How strange: Phase diagrams with 3 critical points

Jacquelyn Noronha-Hostler

Illinois Center for Advanced Studies of the Universe

Based on:

$T = 0$ Cruz Camacho et al, *Phys.Rev.D* 111 (2025) 9, 094030;

$T > 0$ Reinke Pelicer, Cruz-Camacho, Kumar, Dexheimer, JNH, to appear soon

muses

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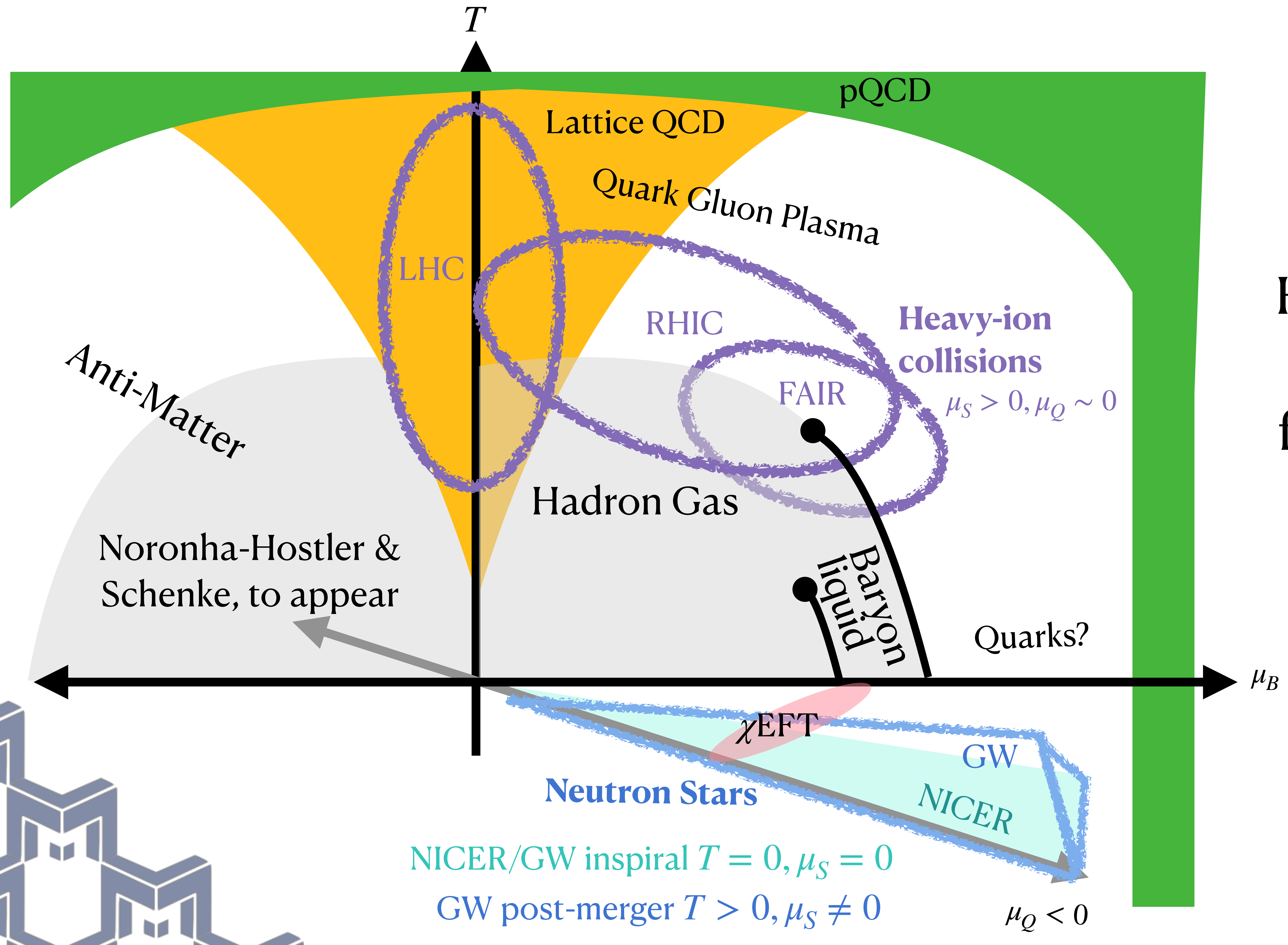
Mapping out the QCD phase diagram



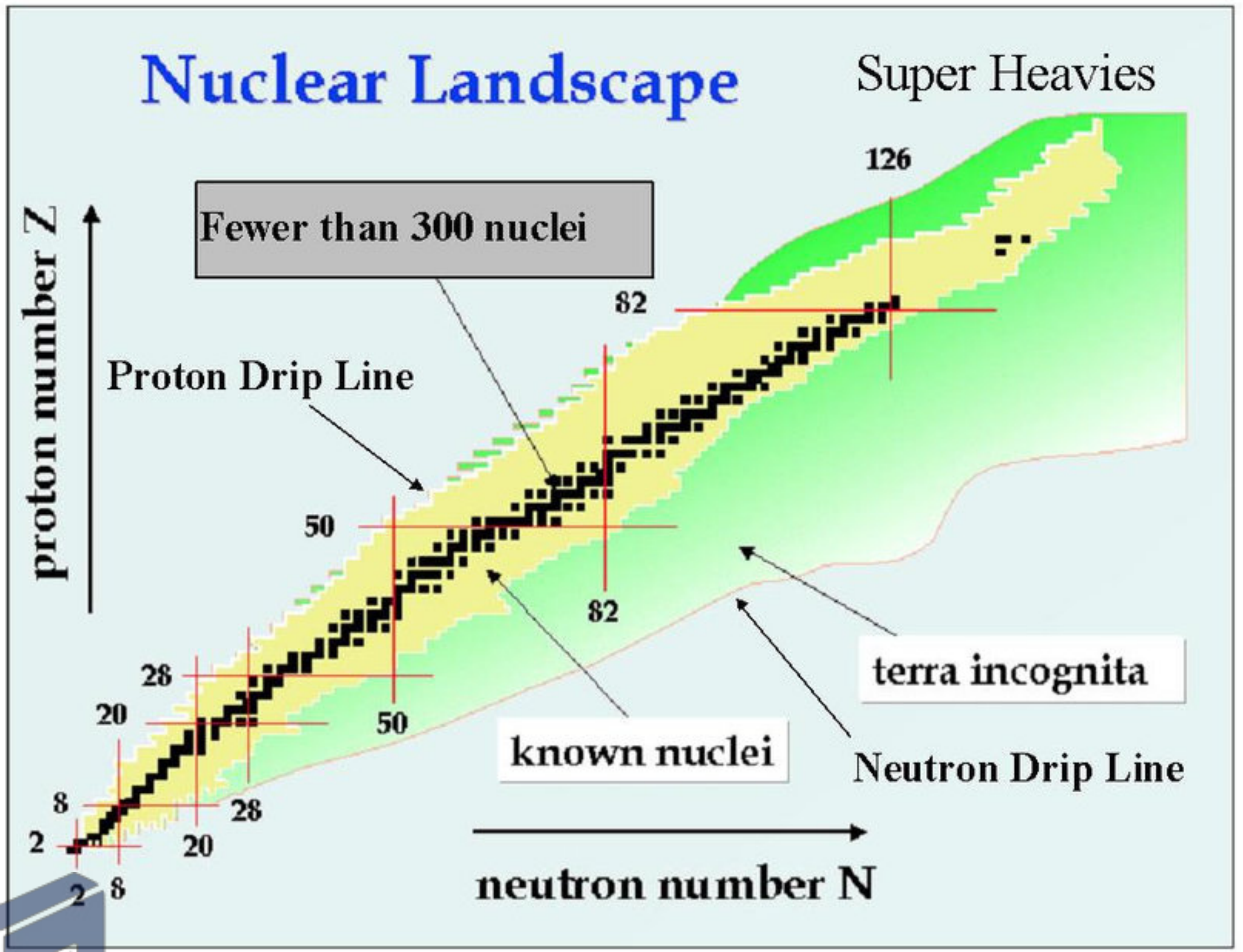
How do we describe the equation of state (EOS) out the regime of first principle calculations (lattice QCD, pQCD, χEFT)?

Instead of waiting on direct QCD calculations

Microscopic models build in symmetries/properties of QCD that are solvable

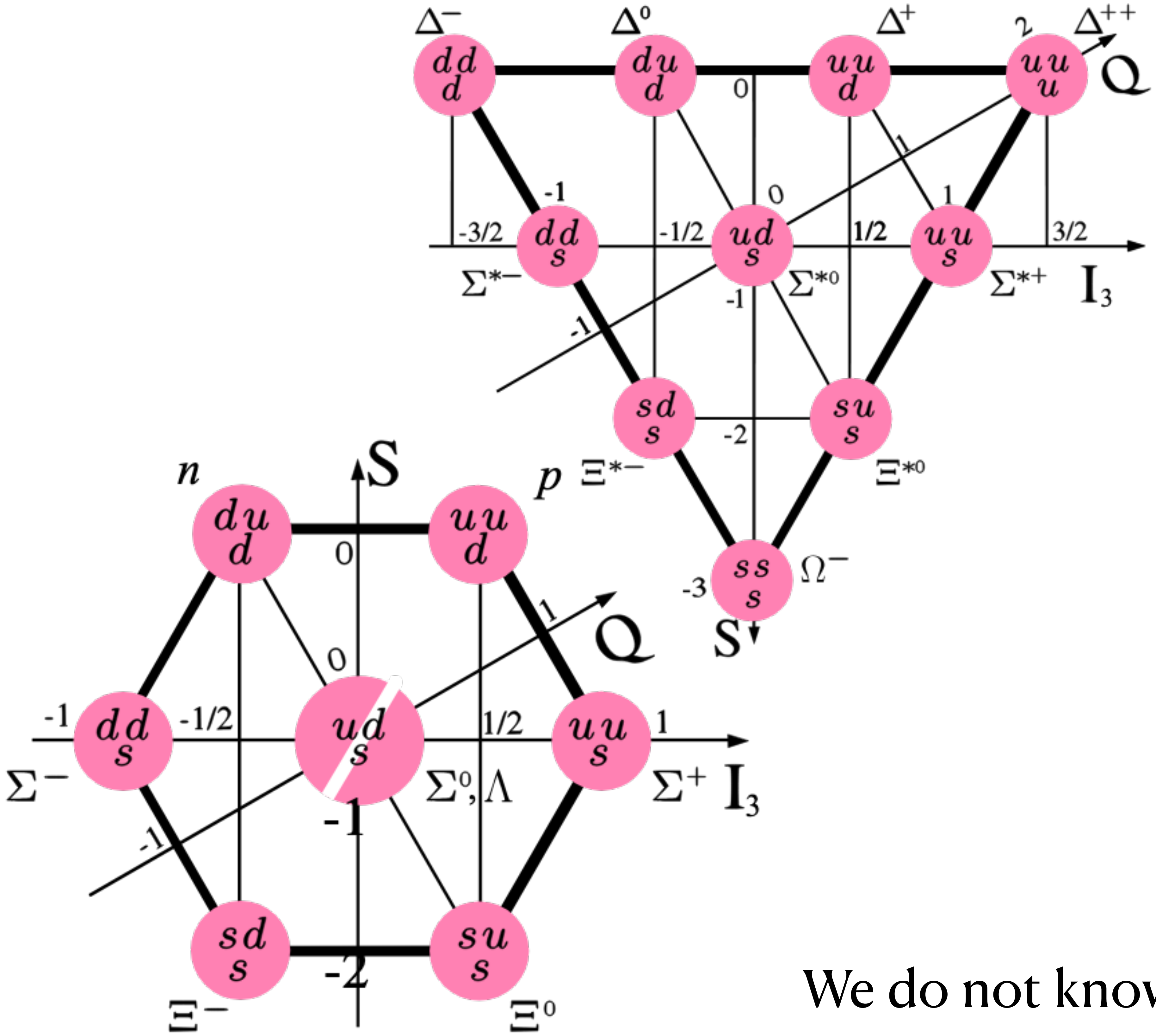
Low n_B

Nuclei before the liquid-gas phase transition



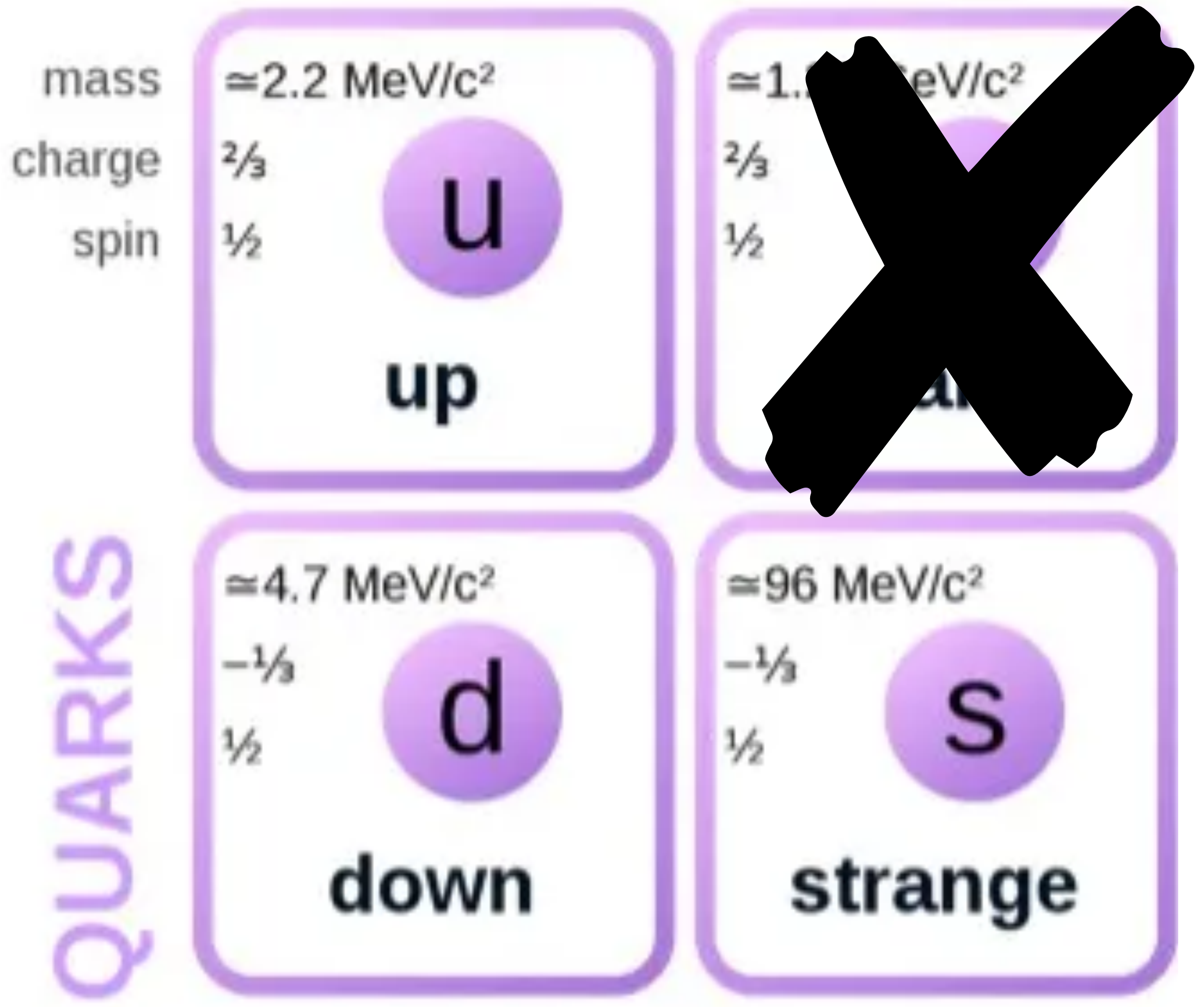
Intermediate n_B

Hadrons (Baryons) before the deconfinement transition



High n_B

Quarks at the highest n_B



We do not know if quarks will appear before the maximum n_B^{max} possible for a neutron star, otherwise they might just be inside black holes!

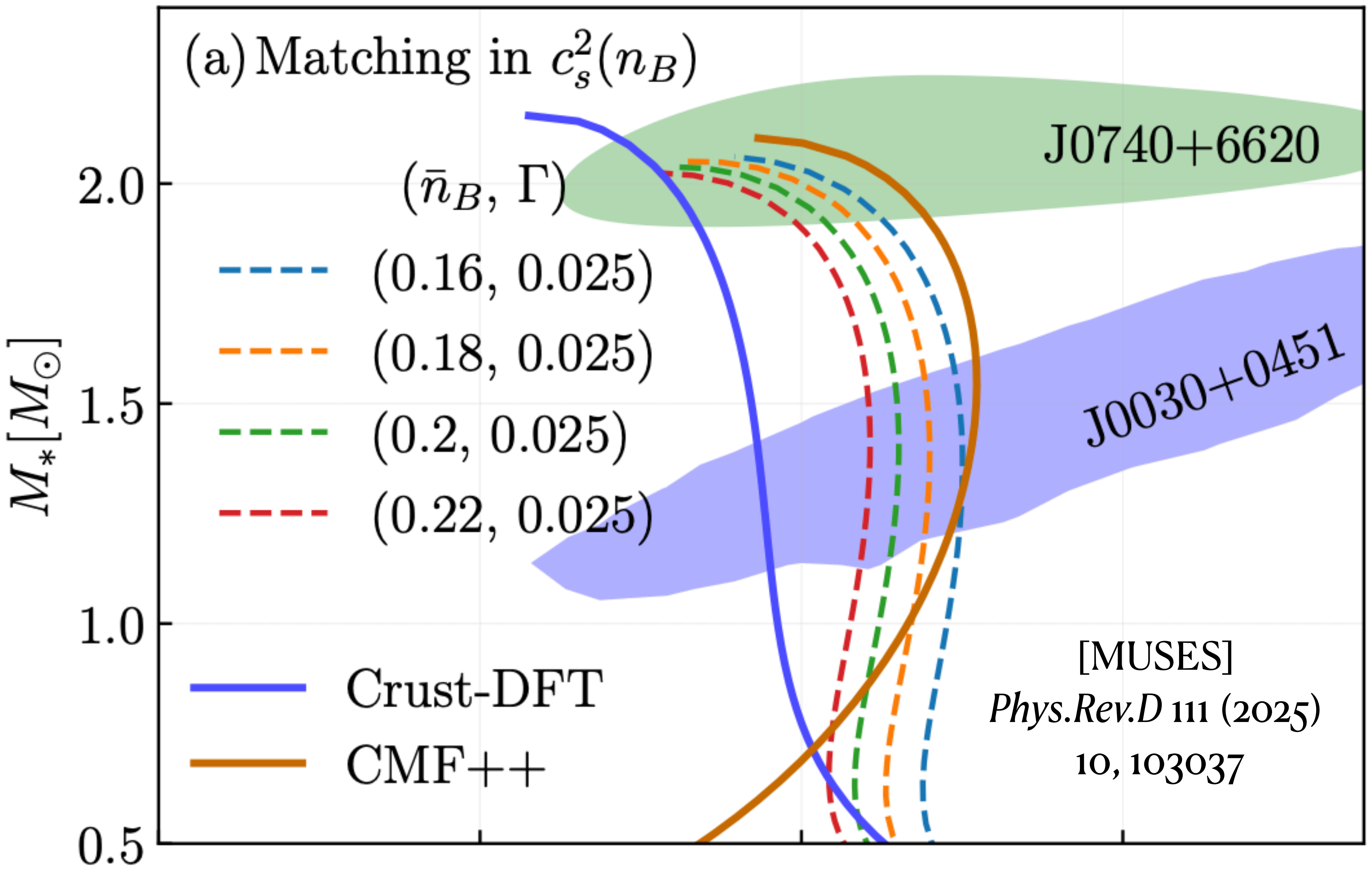
Chiral Mean Field (CMF) model

Dexheimer & Schramm *Astrophys.J.* 683 (2008) 943-948

- Includes full SU(3) baryon octet and decuplet, quarks, Polyakov loop to describe deconfinement phase transition
- Mesons mediate interactions (includes self-interactions)
- Calculable in the full $\left\{T, \mu_B, \mu_S, \mu_Q\right\}$ plane
- Can reproduce quenched QCD results

  Kumar et al, *Phys.Rev.D* 109 (2024) 7, 074008
- Reproduces vacuum masses, saturation properties, astrophysical constraints
- 20-50 free parameters (see Cruz Camacho Tues)

  Cruz Camacho et al, 2603.16019 [nucl-th]

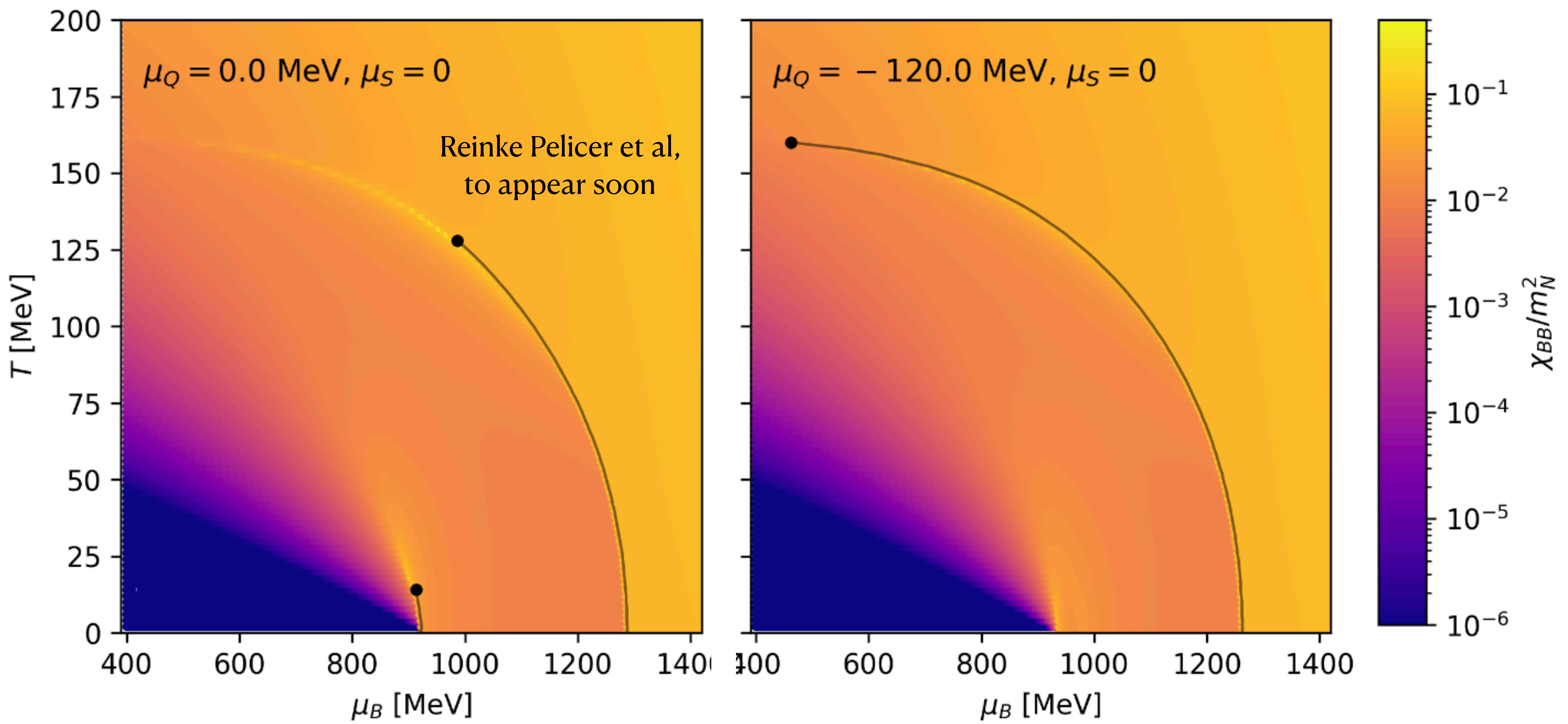


[MUSES] *Phys.Rev.D* 111 (2025) 10, 103037

Open-source, fast, modular c++ version (CMF++) found here! https://ce.musesframework.io/

Caveat: interpretation of these results

- CMF with octet+decuplet+quarks has ~50 parameters+ Ansätze for the couplings
- C4 coupling produces significantly fewer hyperons and has the "standard" QCD phase diagram with just the liquid gas and deconfinement phase transitions.



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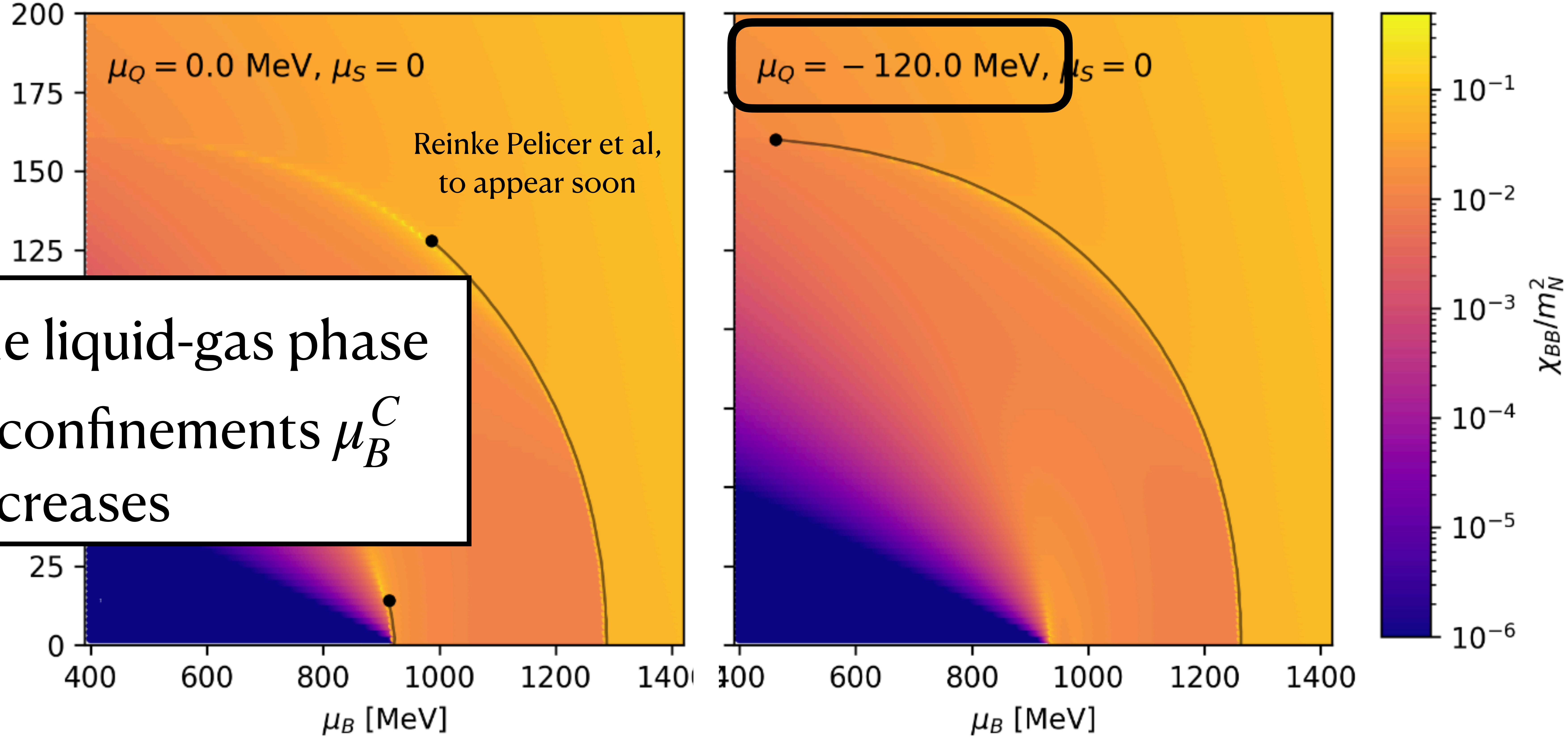
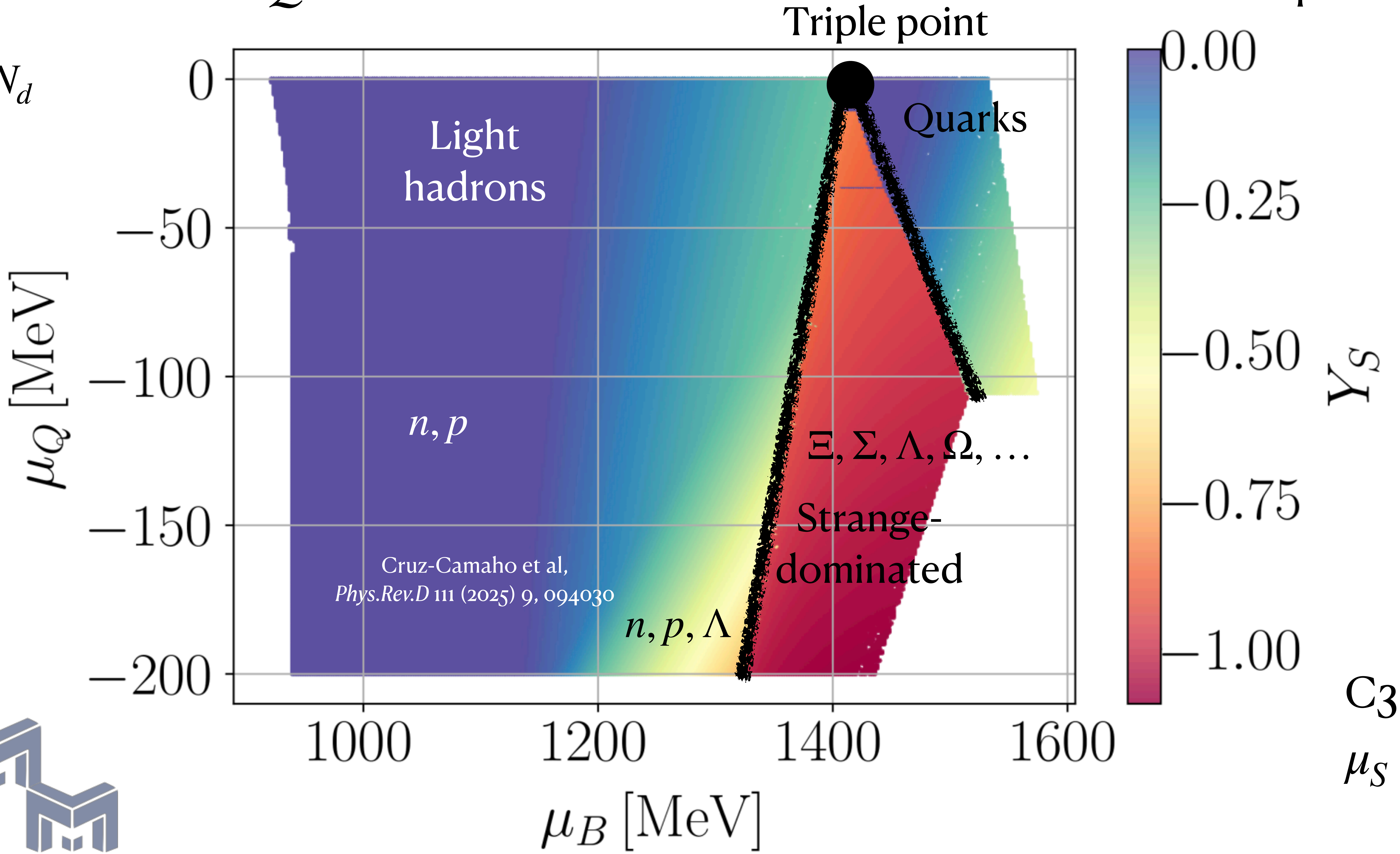


At finite μ_Q the liquid-gas phase vanishes, deconfinements μ_B^C decreases

At finite $\mu_Q < 0$, similar to heavy-ions!

Example $T = 0$ phase diagram

$N_u = N_d$



C3 coupling

$\mu_S = 0$

At finite $\mu_Q < 0$, similar to heavy-ions!

Example $T = 0$ phase diagram

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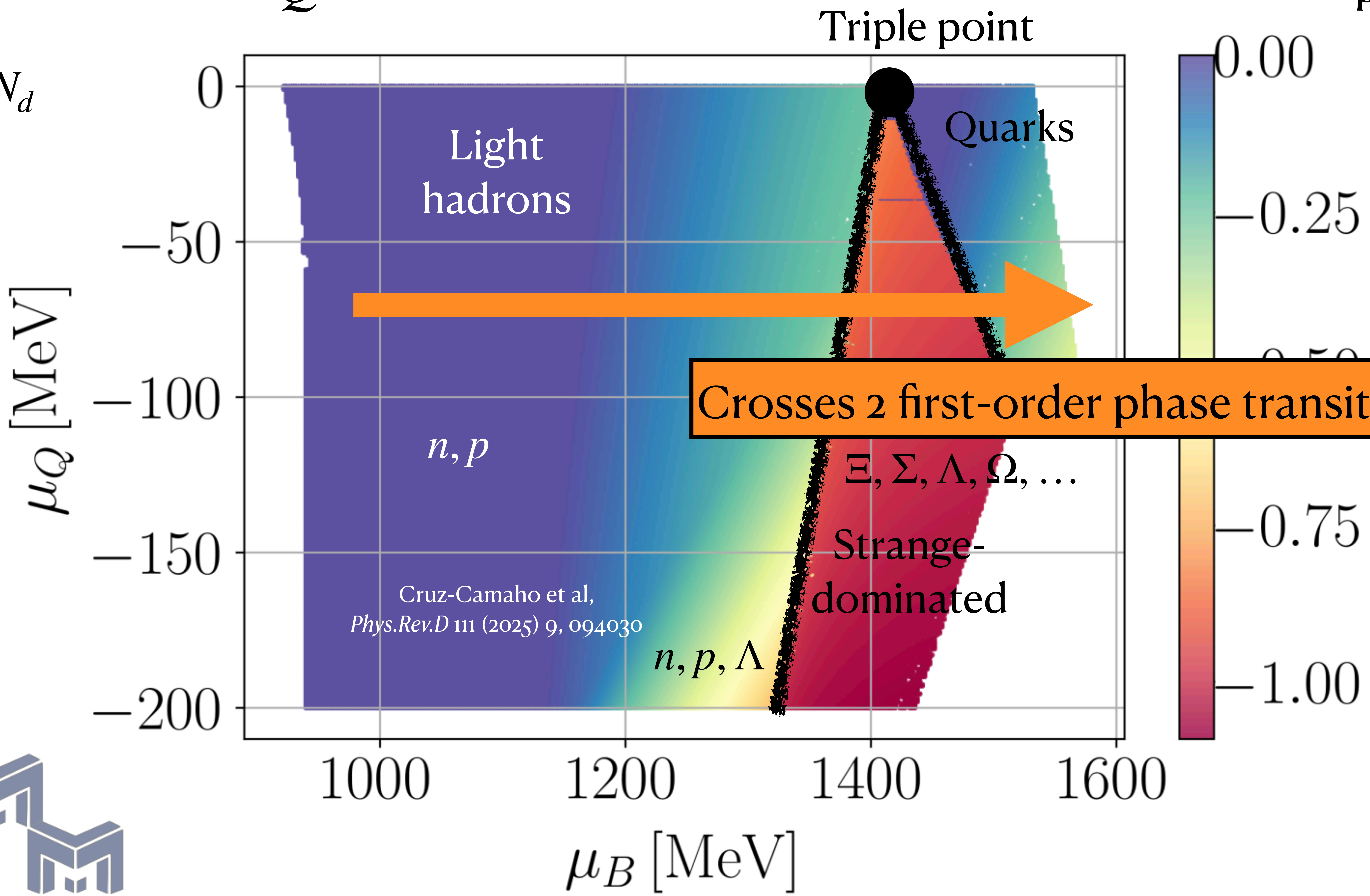
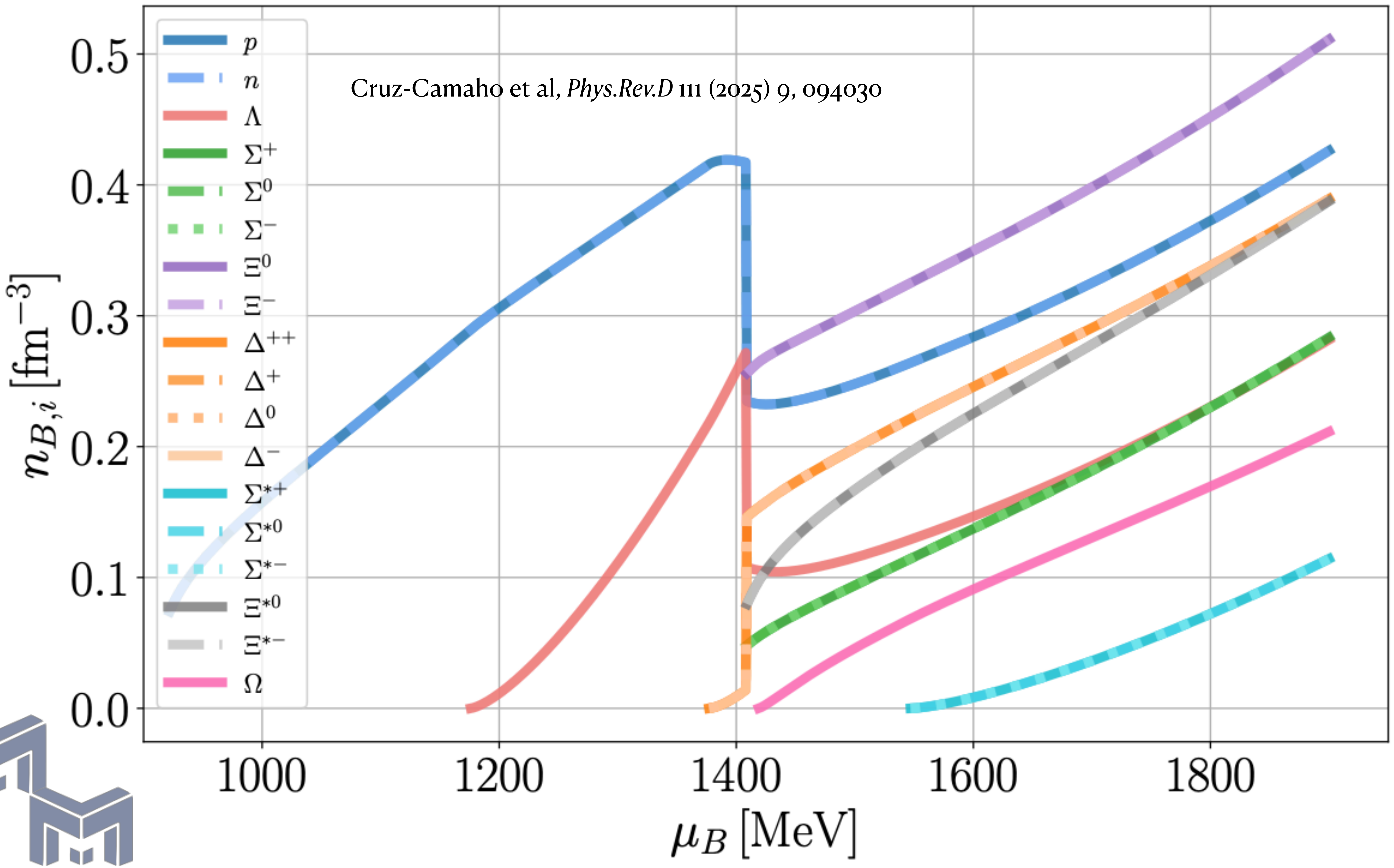


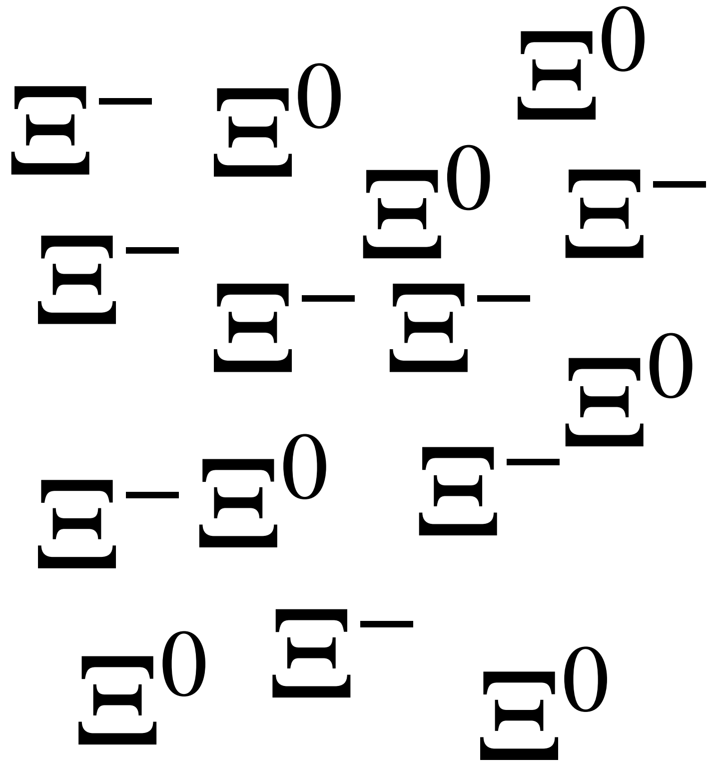
Crosses 2 first-order phase transitions!

C3 coupling

$\mu_S = 0$

What's happens inside the strangeness dominated phase?

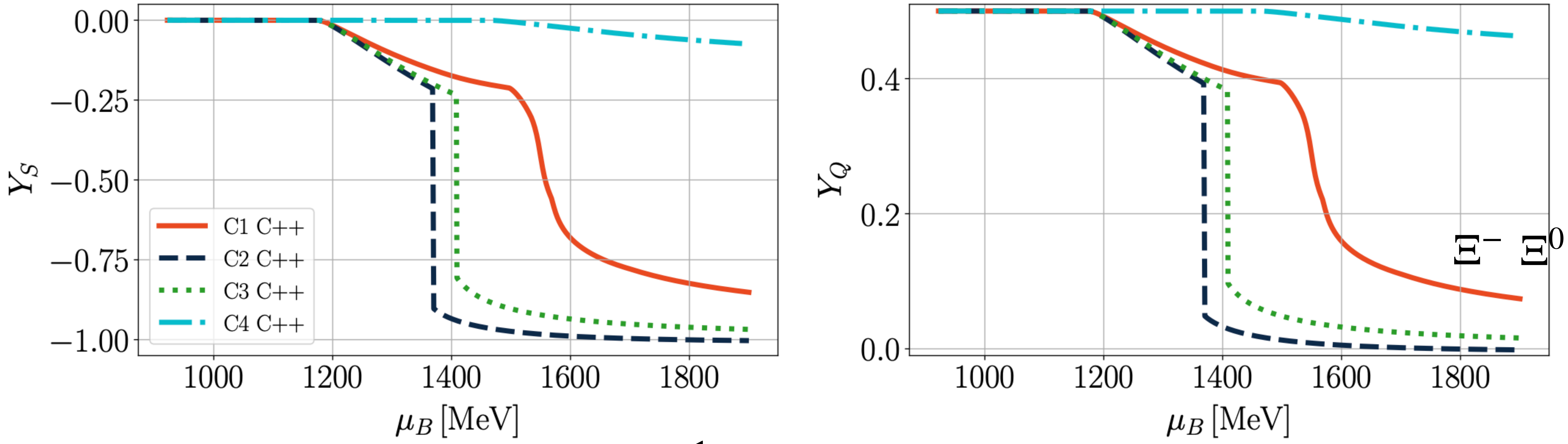




$S = -2, Q = 0, -1$

What's the order parameter?

Isospin restoration, but not where you'd expect it!



Isospin asymmetry: $I_3 = Q - \frac{1}{2}(B + S)$ Gell-Mann–Nishijima formula

Divide by B, $Y_{I_3} = Y_Q - \frac{1}{2}\left(1 + Y_S\right)$.

See Yumu Yang Weds

When $Y_S \to -2$, $Y_Q \to -1/2$, then $Y_{I_3} = 0$ (isospin restored!)

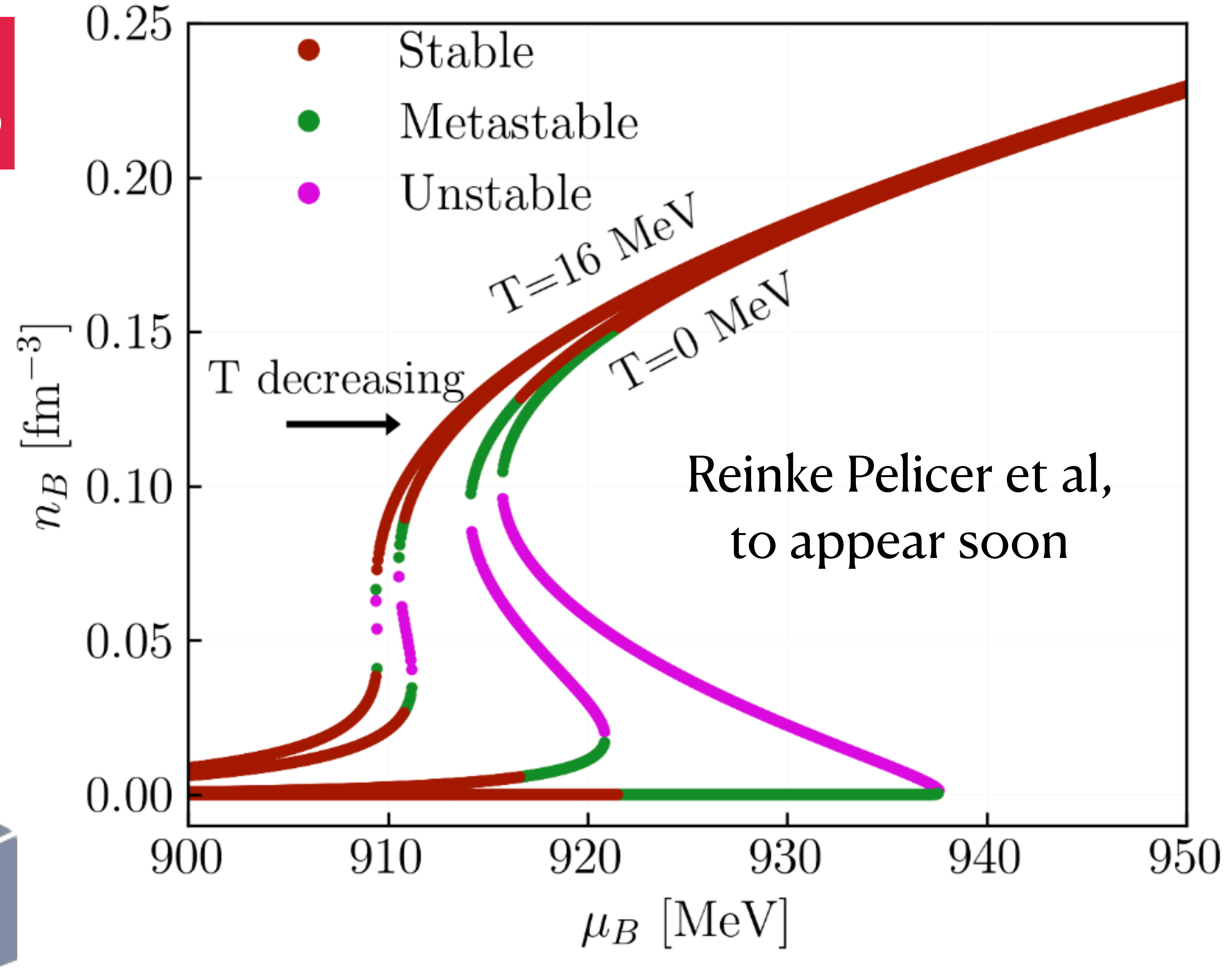
What about finite T?

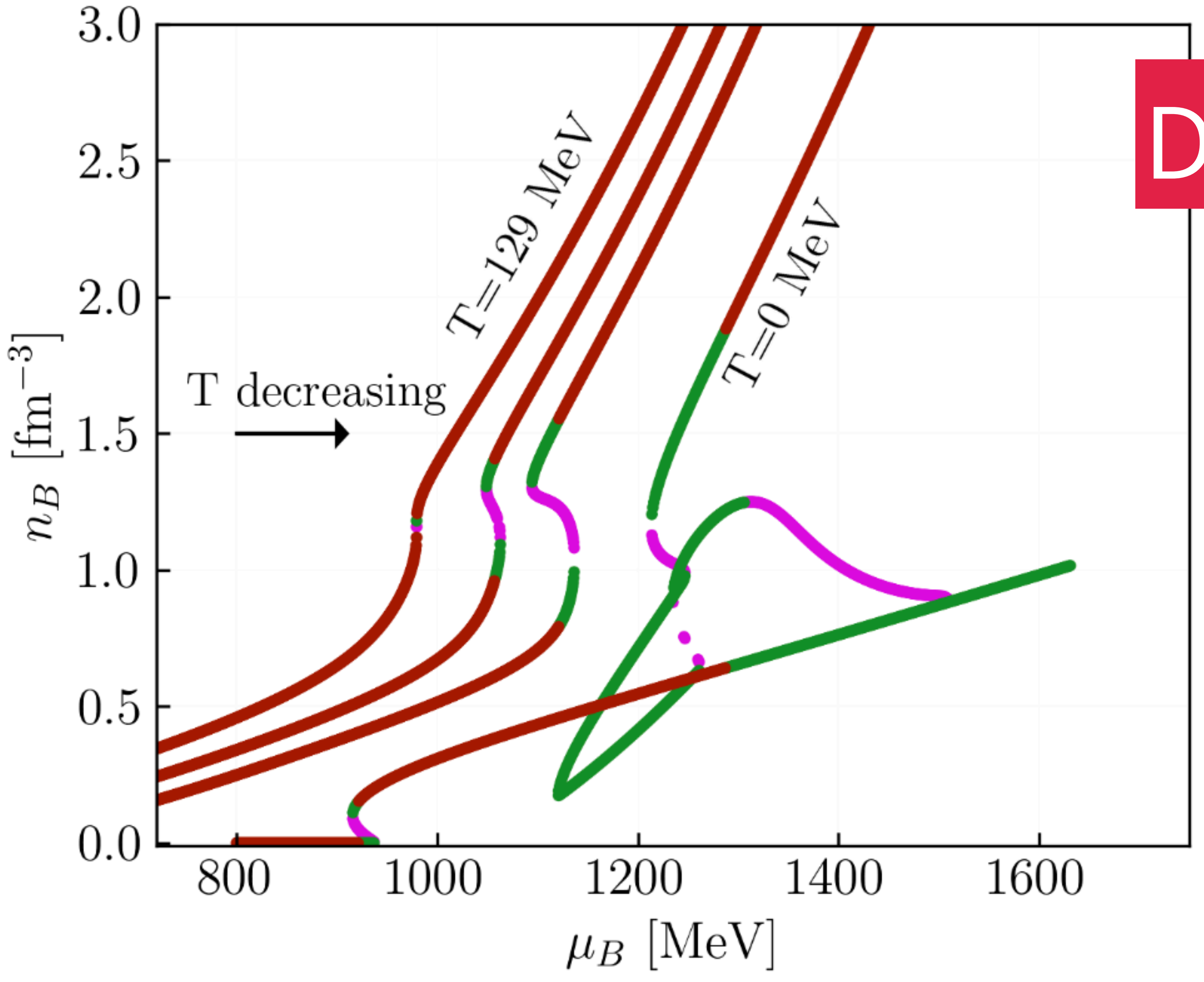
- New challenge: determining stable, metastable, unstable phases. Must be a positive matrix:

$$M = \begin{bmatrix} \frac{\partial s}{\partial T}\Big|_{\vec{\mu}} & \frac{\partial s}{\partial \mu_B}\Big|_{T,\mu_S,\mu_Q} & \frac{\partial s}{\partial \mu_S}\Big|_{T,\mu_B,\mu_Q} & \frac{\partial s}{\partial \mu_Q}\Big|_{T,\mu_B,\mu_S} \\ \frac{\partial n_B}{\partial T}\Big|_{\vec{\mu}} & \chi_2^B & \chi_{11}^{BS} & \chi_{11}^{BQ} \\ \frac{\partial n_S}{\partial T}\Big|_{\vec{\mu}} & \chi_{11}^{SB} & \chi_2^S & \chi_{11}^{SQ} \\ \frac{\partial n_Q}{\partial T}\Big|_{\vec{\mu}} & \chi_{11}^{QB} & \chi_{11}^{QS} & \chi_2^Q \end{bmatrix}$$

- Calculate susceptibilities in 4D phase space

Liquid-gas





Deconfinement

C4 coupling

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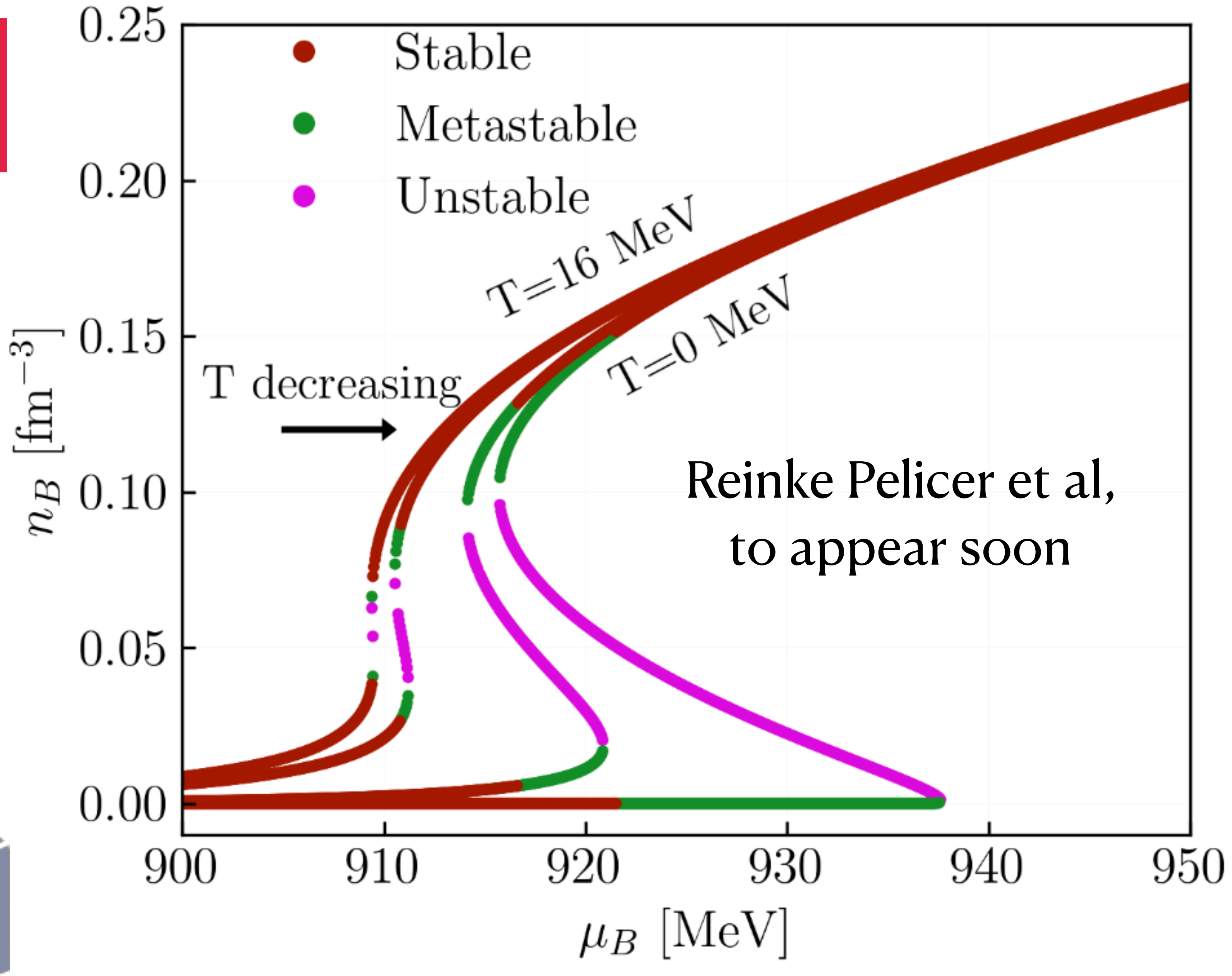
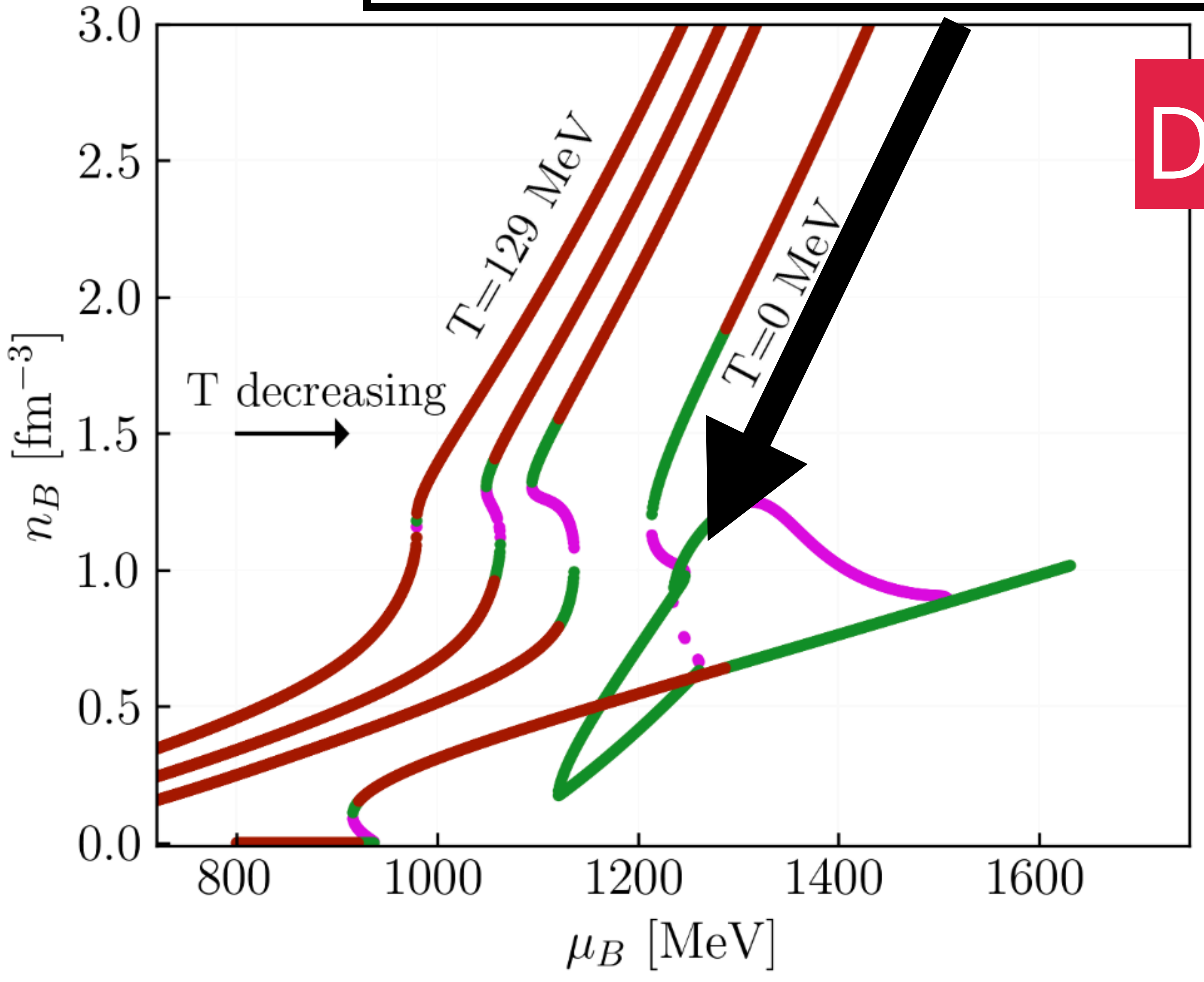
- Calculate susceptibilities in 4D phase space

Strangeness dominated metastable phase in between quarks and hadrons

Liquid-gas





Deconfinement

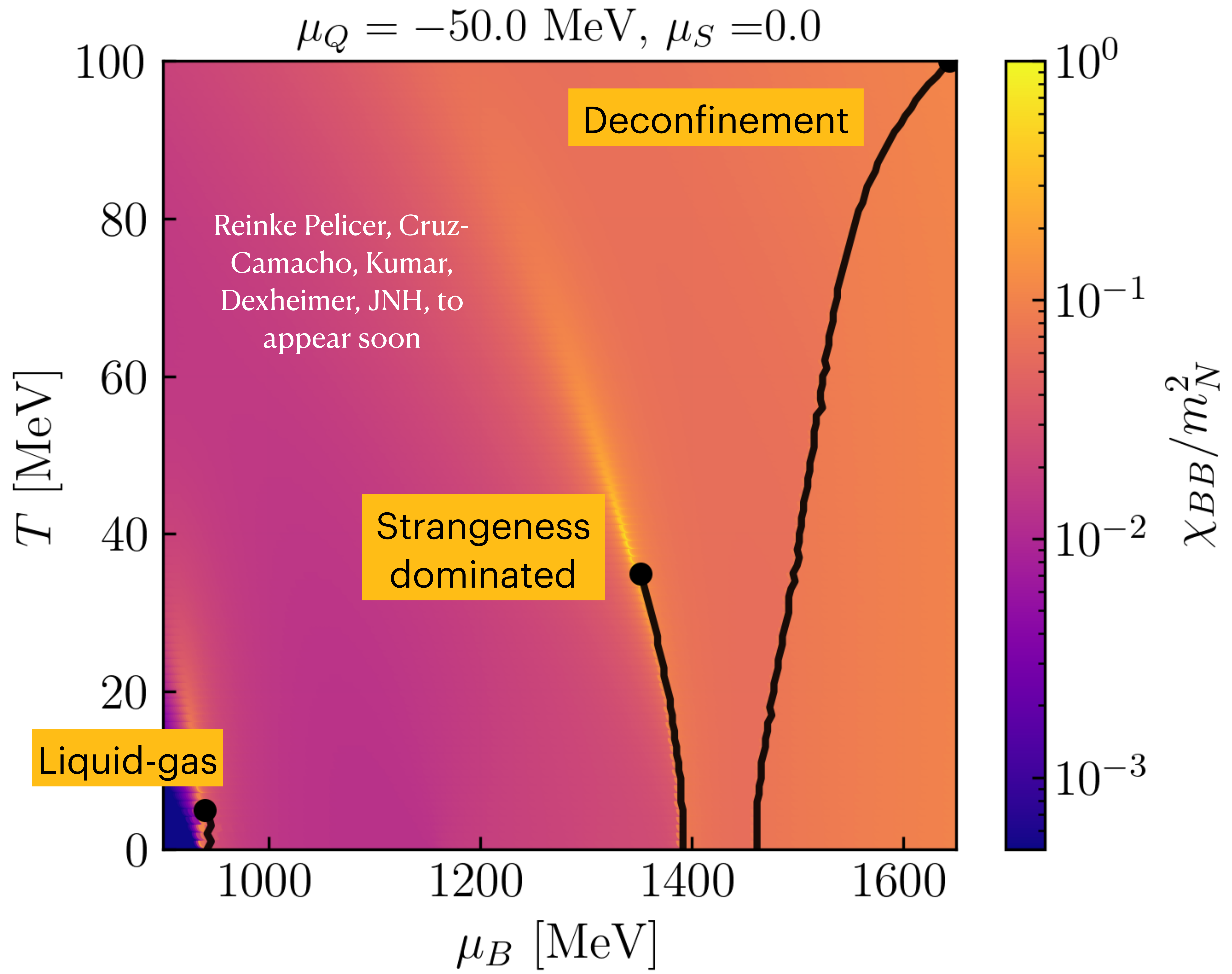
C_4 coupling

Turn up the heat!

We have 3 critical points!

We could see this for isospin asymmetric systems at very low $\sqrt{s_{NN}}$ or binary neutron star mergers

Note this is mean-field theory, so we need to think about the universality class some more.



Can you give me a better intuition for μ_S?

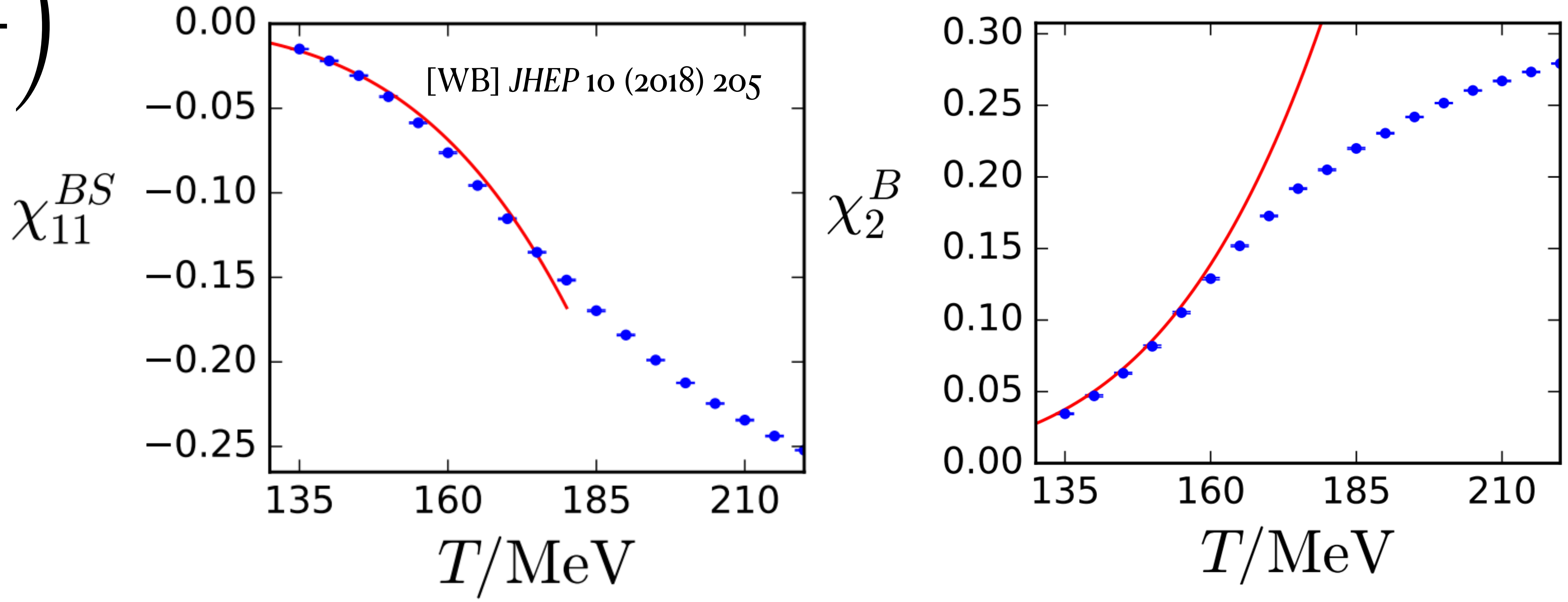
Sure!

Using a Taylor's series (a la lattice QCD) up to $\mathcal{O}(\mu/T)^2$:

$$n_S(T, \mu_B, \mu_S) = \chi_s^2 \left(\frac{\mu_s}{T}\right) + \chi_{BS}^{11} \left(\frac{\mu_B}{T}\right)$$

For strangeness neutrality $n_S = 0$,

so $\mu_S = -\dfrac{\chi_S^2}{\chi_{BS}^{11}} \mu_B$



Since $\chi_S^2 > 0$ (for stability) and $\chi_{BS}^{11} < 0$ (since $S = -1$) then μ_S has the same sign as μ_B for strangeness neutrality

At finite $\mu_S \neq 0$, tricritical point

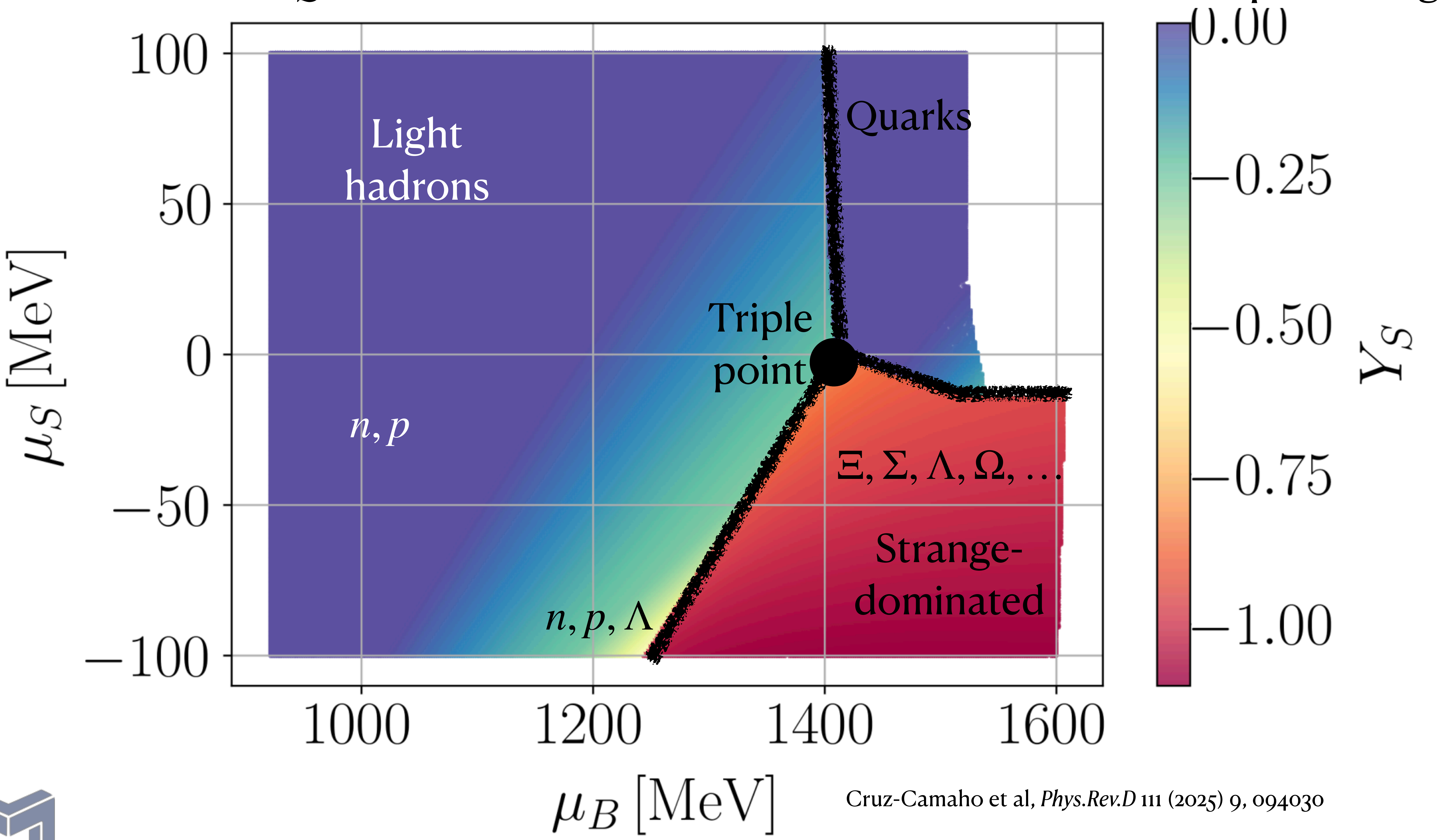
Example $T = 0$ phase diagram

Heavy-ions

$\mu_S > 0$

Electroweak equilibrium

$\mu_S = 0$



Cruz-Camaho et al, *Phys.Rev.D* 111 (2025) 9, 094030

Strangeness-dominated phase in neutron stars?

Need fluctuations in μ_S

Due to long time scales in neutron stars, they are in electroweak equilibrium i.e.

$$n \to p + e^- + \bar{\nu}_e$$

$$\Lambda \to p + e^- + \bar{\nu}_e$$

that leads to

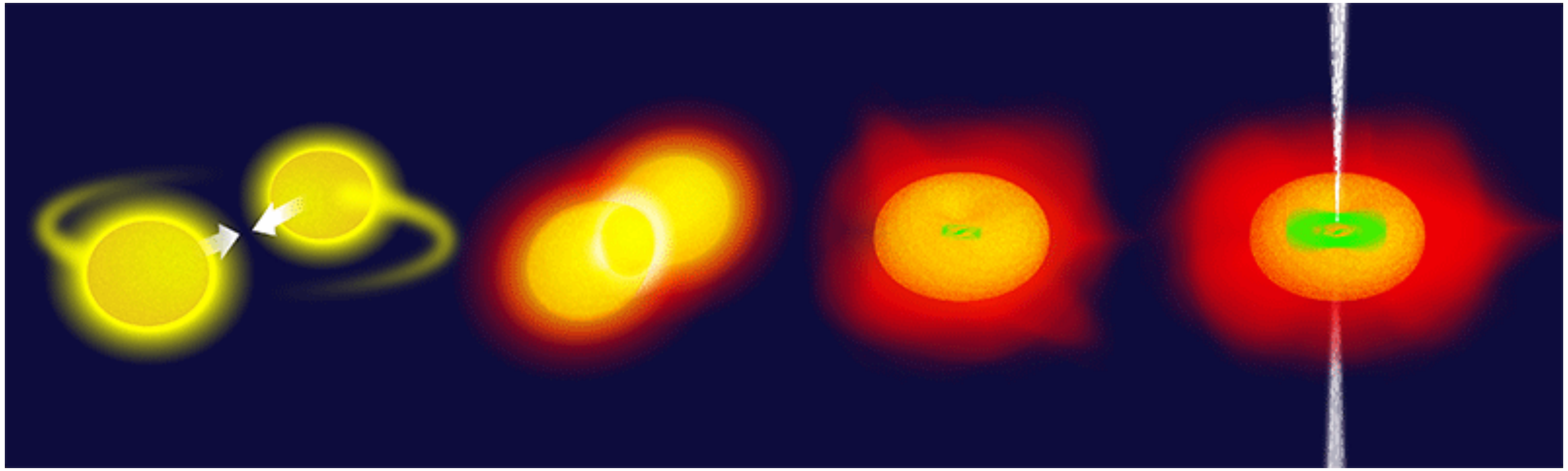
$$\mu_n = \mu_p + \mu_e$$

$$\mu_S = 0$$

However, binary neutron stars mergers can *perturb* the system out-of-electroweak-equilibrium

Measurements: Hegade *Nature Astron.* 8 (2024) 10, 1277-1283; *Phys.Rev.D* 108 (2023) 10, 10 ; *Phys.Rev.D* 110 (2024) 4, 044041; *Phys.Rev.D* 109 (2024) 10, 104064

Calculations: Alford et al, *Phys.Rev.D* 110 (2024) 6, L061303;
Phys.Rev.D 108 (2023) 8, 083019;
Phys.Rev.C 109 (2024) 5, 055803;
Phys.Rev.D 104 (2021) 10, 103027;
Phys.Rev.C 103 (2021) 4, 045810;
Phys.Rev.D 100 (2019) 10, 103021
Most et al, *Astrophys.J.Lett.* 967 (2024) 1, L14;
Mon.Not.Roy.Astron.Soc. 509 (2021) 1, 1096-1108
Yang et al, *Phys.Rev.Lett.* 135 (2025) 22, 222702;
Phys.Rev.C 109 (2024) 1, 015805



Coupled electroweak rate equations

Storbacka, Wu, Dong, Cruz Camacho, Haber, Reinke Pelicer, Dexheimer, Most, Noronha-Hostler, to appear

Rate equations:

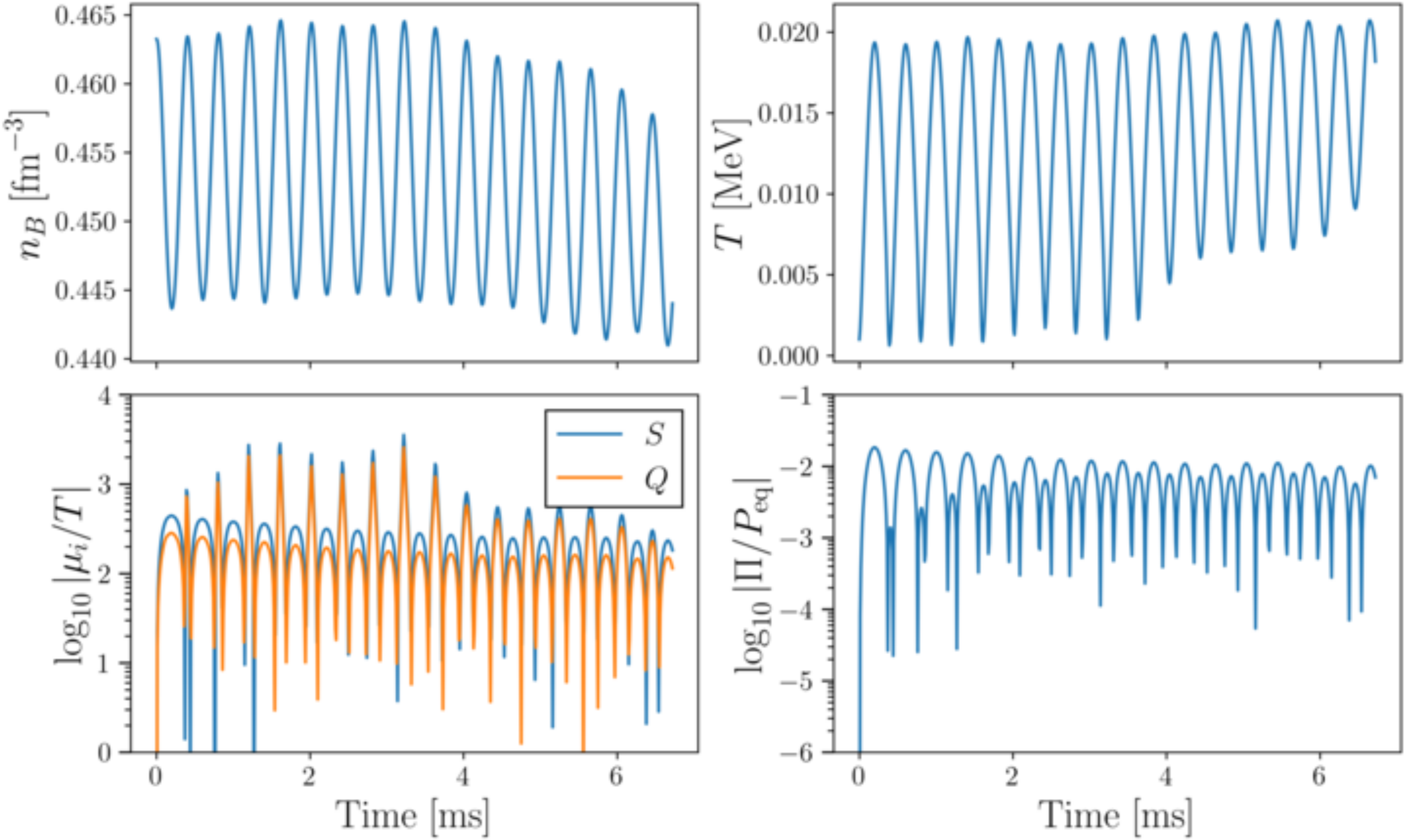
$\dot{n} = \Gamma \left[\text{gain} - \text{loss}\right] + \ldots$

for each particle

In medium reaction rates

Equation of state

$\left\{ T, \mu_B, \mu_S, \mu_Q \right\}$

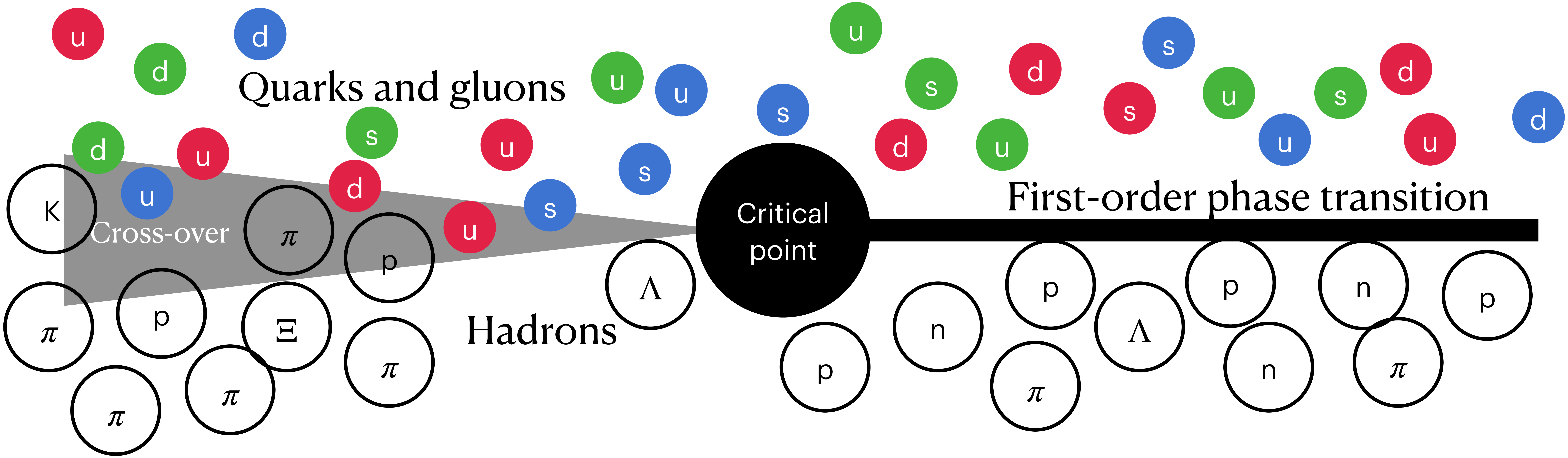


Conclusions and Outlook

- Using a chiral mean field model, we find a new possible phase of matter at large densities: strangeness dominated
- This phase of matter is associated with isospin symmetry restoration
- Depending on the parameters of the equation of state, this phase may either be a metastable phase between light hadrons and deconfiment or a first-order phase transition with a triple point and critical point
- Now possible to explore coupled out-of-electroweak equilibrium in binary neutron star mergers, find large fluctuations in μ_Q, μ_S !

Searching for the QCD critical point

Boundary between a first-order phase transition and cross-over



Finding the QCD critical point would be the first...

1. in a fundamental theory of nature,
2. in a truly ultra-relativistic many-body system,
3. accessed through relativistic hydrodynamic evolution