

# Nuclear Equation of State Modifications from Spacetime Structure

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## Extra dimensions

The **Kaluza–Klein** theory is the first, **simplest** model considering extra dimensions. It **adds** a fifth compactified **spatial dimension** to spacetime: it is essentially **curled up into a microscopic circle** exhibiting a  $U(1)$  symmetry and natural quantization. Originally it was meant to unify gravity with electromagnetism in a geometrical way, but its extensions also consider modeling dark matter, dark energy or solving the hierarchy problem. It features a **scalar field** as well, which is connected to the size of the extra dimension.

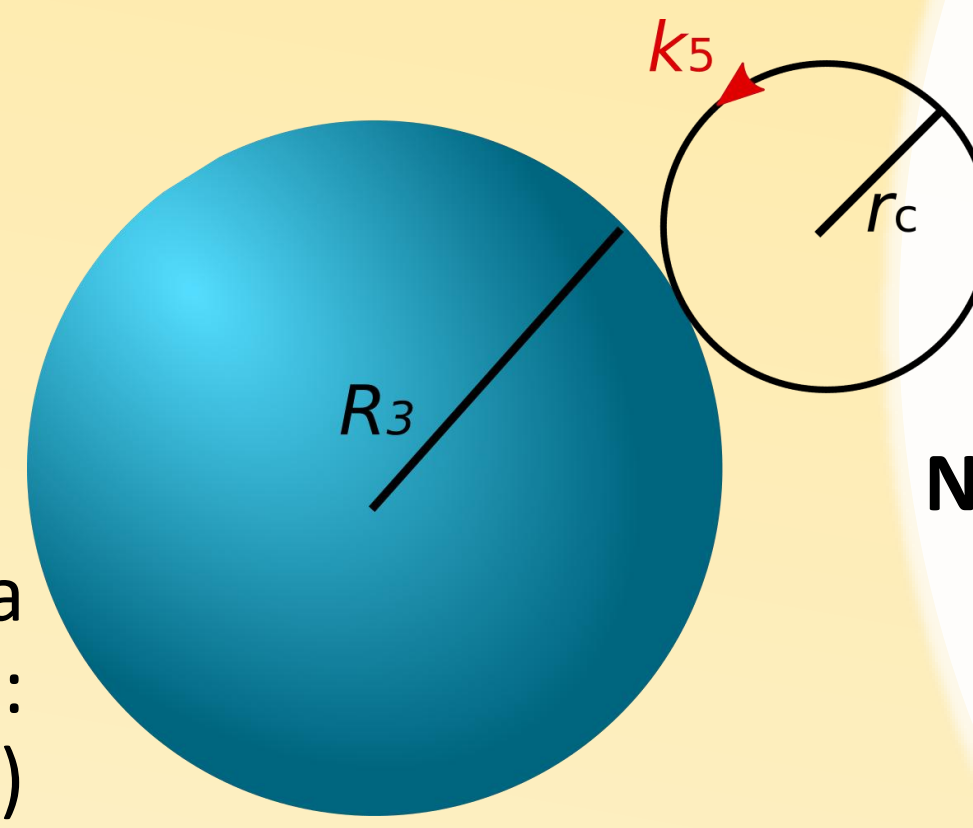
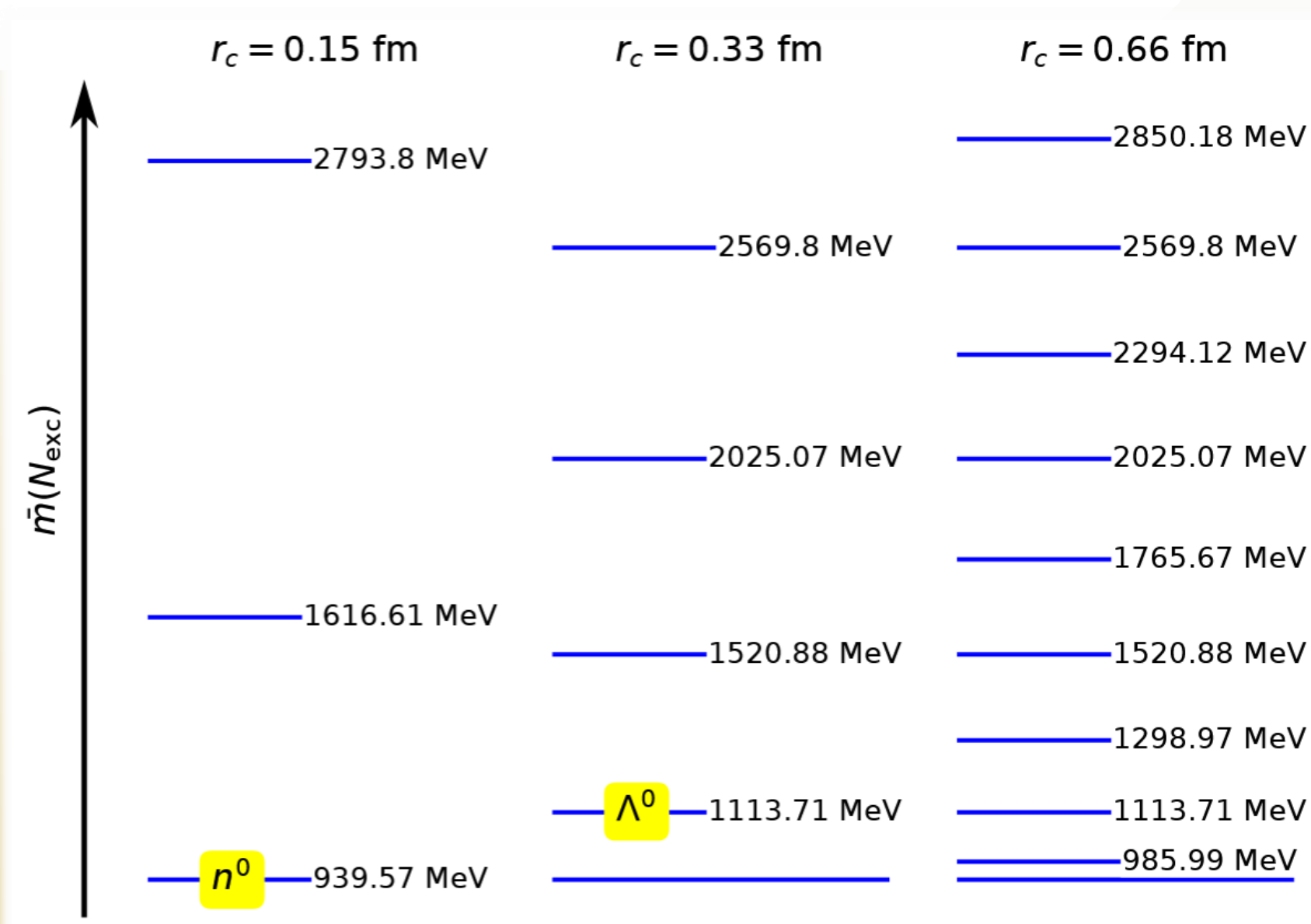
$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + \kappa^2 \Phi^2 A_\alpha A_\beta & \kappa \Phi^2 A_\alpha \\ \kappa \Phi^2 A_\beta & \Phi^2 \end{bmatrix}$$

## Strangeness as a consequence of geometry

The additional  **$U(1)$  symmetry** of the Kaluza–Klein theory gives rise to a **conserved charge**. Effectively, it can be thought of as **strangeness**. From a phenomenological point of view, particles **moving in the extra dimension** gain **quantized momenta** inversely proportional to the size of the extra dimension.

$$E = \sqrt{k^2 + k_5^2 + m^2} = \sqrt{k^2 + \left(\frac{N_{\text{exc}}}{r_c}\right)^2 + m^2} = \sqrt{k^2 + m_{\text{eff}}^2}$$

From a 4-dimensional point of view, this can be thought of as particles with a new **hadronic mass spectrum**. If the size of the extra dimension is fixed to 0.33 fm, the first excited state of the neutron is the **lambda baryon**.



The **equation of state** of dense nuclear matter is an open question. At **high energies**, perturbative **QCD** or lattice calculations allow for a precise modeling, while at **low temperature nuclear physics** and laboratory experiments provide reliable data.

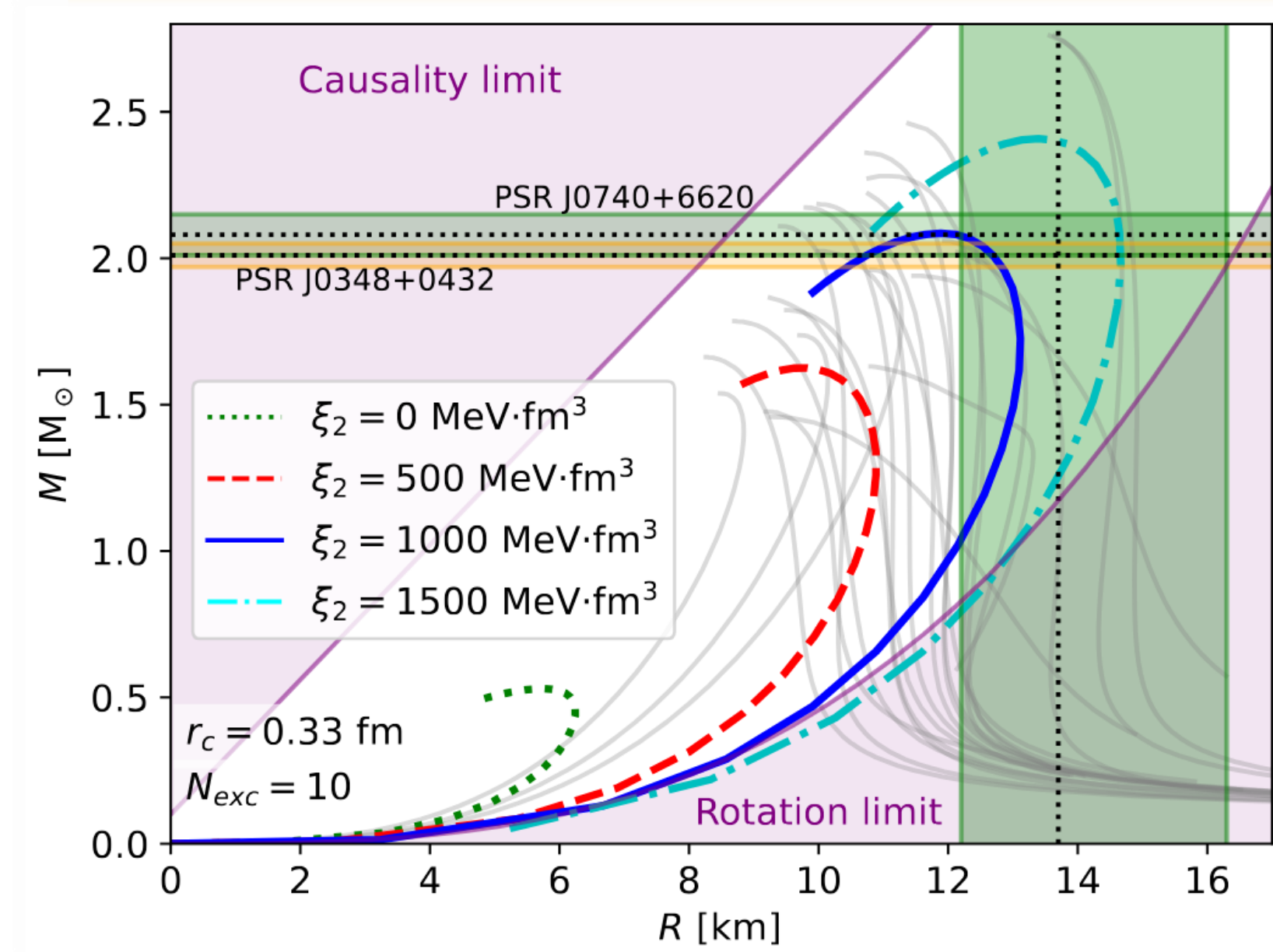
## But what happens in between?

**Neutron stars** provide a natural laboratory for observational tests, and a multitude of dense matter equations of state exist in the literature. However, could **gravity** have a significant **effect on microscopic physics**? What if the **Structure of spacetime** is not how the theory of general relativity describes it?

## Observations: neutron star mass and radius

We used a **zero-temperature Fermi gas** equation of state to model neutron stars, which consists of the neutron as base and its 5-dimensional excitations. The model parameters are the radius of the extra dimension,  $r_c$ , the maximum excitation number,  $N_{\text{exc}}$  and the strength of a **repulsive interaction**,  $\xi$  with a **linear potential** modelling the strong force.

When **multiple extra dimensions** are considered, effects become more enhanced.

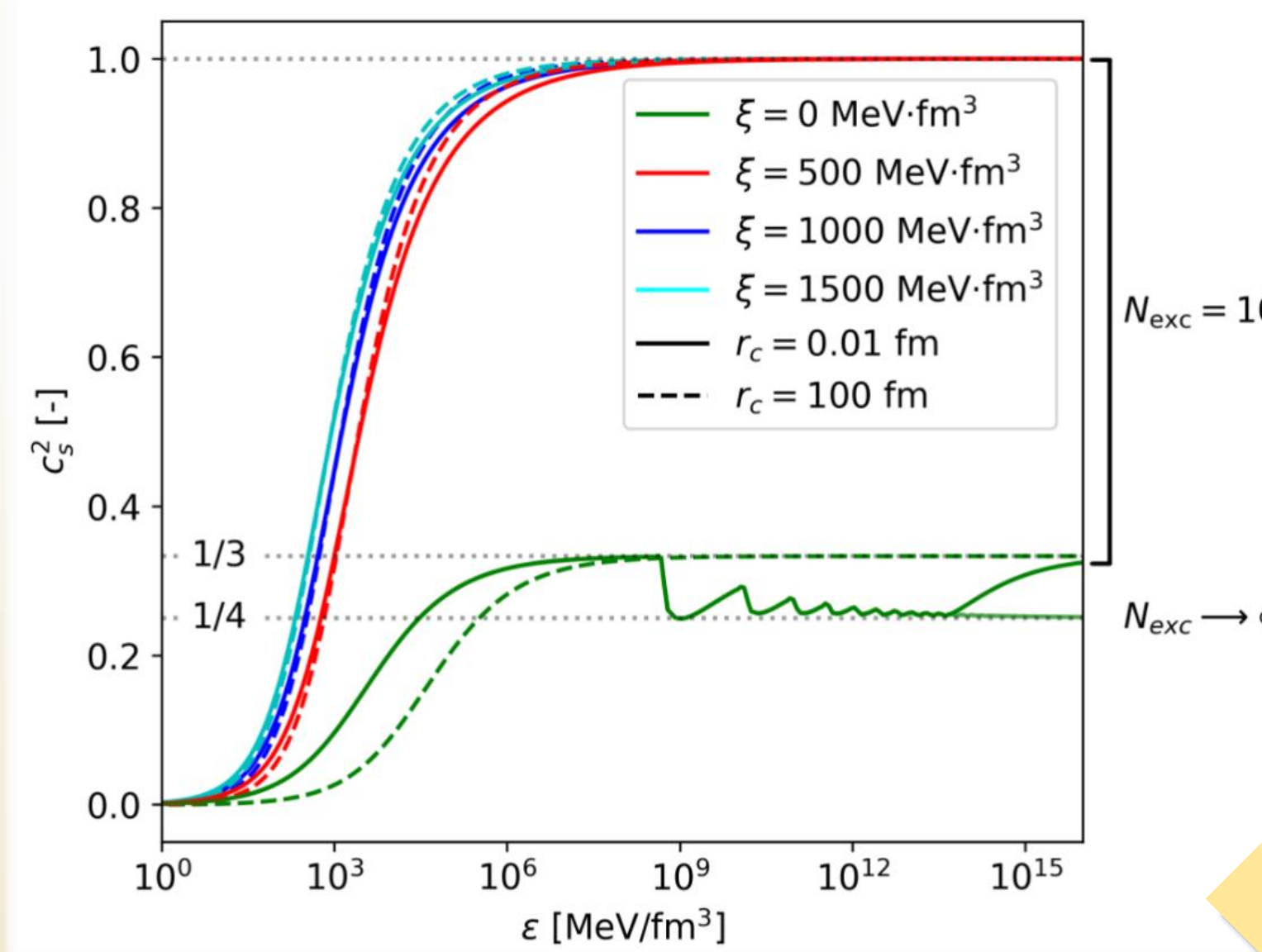


A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "The effect of multiple extra dimensions on the maximal mass of compact stars in Kaluza-Klein space-time", IJMPA 40, 2542004 (2025)



## Observations: speed of sound

The **conformal limit** of the speed of sound behaves differently in an extra-dimensional spacetime, since its square approaches  $1/d$ , where  $d$  is the number of spatial dimensions.

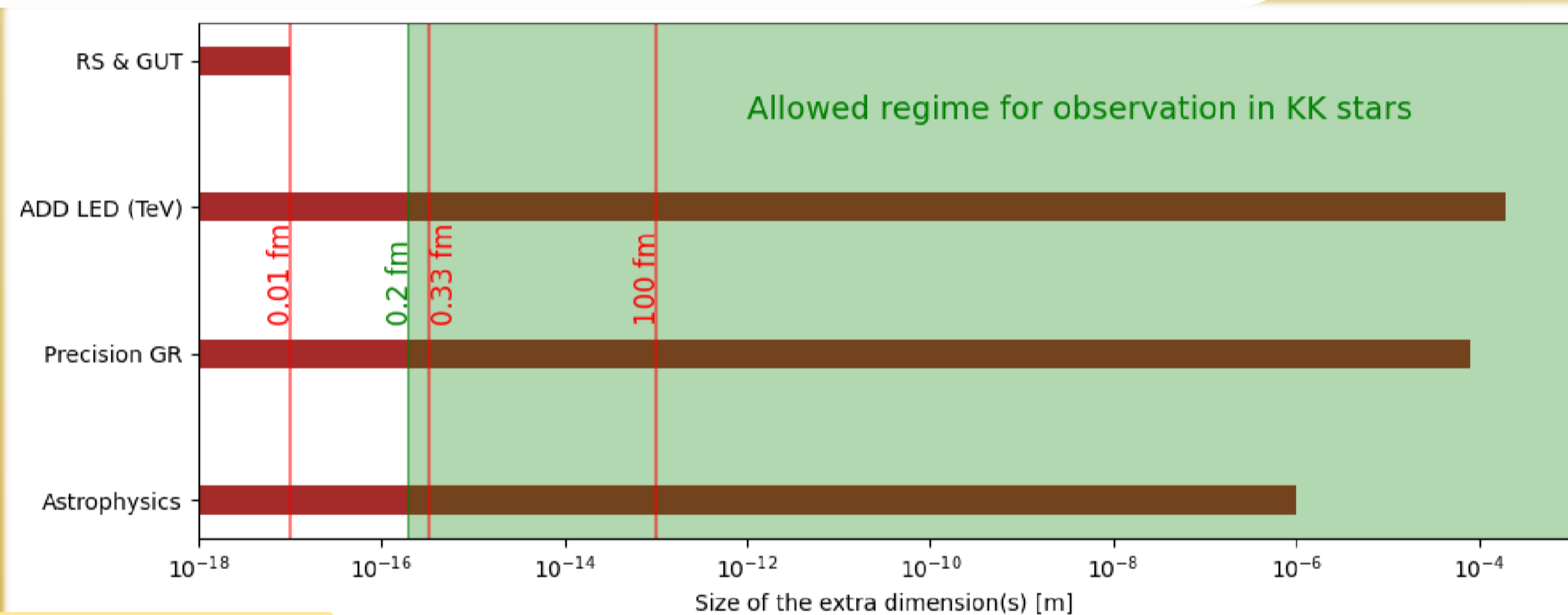


A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Speed of sound in Kaluza-Klein Fermi gas", APPB Vol. 18 (2025), No. 6, article 6-A1,



## Reason of possible constraints

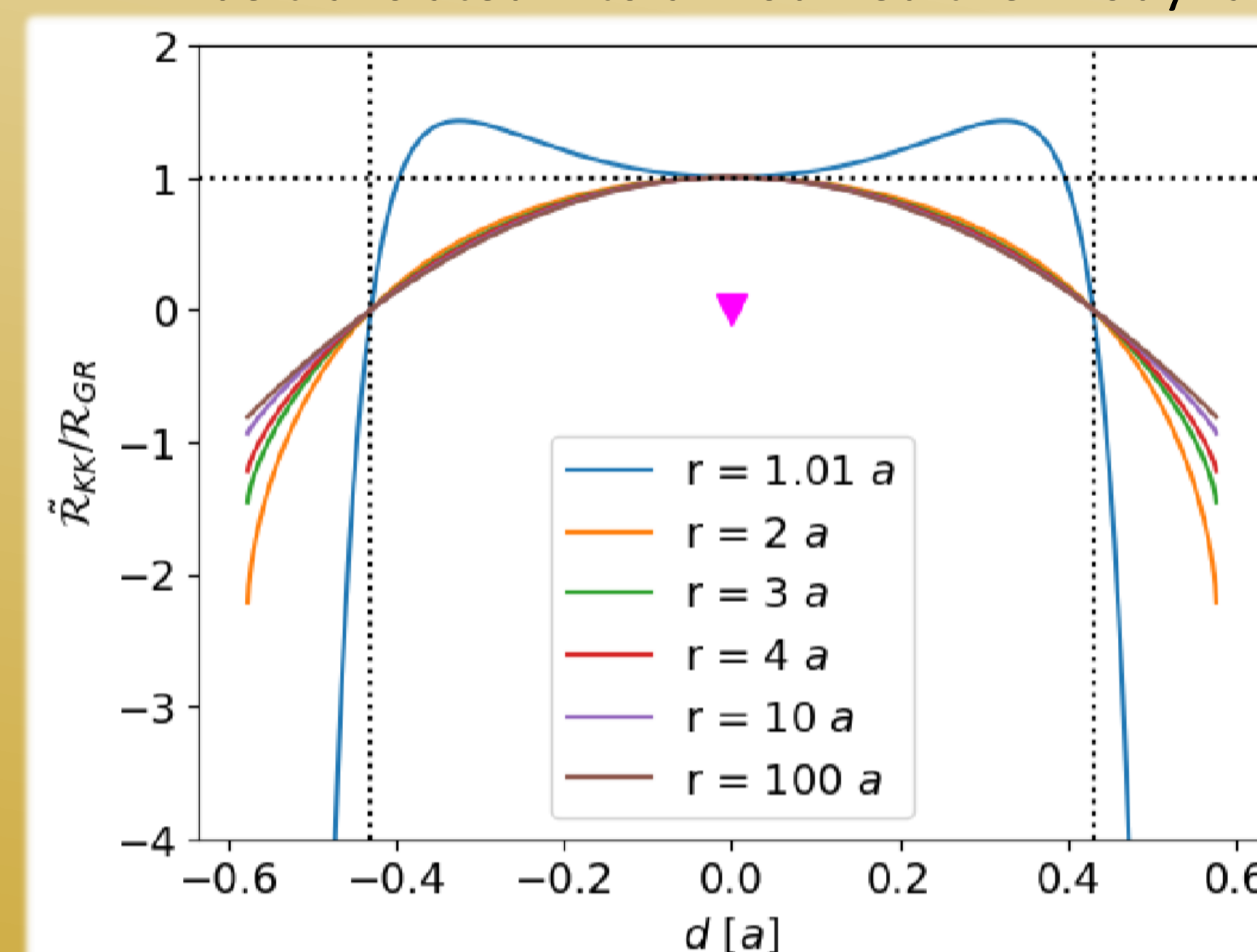
When the size of the extra dimension is **larger than about 0.2 fm**, it affects the maximal mass of neutron stars. Thus, with the advance of observations, and realistic modelling it could be possible to **constrain** the size of the fifth dimension.



A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Application of Kaluza-Klein Theory in Modeling Compact Stars: Exploring Extra Dimensions", MNRAS Vol. 536, 816 (2024)

## The effect of strong gravity on microscopic physics

When writing a dense matter nuclear equation of state, the effects of gravity are generally neglected. However, the spacetime **curvature** caused by **compact astronomical objects**, such as neutron stars could be enough to cause an observable effect in the collective behavior of particles. Both the **uncertainty** and the **dispersion relations** of massive particles **get a correction** due to the curvature of spacetime, which can be translated into a modified thermodynamical description.



$$\sigma_x \sigma_p \geq \frac{\hbar}{2} + f(\mathcal{R})$$

$$p^\mu p_\mu = k^\mu k_\mu + f(\mathcal{R}) = m_{\text{eff}}^2$$

A. Horváth, A. Wojnar, G.G. Barnaföldi: "The effects of strong gravity on the dispersion relation of massive particles in the Kaluza-Klein theory" (2025)



## Acknowledgement

Authors gratefully acknowledge the Hungarian National Research, Development and Innovation Office (NKFIH) under Contracts No. OTKA K147131 and ADVANCED\_25 K153456, No. NKFIH 2024-1.2.5-TÉT-2024-00022, 2025-1.1.5-NEMZ\_KI-2025-00005 and Wigner Scientific Computing Laboratory (WSCLAB). Author A.H. is supported by the DKÖP program of the Doctoral School of Physics of Eötvös Loránd University, the EU Erasmus fellowship and the EU COST Actions BridgeQG (CA23130) and FuSe (CA24101). E.F.-D. also received funding from the NKFIH excellence grant TKP2021-NKTA-64 and HUN-REN. A.W. acknowledges financial support from MICINN (Spain) Ayuda Juan de la Cierva - incorporación 2020 No. IJC2020-044751-I and from the Spanish Agencia Estatal de Investigación Grant No. PID2022-138607NB-I00, funded by MCIN/AEI/10.13039/501100011033, EU and ERDF A way of making Europe.

