22-27 March 2026, UCLA, Los Angeles, CA

Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali del Sud

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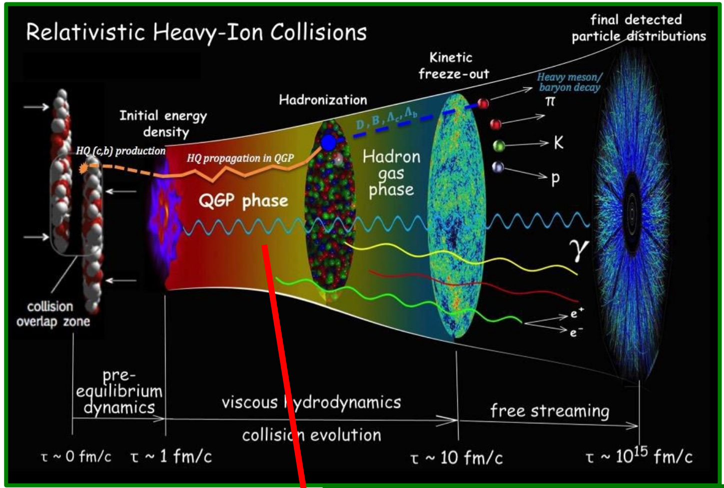
Testing the compatibility of lQCD spatial diffusion coefficient by mean of experimental open heavy flavor observables: R_{AA}, v_2, and v_3

In collaboration with: V. Minissale, S. Plumari, V. Greco

Dipartimento di Fisica e Astronomia 'E.Majorana'- Università degli Studi di Catania

INFN -Laboratori Nazionali del Sud (LNS)

Basic scales of charm and bottom quarks



Charm M_c≈1.3 GeV and Bottom M_b≈4.2 GeV

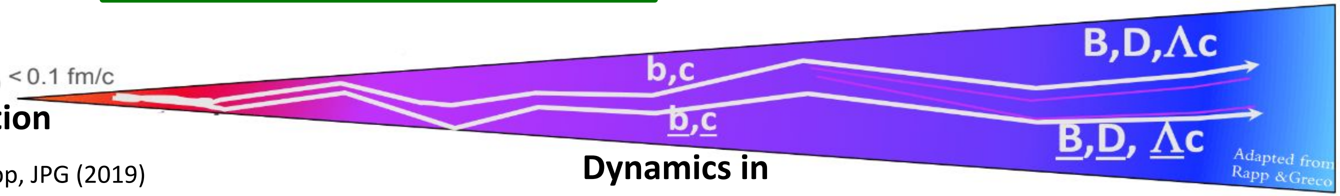
- $m_{c,b} >> \Lambda_{QCD}$
  pQCD initial production →Initial dN/dp_T known
- $m_{c,b} >> T_{RHIC,LHC}$
  negligible thermal production
- $\tau_0 < 0{,}08$ fm/c << τ_{QGP}
- $\tau_{th} \approx \tau_{QGP} >> \tau_{g,q}$

They experience the full evolution of the QGP.

They carry more informations with respect to their light counterparts.

One of the main probes to signal the <u>Quark-Gluon Plasma</u> properties:
Heavy Quarks

Initial $\tau_0 < 0.1$ fm/c
production



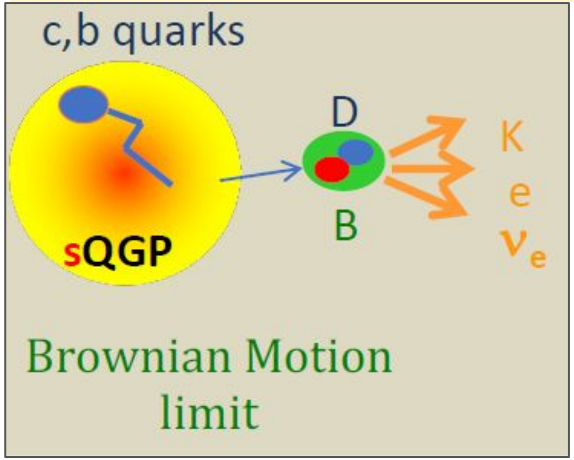
Dynamics in QGP

Hadronization:
Final hadron Spectra and observables

Reviews:
1. - F. Prino and R. Rapp, JPG (2019)
2. - X. Dong and VG, Prog.Part.Nucl.Phys. (2019)
3. - Jiaxing Zhao et al., Prog.Part.Nucl.Phys. (2020)

Standard Dynamics of Heavy Quarks in the QGP

m_{HQ} >> gT soft scattering → Brownian motion **Fokker-Planck approach**



$$\frac{\partial f_{c,b}}{\partial t} = \underset{\text{drag}}{\gamma} \frac{\partial (p f_{c,b})}{\partial p} + \underset{\text{diffusion}}{D_p} \frac{\partial^2 f_{c,b}}{\partial p^2}$$

$$\langle p \rangle = p_0 e^{-\gamma t}$$

$$\langle \Delta p^2 \rangle = 3D_p / \gamma (1 - e^{-2\gamma t})$$

$$\gamma = \int d^3k |M(k,p)|^2 p$$

$$D = \frac{1}{2} \int d^3k |M(k,p)|^2 p^2$$

$|M|^2$ scattering matrix from: pQCD, Quasi particle model (QPM), T-Matrix, etc.

$$D_s = \frac{T}{M\gamma} = \frac{T^2}{D_p} = \frac{T}{M}\tau_{th}$$

Space diffusion coefficient

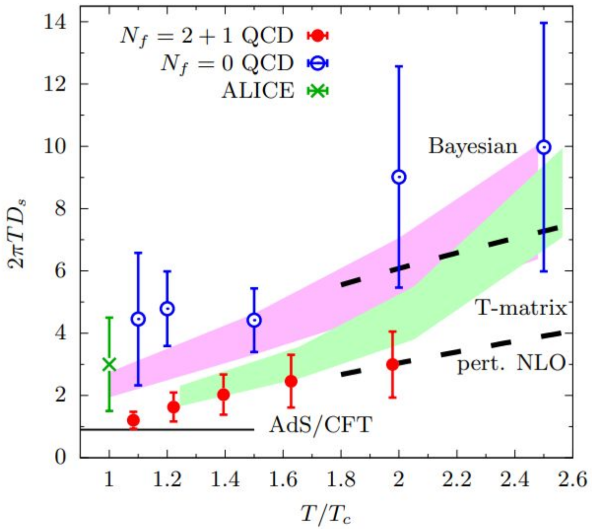
Fluctuation-dissipation theorem: $D_p = TE\gamma$

- ★ Brownian motion challenged for charm (m_c ~ 3 T~ gT) → Relativistic Boltzmann dynamics
- ★ This is the main set up at least at p_T < 6-8 GeV (p_T >10 GeV radiative E loss, jet physics)

Diffusion Transport Coefficient from lQCD

Spatial diffusion coefficient from lattice QCD:
Extract Spectral function ρ_E corresponding to the E field correlation functions
→ Kubo formula in the **p→0 limit**:

$$\frac{D_p}{T^3} = \lim_{\omega\to 0} \frac{T\rho_E(\omega)}{\omega} \Rightarrow D_s = \frac{T^2}{D_p} = \frac{T}{M_Q\gamma} = \frac{T}{M_Q}\tau_{th}$$



From quenched Nf=0 to not quenched (Nf=2+1) QCD (2023-24)

- $2\pi TD_S$ smaller compared to previous quenched (Nf=0) lQCD estimates (about 30 times smaller than LO pQCD estimation)
- For T/Tc ≈1 close to AdS/CFT. Agreement with pQCD (NLO) at high T

$(2\pi T)D_s \approx 4$ at $T_C \to$ $\tau_{th} \approx$ **5-6 fm/c for quenched**

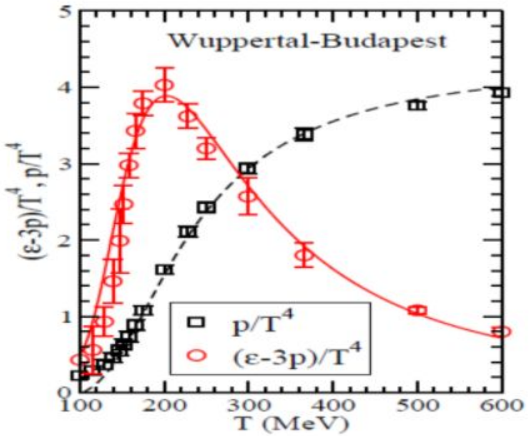
$(2\pi T)D_s \approx 1$ at $T_C \to$ $\tau_{th} \approx$ **1.5 fm/c for unquenched**

New data: faster thermalization of HQs wrt phenomenological models
→**gives predictions for HF observables in agreement with data?**

CATANIA MODEL: QUASI-PARTICLE MODEL AND TRANSPORT THEORY

Quasi Particle Model (QPM) fitting lQCD

Non perturbative dynamics → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lQCD thermodynamics**



$$m_g^2(T) = \frac{2N_c}{N_c^2-1} g^2(T)\,T^2$$
$$m_q^2(T) = \frac{1}{N_c} g^2(T)\,T^2$$

→ **Thermal masses of gluons and light quarks**

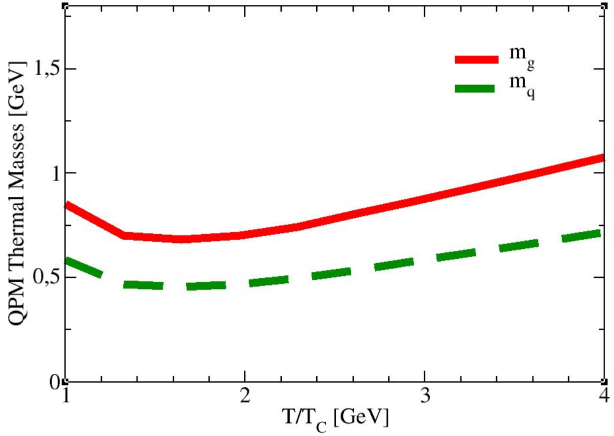
g(T) from a fit to ε from lQCD data → good reproduction of P, ε-3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57 T_c

T→ T_C enhancement of the coupling

N_f=2+1
Bulk:
u,d,s



S. Plumari et al, *Phys.Rev.D* 84 (2011) 094004
H. Berrehrah,, PHYSICAL REVIEW C **93**, 044914 (2016)

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_q(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_q(x,p)=C[f_q,f_g]$$

$$p^{\mu}\partial_{\mu}f_g(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_g(x,p)=C[f_q,f_g]$$

Free-streaming

field interaction
$\varepsilon - 3p \neq 0$

Collision term
gauged to some η/s≠ 0

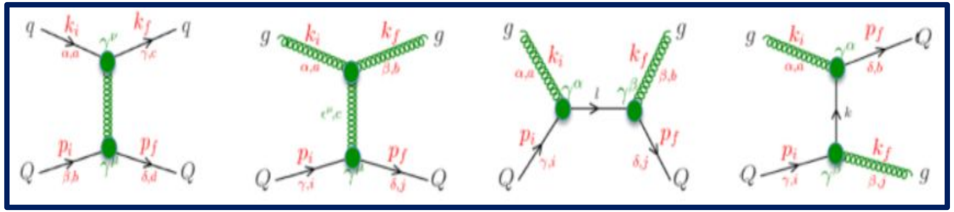
Equivalent to viscous hydro at η/s ≈ 0.1

HQ evolution

$$p^{\mu}\partial_{\mu}f_Q(x,p)=C[f_q,f_g,f_Q]$$

$$C[f_q,f_g,f_Q]=\frac{1}{2E_1}\int\frac{d^3p_2}{2E_2(2\pi)^3}\int\frac{d^3p_1'}{2E_1'(2\pi)^3}$$
$$\times[f_Q(p_1')f_{q,g}(p_2')-f_Q(p_1)f_{q,g}(p_2)]$$
$$\times|M_{(q,g)\to Q}(p_1p_2\to p_1'p_2')|$$
$$\times(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$$

Feynman diagrams at first order pQCD for HQs-bulk interaction:



Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_{q}(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_{q}(x,p)=C[f_{q},f_{g}]$$

$$p^{\mu}\partial_{\mu}f_{g}(x,p)+m(x)\partial^{x}_{\mu}m(x)\partial^{\mu}_{p}f_{g}(x,p)=C[f_{q},f_{g}]$$

Free-streaming

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HQ evolution

$$p^{\mu}\partial_{\mu}f_{Q}(x,p)=C[f_{q},f_{g},f_{Q}]$$

Coalescence + fragmentation hadronization ($\varepsilon \rightarrow \varepsilon_c$)

$$\frac{dN_h}{d^2p_h}=\sum_f\int dz\,\frac{dN_f}{d^2p_f}D_{f\rightarrow h}(z)$$

fragmentation function

$$\frac{dN_{Hadron}}{d^2p_T}=g_H\int\prod_{i=1}^{n}p_i\cdot d\sigma_i\,\frac{d^3p_i}{(2\pi)^3}f_q(x_i,p_i)\,f_W(x_1,\dots,x_n;p_1,\dots,p_n)\,\delta(p_T-\sum_i p_{iT})$$

Wigner function

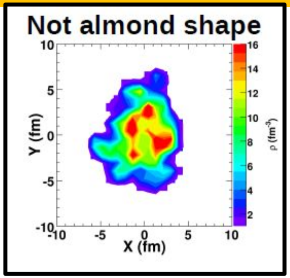
$P_{coal}=1$ for $p=0$

For more details see V. Minissale talk (Wed 25 at 8.45)

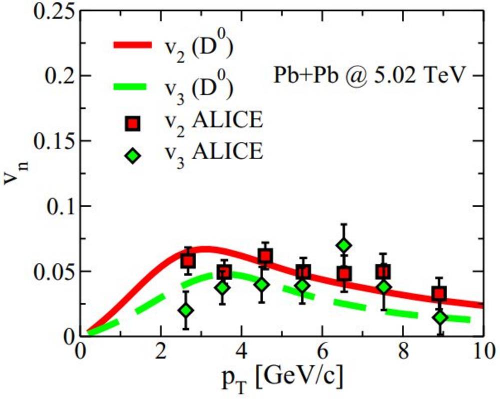
Catania QPM: some prediction for charm…

In a event-by-event approach

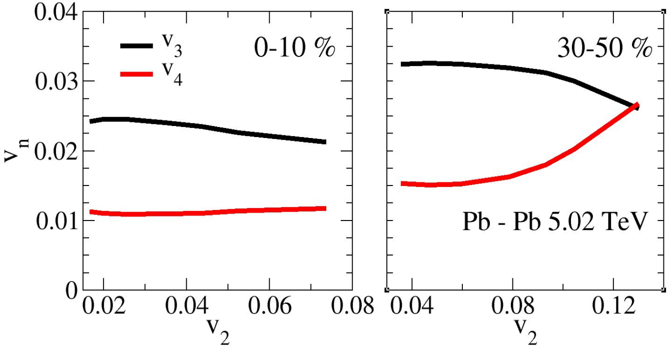
Monte Carlo Glauber
for initial condition of partons



S.Plumari et al, *Phys.Rev.C* 92 (2015) 5



v_n-v_m →similar correlation for charm wrt bulk



ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Catania QPM: some prediction for charm…

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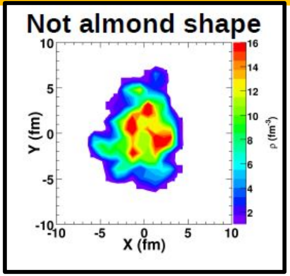
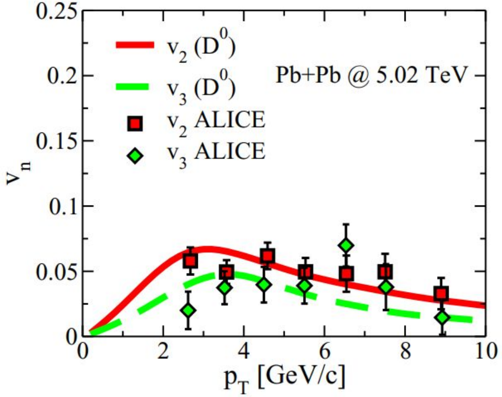


S.Plumari et al, *Phys.Rev.C* 92 (2015) 5



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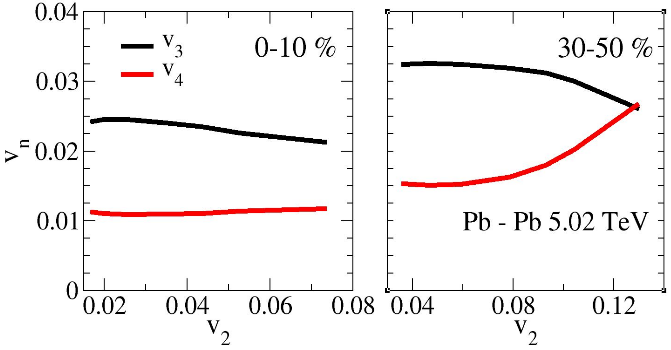
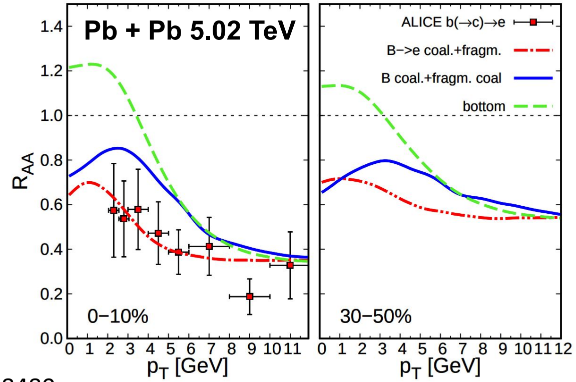
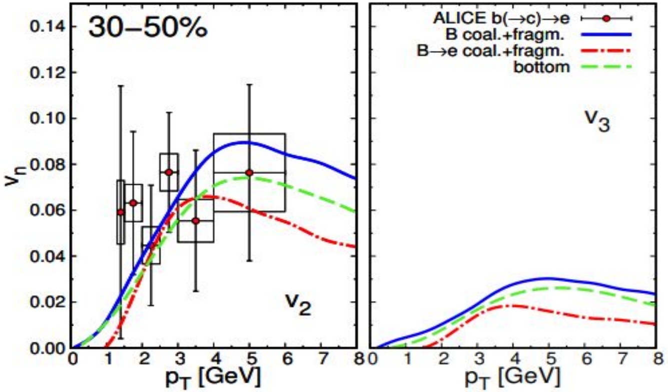


ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

and bottom…

- Prediction for B meson R_{AA} and v_n
- R_{AA} and v_n of electrons from semileptonic B meson decay





M.L. Sambataro et al., *Phys.Lett.B* 849 (2024) 138480
M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Data from: ALICE coll., arxiv:2211.13985

QPM extension: QPMp vs QPM

QPM → no masses evolution wrt the momentum → quark susceptibilities **understimated**

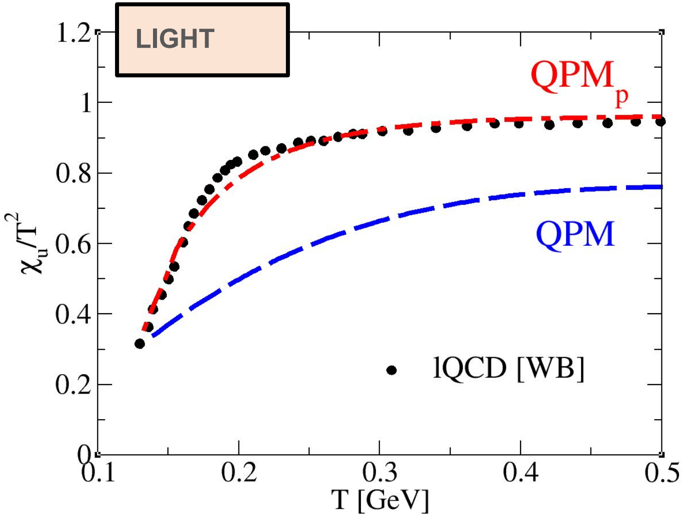
QPMp → following the model developed by PHSD group

H. Berrehrah, W. et al., Phys.Rev.C 93, 044914 (2016).
C. S. Fischer, J. Phys. G 32, R253 (2006).
M.L. Sambataro et al, Eur.Phys.J.C 84 (2024) 9, 881

$$M_g(T,\mu_q,p) = \left(\frac{3}{2}\right)\left(\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_g(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2} + m_{\chi g}$$

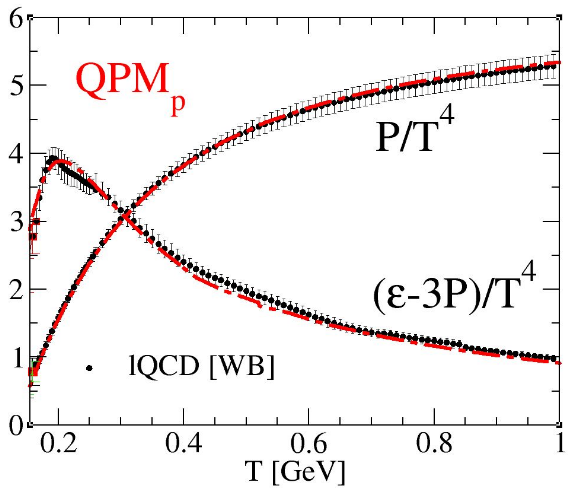
$$M_{q,\bar{q}}(T,\mu_q,p) = \left(\frac{N_c^2-1}{8N_c}g^2(T^\star/T_c(\mu_q))\left[T^2 + \frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_q(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2} + m_{\chi q}$$

Momentum dependent factors

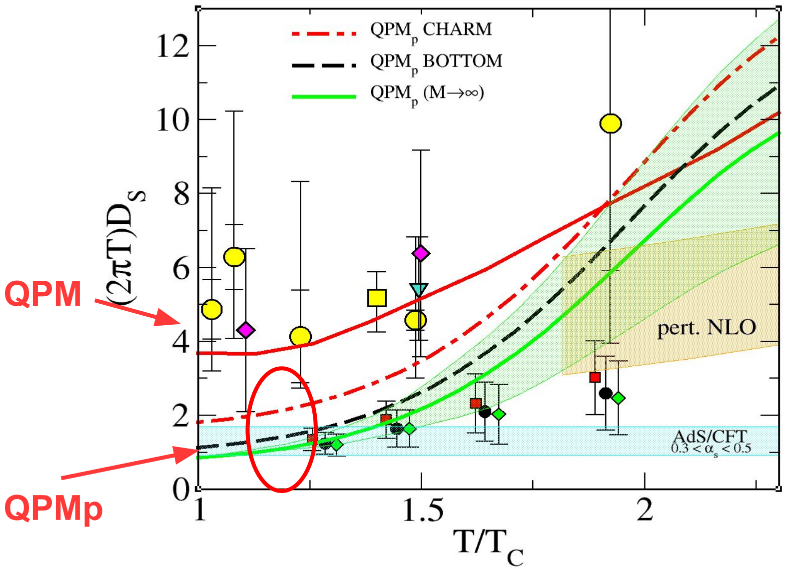


QPMp describes both **EoS** and **quark susceptibilities χ_q , χ_s** which are understimated in the standard QPM approach.

Pressure, trace anomaly **including charm**
Nf=2+1+1



QPMp – spatial diffusion coefficient D_s



M.L. Sambataro et al, Eur.Phys.J.C 84 (2024) 9, 881

lQCD quenched

- lQCD [Banerjee et al. (2011)]
- lQCD [Kaczmarek (2014)]
- lQCD [Francis (2015)]
- lQCD [Brambilla (2020)]

w dynamical fermions

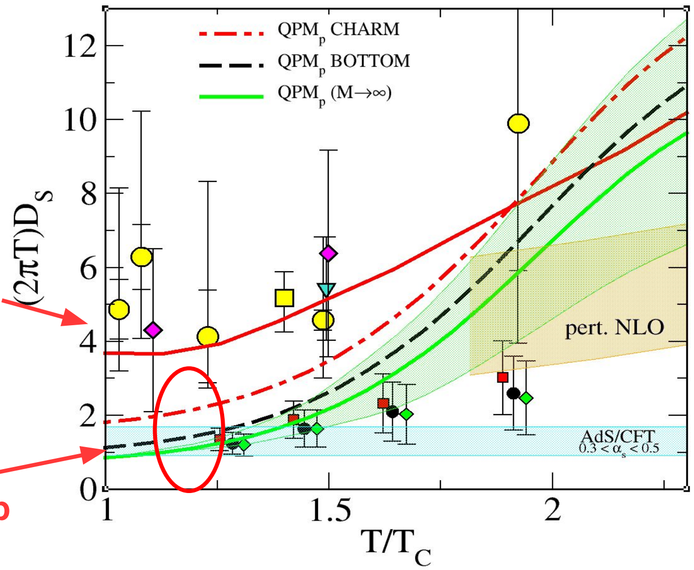
- lQCD [Altenkort(2024)] charm
- lQCD [Altenkort(2024)] bottom
- lQCD [Altenkort(2024)] (M→∞)

Going from QPM to QPMp…

T/T_c <1.6 → strong non-perturbative behaviour of D_S.
bottom: closer to D_S lQCD

but still a significant deviation wrt the lQCD data!

QPMp – spatial diffusion coefficient D_s



QPM

QPMp

M.L. Sambataro et al, Eur.Phys.J.C 84 (2024) 9, 881

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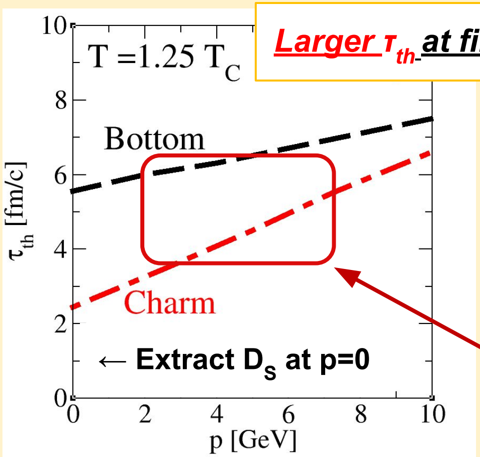
bottom: closer to D_S lQCD

but still a significant deviation wrt the lQCD data!

$$D_s = \frac{T}{M\,\gamma} = \frac{T}{M}\tau_{th}$$

in the p → 0 limit

- τ_{th}(c , p=0) ~ 4 fm/c (QPM) → 2.5 fm/c (QPMp)
- τ_{th}(b , p=0) ~ 9 fm/c (QPM) → 5.5 fm/c (QPMp)



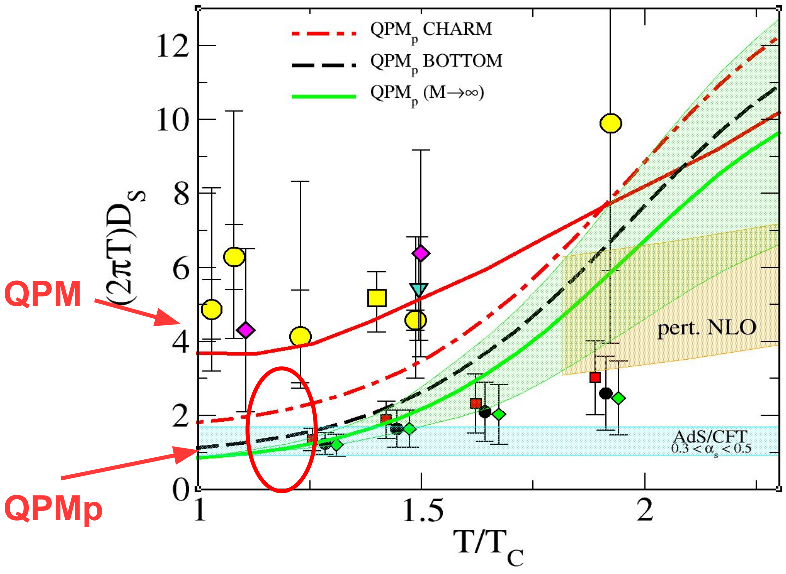
Larger τ_{th} at finite momentum

(p~5 GeV)

About ~ 40% for charm
~ 20% for bottom
in **QPMp**

Region explored from phenomenology to determine D_S

QPMp – spatial diffusion coefficient D_S



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- **τ_{th}(c , p=0) ~ 4 fm/c (QPM) → 2.5 fm/c (QPMp)**
- **τ_{th}(b , p=0) ~ 9 fm/c (QPM) → 5.5 fm/c (QPMp)**

Can this new D_S(T) generate predictions for R_{AA}, v_n in agreement to experimental data?

Event-by-event Langevin : R_{AA} and v_2 from D_s (p,T)

$$dx_j = \frac{p_j}{E} dt$$

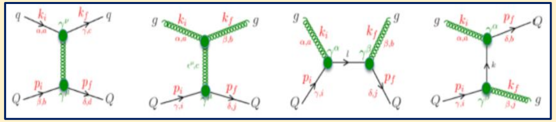
$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk} \rho_k$$

Fluctuation-dissipation theorem :

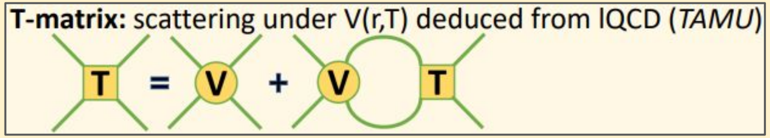
$$D_p(p) = A(p)E(p)T$$

$$\tau_{th}(p) \approx A(p)^{-1}$$

QPMp: g(T) from a fit to lQCD-EoS + tree level diagrams to evaluate **$A(p)=\int d^3k\, |M(k,p)|^2\, p$**



lQCD + pTAMU:



harder p dep. similar to $\tau_{th}(p) \propto E^{-1}$(FDT)

or lQCD + τ_{th}(p) const

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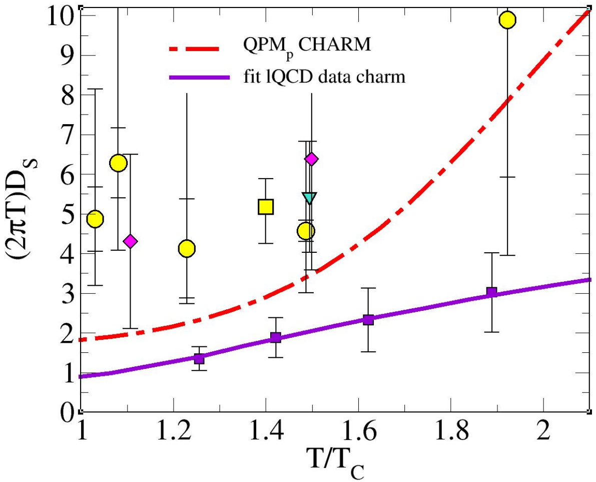
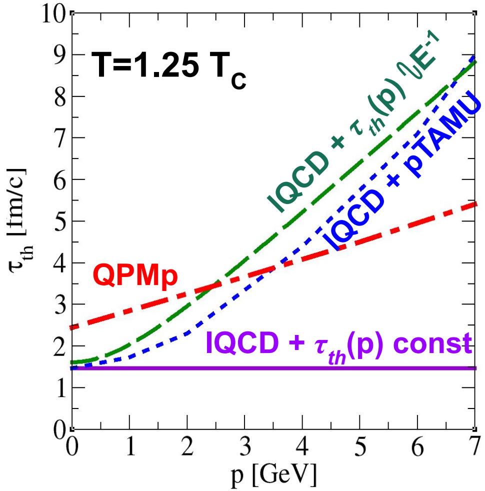
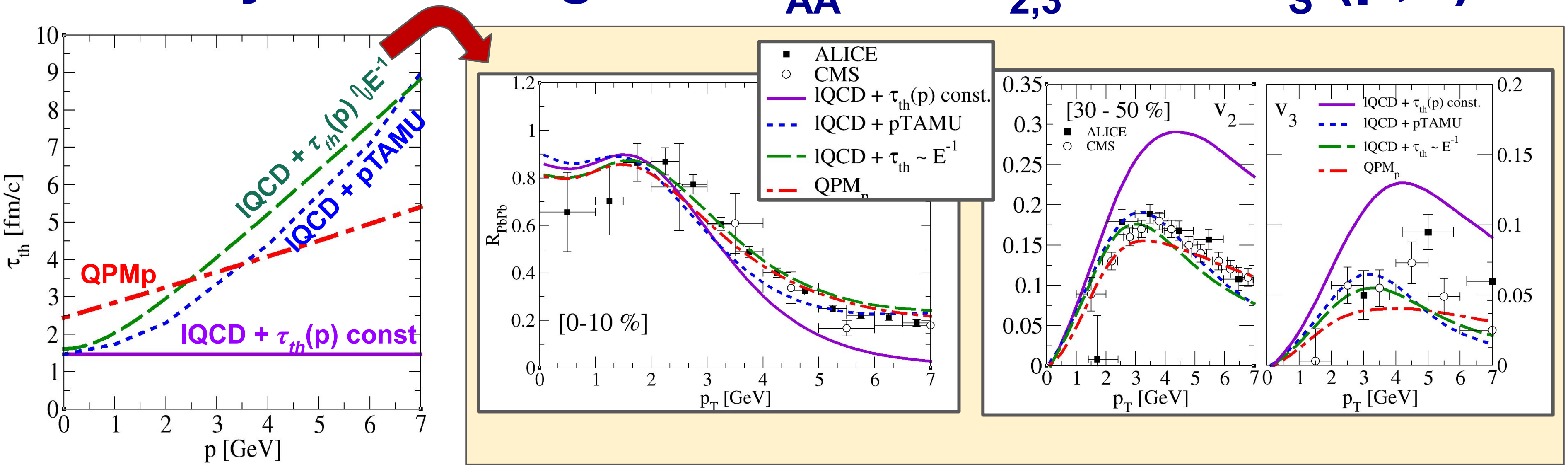
$$D_s = \frac{T}{M\,\gamma} = \frac{T}{M}\,\tau_{th}$$

in the p → 0 limit





Event-by-event Langevin: R_{AA} and $v_{2,3}$ from D_s (p,T)



- D_s(T) from lQCD without p dependence → fails to describe the experimental data!

- QPMp/lQCD + pTAMU **significant difference in τ_{th}** → different D_s in the two models → but **comparable R_{AA} and v_2**

at 1.8 T_C: $(2\pi T)D_s \approx 2.5$ (**TAMU**)

$(2\pi T)D_s \approx 5.5$ (**QPMp**)

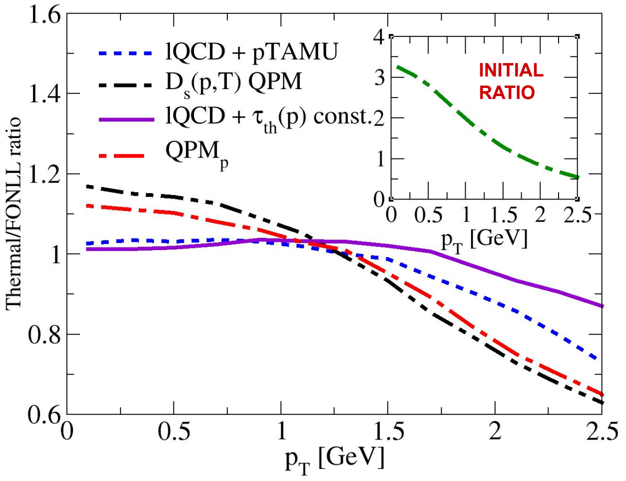
-Based on R_{AA} and v_2 no significant discrimination **QPMp** vs **lQCD + pTAMU** → **additional observables, light ion systems…**

- v_3 needs high precision data

D_s(T) from lQCD-NREFT taken from Phys.Rev.Lett. 132 (2024) 5, 051902
A(p) drag of TAMU taken from Eur.Phys.J.A 60 (2024) 4, 92

M.L.sambataro, et al. *Phys.Lett.B* 872 (2026) 140049

Does small D_s imply equilibrium for charm in PbPb?



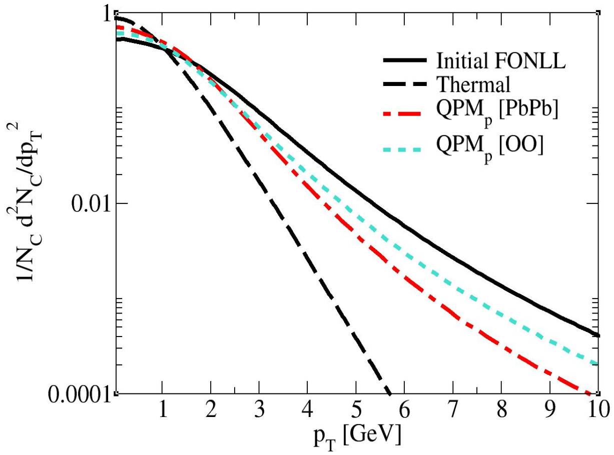
In PbPb:

RATIO → $\frac{\text{final charm dN/dpt starting from non-equilibrium FONLL}}{\text{final charm dN/dpt starting from thermal distribution}}$

D_s from QPM ≈ factor 3 greater wrt D_s from lQCD

- Deviation of the order of 20 % in the low p_T region for $(2\pi T)D_s \approx 3$ from QPM
- At low p_T ratio close to unity for $(2\pi T)D_s \approx 1$ → loss of sensitivity for D observables
  → **dynamical attractor** may be reached
  BUT even in lQCD+pTAMU at p_T>1.5 GeV significant deviation from equilibrium

QPMp – moving to smaller collision system



- **Smaller system:** larger thermalization time wrt fireball lifetime
  → high pt distribution close to non-equilibrium pQCD initial distribution.
- From PbPb to OO → **flattening of the charm quark spectra**.

Heavy vs Light Ion:

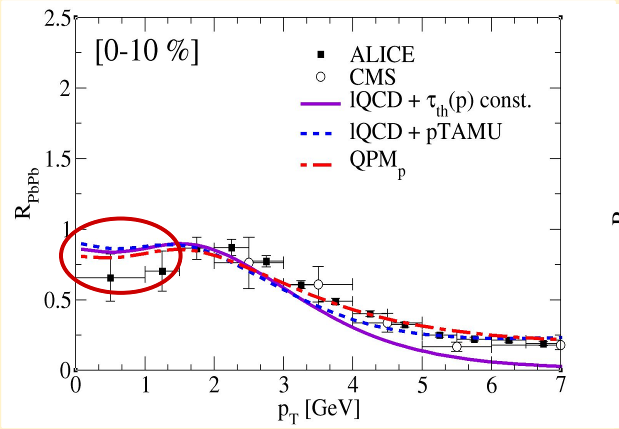
→ **In PbPb:** at p_T<2 GeV

loss of information on initial condition for D_s(lQCD)

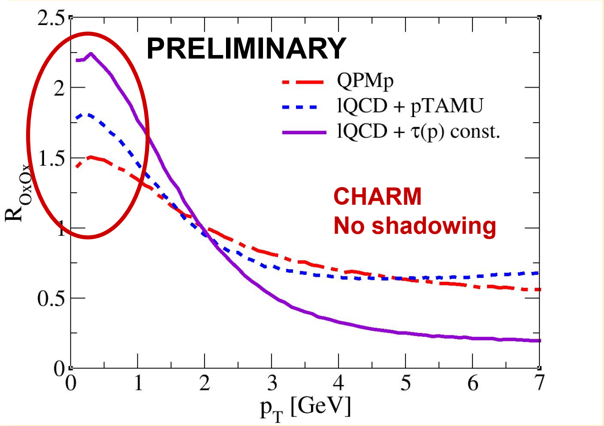
No discrimination **QPMp**/**lQCD**

→ **In OxOx**: Stronger sensitivity to interaction in the low p_T region.

Pb + Pb Heavy Ion



O + O Light Ion



Conclusions

- **QPMp**
  Good agreement with experimental R_{AA}, v_2 and v_3.
  → Decrease of D_s at small T wrt QPM but still larger than lQCD Ds for charm
  Comparison D_s from model/phenomenology : p-dependence very relevant
  → $D_s(T)$ from lQCD w/o p dependence totally out of phenomenology

- No significant discrimination between **QPMp** and **lQCD + pTAMU** based only on R_{AA}, v_2 in PbPb (v_3 need data with smaller uncertainties)
  → Moving to light Ion→ shorter lifetime → more non-equilibrium

- From lQCD $\tau_{th} \approx 1$ fm/c at T_C smaller than typical lifetime of the fireball in PbPb
  → Loss of information on initial condition at small p_T, but at p_T>2 GeV even **lQCD+pTAMU** deviation from equilibrium

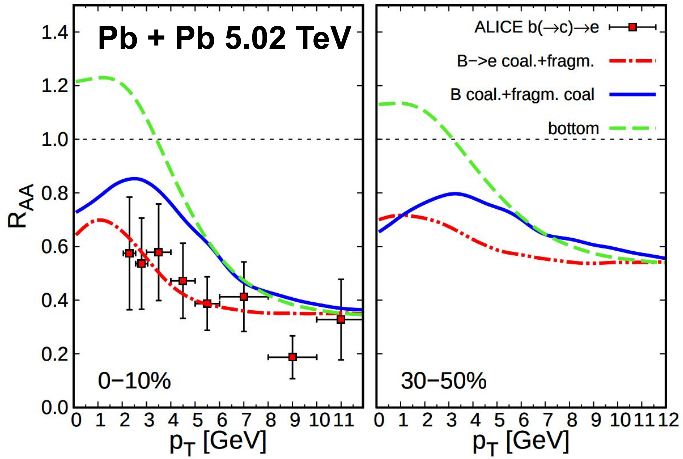
Thanks for the attention!

Back-up slides

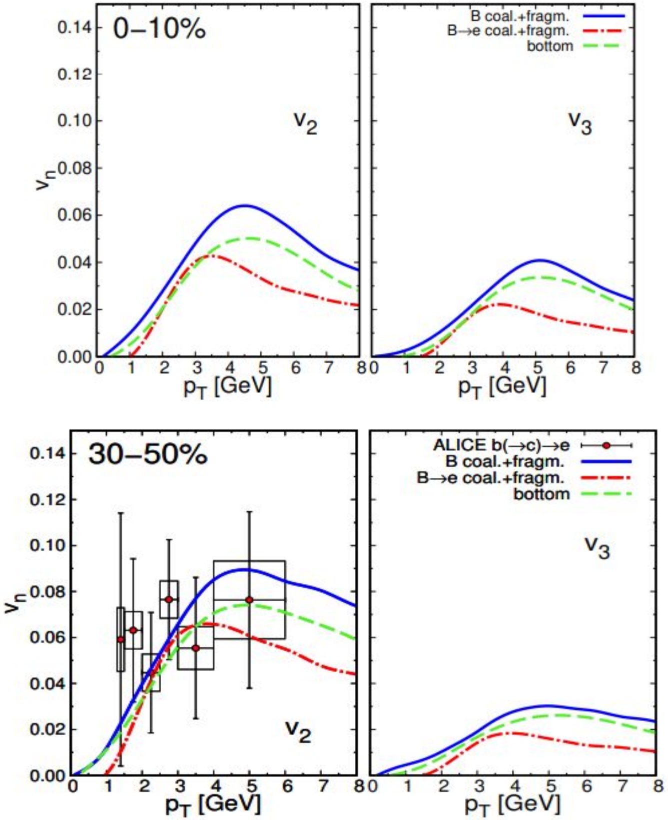
...and bottom [R_{AA} and $v_{(n=2,3)}$]

Hadronization with coalescence + fragmentation model

- Prediction for B meson R_{AA} and v_n
- R_{AA} and v_n of electrons from semileptonic B meson decay



No parameters changed with respect to charm dynamics (only M_b)



M.L. Sambataro et al., *Phys.Lett.B* 849 (2024) 138480

Data from: ALICE coll., arxiv:2211.13985

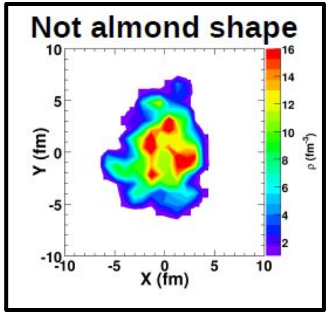
Compared to charm quark:

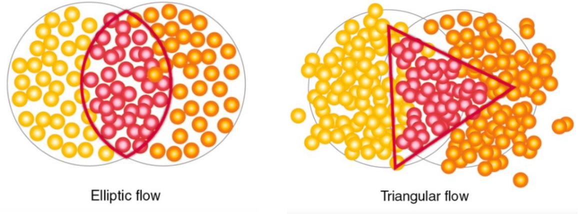
- Efficiency of conversion of ε_2 :
  15% smaller for v_2 in most central collisions.
  40% smaller for v_2 at 30−50% centrality.
- Efficiency of conversion of ε_3 :
  30% smaller for v_3 at both 0-10% and 30-50% centralities.

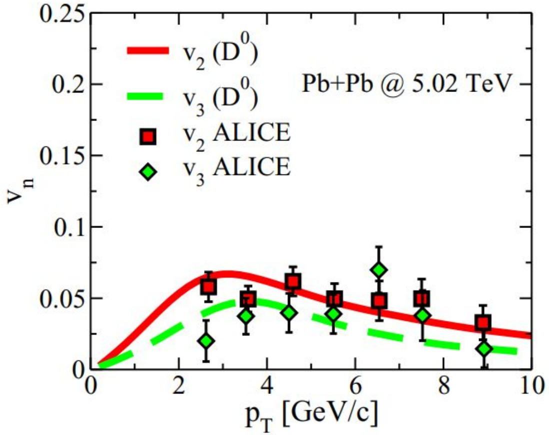
Catania QPM: some prediction for charm…

In a event-by-event approach

Monte Carlo Glauber for initial condition of partons
S.Plumari et al, *Phys.Rev.C* 92 (2015) 5





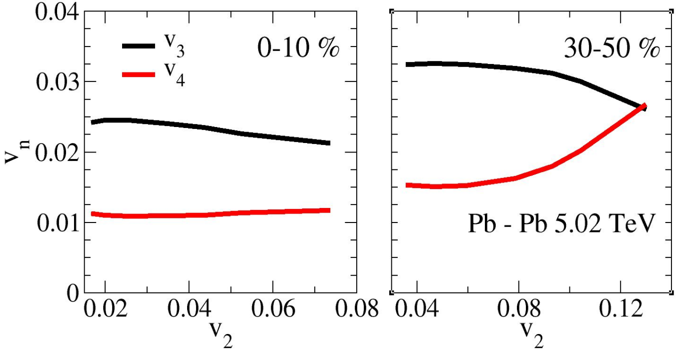


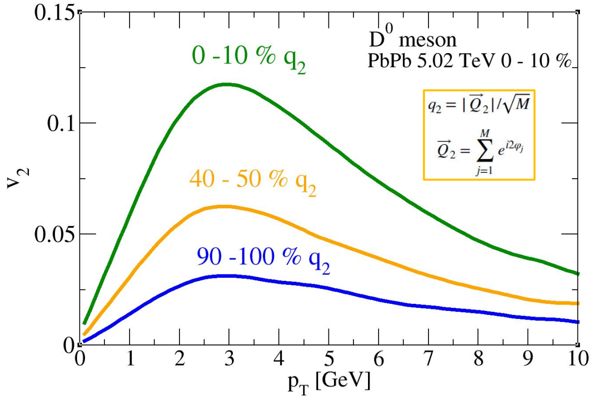
ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

- **Event-Shape Engeenering Technique:**

Prediction for similar correlation for hard particles wrt bulk

Predictions for D mesons correlations





M.L. Sambataro, et al., *Eur.Phys.J.C* 82 (2022)

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_q(x,p)+m(x)\partial_{\mu}^{x}m(x)\partial_{p}^{\mu}f_q(x,p)=C[f_q,f_g]$$

$$p^{\mu}\partial_{\mu}f_g(x,p)+m(x)\partial_{\mu}^{x}m(x)\partial_{p}^{\mu}f_g(x,p)=C[f_q,f_g]$$

Free-streaming

field interaction
$\varepsilon - 3p \neq 0$

Collision Integral gauged to reproduce viscous Hydro at fixed η/s by means of Chapman-Enskog

$$\sigma(n(\vec{x}),T)=\frac{1}{15}\frac{\langle p\rangle_0}{g(a)n(\vec{x})}\frac{1}{\eta/s}$$

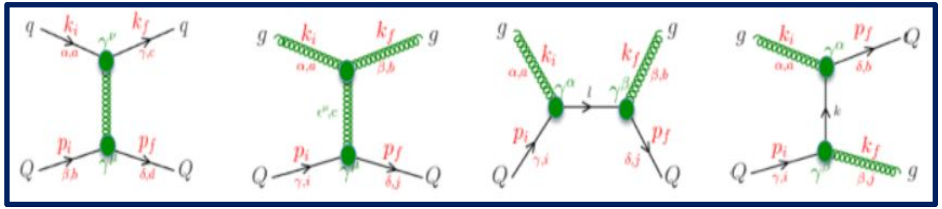
Equivalent to viscous hydro at $\eta/s \approx 0.1$

HQ evolution

$$p^{\mu}\partial_{\mu}f_Q(x,p)=C[f_q,f_g,f_Q]$$

$$C[f_q,f_g,f_Q]=\frac{1}{2E_1}\int\frac{d^3p_2}{2E_2(2\pi)^3}\int\frac{d^3p_1'}{2E_1'(2\pi)^3}$$
$$\times[f_Q(p_1')f_{q,g}(p_2')-f_Q(p_1)f_{q,g}(p_2)]$$
$$\times|M_{(q,g)\to Q}(p_1p_2\to p_1'p_2')|$$
$$\times(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$$

Feynmann diagrams at first order pQCD for HQs-bulk interaction:



Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

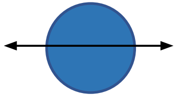
Event-Shape-Engineering (ESE) technique

Pb-Pb 5,02 TeV

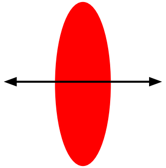
Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

$$q_2 = |\vec{Q}_2|/\sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^{M} e^{i2\varphi_j}$$



20 % small q_2

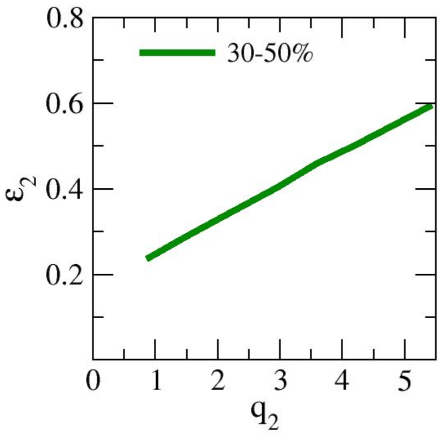


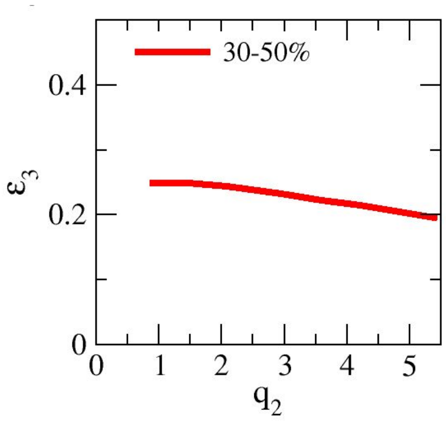
20 % large q_2

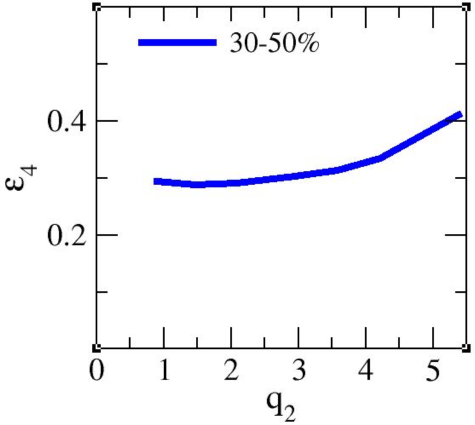
Large $q_2 \rightarrow$ large ε_2

$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_\perp^n \sin(n\varphi) \rangle}{\langle r_\perp^n \cos(n\varphi) \rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$







Anti-correlation between ε_2 and ε_3

Non-linear correlation between ε_2 and ε_4

M.L. Sambataro et al., *Eur.Phys.J.C* 82 (2022) 9, 833

Event-shape-engeenering technique

Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

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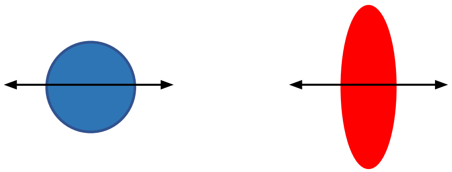
$$\vec{Q}_2 = \sum_{j=1}^{M} e^{i2\varphi_j}$$



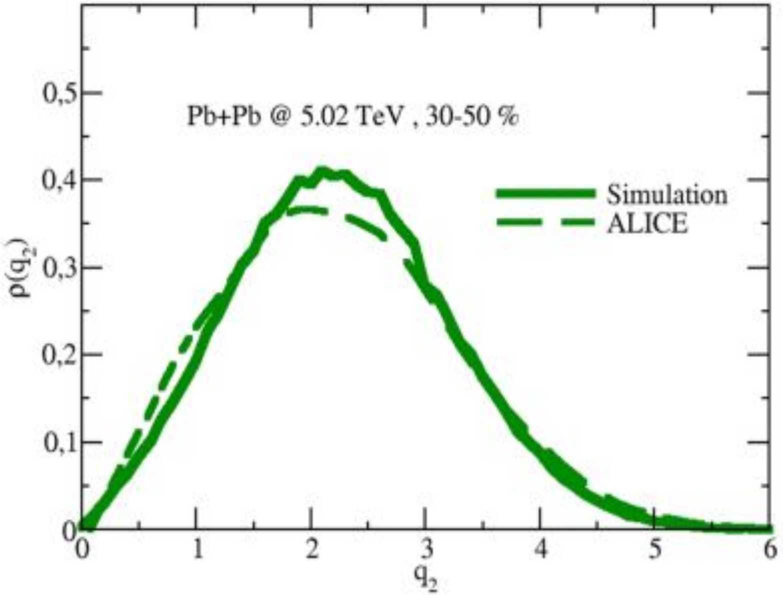
20 % small q_2 20 % large q_2

Large $q_2 \rightarrow$ large ε_2

$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_\perp^n \sin(n\varphi) \rangle}{\langle r_\perp^n \cos(n\varphi) \rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$

M.L. Sambataro et al., *Eur.Phys.J.C* 82 (2022) 9, 833



Discrepancy between selected v_2 and unbiased one ∼ 50%

Event-shape-engeenering technique

Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

$$q_2 = |\vec{Q}_2| / \sqrt{M}$$

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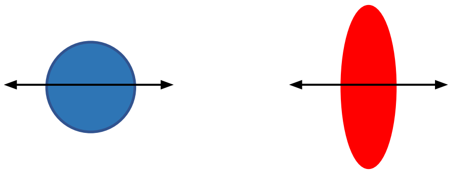


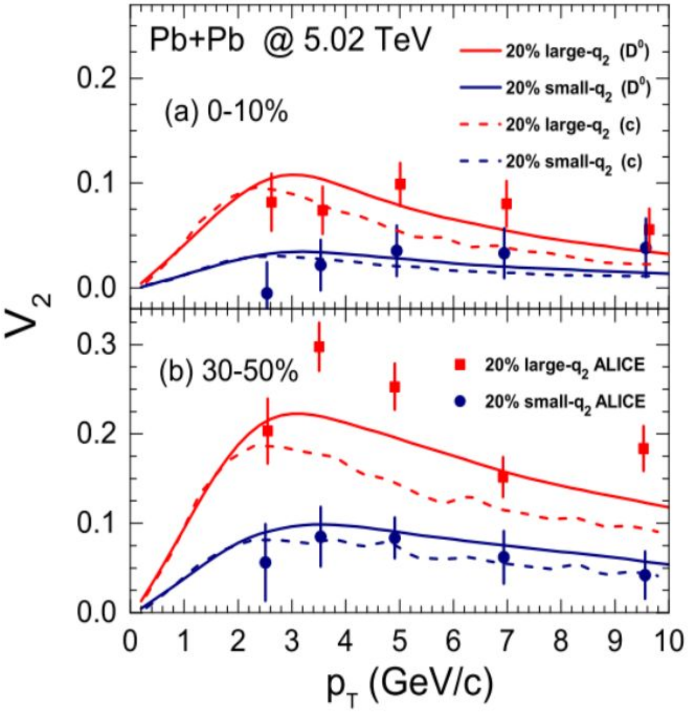
20 % small q_2 20 % large q_2

Large $q_2 \rightarrow$ large ε_2

$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_\perp^n \sin(n\varphi) \rangle}{\langle r_\perp^n \cos(n\varphi) \rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$

M.L. Sambataro et al., *Eur.Phys.J.C* 82 (2022) 9, 833

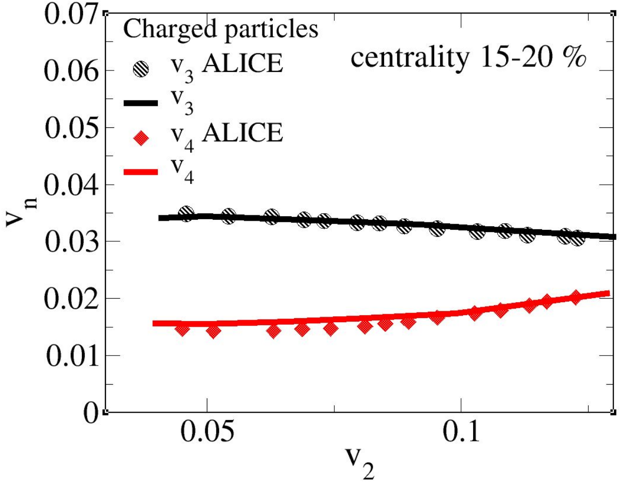


Discrepancy between selected v_2 and unbiased one ~ 50%

ESE: $v_n - v_m$ correlations

Charged particles

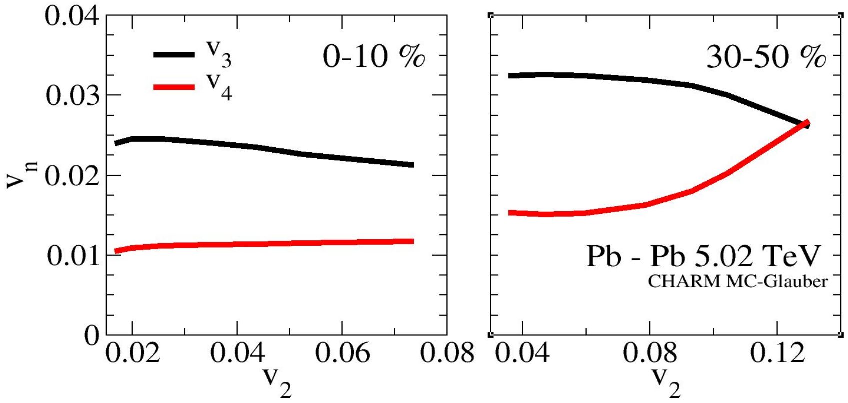
Pb-Pb 2,76 TeV



Correlations between the ε_n and ε_m present in the initial geometry → correlations between flow harmonics different orders, i.e. correlations v_n and v_m

M.L. Sambataro et al., *Eur.Phys.J.C* 82 (2022) 9, 833

Predictions for D mesons



→ Similar correlation between v_n of soft and hard particles.

Data taken from: S. Mohapatra *Nucl.Phys.A* 956 (2016) 59-66

Hybrid Hadronization Model for HQs

✔ **COALESCENCE**: Formula developed for the light sector **[Greco, Ko, Levai PRL 90 (2003)]**

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = g_H \int \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \, f_{q_i}(x_i, p_i) \, f_W(x_1 \ldots x_n; p_1 \ldots p_n) \, \delta\left(\boldsymbol{P_T} - \sum_i^n p_{T,i}\right)$$

Statistical Factor
Color-spin-isospin

Parton Distribution Functions
(after Boltzmann evolution)

Hadron Wigner Function
(parameters fix according to quark model)
C.-W. Hwang, EPJ C23, 585 (2002)
C. Albertus et al., NPA 740, 333 (2004)

✔ **FRAGMENTATION**: HQs that do not undergo to Coalescence

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = \sum_f \int dz \frac{dN_f}{d^2 p_T} \frac{D_{f\to H}(z)}{z^2}$$

We use Peterson parametrization: $D_H(z) \propto \left[z\left(1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}\right)^2\right]^{-1}$ *Peterson et al. PRD 27 (1983) 105*

Parameter ε_c tuned to reproduce *D* and *B* meson spectra in pp collisions.

Plumari, Minissale, Das , Coci , Greco, EPJ C 78 (2018) no.4

Non-perturbative effects: impact of off-shell dynamics QPM vs. DQPM

- Partons are dressed by non-perturbative spectral functions:

$$A_i^{BW}(m_i) = \frac{2}{\pi}\frac{m_i^2\gamma_i^*}{(m_i^2 - M_i^2)^2 + (m_i\gamma_i^*)^2}$$

$$C[f] = \int dm_i A(m_i) \int dm_f A(m_f) \times \frac{1}{2E_p}\int \frac{d^3\mathbf{q}}{2E_q(2\pi)^3}\int \frac{d^3\mathbf{q}'}{2E_{q'}(2\pi)^3}\int \frac{d^3\mathbf{p}'}{2E_{p'}(2\pi)^3} \times \frac{1}{\gamma_Q}\sum |\mathcal{M}_Q|^2 (2\pi)^4\delta^4(p+q-p'-q') \times [f(\mathbf{p}')\hat{f}(\mathbf{q}', m_f) - f(\mathbf{p})\hat{f}(\mathbf{q}, m_i)]$$

→ Evaluated in DQPM approach

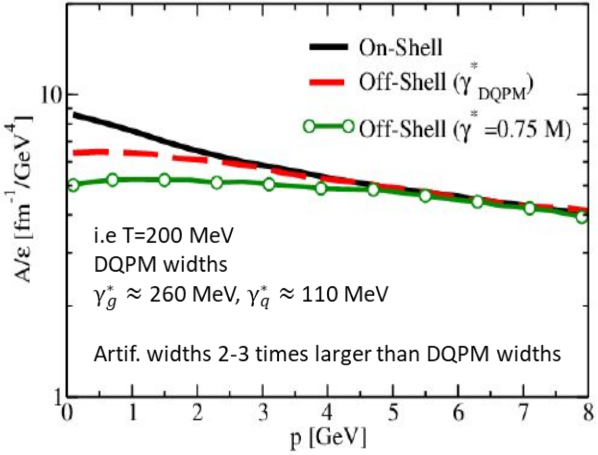
For references: W. Cassing, Nucl.Phys. A831, 215
E. Bratkovskaya, Nucl.Phys. A856, 162
H. Berrehrah, Phys. Rev. C 89(5), 054901
M.L. Sambataro et al., *Eur.Phys.J.C* 80 12, 1140

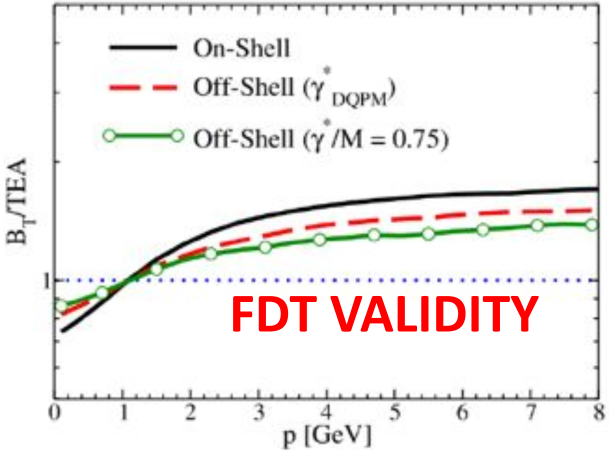
Off-shell≈ PHSD but also larger widths!

Bulk is not with the same energy density
The energy density of off-shell case is smaller

- Transport coefficient scales with energy density of the system ε
- Larger breaking for low p region ($p \lesssim$2-3 GeV/c)
  → **larger off-shell effects**
  →30-40% decreasing drag

BOX CALCULATION [T=200 MeV] FOR CHARM





QPM extended – QPMp and m_c (T)

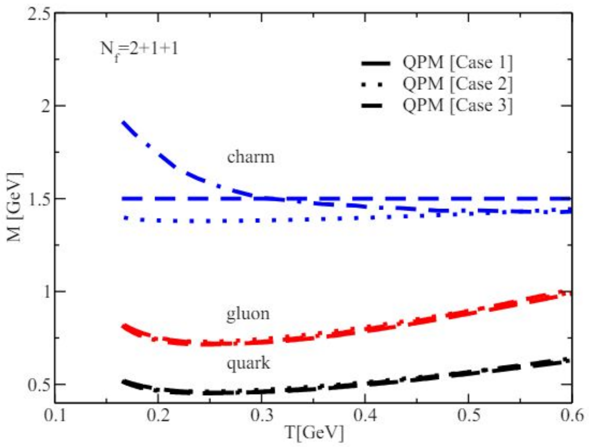
we have also extended our quasi-particle model approach for **Nf = 2+1** to **Nf = 2 + 1 + 1** where the **charm quark is included**

Temperature parametrization for charm mass

Case 1: $m_c = 1.5\ GeV$

Case 2: $m_c^2 = m_{c0}^2 + \frac{N_c^2 - 1}{8N_c} g^2 [T^2 + \frac{\mu_c^2}{\pi^2}]$

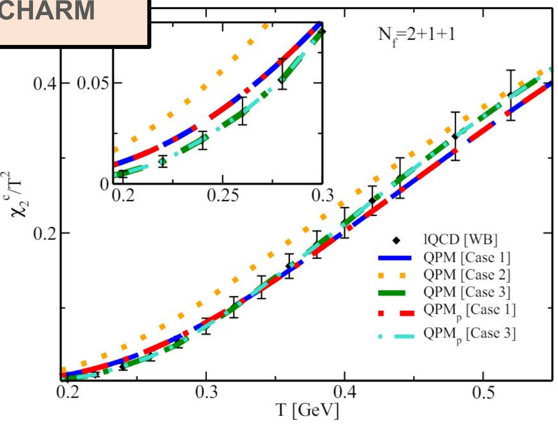
Case 3: m_c fixed by $\chi_2^c = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_i^2}$



QUARK SUSCEPTIBILITIES

$$\chi_{u,s,c} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_{u,s,c}^2}$$

CHARM



- lQCD data overestimated for T≈0.2-0.3 GeV with constant m_C.

- **Disfavored**: increasing m_C(T) and m_C smaller than 1.5 GeV

- Susceptibility implies a decreasing m_C(T) from 1.9 at T_C down to 1.5 at T_C .

M.L. Sambataro et al. e-Print: 2404.17459