Strangeness in Quark Matter

Merging multidimensional equations of state of strongly interacting matter via a statistical mixture

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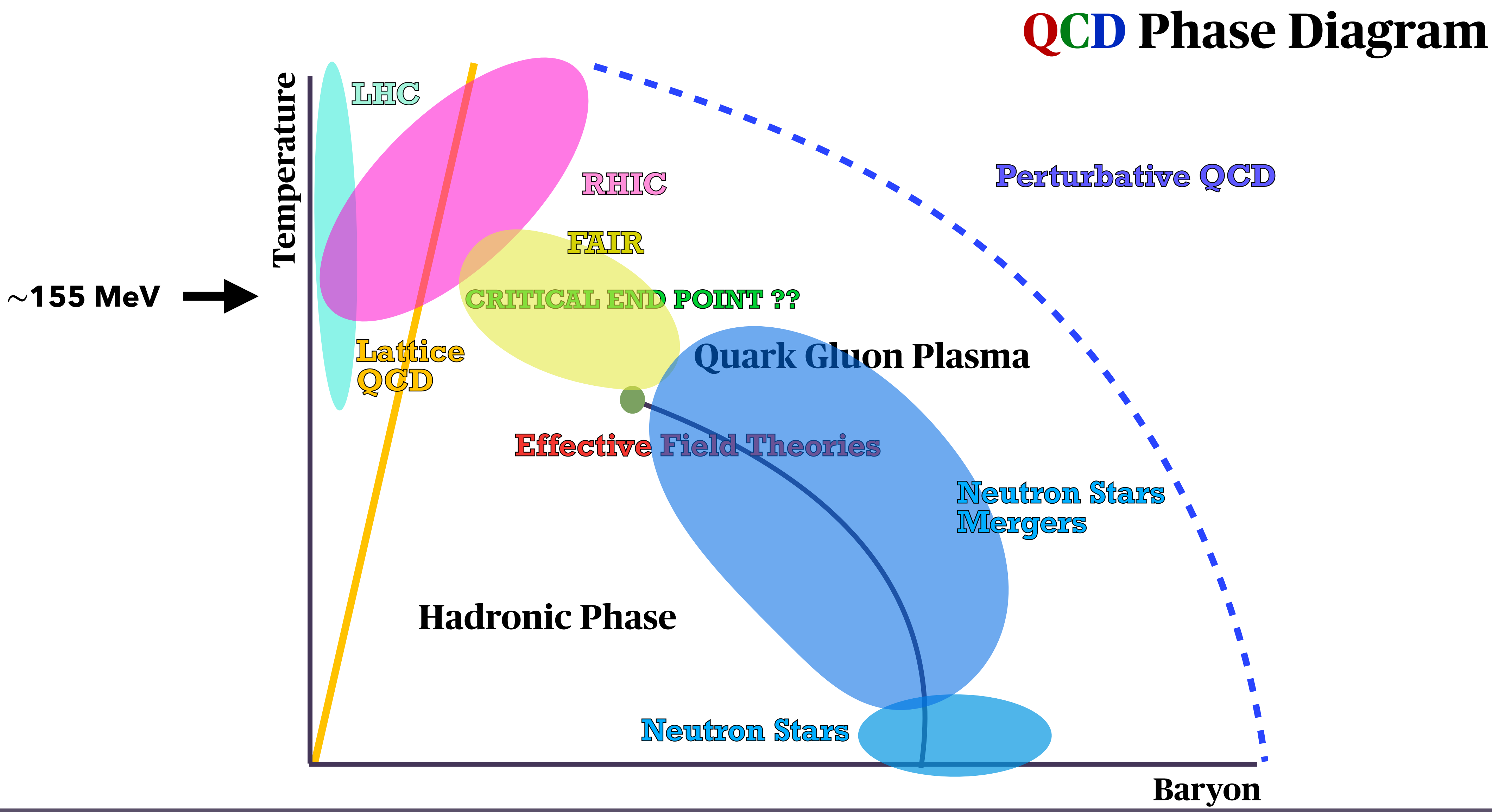
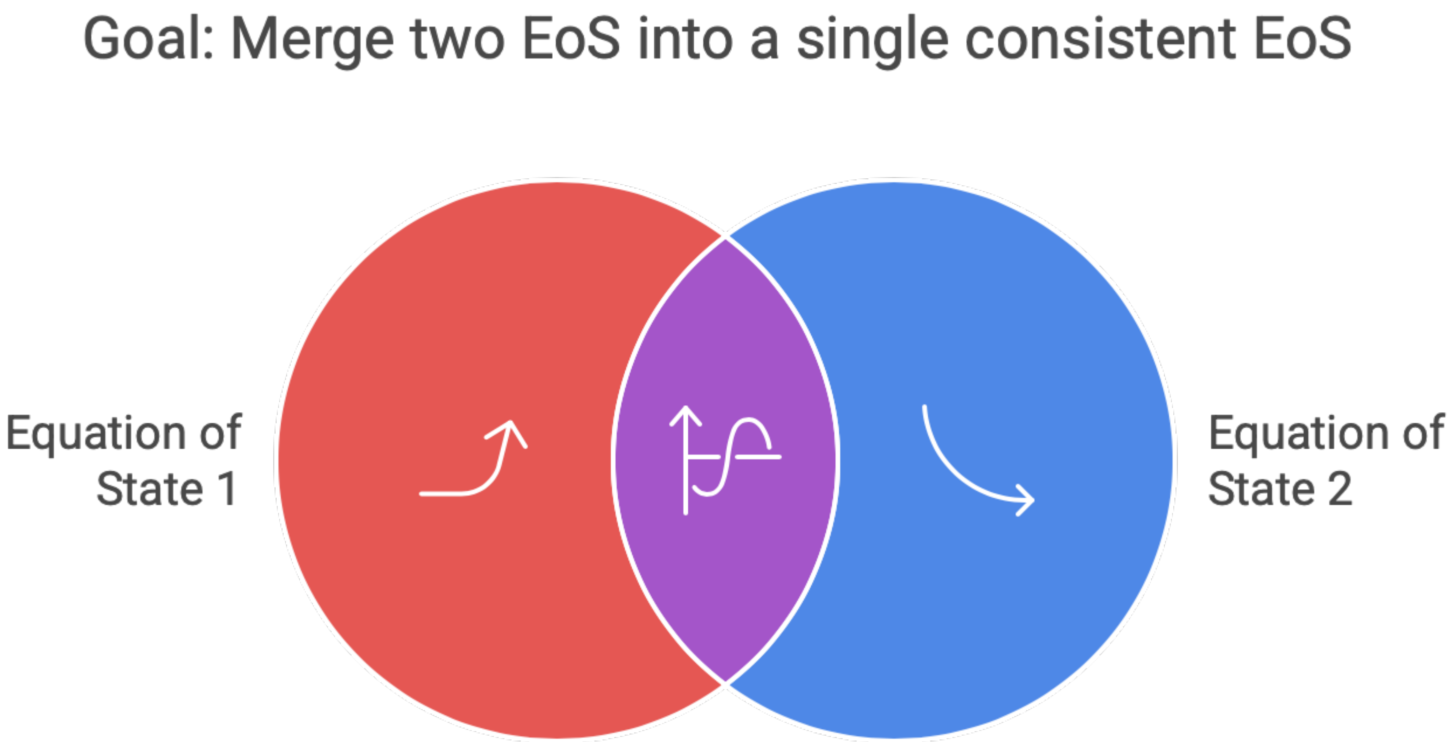

QCD Phase Diagram
Temperature
~155 MeV
LHC
RHIC
FAIR
CRITICAL END POINT ??
Lattice
QCD
Quark Gluon Plasma
Perturbative QCD
Effective Field Theories
Neutron Stars
Mergers
Hadronic Phase
Neutron Stars
Baryon

MERGING EOS THROUGH A STATISTICAL MIXTURE



OUTLINE

- **Mering Procedure**
  - **Switching Function**
  - **Ideal Mixture**
  - **Phase Transitions**
- **Proof of principle**
  - **Holographic**
  - **HRGvdW**
- **Results**
- **Features**
- **Take aways**

Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

Merging multidimensional equations of state of strongly interacting matter via a statistical mixture

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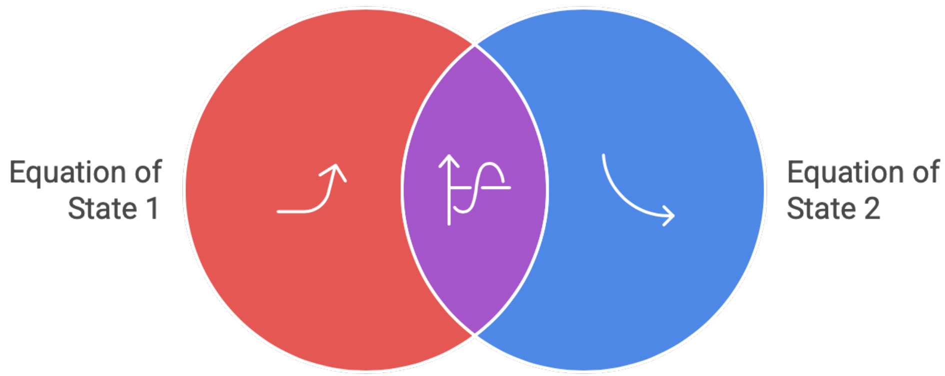
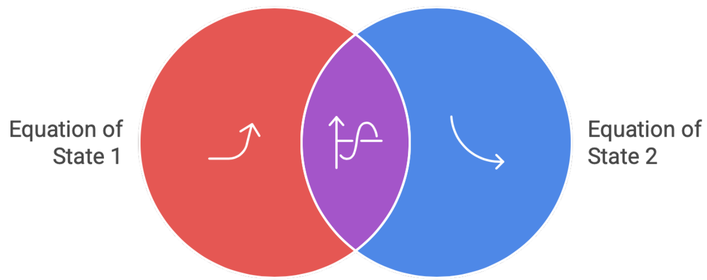
We introduce a general method to merge multidimensional equations of state (EoSs) by combining them in a two-fluid equilibrium statistical mixture in the grand canonical ensemble. The merged grand potential density ω is built directly from the input EoSs and the fluid fractions are fixed by minimizing ω at fixed temperature T and baryon chemical potential μ_B. Thermodynamic consistency and stability are guaranteed as all thermodynamic quantities are consistently derived from a single merged grand potential $\omega(T, \mu_B)$ with the correct convexity properties. Our method can accommodate a first-order phase transition and a critical endpoint with mean-field critical exponents. We use this method to merge a van der Waals Hadron–Resonance–Gas EoS with a holographic Einstein–Maxwell–Dilaton EoS that has a critical point and a first-order line. The result is a single EoS, spanning hadronic and deconfined matter over a broad range in (T, μ_B), which can be readily used in heavy-ion hydrodynamic simulations. Our merging method can be generalized to consider a higher dimensional phase diagram (e.g., by considering more chemical potentials) and more than two input EoSs.

Goal: Merge two EoS into a single consistent EoS



MERGING PROCEDURE : *SWITCHING FUNCTION*

Goal: Merge two EoS into a single consistent EoS



$$P = p\,P_1 + (1-p)\,P_2$$

$$p \in [0,1]$$

P_1 : **Pressure for EoS$_1$**

P_2 : **Pressure for EoS$_2$**

This *does* make P continuous but that *does not* suffice for thermodynamics

Second derivatives $\Longrightarrow$ Susceptibilities can be *negative*

Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

MERGING PROCEDURE

We need :

Continuous Pressure ☑

Convexity of Pressure in intensive variables $\Longrightarrow$ **Stability**

MERGING PROCEDURE

Treat $p \in [0,1]$ as an *internal order-parameter* such that for $p \to 0$ phase 1 dominates and for $p \to 1$ phase 2 dominates

p is solved through the equilibrium conditions:

$$\frac{\partial P}{\partial p} = 0 \,, \quad \frac{\partial^2 P}{\partial p^2} \leq 0 \,.$$

Phase with the maximum pressure will dominate

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MERGING PROCEDURE

$$P = p\,P_1 + (1-p)\,P_2 + a\,S_{\text{mix}}(p)$$

Shannon entropy

$$S_{\text{mix}}(p) \equiv -\left[p\ln p + (1-p)\,\ln(1-p)\right]$$

Ensures stable curvature in p

$$\frac{\partial^2}{\partial p^2}[a\,S_{\text{mix}}] = -a\left(\frac{1}{p} + \frac{1}{1-p}\right) < 0$$

For correct dimensions we choose

$$a = \frac{T}{\Delta V}$$

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MERGING PROCEDURE : *IDEAL MIXTURE*

At Equilibrium

$$\bar{p} = \frac{1}{1 + e^{-(P_1 - P_2)/a}}, \quad 1 - \bar{p} = \frac{1}{1 + e^{(P_1 - P_2)/a}}$$

For $a \to 0$, $\bar{p}$ becomes a step function $\Longrightarrow$ first-order phase transition

For $a \to T/\Delta V \Longrightarrow$ smooth crossover

$$P = \frac{T}{\Delta V} \log \left(e^{P_1 \Delta V/T} + e^{P_2 \Delta V/T} \right)$$

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MERGING PROCEDURE

We need :

Continuous Pressure ✓

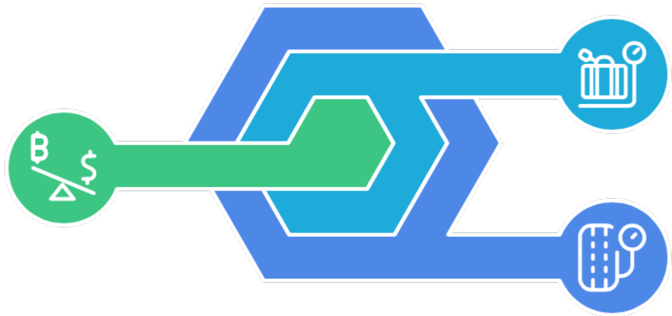
Convexity of Pressure in intensive variables $\Longrightarrow$ Stability ✓

Create Phase transition and critical point

MERGING PROCEDURE : *PHASE TRANSITION*

We can add an interaction term

$$P = p\,P_1 + (1-p)\,P_2 - a\left[p\ln p + (1-p)\;\ln(1-p)\right] - b\,p(1-p)$$



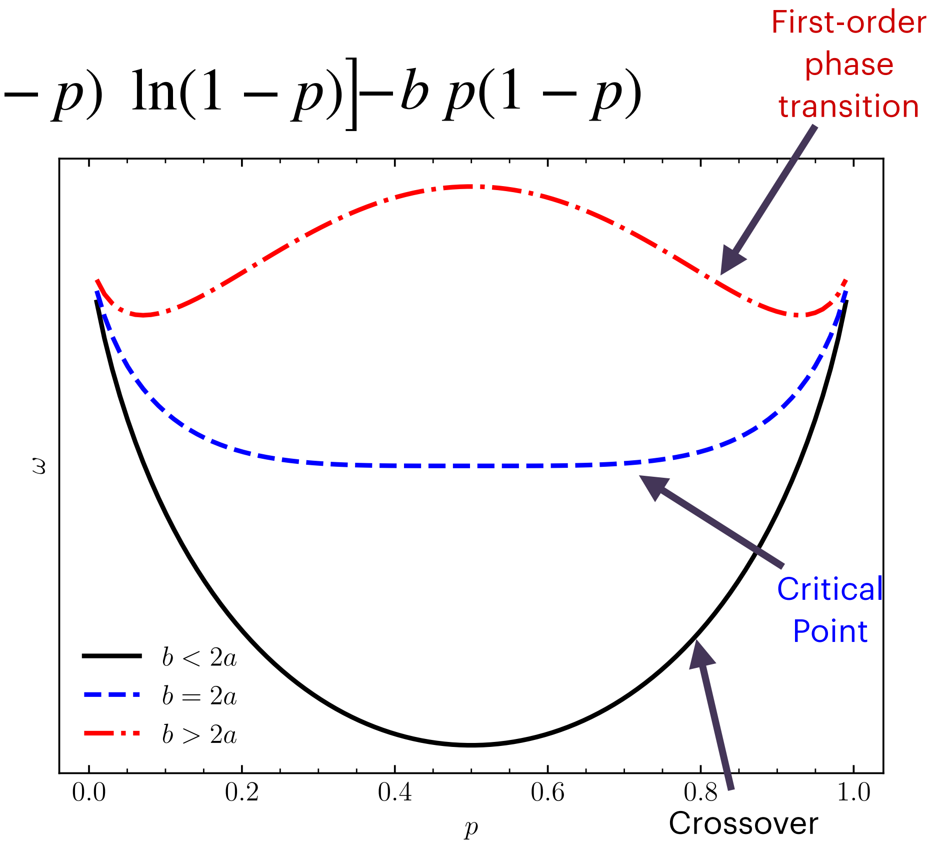
Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

MERGING PROCEDURE : *PHASE TRANSITION*

$$P = p\,P_1 + (1-p)\,P_2 - a\left[p\ln p + (1-p)\,\ln(1-p)\right] - b\,p(1-p)$$

- $b < 2a$ or $b = 0 \Longrightarrow$ **Crossover**
- $b > 2a \Longrightarrow$ **First-order phase transition**
- $b = 2a \Longrightarrow$ **Critical point**

$$T_c = b\frac{\Delta V}{2}$$



Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

MERGING PROCEDURE : *THERMODYNAMICS*

We can find the probability weight by,

$$\left(\frac{\partial P}{\partial p}\right)_{T,\mu_B} = P_1 + P_2 - a\ln\frac{p}{1-p} - (1-2p)b = 0,$$

and,

$$\left(\frac{\partial^2 P}{\partial p^2}\right)_{T,\mu_B} = \frac{a}{p(1-p)} - 2b < 0.$$

All thermodynamic quantities follow directly from pressure.

Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

MERGING PROCEDURE

have

We ~~need :~~

Continuous Pressure ☑

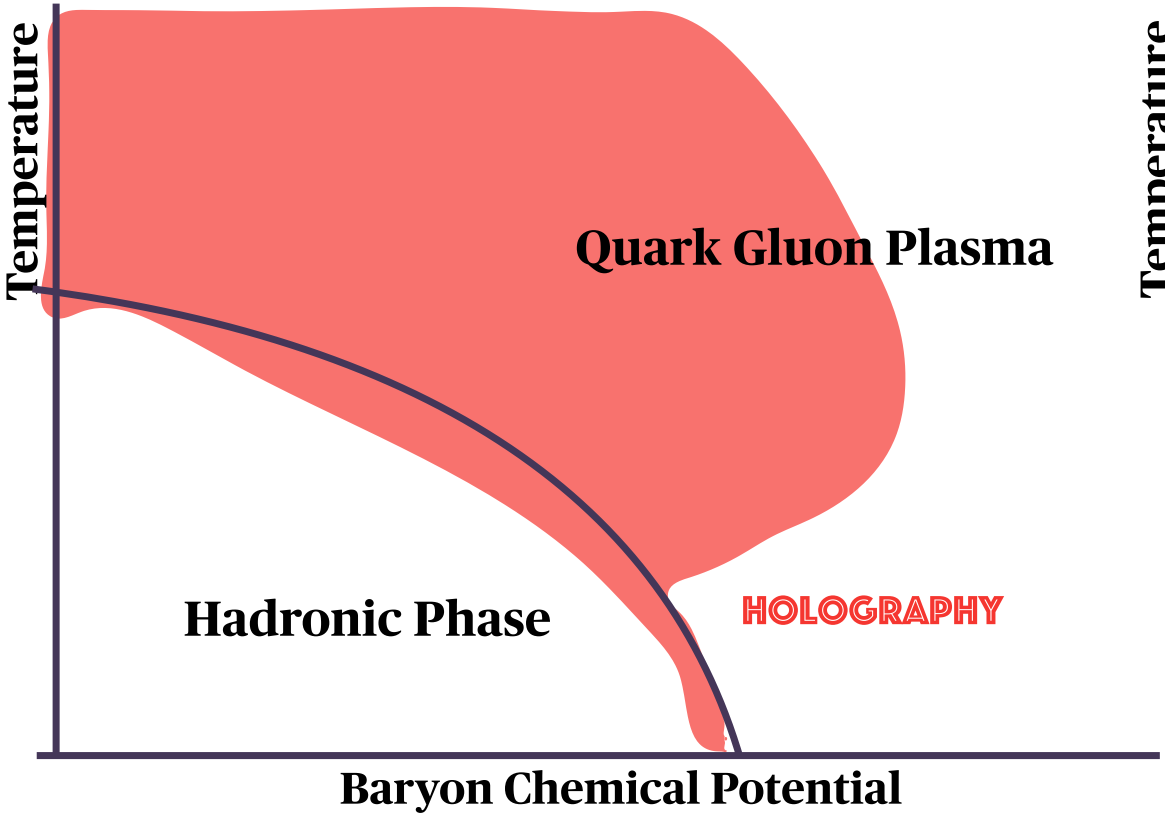
Convexity of Pressure in intensive variables $\Longrightarrow$ Stability ☑

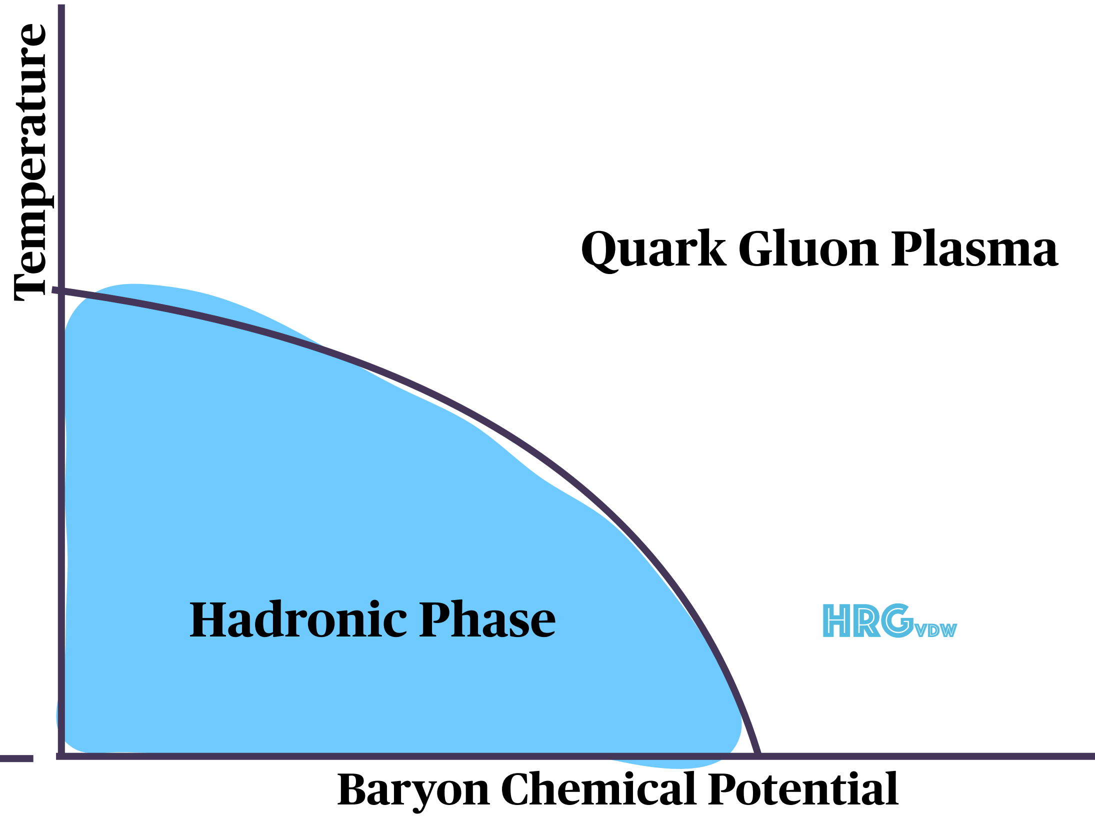
Create Phase transition and critical point ☑

PROOF OF PRINCIPLE

REGION OF VALIDITY *

* This is just an artist's rendition





Holography: Einstein Maxwell Dilaton

Models QGP by solving the black brane ansatz in AdS_5.

$$S_{bulk} = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5} d^5x\,\sqrt{-g}\left[R - \frac{(\partial\phi)^2}{2} - V(\phi) - \frac{f(\phi)F_{\mu\nu}^2}{4}\right]$$

Fit to lattice

Finite temperature (thermal) state of the boundary QFT = **AdS black brane**

R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD 96 (2017)
J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD 104 (2021)
Maldacena, et.al
O. DeWolfe, S. S. Gubser and C. Rosen, PRD 83 (2011)

van der Waals Hadron Resonance Gas

Treats matter as a gas of hadrons and resonances.

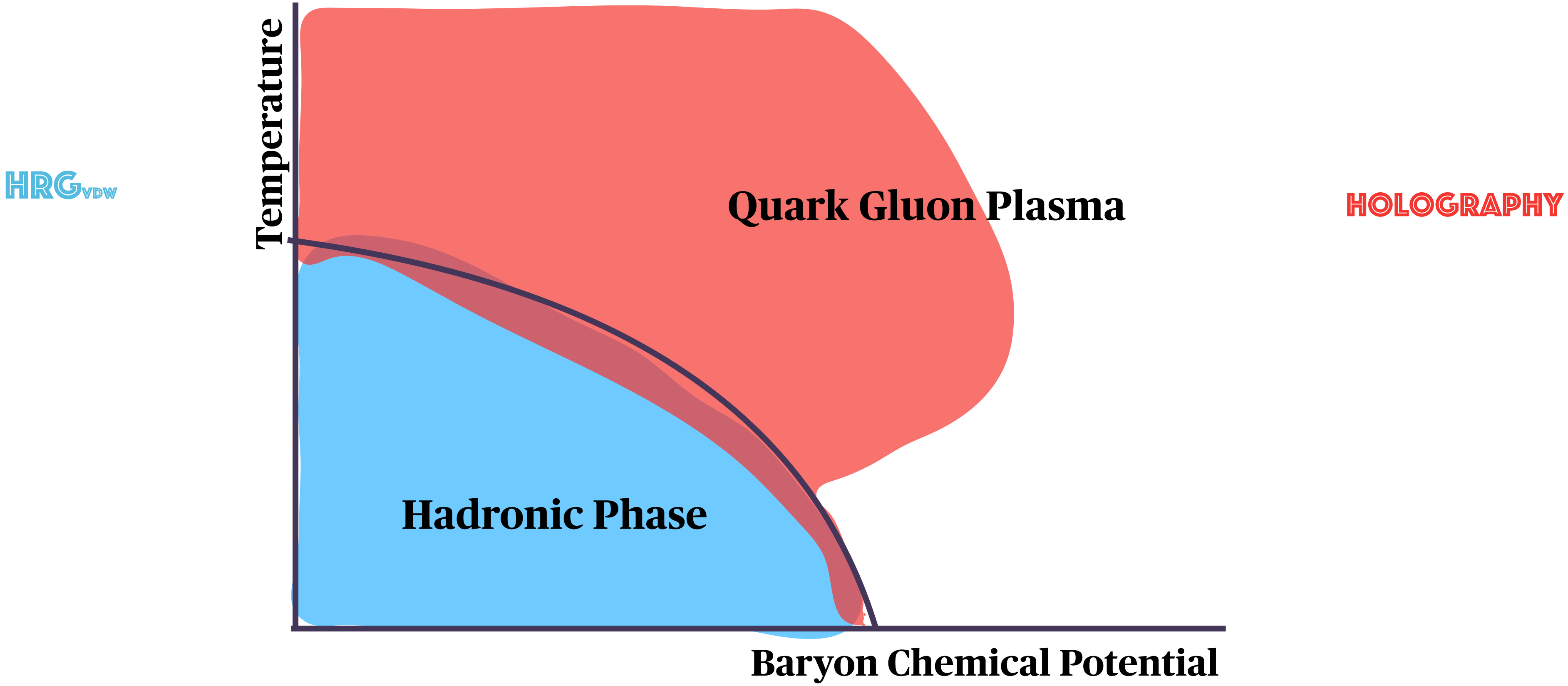
$$P(T,\mu) = P_{\text{mesons}}(T,\mu) + P_{\text{baryons}}(T,\mu) + P_{\text{antibaryons}}(T,\mu)$$

Repulsive excluded volume

Attractive mean field and repulsive excluded-volume

Volodymyr Vovchenko, Mark I. Gorenstein, Horst Stoecker Phys. Rev. Lett. 118, 182301
Volodymyr Vovchenko, Anton Motornenko, Paolo Alba, Mark I. Gorenstein, Leonid M. Satarov, Horst Stoecker Phys. Rev. C 96, 045202
J. Steinmer, et.al. J.Phys.G38:035001,2011

REGION OF VALIDITY *



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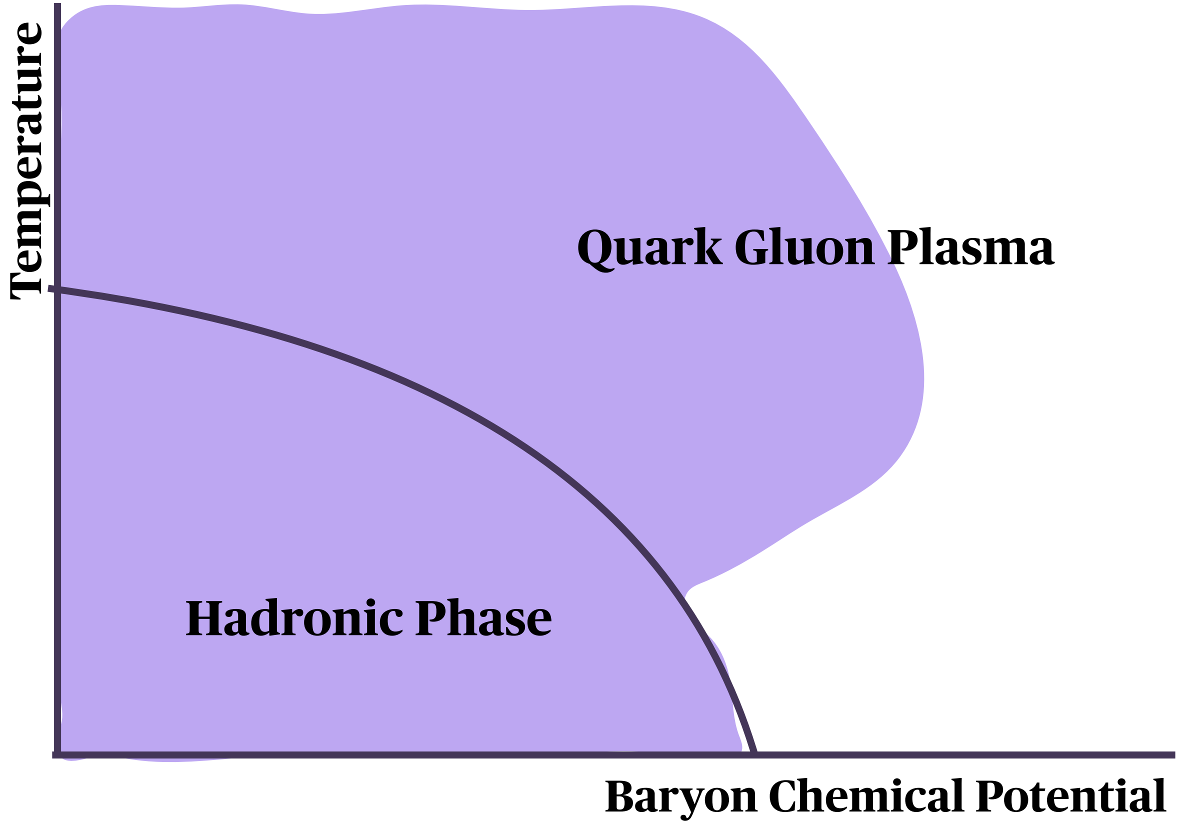
REGION OF VALIDITY *

MERGED EQUATION OF STATE



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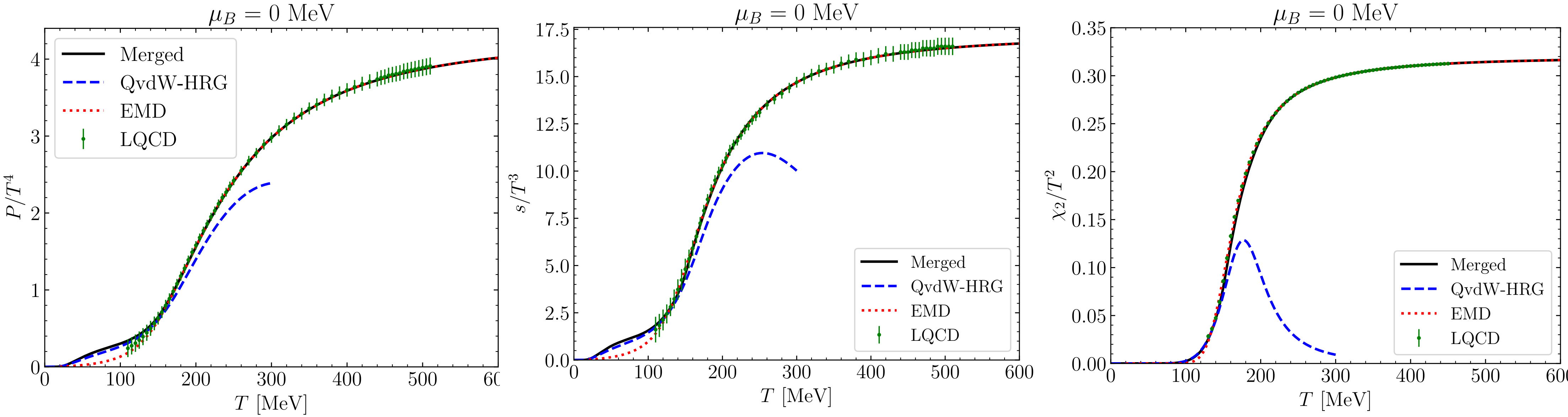
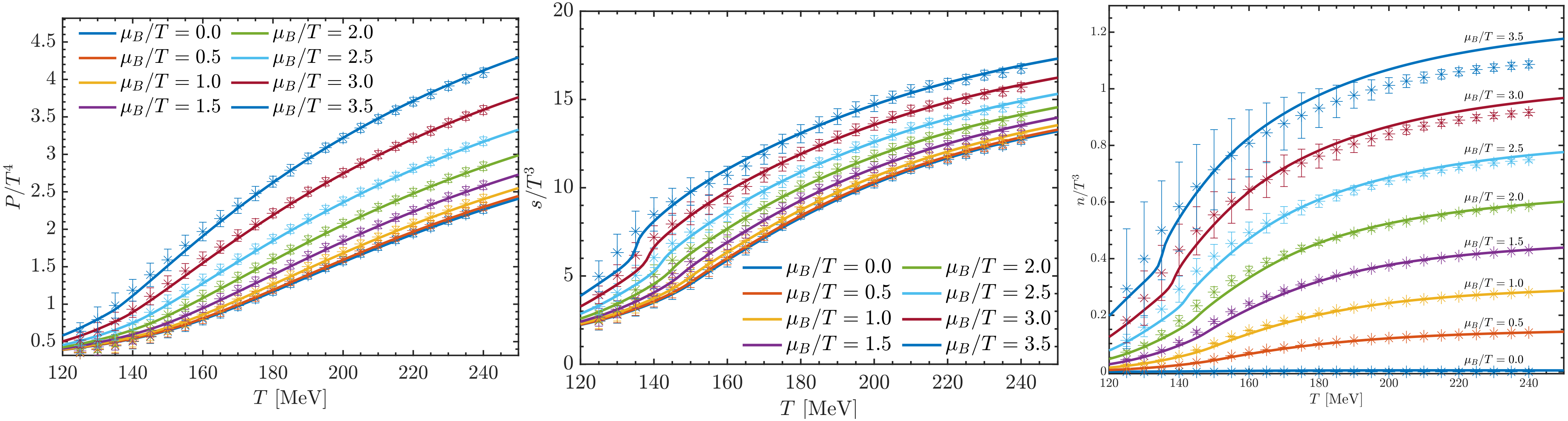
Results

AGREES WITH LQCD



Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

AGREES WITH LQCD T'-EXPANSION



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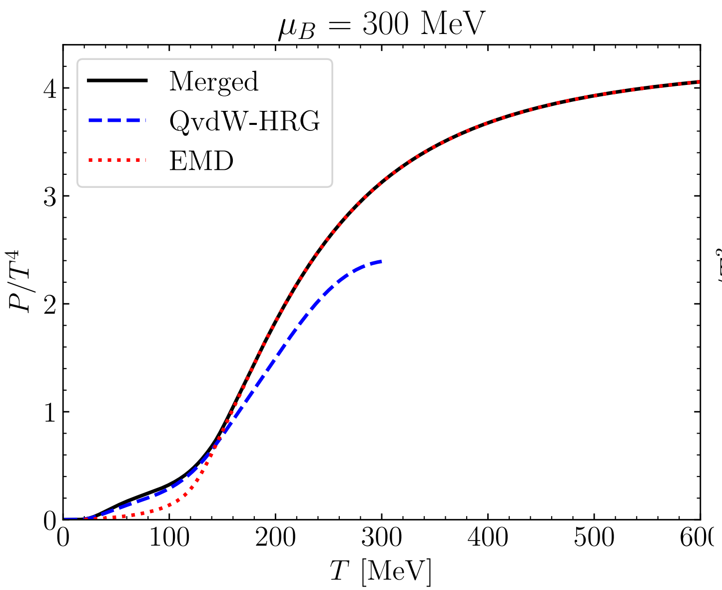
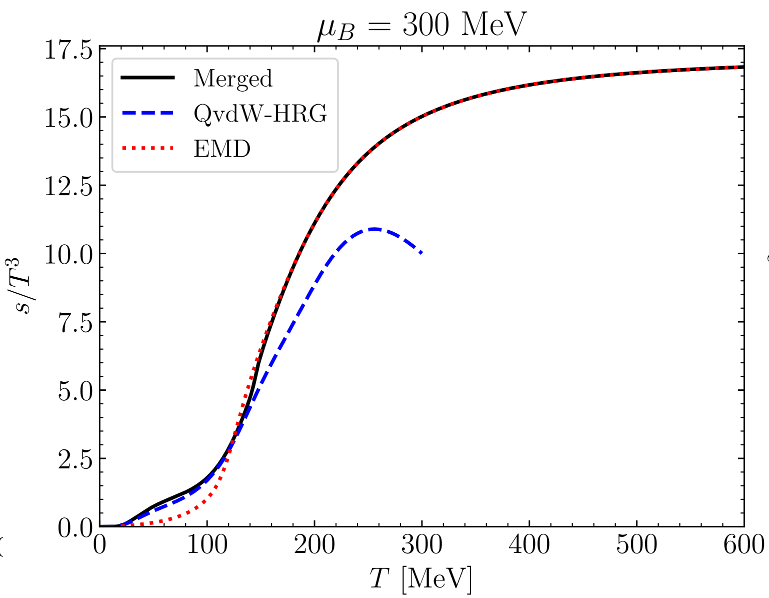
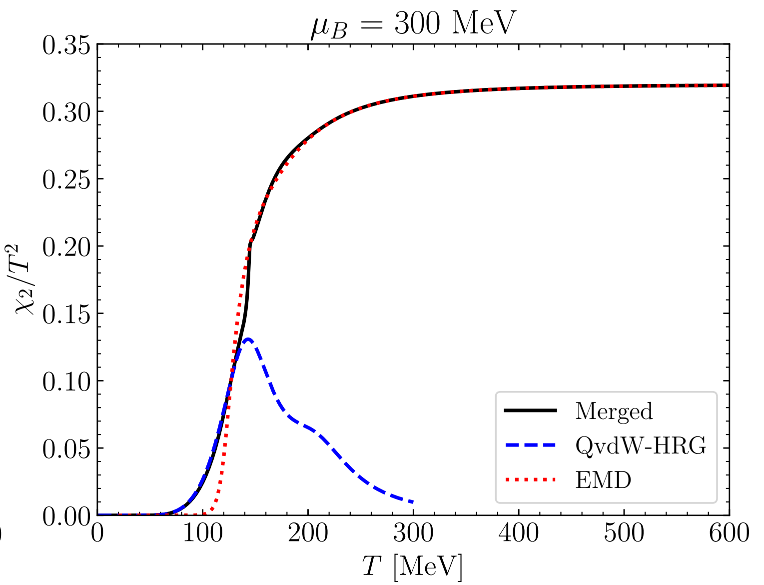
CROSSOVER REGION







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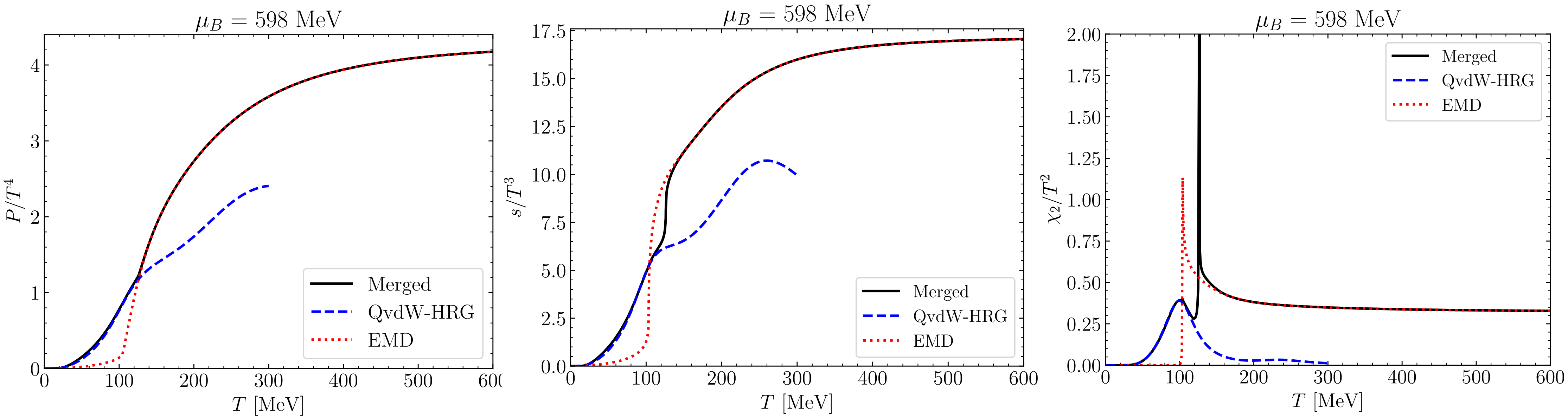
CRITICAL POINT



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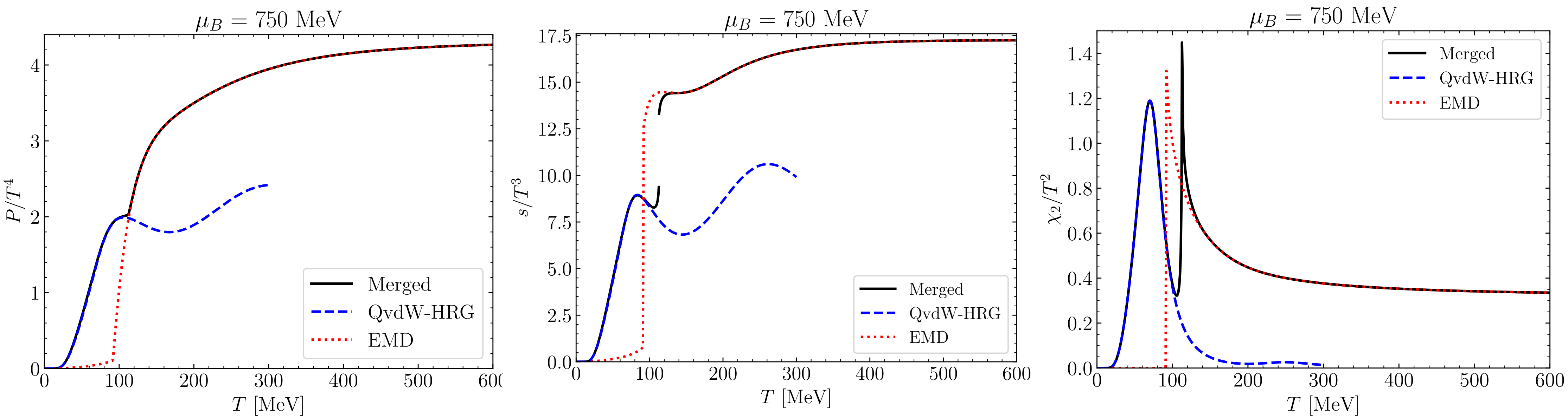
FIRST-ORDER PHASE TRANSITION



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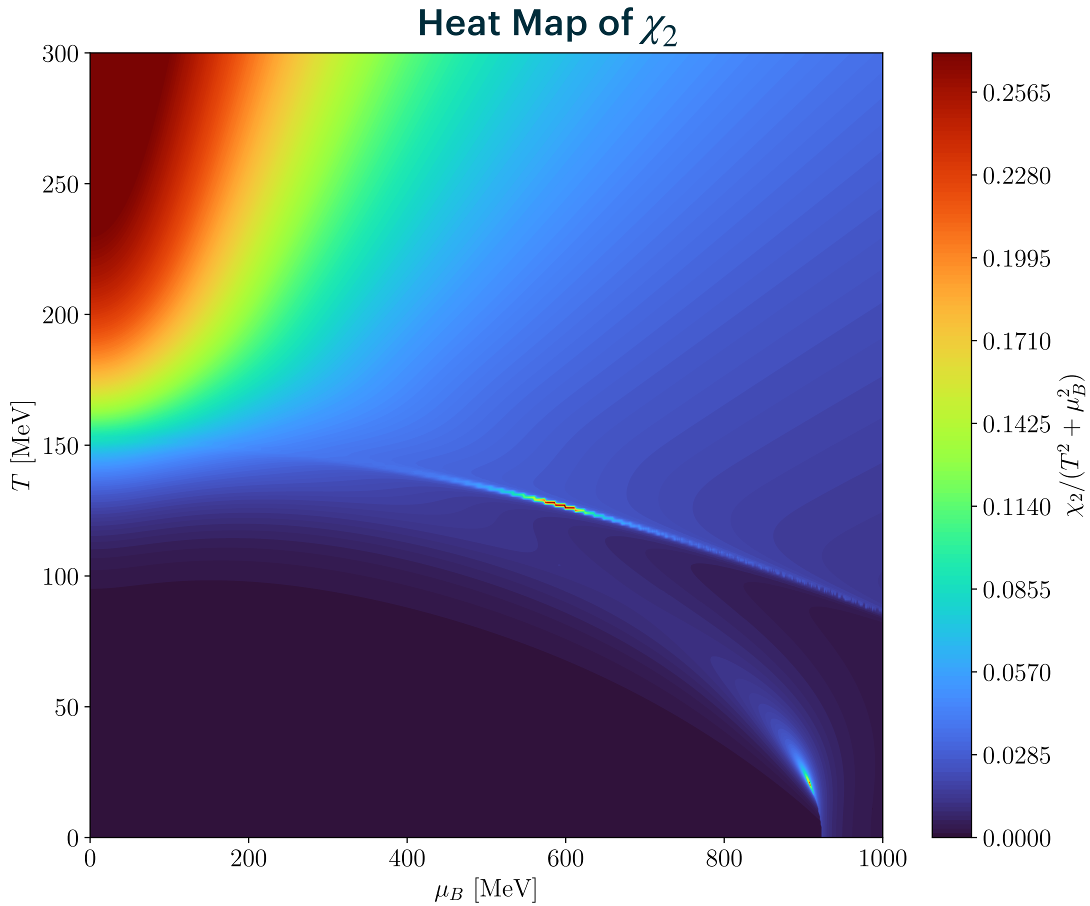
ROBUST FEATURE: BOTH PHASE TRANSITIONS



Normalization

$$\frac{\chi_2}{T^2+\mu_B^2}$$

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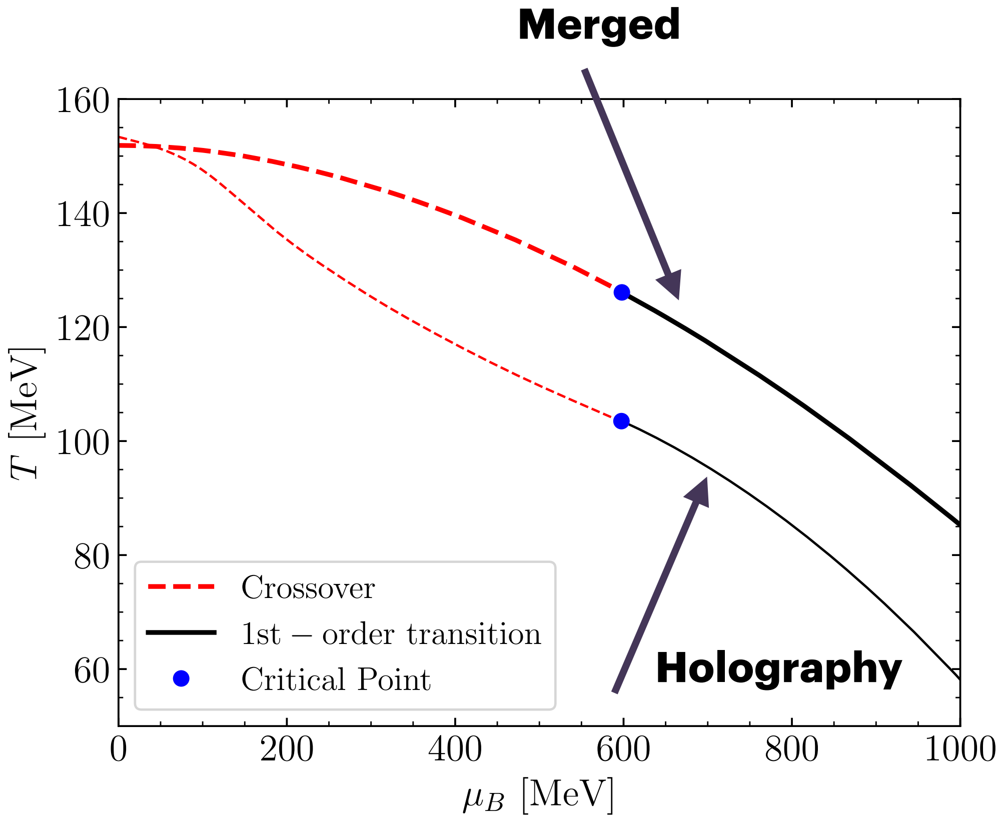
PHASE TRANSITION LINE

We fix $\mu_{Bc} = 598$ MeV from Holographic model and obtain $T_c = 126.1$ MeV



Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]

Features

- Stable Equation of State.
- Ensures Thermodynamic consistency.
- All thermodynamic quantities follow directly and analytically from Pressure.
- Accommodate multiple chemical potentials.
- First-order phase transition and a critical point

Take Away

- Why? There is no single framework that can be used as a unified description across the entire QCD phase diagram.
- What? We create a model-independent way to merge 2 (or more) EoSs
- How? Through a Statistical Mixture Framework.
- Where? Merge Holographic EoS (models QGP) with QvdW-HRG EoS (Hadrons). *Y. Yang, PG, et.al arXiv:2601.07987 [nucl-th]*
- It is the only equation of state with such a broad region of validity.

THANK YOU FOR YOUR ATTENTION TO THIS (DENSE) MATTER

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