}(\theta) + \hat{\rho}(\theta)\hat{\Lambda}_\theta\right) - \hat{\rho}(\theta)\langle\hat{\Lambda}_\theta\rangle$$

$$\partial_\theta\left(\partial_t \hat{\rho}(t,\theta)\right) = \partial_t\left(\partial_\theta \hat{\rho}(t,\theta)\right)$$

Schwarz' theorem: symmetry of second partial derivatives

V. López-Pardo, A.R. arXiv:2506.23600

choice of experimentally accessible operators

$$\hat{\Lambda}_\theta = \sum_i c_\theta^{(i)} \hat{A}_i$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat{\rho}(t,\theta)\, \hat{A}_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat{\rho}(t,\theta)\, \hat{A}_j\right]$$

$$\sum_i M_{ji}\, c_\theta^{(i)} = D_j$$

Explicit construction of SLD via solution of linear system of equations

Application to Quarkonium – pNRQCD OQS

- As a first step: optimal estimation of local properties – transport coefficients

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_i^{S\leftrightarrow O} = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \quad L_i^{O\leftrightarrow O} = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

N. Brambilla et. al. PRD100 (2019), 054025

governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c}\int_0^\infty dt \left\langle \left\{ E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right\} \right\rangle$$ heavy quarkonium diffusion constant

$$\gamma \propto -\frac{i}{6N_c}\int_0^\infty dt \left\langle \left[E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right] \right\rangle$$ potential correction

Application to Quarkonium – pNRQCD OQS

- As a first step: optimal estimation of local properties – transport coefficients

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[\tilde{H}_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_k \gamma_k \Big(L_k \rho_{Q\bar{Q}} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar{Q}} - \frac{1}{2} \rho_{Q\bar{Q}} L_k^\dagger L_k \Big)$$

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_i^{S\leftrightarrow O} = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \quad L_i^{O\leftrightarrow O} = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

N. Brambilla et. al. PRD100 (2019), 054025

governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c} \int_0^\infty dt \left\langle \{ E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \} \right\rangle$$ heavy quarkonium diffusion constant

$$\gamma \propto -\frac{i}{6N_c} \int_0^\infty dt \left\langle [E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0})] \right\rangle$$ potential correction

Application to Quarkonium – pNRQCD OQS

- As a first step: optimal estimation of local properties – transport coefficients

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_i^{S\leftrightarrow O} = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \quad L_i^{O\leftrightarrow O} = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

N. Brambilla et. al. PRD100 (2019), 054025

governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c}\int_0^\infty dt \left\langle \left\{ E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right\} \right\rangle$$ heavy quarkonium diffusion constant

$$\gamma \propto -\frac{i}{6N_c}\int_0^\infty dt \left\langle \left[E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right] \right\rangle$$ potential correction

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[\tilde{H}_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_k \gamma_k \left(L_k \rho_{Q\bar{Q}} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}} L_k^\dagger L_k \right)$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat{\rho}(t,\theta)\, \hat{A}_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat{\rho}(t,\theta)\, \hat{A}_j\right]$$

Application to Quarkonium – pNRQCD OQS

- As a first step: optimal estimation of local properties – transport coefficients

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_i^{S\leftrightarrow O} = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \quad L_i^{O\leftrightarrow O} = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

N. Brambilla et. al. PRD100 (2019), 054025

governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c}\int_0^\infty dt \left\langle \left\{ E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right\} \right\rangle$$ heavy quarkonium diffusion constant

$$\gamma \propto -\frac{i}{6N_c}\int_0^\infty dt \left\langle \left[E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right] \right\rangle$$ potential correction

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[\tilde{H}_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_k \gamma_k \left(L_k \rho_{Q\bar{Q}} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}} L_k^\dagger L_k \right)$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat{\rho}(t,\theta)\, \hat{A}_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat{\rho}(t,\theta)\, \hat{A}_j\right]$$

$$\begin{aligned} M_{ij} &= +\tfrac{i}{2}\left\langle \{\hat{A}_j, [\hat{H}, \hat{A}_i]\} \right\rangle \\ &+ \tfrac{1}{2}\sum_n \left(\left\langle \hat{C}_n^+ \hat{A}_j\,,\, [\hat{A}_i, \hat{C}_n] \right\rangle + \left\langle [\hat{C}_n^+, \hat{A}_i]\, \hat{A}_j \hat{C}_n \right\rangle - \tfrac{1}{2}\left\langle [\hat{A}_j, [\hat{A}_i, \hat{\Gamma}_n]] \right\rangle \right) \\ &- \langle \hat{A}_j \rangle \left(-i\left\langle \{\hat{A}_i, \hat{H}\} \right\rangle + \sum_n \left(\left\langle \hat{C}_n^+ \hat{A}_i \hat{C}_n \right\rangle - \tfrac{1}{2}\left\langle \{\hat{A}_i, \hat{\Gamma}_n\} \right\rangle \right) \right) \end{aligned}$$

$$D_j = -\tfrac{i}{2}\begin{pmatrix} \left\langle [\hat{A}_j^s, \hat{r}^{\,2}] \right\rangle & 0 \\ 0 & \left\langle [\hat{A}_j^o, \hat{r}^{\,2}] \right\rangle \Omega_o \end{pmatrix}$$

Application to Quarkonium – pNRQCD OQS

- As a first step: optimal estimation of local properties – transport coefficients

pNRQCD & strongly coupled medium

Non-perturbative medium but Coulombic bound states

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_O \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$L_i^{S\leftrightarrow O} = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix} \quad L_i^{O\leftrightarrow O} = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

N. Brambilla et. al. PRD100 (2019), 054025

governed by two (static) transport coefficients:

$$\kappa \propto \frac{1}{6N_c}\int_0^\infty dt \left\langle \left\{ E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right\} \right\rangle$$ heavy quarkonium diffusion constant

$$\gamma \propto -\frac{i}{6N_c}\int_0^\infty dt \left\langle \left[E^{a,i}(t,\mathbf{0}), E^{a,i}(t,\mathbf{0}) \right] \right\rangle$$ potential correction

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[\tilde{H}_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \sum_k \gamma_k \left(L_k \rho_{Q\bar{Q}} L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}} L_k^\dagger L_k \right)$$

$$\mathrm{Tr}\left[\partial_\theta \partial_t \hat{\rho}(t,\theta)\, \hat{A}_j\right] = \mathrm{Tr}\left[\partial_t \partial_\theta \hat{\rho}(t,\theta)\, \hat{A}_j\right]$$

$$\begin{aligned} M_{ij} = &+\tfrac{i}{2}\left\langle \{\hat{A}_j, [\hat{H}, \hat{A}_i]\} \right\rangle \\ &+ \tfrac{1}{2}\sum_n \left(\left\langle \hat{C}_n^+ \hat{A}_j\,,\, [\hat{A}_i, \hat{C}_n] \right\rangle + \left\langle [\hat{C}_n^+, \hat{A}_i]\, \hat{A}_j \hat{C}_n \right\rangle - \tfrac{1}{2}\left\langle [\hat{A}_j, [\hat{A}_i, \hat{\Gamma}_n]] \right\rangle \right) \\ &- \langle \hat{A}_j \rangle \left(-i \left\langle \{\hat{A}_i, \hat{H}\} \right\rangle + \sum_n \left(\left\langle \hat{C}_n^+ \hat{A}_i \hat{C}_n \right\rangle - \tfrac{1}{2}\left\langle \{\hat{A}_i, \hat{\Gamma}_n\} \right\rangle \right) \right) \end{aligned}$$

$$D_j = -\tfrac{i}{2}\begin{pmatrix} \left\langle [\hat{A}_j^s, \hat{r}^2] \right\rangle & 0 \\ 0 & \left\langle [\hat{A}_j^o, \hat{r}^2] \right\rangle \Omega_o \end{pmatrix}$$

$$\hat{A}_i = \{ |\Upsilon(1S)\rangle\langle\Upsilon(1S)|, |\Upsilon(2S)\rangle\langle\Upsilon(2S)|, |\Upsilon(3S)\rangle\langle\Upsilon(3S)|, |\chi_i(1P)\rangle\langle\chi_i(1P)| \}$$

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

SLD for transport coefficient γ

```
{{cY1S → 0. - 0.00348522 cC10 - 0.00348522 cC11 - 0.00348522 cC12 - 0.261354 cY2S - 0.216845 cY3S}}
```

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

SLD for transport coefficient γ

{{cY1S → 0. - 0.00348522 cC10 - 0.00348522 cC11 - 0.00348522 cC12 - 0.261354 cY2S - 0.216845 cY3S}}

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

SLD for transport coefficient γ

{{cY1S → 0. - 0.00348522 cC10 - 0.00348522 cC11 - 0.00348522 cC12 - 0.261354 cY2S - 0.216845 cY3S}}

SLD for transport coefficient κ

{{cY1S → 20.4013 - 0.00348522 cC10 - 0.00348522 cC11 - 0.00348522 cC12 - 0.261354 cY2S - 0.216845 cY3S}}

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

SLD for transport coefficient γ

{{cY1S → 0. – 0.00348522 cC10 – 0.00348522 cC11 – 0.00348522 cC12 – 0.261354 cY2S – 0.216845 cY3S}}

SLD for transport coefficient κ

{{cY1S → 20.4013 – 0.00348522 cC10 – 0.00348522 cC11 – 0.00348522 cC12 – 0.261354 cY2S – 0.216845 cY3S}}

First PRELIMINARY insights

- Using the assumptions of singlet dominance and no off-diagonal contributions
- Highly restricted basis: Υ(1S) Υ(2S) Υ(3S) χ_{b012}(1P): system matrix Mij degenerate

SLD for transport coefficient γ

{{cY1S → 0. – 0.00348522 cC10 – 0.00348522 cC11 – 0.00348522 cC12 – 0.261354 cY2S – 0.216845 cY3S}}

SLD for transport coefficient κ

{{cY1S → 20.4013 – 0.00348522 cC10 – 0.00348522 cC11 – 0.00348522 cC12 – 0.261354 cY2S – 0.216845 cY3S}}

- Υ(1S) survival sensitive to κ but determination of γ requires excited states survival

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

coherent dynamics

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x},[\hat{x},\hat{\rho}_{\text{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x},\{\hat{p},\hat{\rho}_{\text{probe}}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics fluctuations dissipation

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics fluctuations dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics — fluctuations — dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\rm T} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

coherent dynamics fluctuations dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\rm T} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\text{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\text{probe}}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics — fluctuations — dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\text{T}} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x},[\hat{x},\hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x},\{\hat{p},\hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics — fluctuations — dissipation

$$\hat{A}_j \in \{\hat{x},\hat{p},\hat{x}^2,\hat{p}^2,\{\hat{x},\hat{p}\}\}$$

$$D_{\rm T} = \left[0,0,0,-2\frac{\gamma}{b},0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\text{probe}} = -\frac{i}{\hbar}[\hat{H}_{\text{probe}}, \hat{\rho}_{\text{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\text{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\text{probe}}\}]$$

coherent dynamics fluctuations dissipation

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\text{T}} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

$$c_T^{(x)} = 0,$$
$$c_T^{(p)} = 0,$$
$$c_T^{(x^2)} = \frac{4c\left(a^4b^2 + 4(bc+\gamma^2)\right)}{a^8b^4 - 16b^2c^2},$$
$$c_T^{(p^2)} = \frac{4}{a^4b - 4c},$$
$$c_T^{(\{x,p\})} = \frac{16c\gamma}{a^8b^3 - 16bc^2}.$$

V. López-Pardo, A.R. arXiv:2506.23600

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\mathrm{probe}} = -\frac{i}{\hbar}[\hat{H}_{\mathrm{probe}}, \hat{\rho}_{\mathrm{probe}}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\mathrm{probe}}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\mathrm{probe}}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics fluctuations dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\mathrm{T}} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

$$c_T^{(x)} = 0,$$
$$c_T^{(p)} = 0,$$
$$c_T^{(x^2)} = \frac{4c\left(a^4b^2 + 4(bc+\gamma^2)\right)}{a^8b^4 - 16b^2c^2},$$
$$c_T^{(p^2)} = \frac{4}{a^4b - 4c},$$
$$c_T^{(\{x,p\})} = \frac{16c\gamma}{a^8b^3 - 16bc^2}.$$

reproduces the known result from explicit density matrix (T in equilibrium)

V. López-Pardo, A.R. arXiv:2506.23600

Quantum Brownian motion (equilibrium)

- SLD for **temperature T** from the Caldeira-Leggett master equation

$$\frac{d}{dt}\hat{\rho}_{\rm probe} = -\frac{i}{\hbar}[\hat{H}_{\rm probe}, \hat{\rho}_{\rm probe}] - \frac{2m\gamma k_B T}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}_{\rm probe}]] - \frac{i\gamma}{\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}_{\rm probe}\}]$$

A.O. Caldeira and A.J. Leggett: Physica 121A (1983) 587

coherent dynamics — fluctuations — dissipation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$$D_{\rm T} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M = \begin{bmatrix} 0 & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma & -8b\langle p^2\rangle\langle x^2\rangle - 2b \\ 0 & 0 & 0 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T\langle p^2\rangle}{b} & 8c\langle p^2\rangle\langle x^2\rangle + 2c \\ 0 & 0 & 2b + 8c\langle x^2\rangle^2 & -8b\langle p^2\rangle^2 - 2c & -\frac{8\gamma T\langle x^2\rangle}{b} - 2\gamma + 8\gamma\langle p^2\rangle\langle x^2\rangle \end{bmatrix}$$

$$c_T^{(x)} = 0,$$
$$c_T^{(p)} = 0,$$
$$c_T^{(x^2)} = \frac{4c\left(a^4b^2 + 4(bc+\gamma^2)\right)}{a^8b^4 - 16b^2c^2},$$
$$c_T^{(p^2)} = \frac{4}{a^4b - 4c},$$
$$c_T^{(\{x,p\})} = \frac{16c\gamma}{a^8b^3 - 16bc^2}.$$

reproduces the known result from explicit density matrix (T in equilibrium)

V. López-Pardo, A.R. arXiv:2506.23600

need to include the {x,p} operator to obtain a well-posed linear system even though $\langle\{x,p\}\rangle_{eq} = 0$

Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T** & **relaxation rate ɣ** from Caldeira-Leggett master equation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T** & **relaxation rate γ** from Caldeira-Leggett master equation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

$M_T = M_\gamma$

$$\begin{bmatrix}
-b\langle\{x,p\}\rangle & -2b\langle p^2\rangle & 0 & 0 & 0 \\
2c\langle x^2\rangle + \gamma\langle\{x,p\}\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} + c\langle\{x,p\}\rangle & 0 & 0 & 0 \\
0 & 0 & -4b\langle x^2\rangle\langle\{x,p\}\rangle & -4b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma & -8b\langle x^2\rangle\langle p^2\rangle - 2b - 2b\langle\{x,p\}\rangle^2 \\
0 & 0 & 4c\langle x^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T}{b}\langle p^2\rangle + 4c\langle p^2\rangle\langle\{x,p\}\rangle & 8c\langle x^2\rangle\langle p^2\rangle + 2c + 2c\langle\{x,p\}\rangle^2 + 8\gamma\langle p^2\rangle\langle\{x,p\}\rangle - \frac{4\gamma T}{b}\langle\{x,p\}\rangle \\
0 & 0 & 8c\langle x^2\rangle^2 + 2b + 4\gamma\langle x^2\rangle\langle\{x,p\}\rangle & -8b\langle p^2\rangle^2 - 2c + 4\gamma\langle p^2\rangle\langle\{x,p\}\rangle + 2c\langle\{x,p\}\rangle^2 - \frac{4\gamma T}{b}\langle\{x,p\}\rangle & 8\gamma\langle x^2\rangle\langle p^2\rangle - \frac{8\gamma T}{b}\langle x^2\rangle - 2\gamma + 8c\langle x^2\rangle\langle\{x,p\}\rangle - 8b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2
\end{bmatrix}$$

Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T** & **relaxation rate γ** from Caldeira-Leggett master equation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

T →

$$D_T = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$M_T = M_\gamma$$

$$\begin{bmatrix}
-b\langle\{x,p\}\rangle & -2b\langle p^2\rangle & 0 & 0 & 0 \\
2c\langle x^2\rangle + \gamma\langle\{x,p\}\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} + c\langle\{x,p\}\rangle & 0 & 0 & 0 \\
0 & 0 & -4b\langle x^2\rangle\langle\{x,p\}\rangle & -4b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma & -8b\langle x^2\rangle\langle p^2\rangle - 2b - 2b\langle\{x,p\}\rangle^2 \\
0 & 0 & 4c\langle x^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T}{b}\langle p^2\rangle + 4c\langle p^2\rangle\langle\{x,p\}\rangle & 8c\langle x^2\rangle\langle p^2\rangle + 2c + 2c\langle\{x,p\}\rangle^2 + 8\gamma\langle p^2\rangle\langle\{x,p\}\rangle - \frac{4\gamma T}{b}\langle\{x,p\}\rangle \\
0 & 0 & 8c\langle x^2\rangle^2 + 2b + 4\gamma\langle x^2\rangle\langle\{x,p\}\rangle & -8b\langle p^2\rangle^2 - 2c + 4\gamma\langle p^2\rangle\langle\{x,p\}\rangle + 2c\langle\{x,p\}\rangle^2 - \frac{4\gamma T}{b}\langle\{x,p\}\rangle & 8\gamma\langle x^2\rangle\langle p^2\rangle - \frac{8\gamma T}{b}\langle x^2\rangle - 2\gamma + 8c\langle x^2\rangle\langle\{x,p\}\rangle - 8b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2
\end{bmatrix}$$

Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T & relaxation rate γ** from Caldeira-Leggett master equation

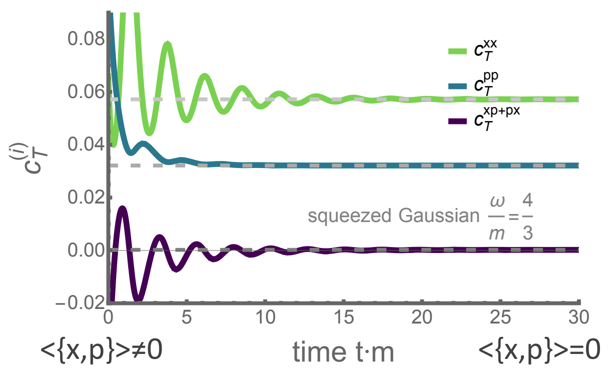
$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x},\hat{p}\}\}$$

T

$$D_T = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$\mathbf{M_T = M_\gamma}$$

$$\begin{bmatrix} -b\langle\{x,p\}\rangle & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle + \gamma\langle\{x,p\}\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} + c\langle\{x,p\}\rangle & 0 & 0 & 0 \\ 0 & 0 & -4b\langle x^2\rangle\langle\{x,p\}\rangle & -4b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma & -8b\langle x^2\rangle\langle p^2\rangle - 2b - 2b\langle\{x,p\}\rangle^2 \\ 0 & 0 & 4c\langle x^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T}{b}\langle p^2\rangle + 4c\langle p^2\rangle\langle\{x,p\}\rangle & 8c\langle x^2\rangle\langle p^2\rangle + 2c + 2c\langle\{x,p\}\rangle^2 + 8\gamma\langle p^2\rangle\langle\{x,p\}\rangle - \frac{4\gamma T}{b}\langle\{x,p\}\rangle \\ 0 & 0 & 8c\langle x^2\rangle^2 + 2b + 4\gamma\langle x^2\rangle\langle\{x,p\}\rangle & -8b\langle p^2\rangle^2 - 2c + 4\gamma\langle p^2\rangle\langle\{x,p\}\rangle + 2c\langle\{x,p\}\rangle^2 - \frac{4\gamma T}{b}\langle\{x,p\}\rangle & 8\gamma\langle x^2\rangle\langle p^2\rangle - \frac{8\gamma T}{b}\langle x^2\rangle - 2\gamma + 8c\langle x^2\rangle\langle\{x,p\}\rangle - 8b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 \end{bmatrix}$$



Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T & relaxation rate ɣ** from Caldeira-Leggett master equation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

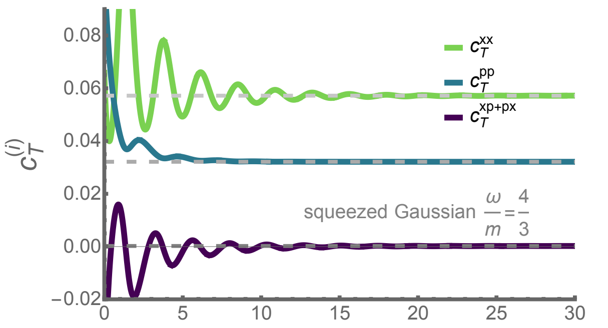
T, ɣ

$$D_{\mathrm{T}} = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$D_{\gamma} = \left[0, 0, 0, -2\frac{T}{b} + 4\left\langle p^2\right\rangle, 2\left\langle\{\hat{x}, \hat{p}\}\right\rangle\right]$$

$M_{\mathrm{T}} = M_{\gamma}$

$$\begin{bmatrix} -b\langle\{x,p\}\rangle & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle + \gamma\langle\{x,p\}\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} + c\langle\{x,p\}\rangle & 0 & 0 & 0 \\ 0 & 0 & -4b\langle x^2\rangle\langle\{x,p\}\rangle & -4b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma & -8b\langle x^2\rangle\langle p^2\rangle - 2b - 2b\langle\{x,p\}\rangle^2 \\ 0 & 0 & 4c\langle x^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T}{b}\langle p^2\rangle + 4c\langle p^2\rangle\langle\{x,p\}\rangle & 8c\langle x^2\rangle\langle p^2\rangle + 2c + 2c\langle\{x,p\}\rangle^2 + 8\gamma\langle p^2\rangle\langle\{x,p\}\rangle - \frac{4\gamma T}{b}\langle\{x,p\}\rangle \\ 0 & 0 & 8c\langle x^2\rangle^2 + 2b + 4\gamma\langle x^2\rangle\langle\{x,p\}\rangle & -8b\langle p^2\rangle^2 - 2c + 4\gamma\langle p^2\rangle\langle\{x,p\}\rangle + 2c\langle\{x,p\}\rangle^2 - \frac{4\gamma T}{b}\langle\{x,p\}\rangle & 8\gamma\langle x^2\rangle\langle p^2\rangle - \frac{8\gamma T}{b}\langle x^2\rangle - 2\gamma + 8c\langle x^2\rangle\langle\{x,p\}\rangle - 8b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 \end{bmatrix}$$



<{x,p}>≠0 <{x,p}>=0

Quantum Brownian motion (squeezed Gaussian)

- SLD for **temperature T & relaxation rate γ** from Caldeira-Leggett master equation

$$\hat{A}_j \in \{\hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2, \{\hat{x}, \hat{p}\}\}$$

T, γ →

$$D_T = \left[0, 0, 0, -2\frac{\gamma}{b}, 0\right]$$

$$D_\gamma = \left[0, 0, 0, -2\frac{T}{b} + 4\left\langle p^2 \right\rangle, 2\left\langle \{\hat{x}, \hat{p}\} \right\rangle\right]$$

$$\mathbf{M_T = M_\gamma}$$

$$\begin{bmatrix} -b\langle\{x,p\}\rangle & -2b\langle p^2\rangle & 0 & 0 & 0 \\ 2c\langle x^2\rangle + \gamma\langle\{x,p\}\rangle & 2\gamma\langle p^2\rangle - \frac{2\gamma T}{b} + c\langle\{x,p\}\rangle & 0 & 0 & 0 \\ 0 & 0 & -4b\langle x^2\rangle\langle\{x,p\}\rangle & -4b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma & -8b\langle x^2\rangle\langle p^2\rangle - 2b - 2b\langle\{x,p\}\rangle^2 \\ 0 & 0 & 4c\langle x^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 & 8\gamma\langle p^2\rangle^2 - \frac{8\gamma T}{b}\langle p^2\rangle + 4c\langle p^2\rangle\langle\{x,p\}\rangle & 8c\langle x^2\rangle\langle p^2\rangle + 2c + 2c\langle\{x,p\}\rangle^2 + 8\gamma\langle p^2\rangle\langle\{x,p\}\rangle - \frac{4\gamma T}{b}\langle\{x,p\}\rangle \\ 0 & 0 & 8c\langle x^2\rangle^2 + 2b + 4\gamma\langle x^2\rangle\langle\{x,p\}\rangle & -8b\langle p^2\rangle^2 - 2c + 4\gamma\langle p^2\rangle\langle\{x,p\}\rangle + 2c\langle\{x,p\}\rangle^2 - \frac{4\gamma T}{b}\langle\{x,p\}\rangle & 8\gamma\langle x^2\rangle\langle p^2\rangle - \frac{8\gamma T}{b}\langle x^2\rangle - 2\gamma + 8c\langle x^2\rangle\langle\{x,p\}\rangle - 8b\langle p^2\rangle\langle\{x,p\}\rangle + 2\gamma\langle\{x,p\}\rangle^2 \end{bmatrix}$$