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Calculation of the Cosmic Trajectories over the QCD phase transition

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with: Francesco Di Clemente, Geetika Yadav, Alessandro Drago, Volodymyr Vovchenko, and Claudia Ratti

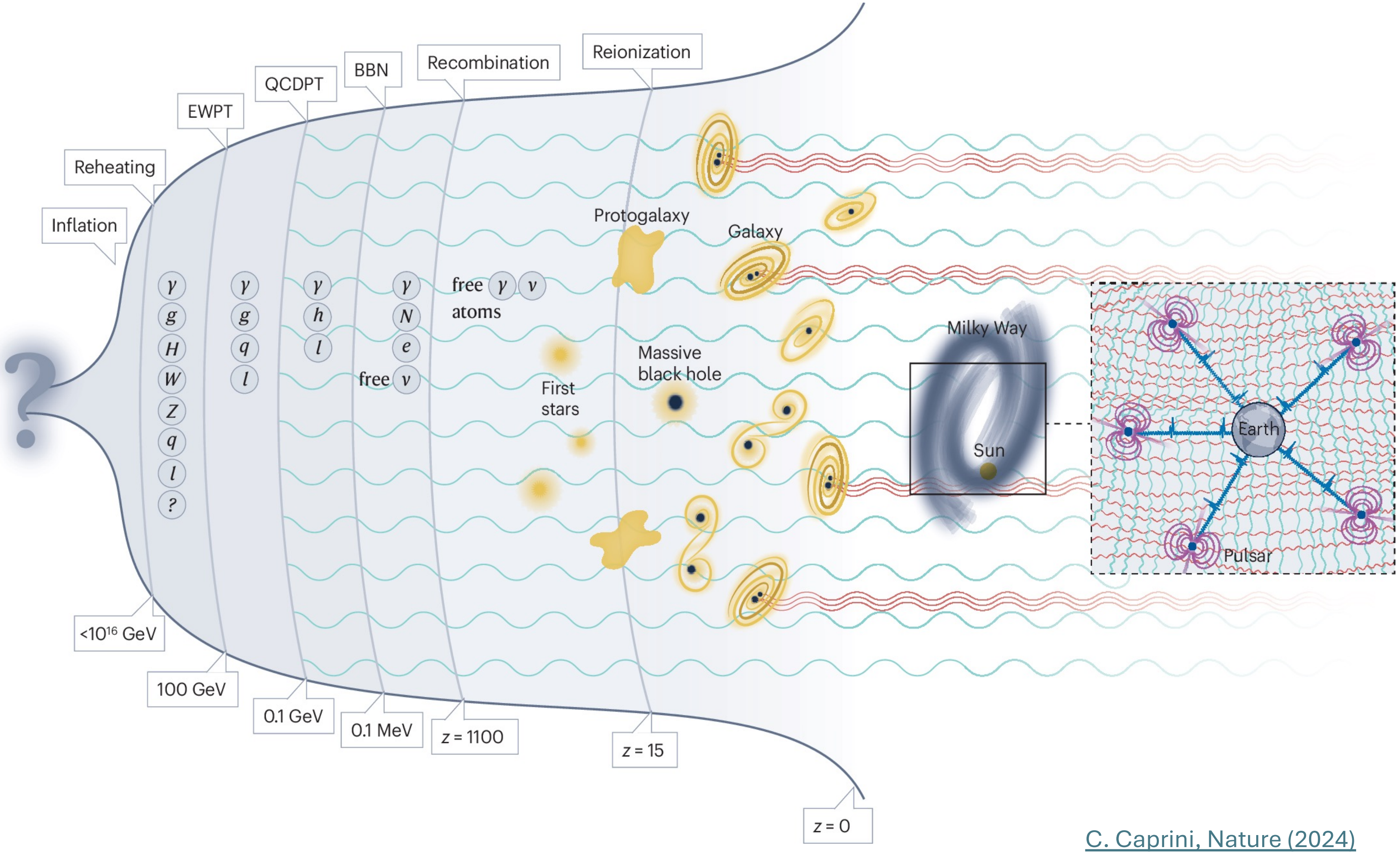
Based on:

- L.F. et al. PRD (2026)
- F. Di Clemente, L.F. et al. eprint 2511.11995 (2025)

Wed 25/03/2026

Cosmological QCD Phase Transition

- QCD Phase Transition (PT) was **thought to happen at 0 μB**
- Recent results show that it is **not necessarily true**!
- The questions we want to answer are:
    - What is the nature of the cosmological **QCD PT**?
    - Was the **gravitational wave background** produced during this epoch?
    - Could **dark matter** have been produced *à la* Witten during the QCD phase transition?



C. Caprini, Nature (2024)

Gravitational Waves Background

From the observation of Pulsar Timing Array (PTA) we measure a **stochastic gravitational–wave (GW) background** in the nHz frequencies.

- The possible sources are:
  - Binary supermassive black-holes
  - Cosmological first order phase transition

S. Pascoli et. al., eprint 2602.02829 (2026); NANO*Grav collab., AJL (2023); W. Ratzinger, P. Schwaller SciPost Phys. (2021); A. Kosowsky et al., PRL (1992)*

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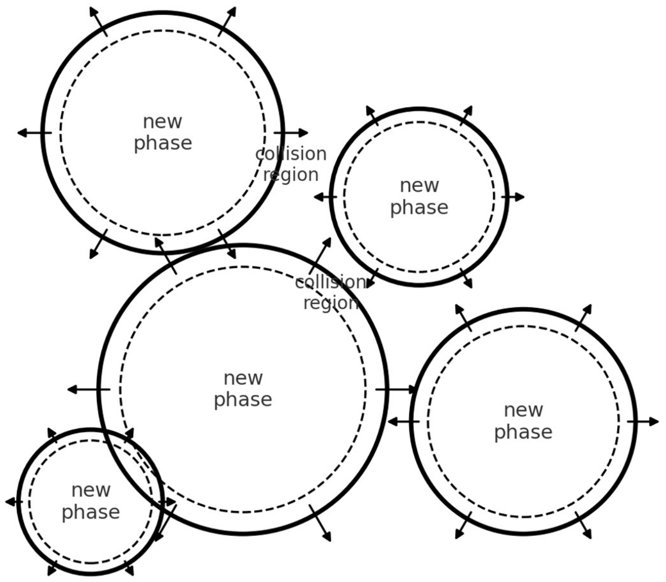

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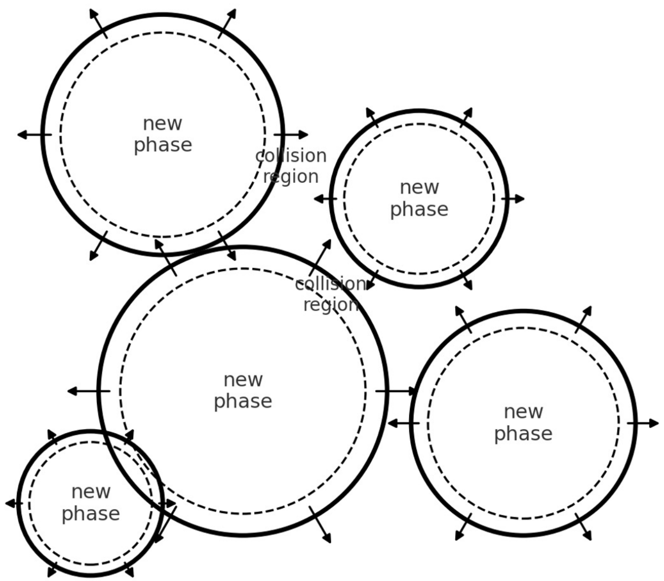
Peak frequency:

$$f_p \approx 1.65\times10^{-7} Hz \left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{GeV}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

$$T_* \approx 10^{-3} - 10^{-2} GeV$$

$$T_* \sim T_n$$



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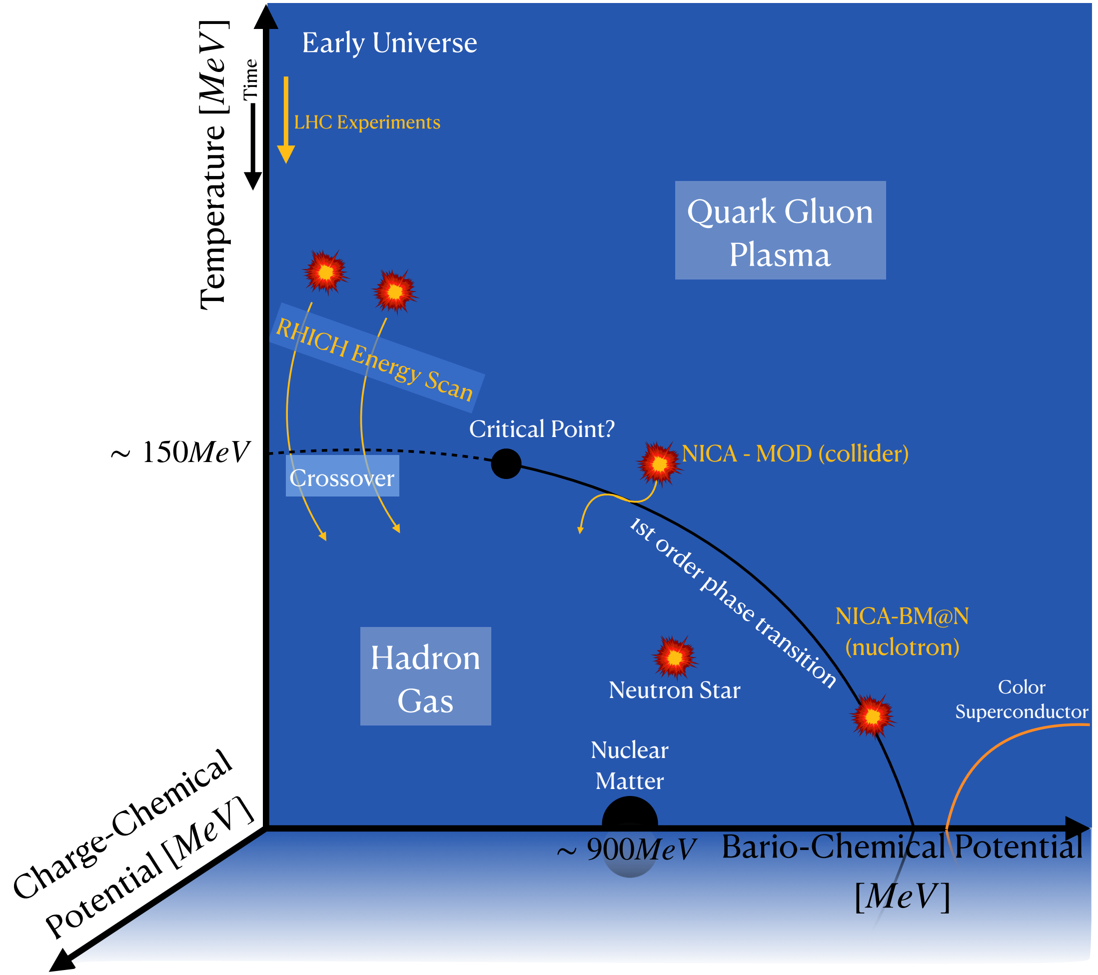
Why cosmic trajectories?

- We need to define cosmologically how the universe **transitioned from the QGP to the Hadronic phase**
- At any moment of its evolution, the Universe is in a specific position on the phase diagram
- All these points together over time form the **"Cosmic trajectory"**
- The position of the trajectory is **defined by the nature of the phase transition**
- Once the free parameters are fixed, **the trajectory is unique**!



QCD Phase diagram in Temperature, Baryon and electric charge chemical potentials

D. J. Schwartz, M. Stuke, IOPScience (2009); M. M. Wygas et al., PRL (2018)); M. M. Wygas et al., PRD (2020); F. Gao et al. PRL (2021); F. Di Clemente, L.F. et al. E-Print (2025); L. Formaggio et al., PRD (2026)

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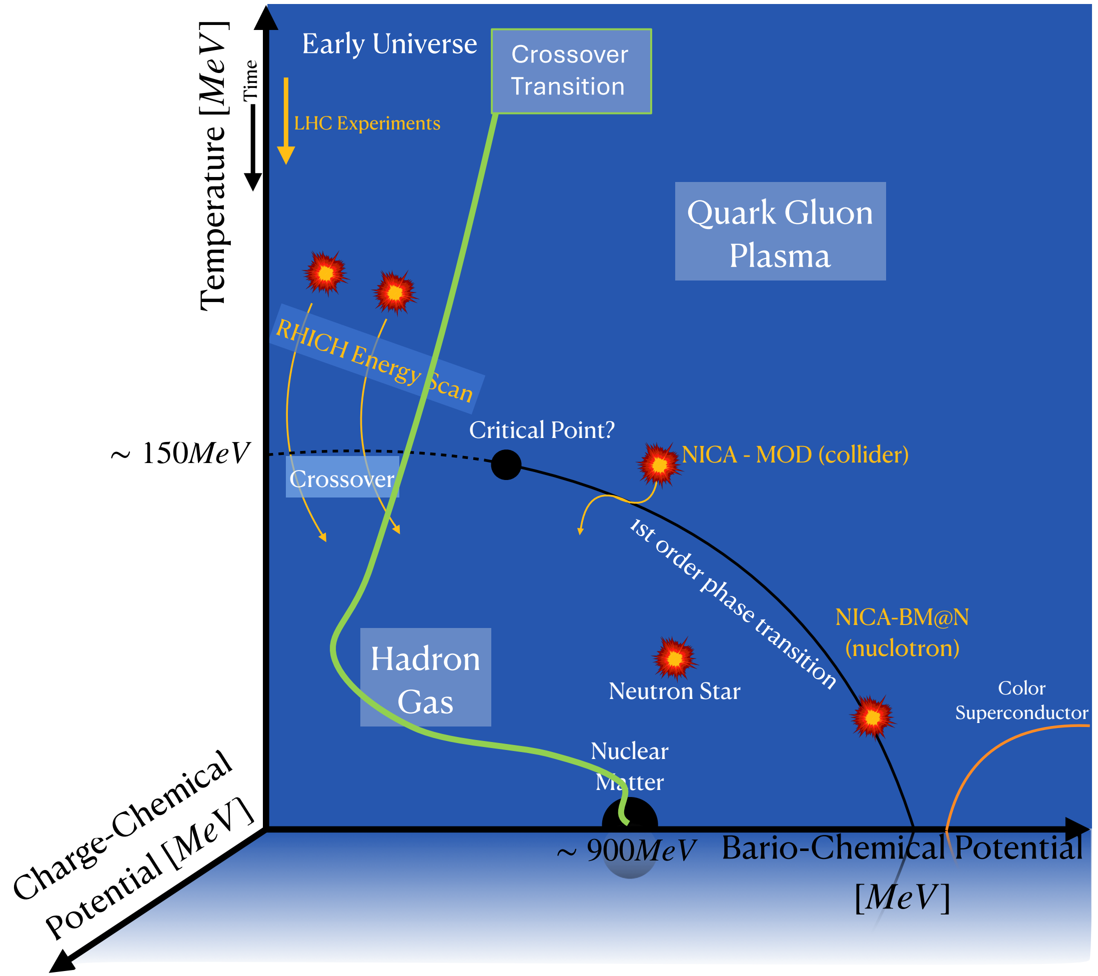

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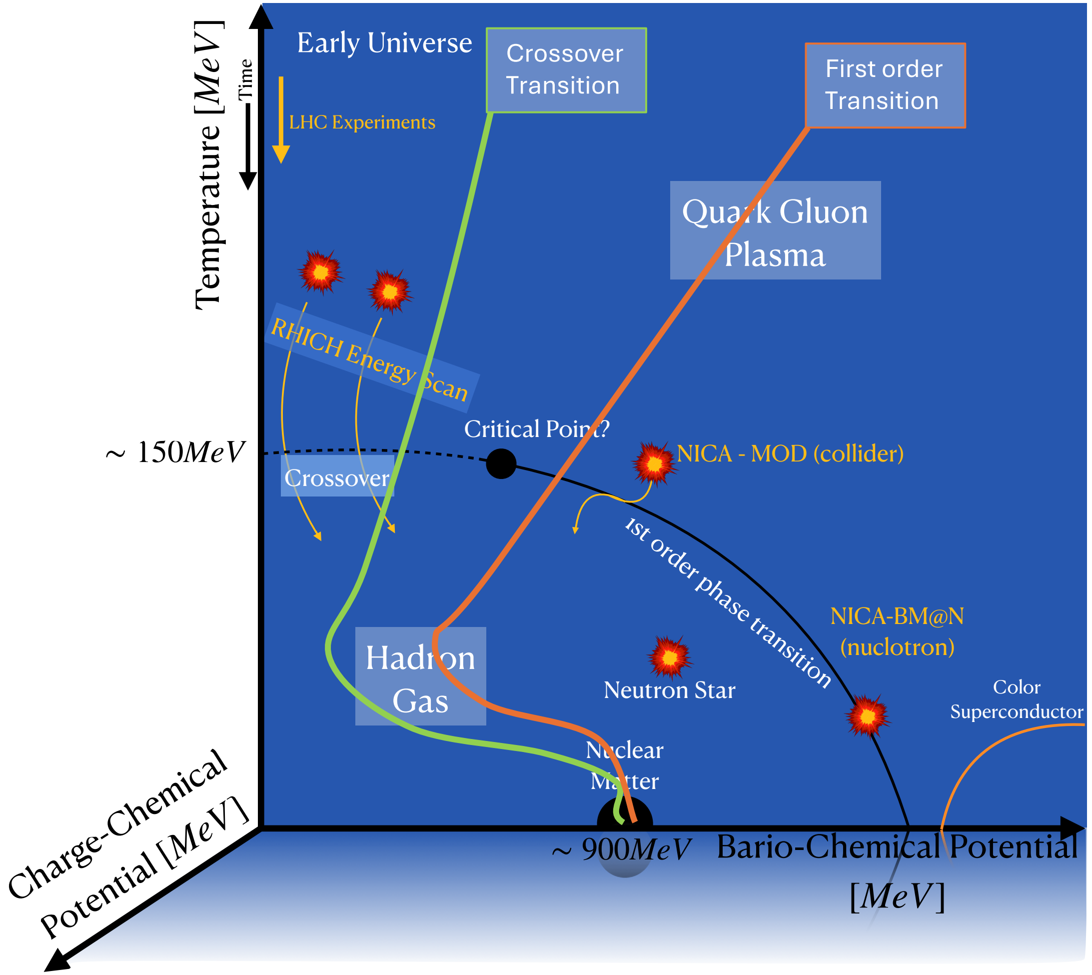

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Numerical Method

To study how the QCD phase transition occurs, we need to calculate the Cosmic Trajectory in the plane of the Early Universe. This involves:

- **Defining a system of equations which depends on the chemical potentials and on the temperature**
- Integrating the EoS into this system
- Solving the system at fixed values of the temperature to calculate the cosmic trajectory by points

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Conservation equations

- Baryogenesis $T_b \simeq 10\ TeV$ → **Fix the baryon asymmetry $\boldsymbol{b}$**
- Electro-Weak phase transition $T_{EW} \simeq 160\ GeV$ → **Fix the three lepton asymmetries $\boldsymbol{l_i}$**
- Neutrino oscillations $T_{osc} \simeq 10\ MeV$ → CP violation processes become efficient
- QCD phase transition $T_{QCD} \simeq 150\ MeV$
- **Charge Neutrality** of the Universe

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Where:

- $b = 8.6 \cdot 10^{-11}$
- $\mathrm{B_i}$: Baryon fraction of species i
- $\mathrm{Q_i}$: Charge fraction of species i
- $\mathrm{\bar{n}_i = n_i(T, \mu) - n_i(T, -\mu)}$: Net number density of species i
- s : total entropy density of the Universe

The three lepton asymmetries l_i are the free parameters of the system

Lepton asymmetries

The main constraints come from the **Big Bang Nucleosynthesis** (BBN) and the **Cosmic Microwave Background** (CMB)

- **BBN:** $|\boldsymbol{l_e}| \leq \mathbf{10^{-3}}$. l_e defines ratio $\frac{n_p}{n_n}$ (strongest constraint) measured at the BBN epoch
- **CMB:** $|\mathbf{l}| \leq \mathbf{10^{-2}}$. Each l_i introduces an extra contribution in the effective density of the universe changing the rate expansion of the universe
- **Neutrino Oscillations:** $\boldsymbol{sin(\theta_{13})}$ of the PMNS matrix defines how the lepton asymmetries are related to each other.

We investigate two principal scenarios:

Symmetric configuration: $\boldsymbol{l_e = l_\mu = l_\tau = \frac{l}{3}}$

Asymmetric configuration:

- electrophobic $\boldsymbol{l_e = 0}$ $\boldsymbol{l_\mu = -l_\tau}$
- Tauphobic $\boldsymbol{l_\mu = -l_e}$ $\boldsymbol{l_\tau = 0}$

G. Mangano et al., PLB (2012)

Free Quark Gluon Plasma Model

Particle		Mass	Spin	g contribution
Quarks	t b	173 GeV 4 GeV	1/2	2x3=6
	c	1 GeV		
	s d u	100MeV 5MeV 2MeV		
Leptons	e μ τ	0.5 MeV 106 MeV 1777MeV	1/2	2
	neutrinos	0 MeV	1/2	1
Gluons	g	0 MeV	1	8x2
Photon	γ	0 MeV	2	2
Vector Boson	W Z	80 GeV 91 GeV	1	3
Higgs Boson	H	125GeV	0	1

The full equation's form is:

$$s = s_{lep} + s_{\gamma} + s_{QCD}$$

$$b = \frac{1}{3}\frac{(\bar{n}_{up} + \bar{n}_{down} + \bar{n}_{strange} + \bar{n}_{charm})}{s}$$

$$0 = -\bar{n}_e - \bar{n}_\mu - \bar{n}_\tau + \frac{2}{3}\bar{n}_{up} - \frac{1}{3}\bar{n}_{down} + \frac{2}{3}\bar{n}_{up} - \frac{1}{3}\bar{n}_{down}$$

$$l_i = \frac{\bar{n}_i}{s} + \frac{\bar{n}_{\nu_i}}{s}$$

- The first attempt to solve the system is to describe each particle as a free particle described by its **proper quantum distribution** (bosons and fermions)

QCD Equation of State

2+1 & 2+1+1 flavors

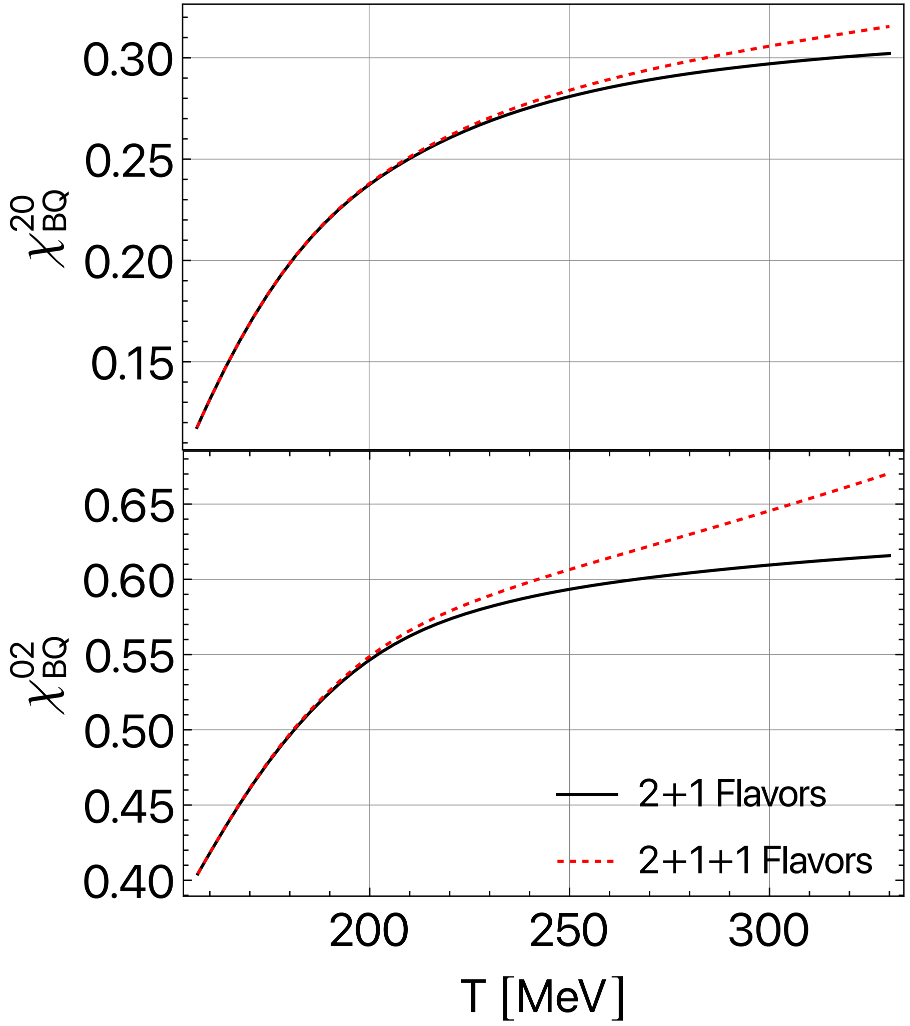
We use a 2+1 flavor lattice QCD-based EoS to describe the QCD contribution. Leptons and photons are still described by their free quantum particle distributions.

$$\frac{p\left(T,\mu_B,\mu_Q,\mu_S,\mu_C\right)}{T^4}=\sum_{i,j,k,m}^{i+j+k+m\leq 4}\frac{1}{i!j!k!m!}\chi_{ijkm}^{BQSC}\left(\frac{\mu_B}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k\left(\frac{\mu_C}{T}\right)^m$$

For $i+j+k\leq 4$

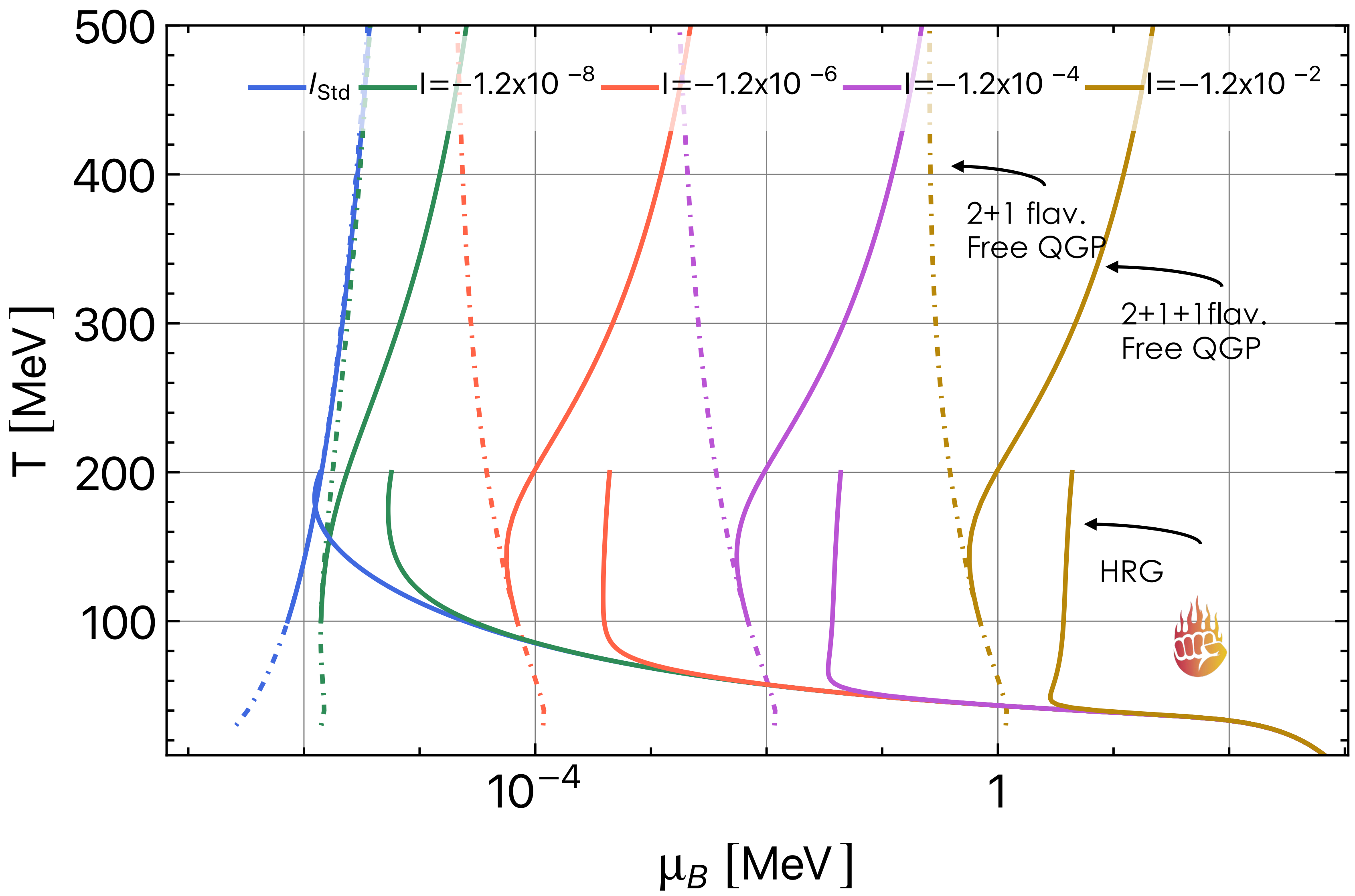
- At low temperatures, this EoS describes the **HRG model**
- For $110MeV < T < 330MeV$, it is fitted over **the Lattice QCD results**
- At high temperature, is imposed the convergence to the **Stephan-Boltzmann limit**

Due to beta equilibrium $\mu_S = 0$ $\mu_C = 0$.



J. Noronha-Hostler et al., PRC (2019); A. Abuali et al., PRD (2025)   Charm data provided by P. Petreczky A. Bazavov et al., PLB (2024)

Free QGP: Symmetric case

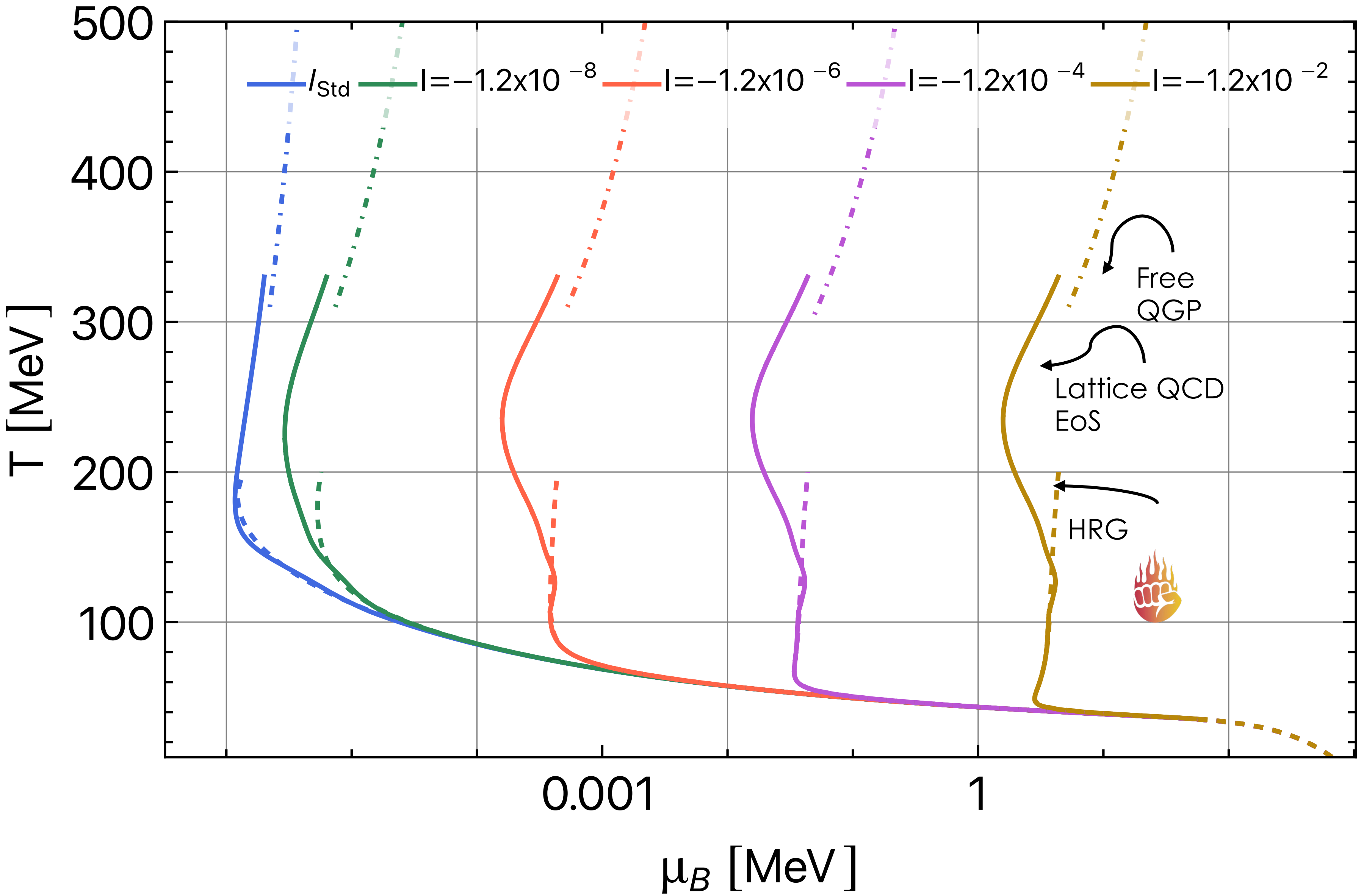


Cosmic trajectories computed for the **symmetric lepton-asymmetry configuration**.

- **Lower solid:** HRG model (Thermal Fist)
- **Upper solid**: 2 + 1 + 1flavor free QGP
- **Dot-dashed**: 2+1 flavor free QGP

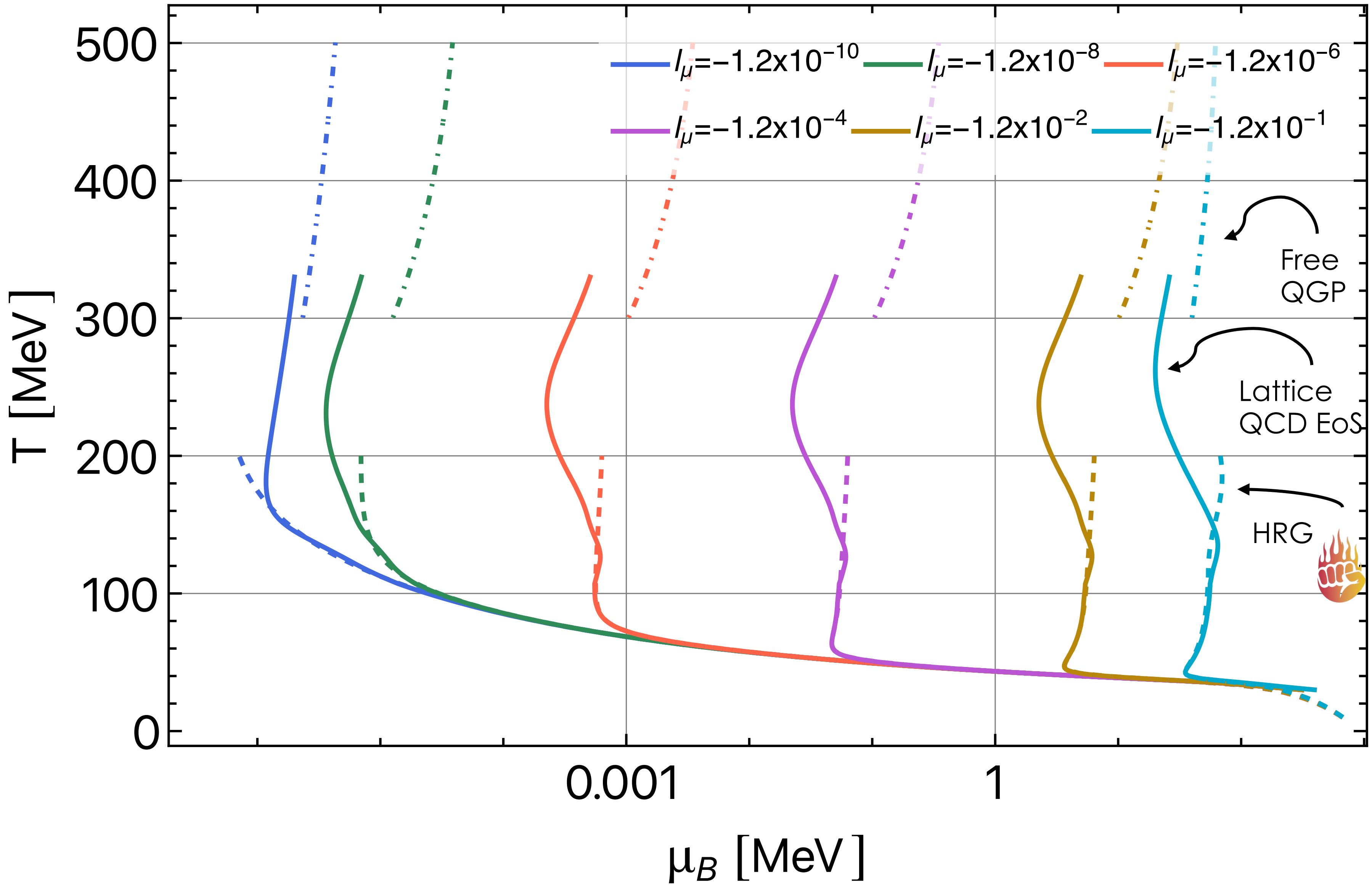
L.F. et al. PRD (2026); Thermal Fist software: V. Vovchenko CPC (2019)

Lattice QCD EoS: 2+1+1 flavors, symmetric case



L.F. et al. PRD (2026)

Lattice QCD EoS: 2+1+1 flavors, asymmetric case



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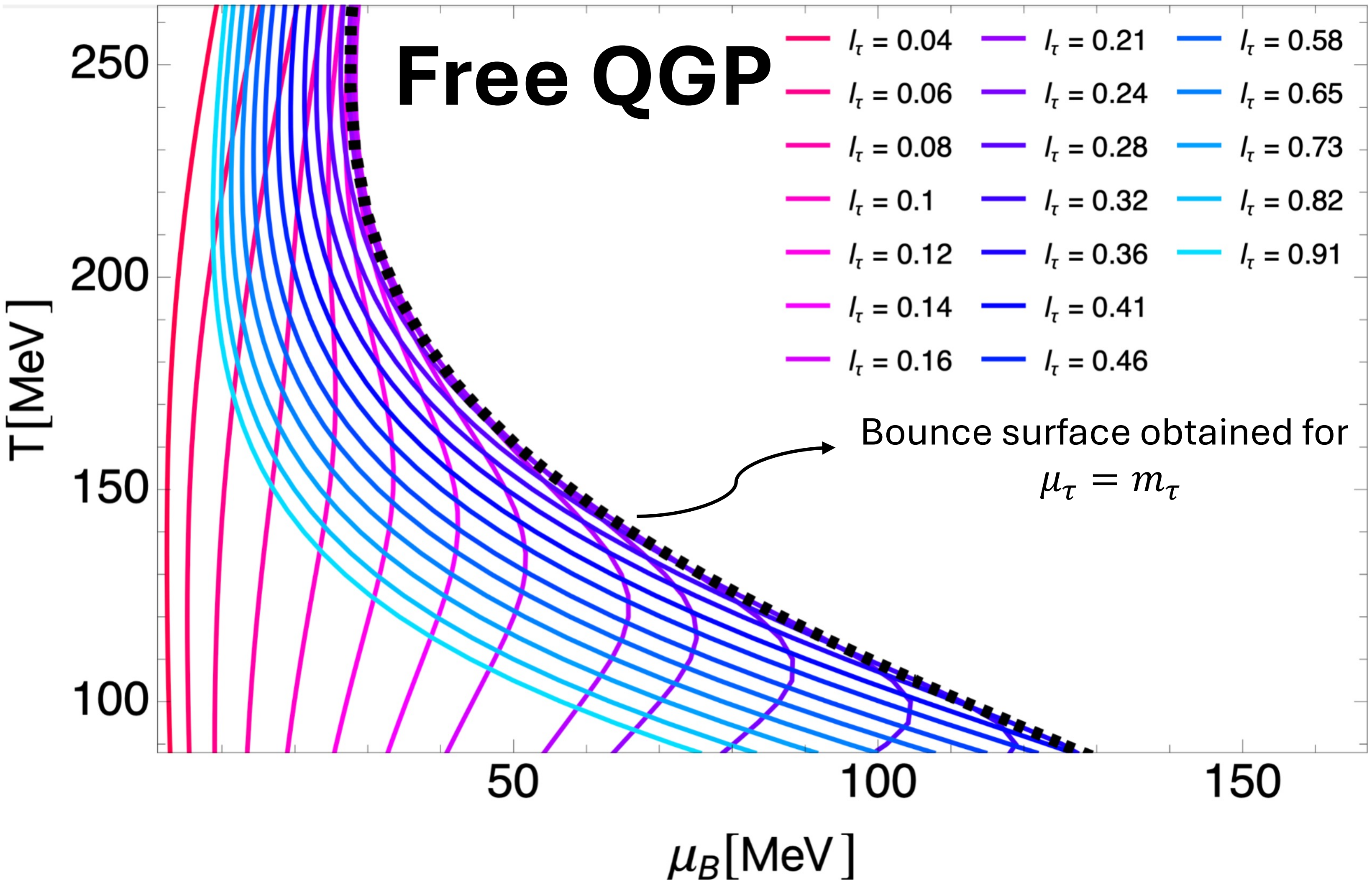
$$0.1 \leq |l_i| \leq 1$$

e-phobic scenario – results



$$l_\tau = -l_\mu, l_e = 0$$

- For **large l_τ**, μ_B decreases: τ^- and μ^+ densities are so large that their **cancellation is more effective**

↓

charge neutrality is handled by **leptons** and the **quark sector does not need to increase μ_B**

Same behavior observed for the HRG and the Lattice QCD EoS!

F. Di Clemente, L.F. et al. eprint 2511.11995 (2025)

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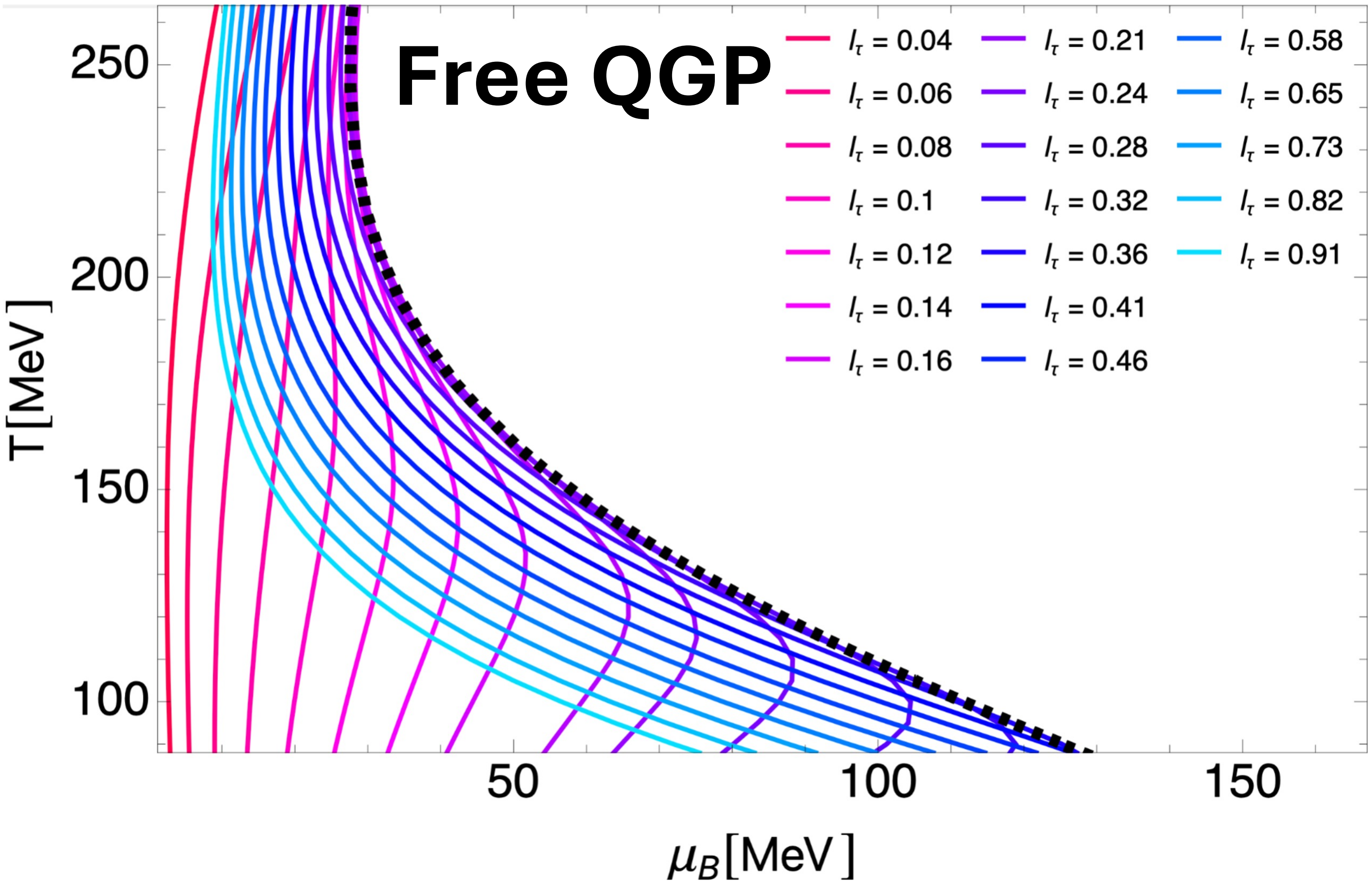

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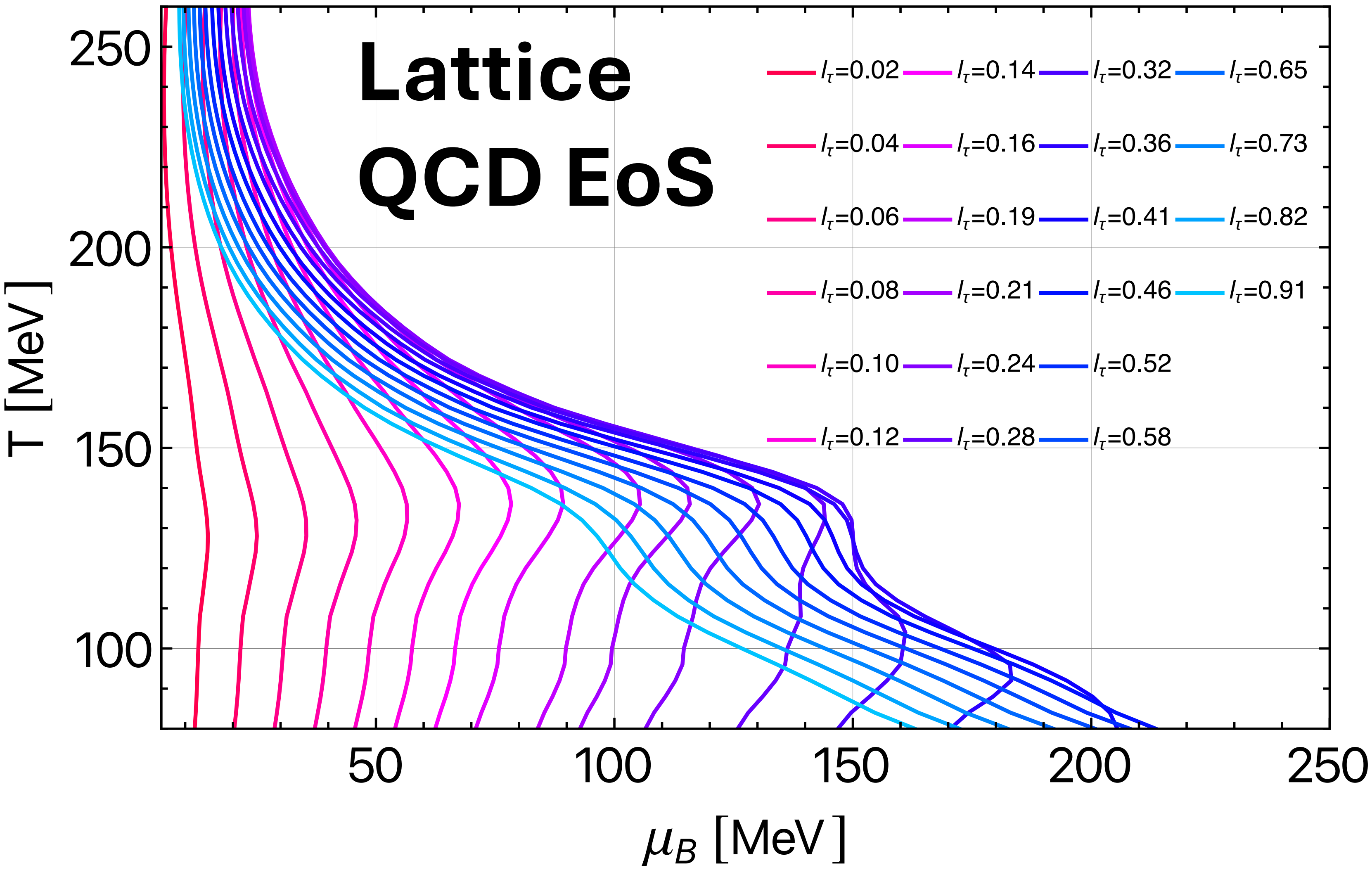

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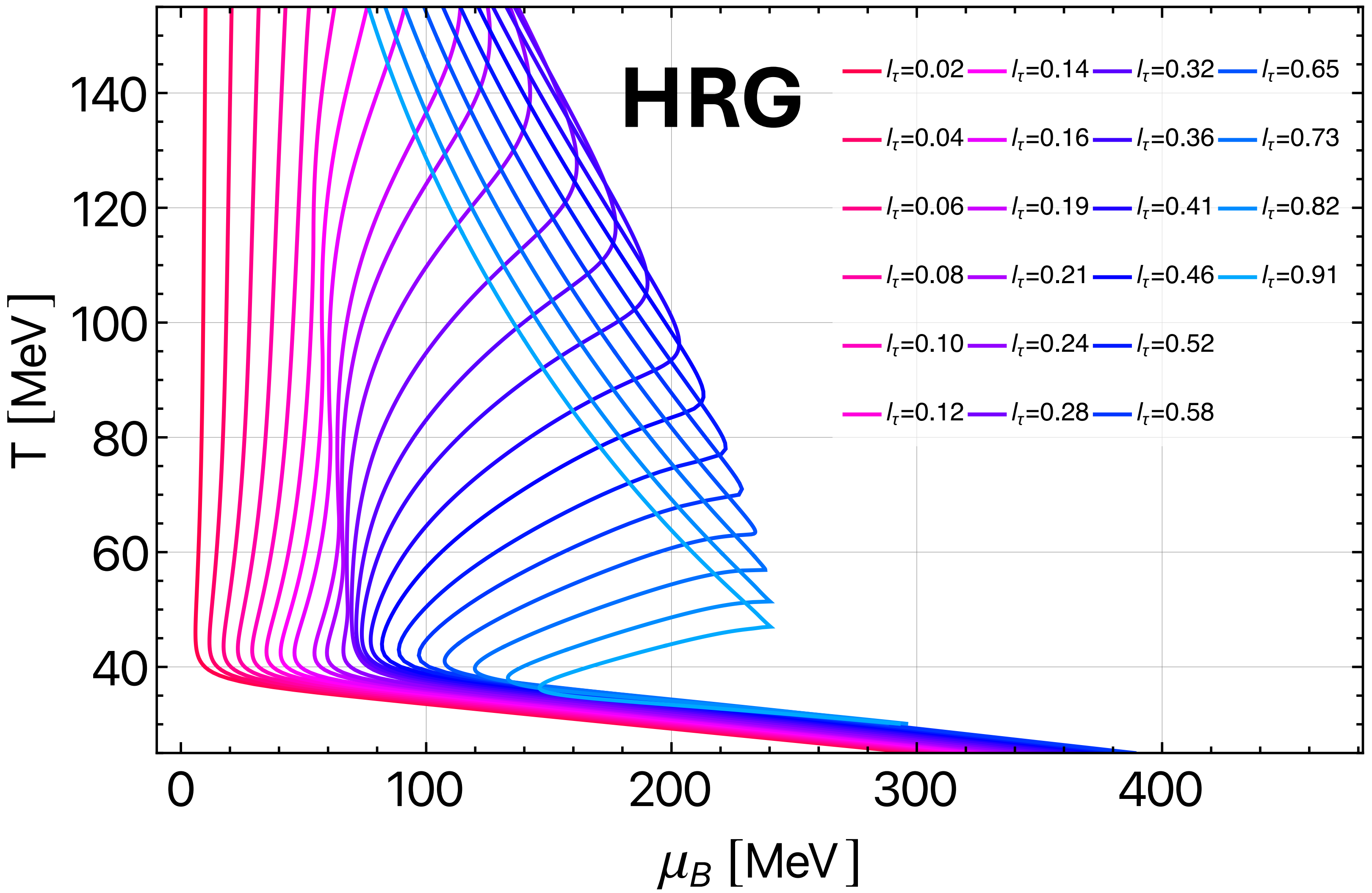

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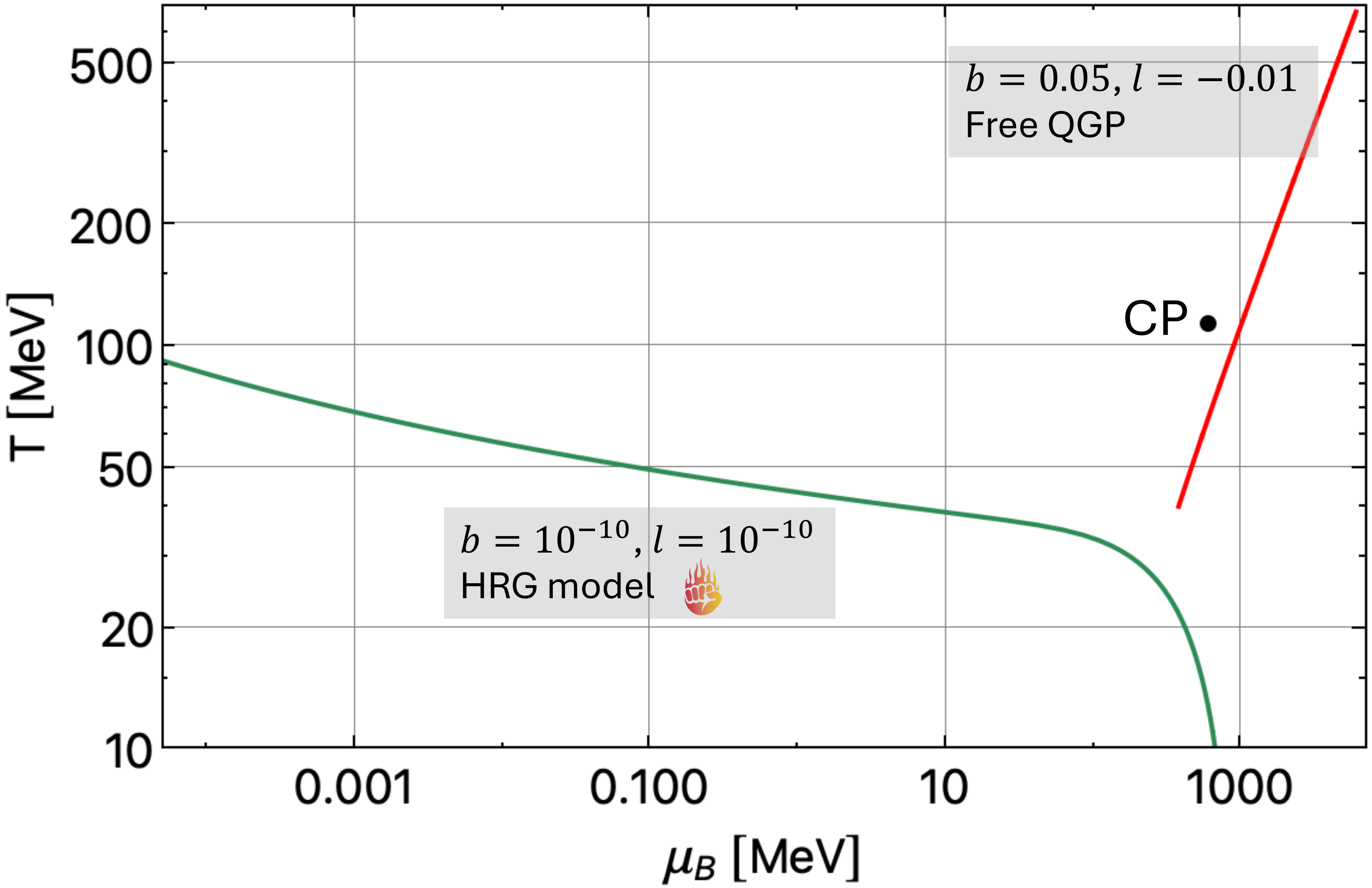
The only possibility for a FOPT: Little inflation

(a sketch)

Little inflation: how to use high baryon asymmetries



Critical point at
$(T, \mu_B) \simeq (114{,}602) MeV$
H. Shah et al., eprint (2024)

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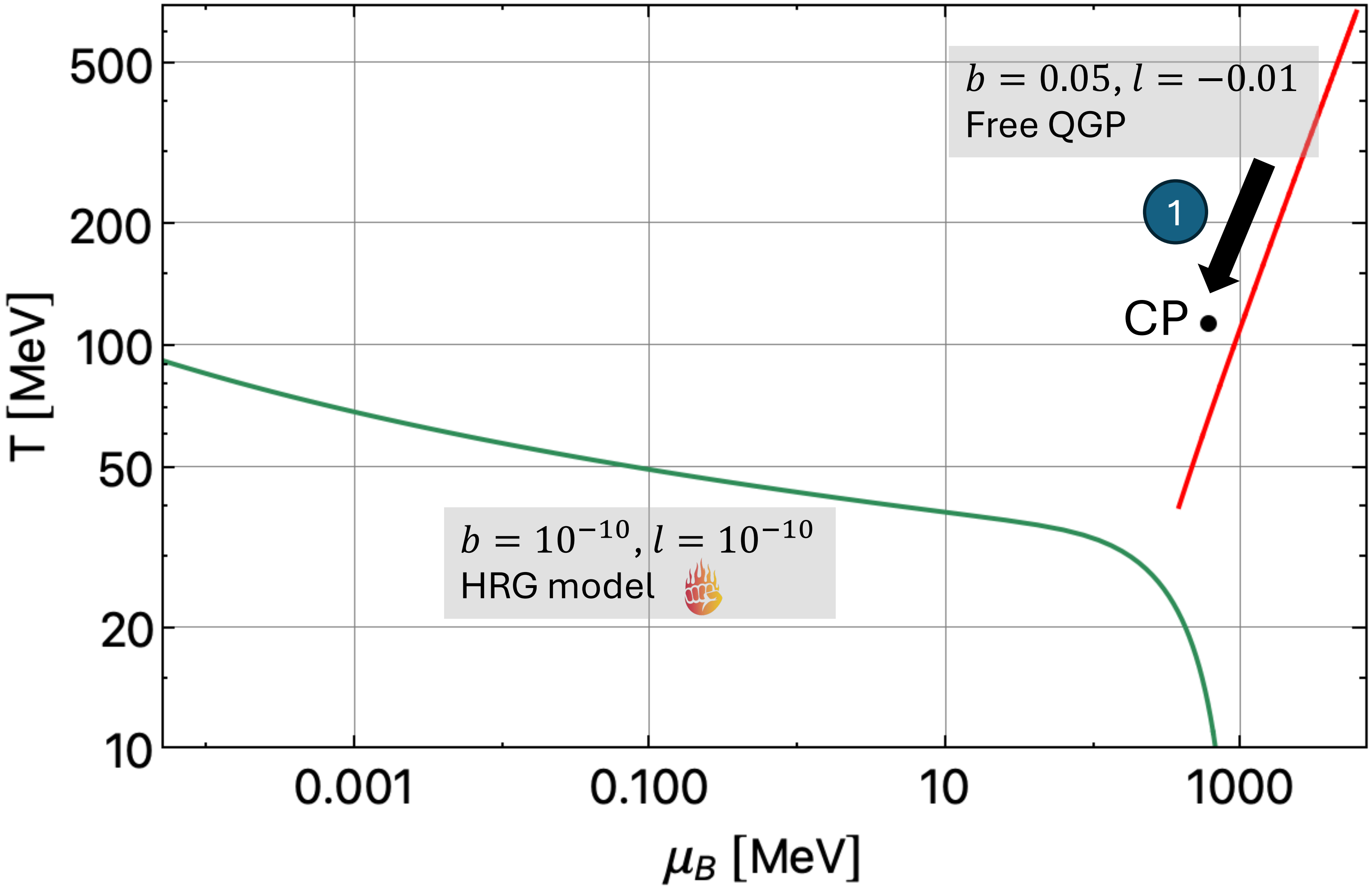

1) Beginning with **high $b \simeq 0.05$.** The universe undercools until

$$\frac{\Gamma}{H} \sim 1$$

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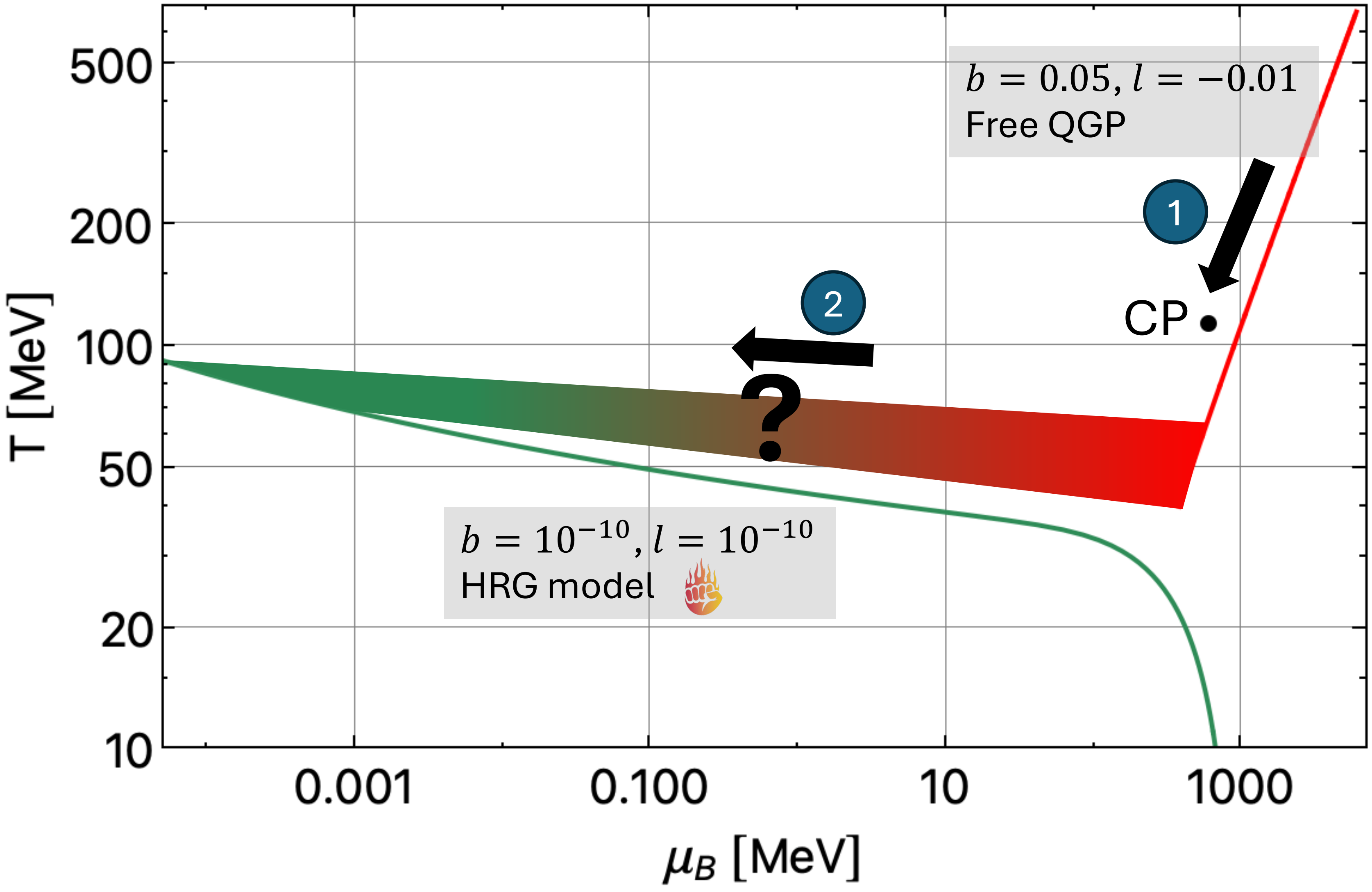

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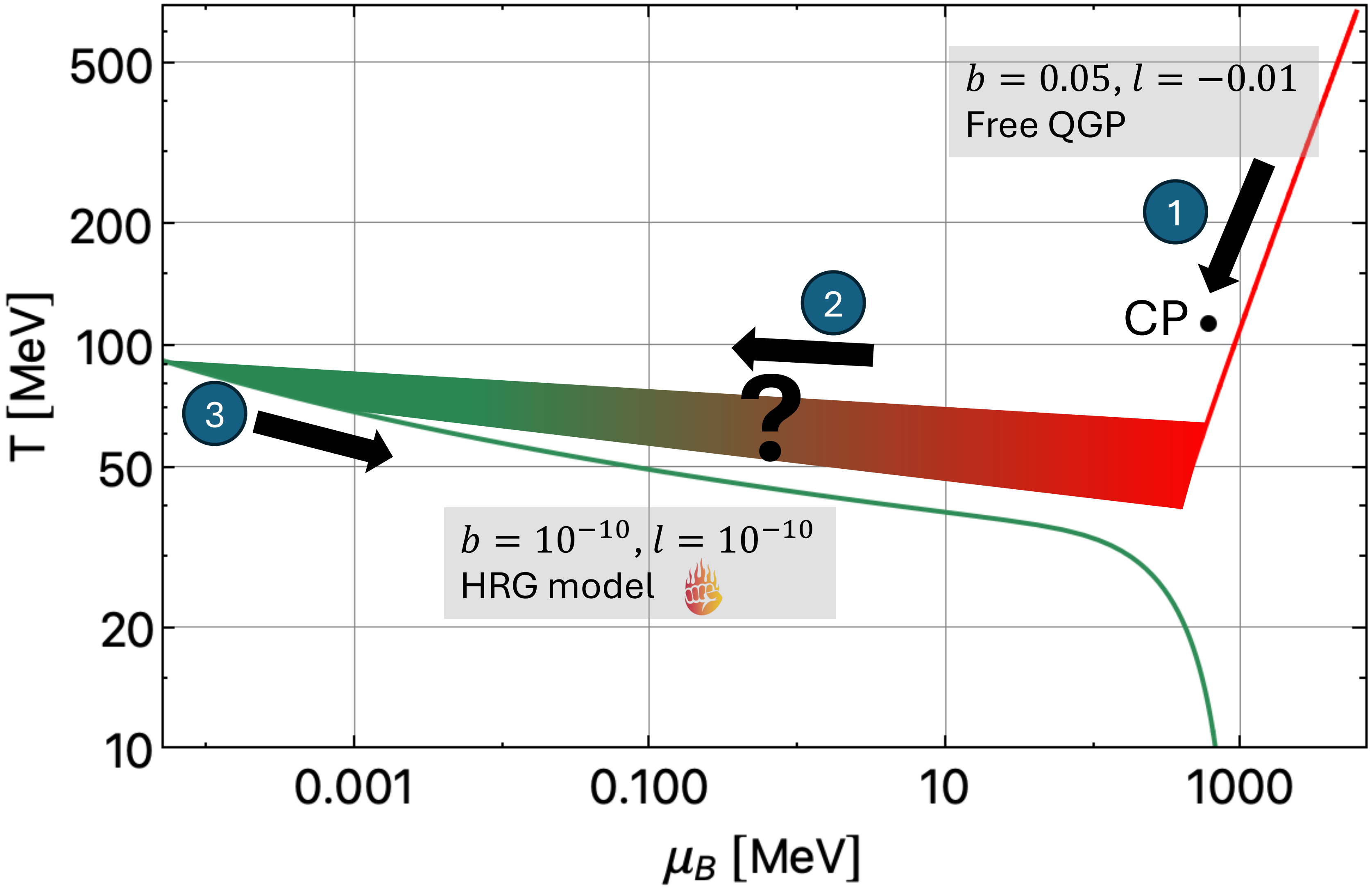

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3) The trajectory now is defined by the hadronic phase

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Conclusions & Further Work

- We studied **how the lepton asymmetries affect the cosmological QCD PT** using a 2+1+1 flavors Lattice QCD EoS
- For standard cosmological scenario **the QCD PT is a crossover**
- We observe an **upper bounds for the value of the cosmological baryon chemical potential**

Further works and investigation:

- Code will be open access via the MUSES calculation engine
- Investigate how the system behaves with a CP at $\mu_Q \neq 0$
- **Little inflation**

muses

THANKS!