Improved Description of Hot and Dense Hadronic Matter in an Extended Chiral Mean Field Model with Medium-Modified Mesons

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On behalf of **Rajesh Kumar**

Collaborator: J. Grefa, K. Maslov, Y. Wang, A. Kumar, R. Rapp, C. Ratti, and V. Dexheimer

muses

NP3M

Center for Nuclear Research

Strangeness in Quark Matter

Los Angeles, CA

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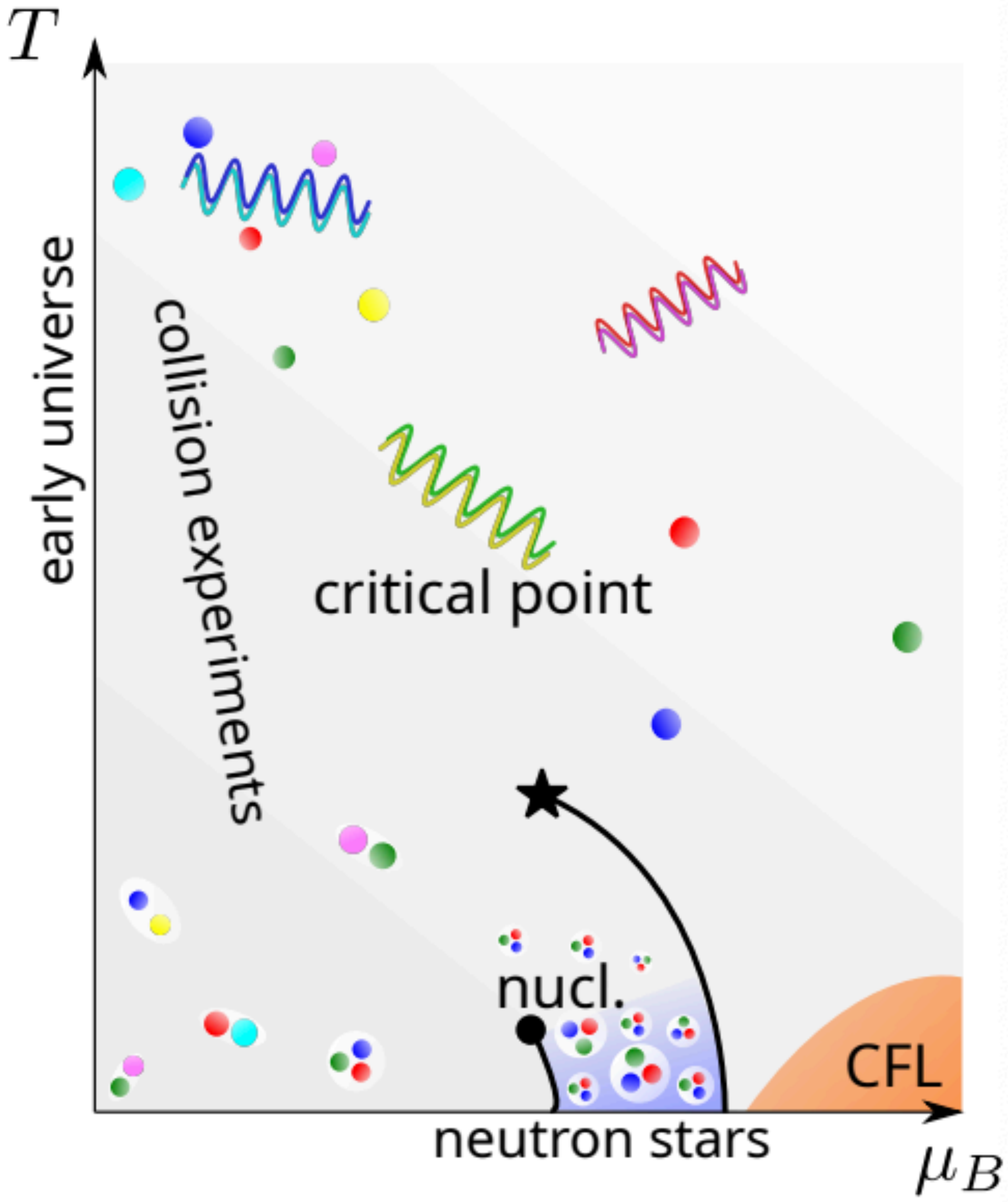
QCD Phase Diagram

- Lattice QCD is reliable at $\mu_B = 0$
- Extrapolations are limited to $\mu_B \sim 3.5T$
  Talk by A. Abuali
- At finite baryon density (relevant for neutron stars and mergers), we rely on **effective models**



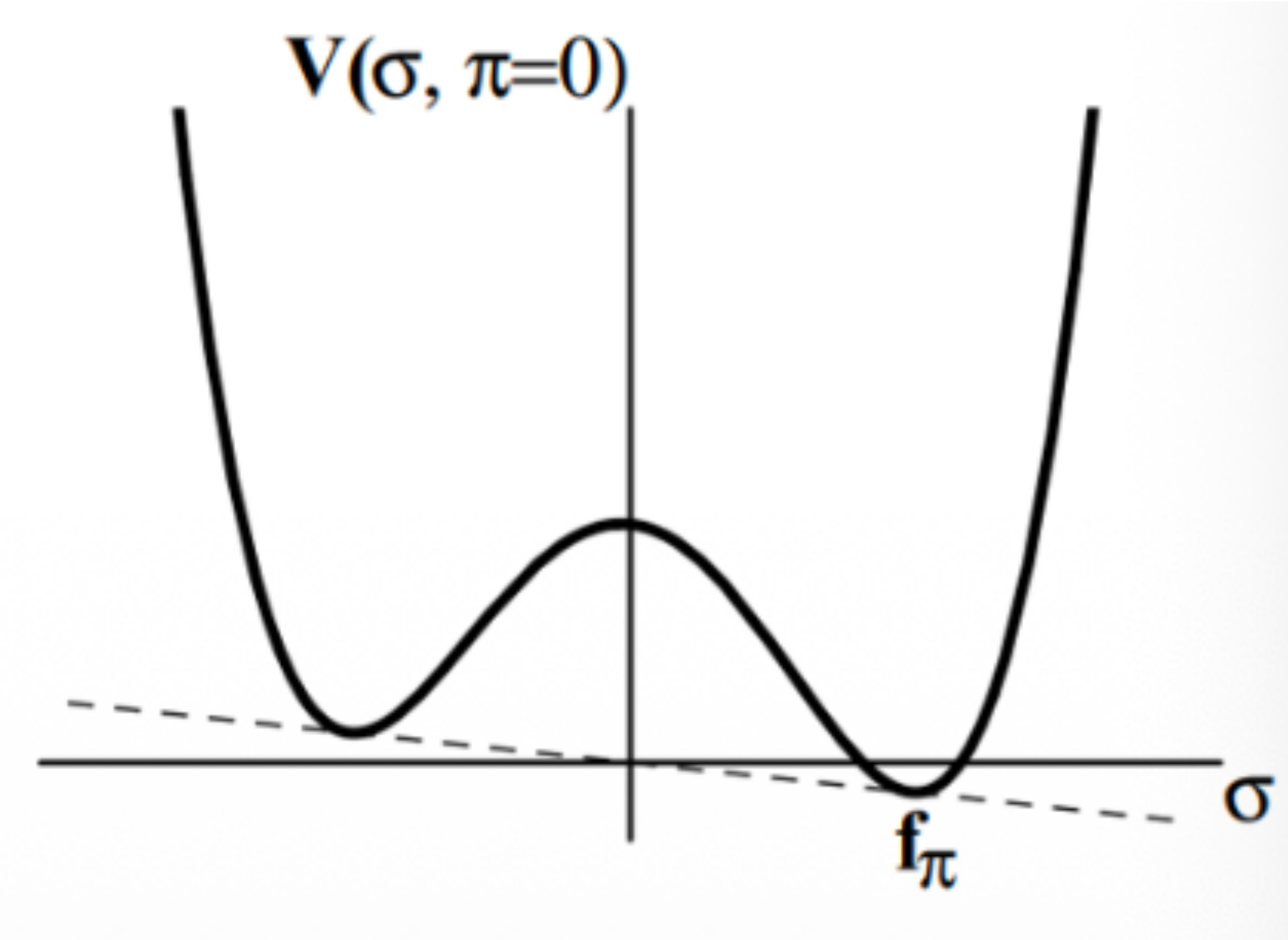
Effective Models for Nuclear Matter

Effective Models for Nuclear Matter

Interpret data ⟷ make predictions

Requirements:

- Chiral symmetry
- Broken scale invariance
- Nuclear matter degrees of freedom and interactions
- Constrained by first-principles results and/or experiments/observations



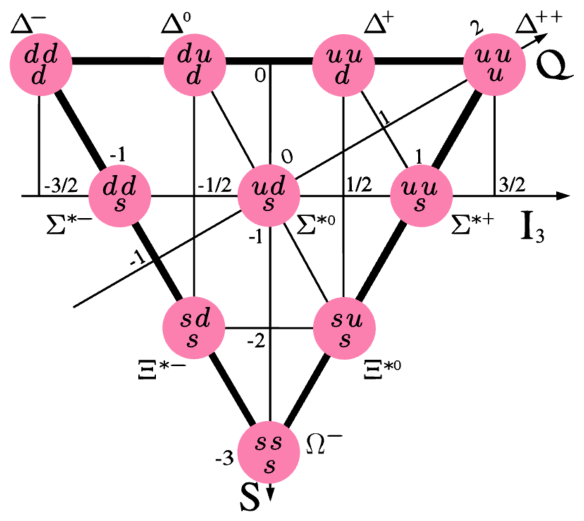
Chiral Mean Field (CMF) Model

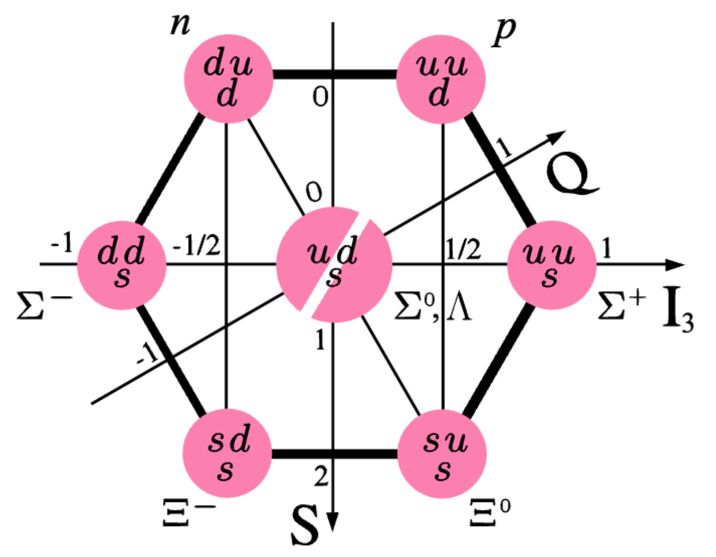
- CMF Lagrangian

$$\mathscr{L}_{\mathrm{CMF}} = \mathscr{L}_{\mathrm{kin}} + \mathscr{L}_{\mathrm{int}} + \mathscr{L}_{\mathrm{scal}} + \mathscr{L}_{\mathrm{vec}} + \mathscr{L}_{\mathrm{esb}} - U_{\Phi}\,.$$

The Polyakov loop–like potential is given by:

$$U_\Phi = \left(a_0 T^4 + a_1 \mu_B^4 + a_2 T^2 \mu_B^2\right)\Phi^2 + a_3 T_0^4 \ln\left(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4\right).$$





- Quarks and hadrons interactions are mediated via the exchange of scalar (σ, ζ and δ) and vector (ω, ϕ and ρ) mesons.

N. Cruz-Camacho, *et al., PRD 111 (2025) 9, 094030*

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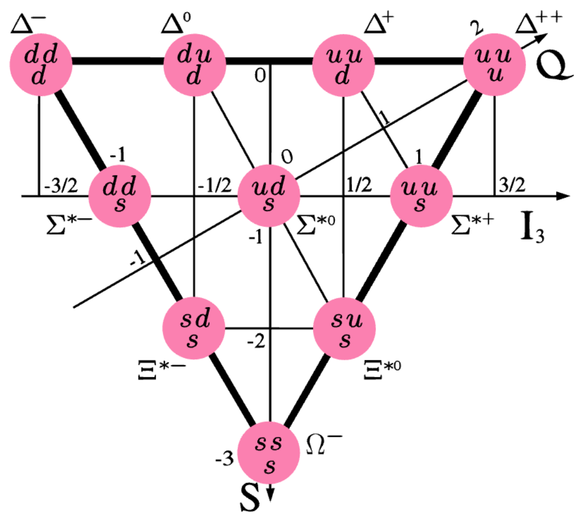

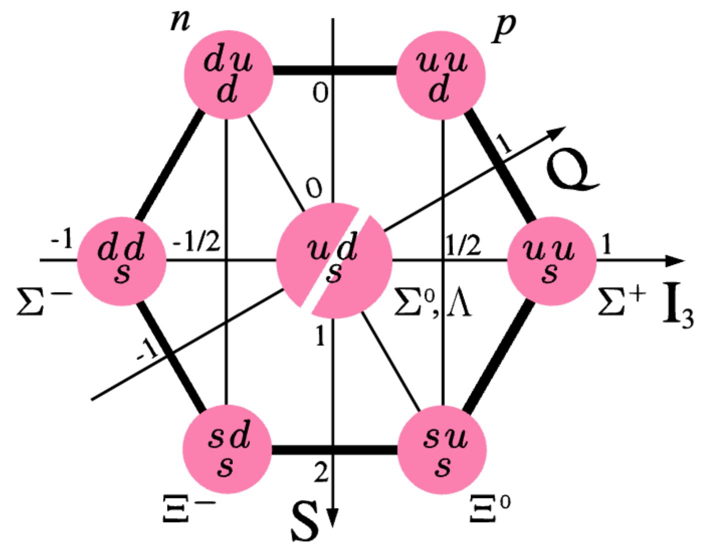

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CMF++
Chiral Mean Field

- In this study, we restrict ourselves to the **hadronic regime**, ignoring quark degrees of freedom, and use the **redefined CMF model RC4**.

R. Kumar, *et al., PRD 109 (2024) 7, 074008*

Chiral Mean Field (CMF) Model

The mean field approximation (MFA)

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- Mesons not dynamical degrees of freedom are replaced by time and space independent expectation value

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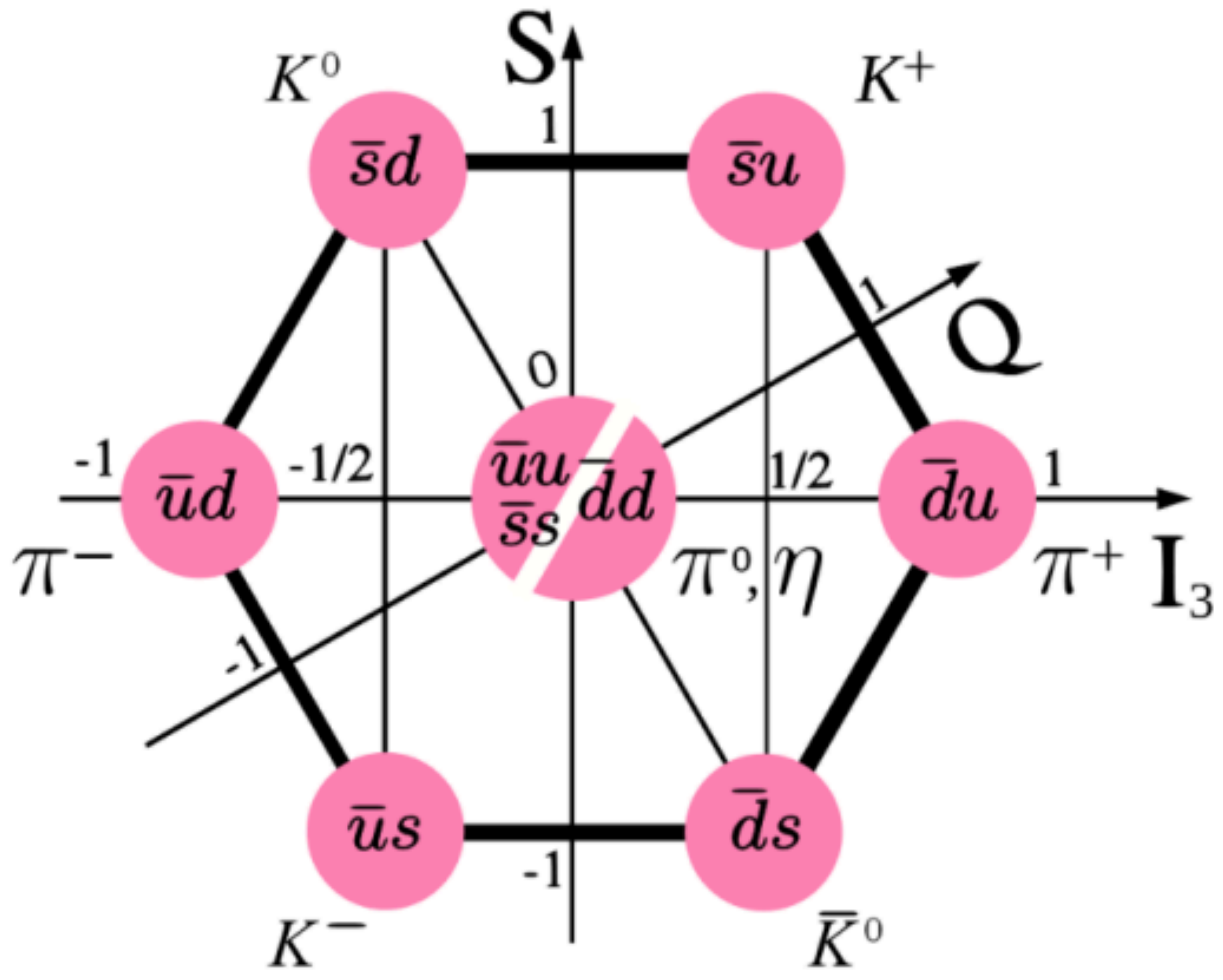
$$< \pi_i > = 0$$



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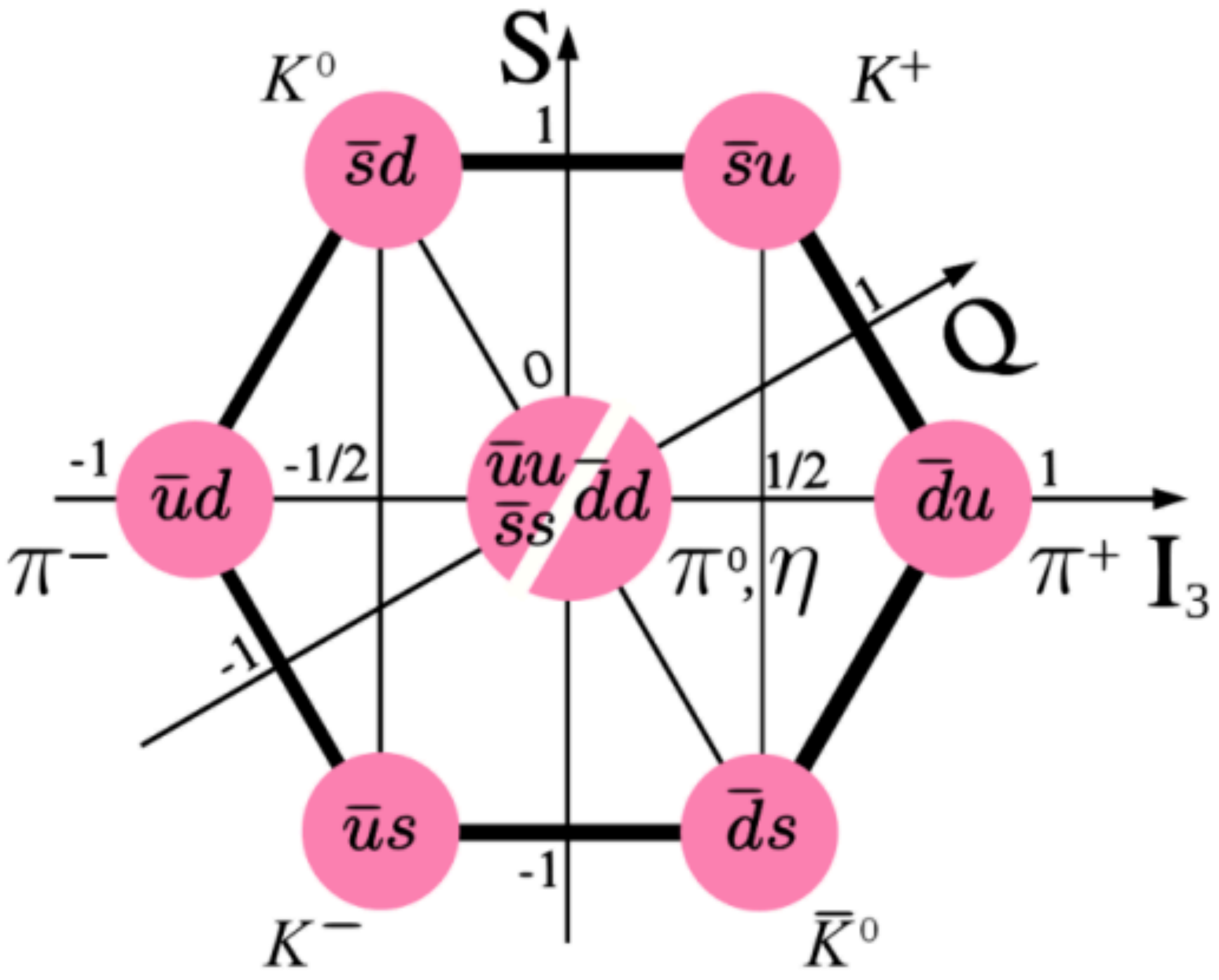

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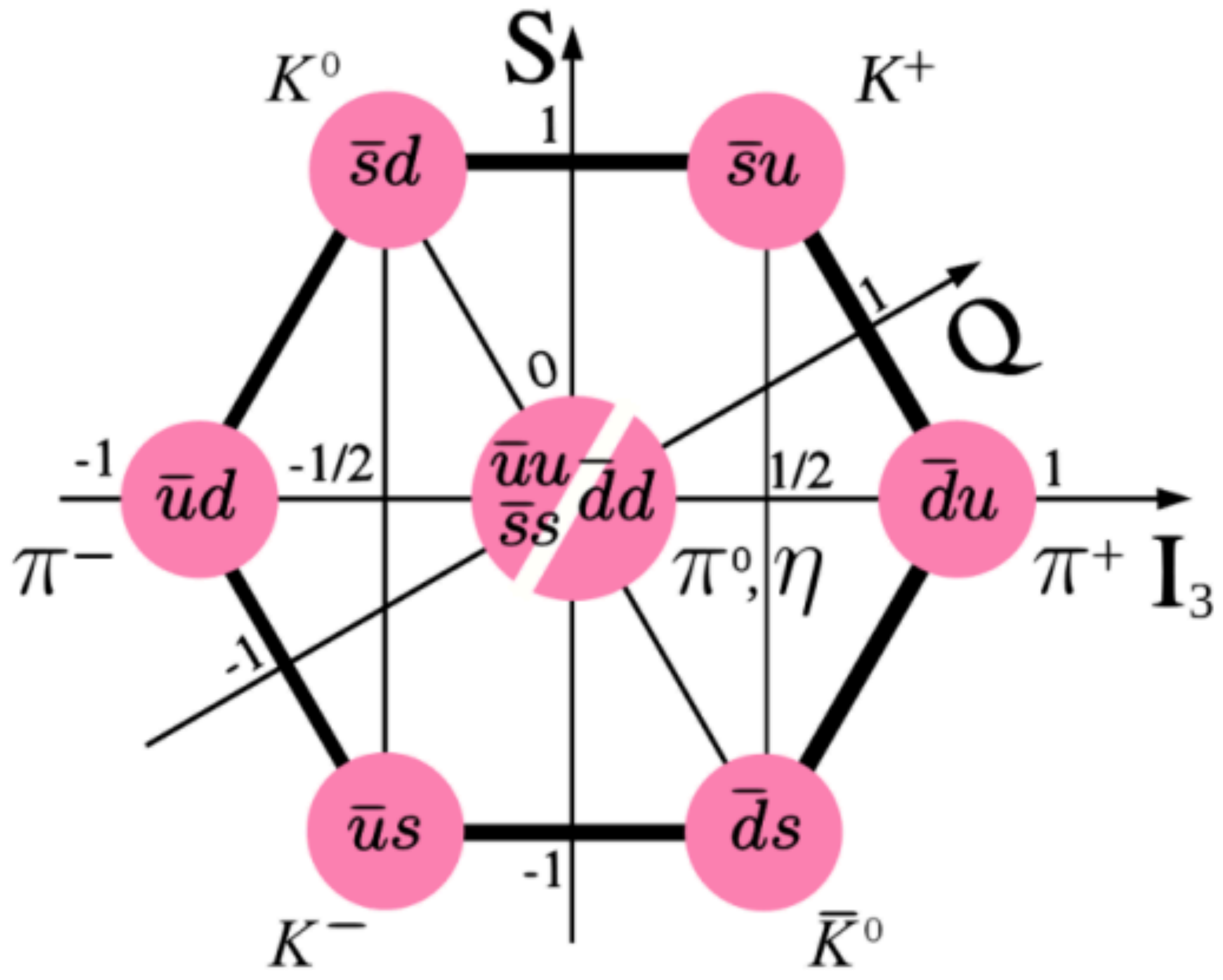

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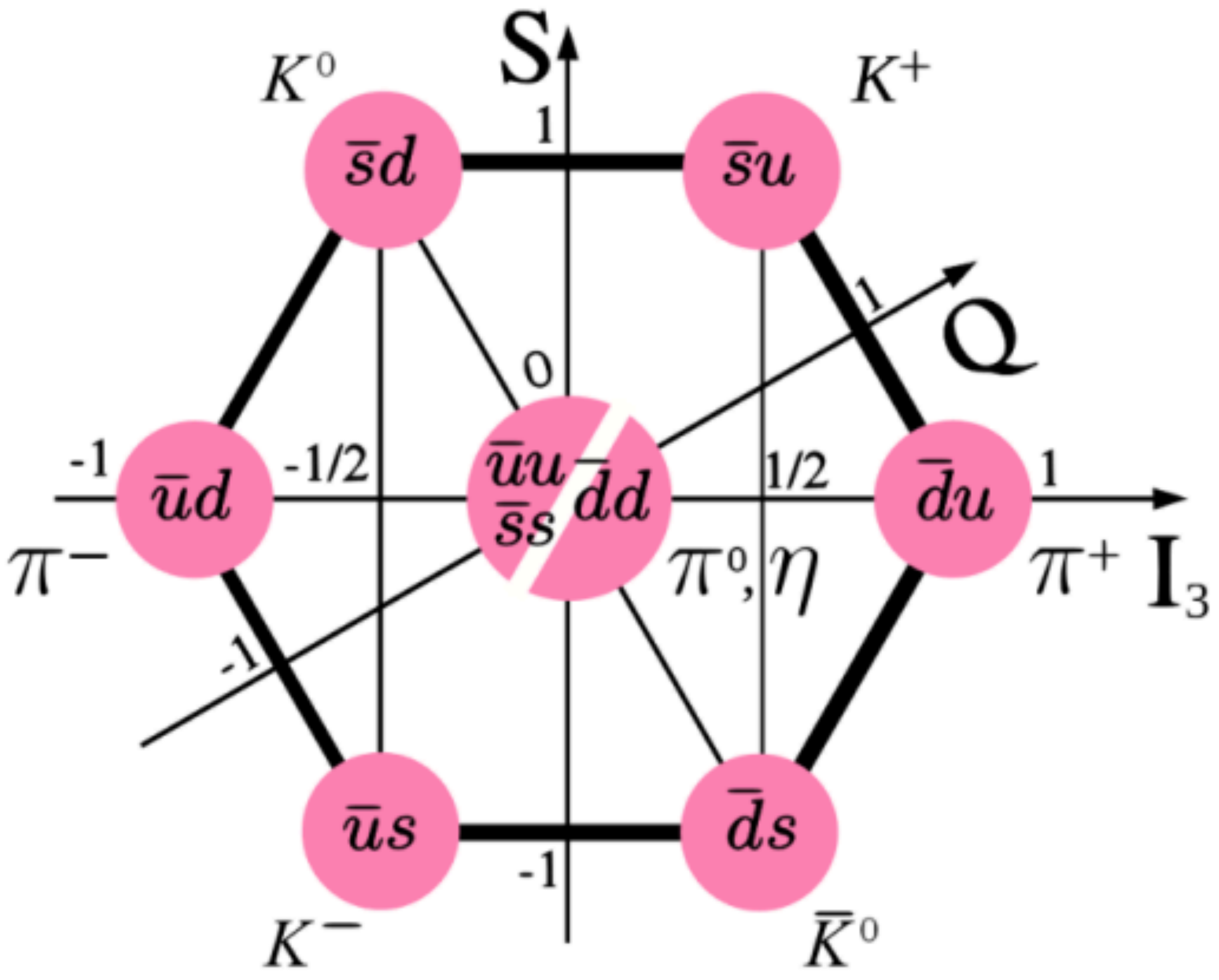


Caveat

- No interacting mesons affects the EoS

V. A. Dexheimer and S. Schramm, *PRC 81, 045201 (2010)*

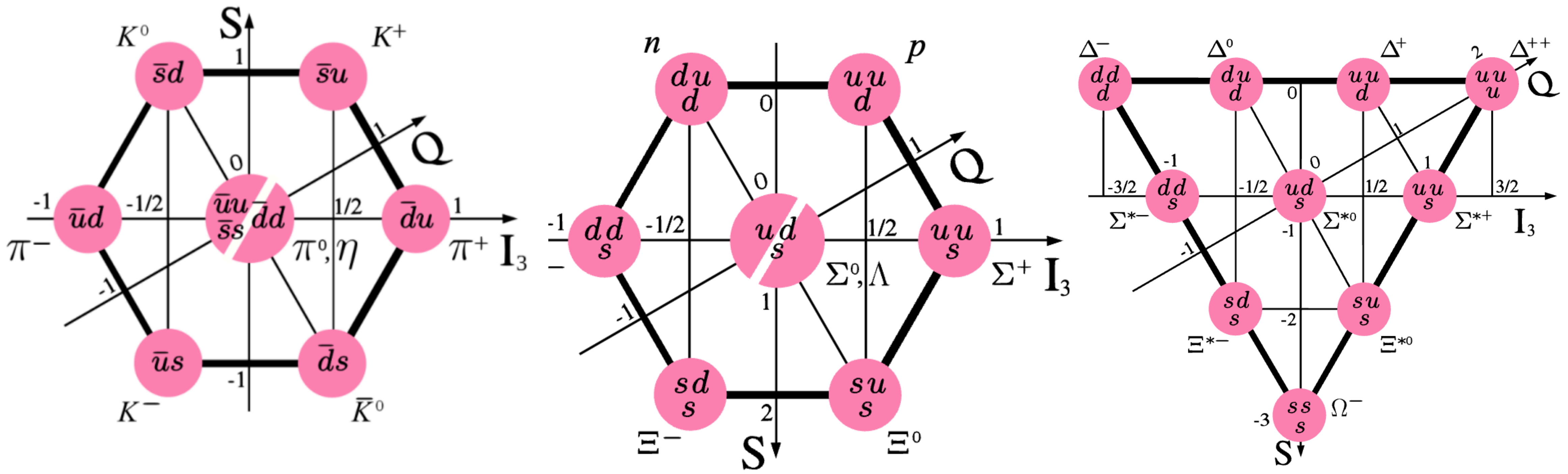
Formalism

Hadronic (H) grand canonical potential

- For a thermal model with baryons and meson

$$\frac{\Omega^H}{V} = U_M + \frac{\Omega^B_{th}}{V} + \frac{\Omega^M_{th}}{V}$$

$$U_M = \mathcal{L}_{\text{vec}} - \mathcal{L}_{\text{scal}} - \mathcal{L}_{\text{esb}} + \mathcal{L}_{\text{vac}}$$



- The inclusion of thermal mesons with field-dependent masses influences the mean-field equation of motion, as they introduce back-reaction terms

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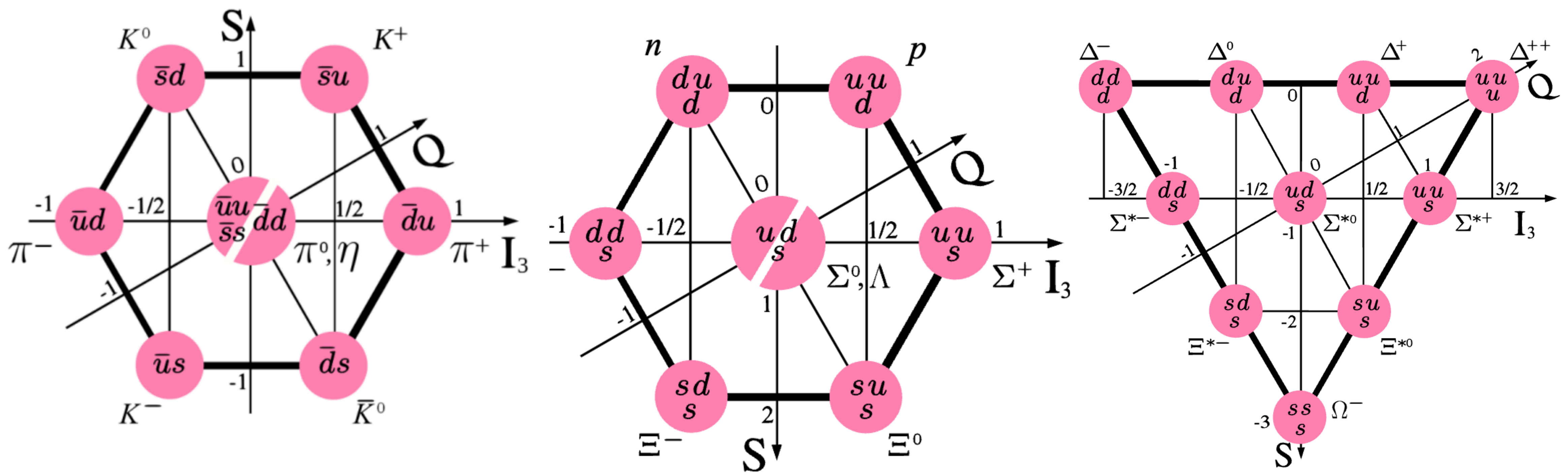

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The feedback in CMF equation of motion

$$\frac{\partial\left(\Omega^H/V\right)}{\partial\vartheta} = \frac{\partial\left(\Omega^{orig}/V\right)}{\partial\vartheta} + \sum_{i\in M} n_s^M \frac{\partial m_i^*}{\partial\vartheta}$$

m_i^* — in medium mass of particle i

$$\varphi = \sigma, \zeta, \delta, \omega, \rho, \phi$$

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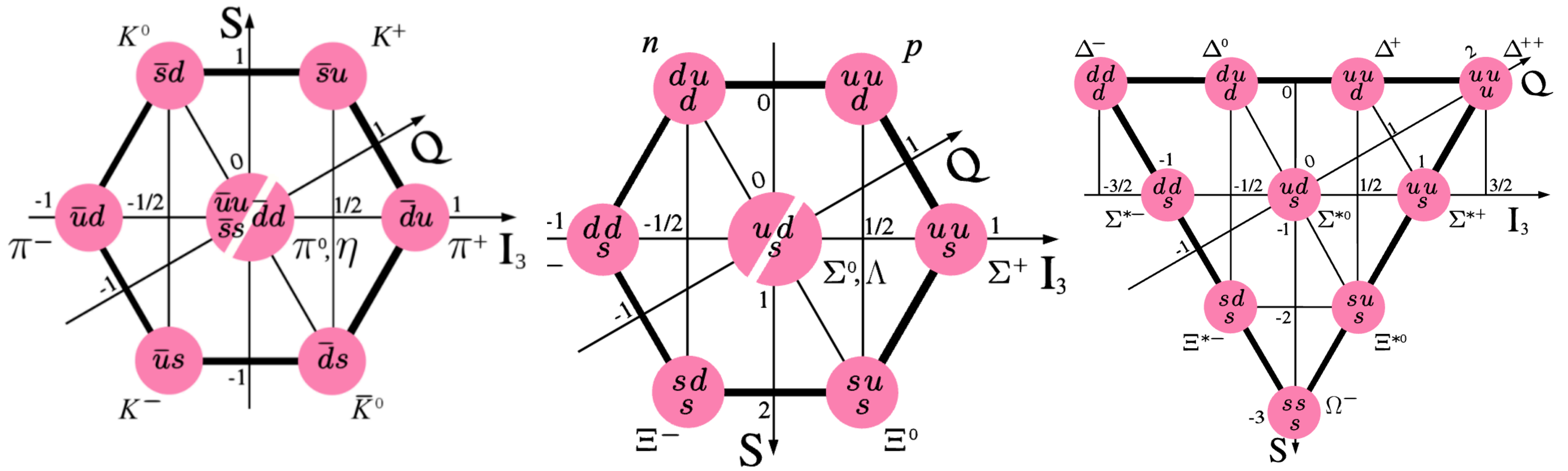

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For non-interacting mesons:

m_i^* — constant $\qquad \frac{\partial m_i^*}{\partial\vartheta} = 0$

R. Kumar *et al.*, PRD 111, 074029 (2025)

Interacting mesons

In-medium meson mass

The spontanesous symmetry-breaking Lagrangian term in the CMF model is written as:-

$$\mathscr{L}^u_{esb} = \left(-\frac{1}{2} m^2_{\eta^0} \mathrm{Tr} Y^2 - \frac{1}{2} \mathrm{Tr} \left[A_p \left(u(X + iY)u + u^\dagger (X - iY) u^\dagger \right) \right] \right)$$

The second derivative of the $\mathscr{L}^u_{esb}$ at its minimum with respect to the respective mesons φ_i, gives the in-medium mass

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The second derivative of the $\mathscr{L}^u_{esb}$ at its minimum with respect to the respective mesons φ_i, gives the in-medium mass

$$m^{*2}_{\varphi_{ij}} = -\lim_{\varphi \to <\varphi>} \frac{\partial^2}{\partial\varphi_i \partial\varphi_j}\mathscr{L}^u_{esb}$$

with $\varphi_i = \pi, \eta, \eta', K$, and for the vacuum expectation for the mesons we consider $< \varphi > = 0$. Similarly, the in medium mass can be obtained for **vector mesons** $(\omega, \rho, K^*, \phi)$ using $\mathscr{L}_{vec}$

R. Kumar *et al.*, PRD 111, 074029 (2025)

In-medium meson mass

Pseudoscalar mesons

$$m^{*2}_{\pi^0/\pi^+/\pi^-} = m_\pi^2 \frac{\sigma}{\sigma_0}$$

$$m^{*2}_{K^+/K^-} = \frac{0.5 m_K^2 \left(2\zeta + \sqrt{2}\,(\delta + \sigma)\right)\left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$$m^{*2}_{K^0/\bar{K}^0} = \frac{0.5 m_K^2 \left(2\zeta + \sqrt{2}\,(-\delta + \sigma)\right)\left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$$m^{*2}_{\eta^8} = \frac{m_\pi^2 \sigma\sigma_0 + \sqrt{2}\zeta\left(\sqrt{2} m_K^2 \left(\sqrt{2}\sigma_0 + 2\zeta_0\right) - 2 m_\pi^2 \sigma_0\right)}{\sigma_0^2 + 4\zeta_0^2}$$

Vector mesons

$$m^{*2}_\omega = m_\omega^2 + 6 g_4 \left(\frac{Z_\phi}{Z_\omega}\right)\phi^2$$

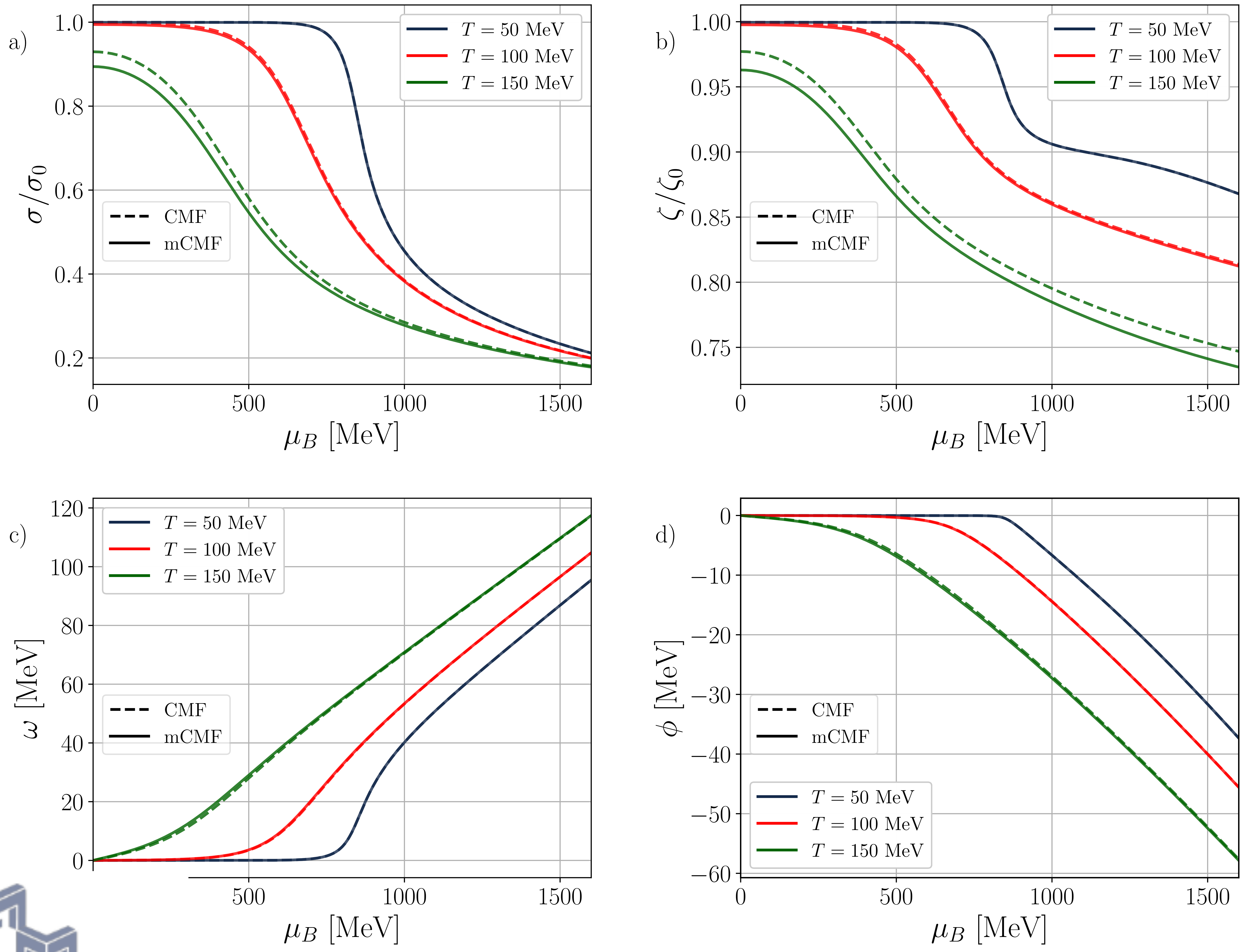
$$m^{*2}_\phi = m_\phi^2 + 6 g_4 \left(\frac{Z_\phi}{Z_\omega}\right)\omega^2$$

- For interacting mesons the masses are not constant, they depend on T and μ_B through the mean fields

$$m_i^* = m_i^*(\varphi(T, \rho_B)) = m_i^*(\varphi(T, \mu_B)) = m_i^*(\sigma, \zeta, \delta, \omega, \rho, \phi)$$

R. Kumar *et al.*, PRD 111, 074029 (2025)

Meson Mean fields in isospin symmetric matter

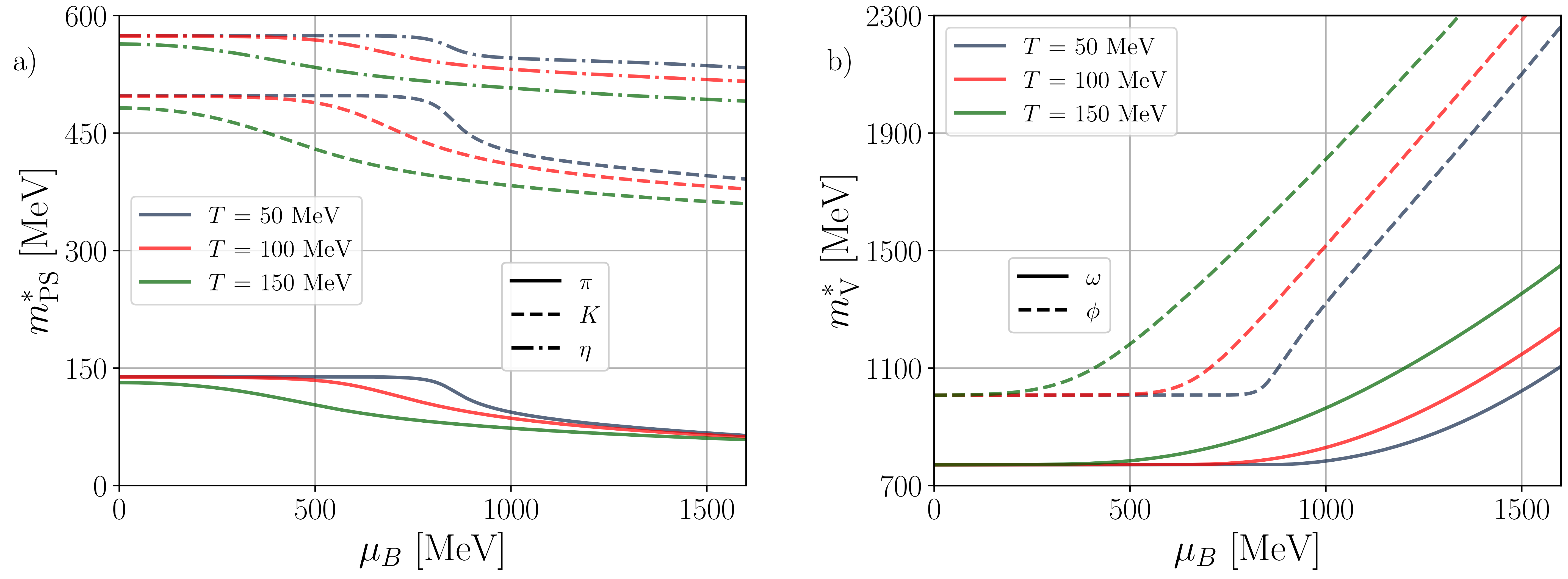


- Increasing T or μB modifies mean fields

R. Kumar *et al.*, PRD 111, 074029 (2025)

pseudoscalar meson mass and particle population

In medium masses



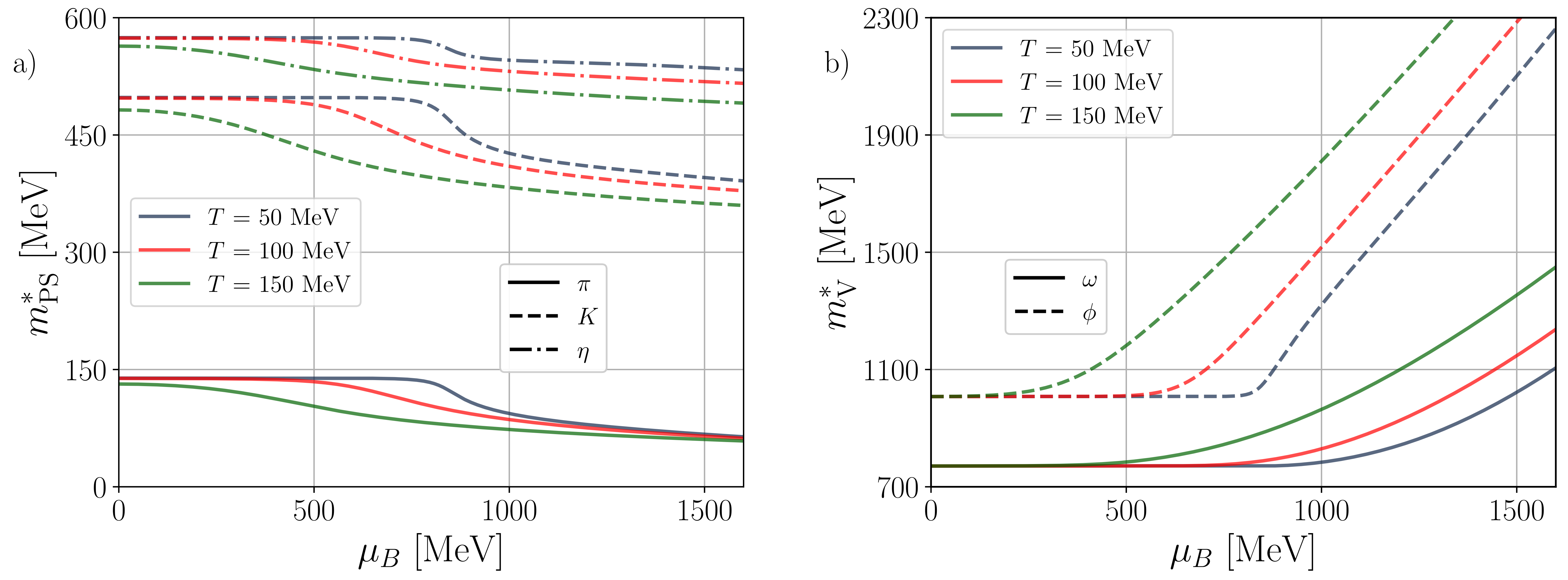
- Increasing T or μB modifies mean fields
- Mean fields modify meson masses
- Meson masses reveals the onset of chiral symmetry restoration

$$m^{*2}_{\pi^0/\pi^+/\pi^-} = m_\pi^2 \frac{\sigma}{\sigma_0}$$

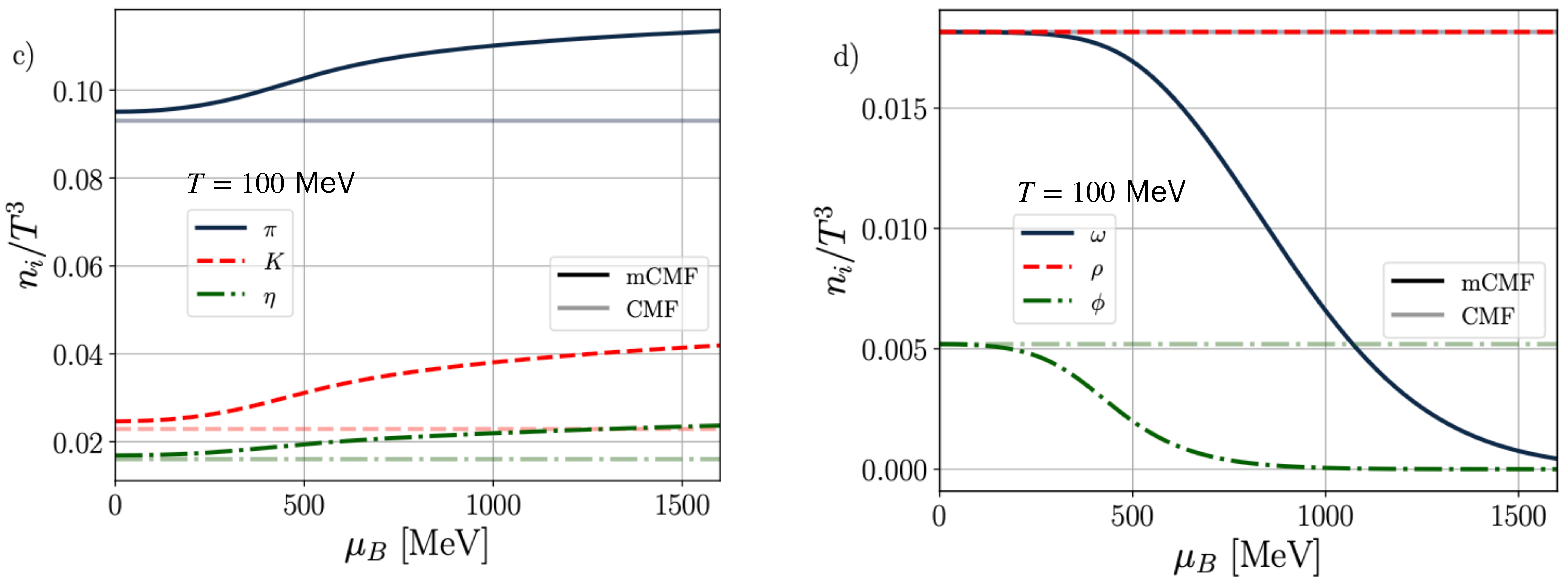
R. Kumar *et al.*, PRD 111, 074029 (2025)

pseudoscalar meson mass and particle population

In medium masses



Particle population



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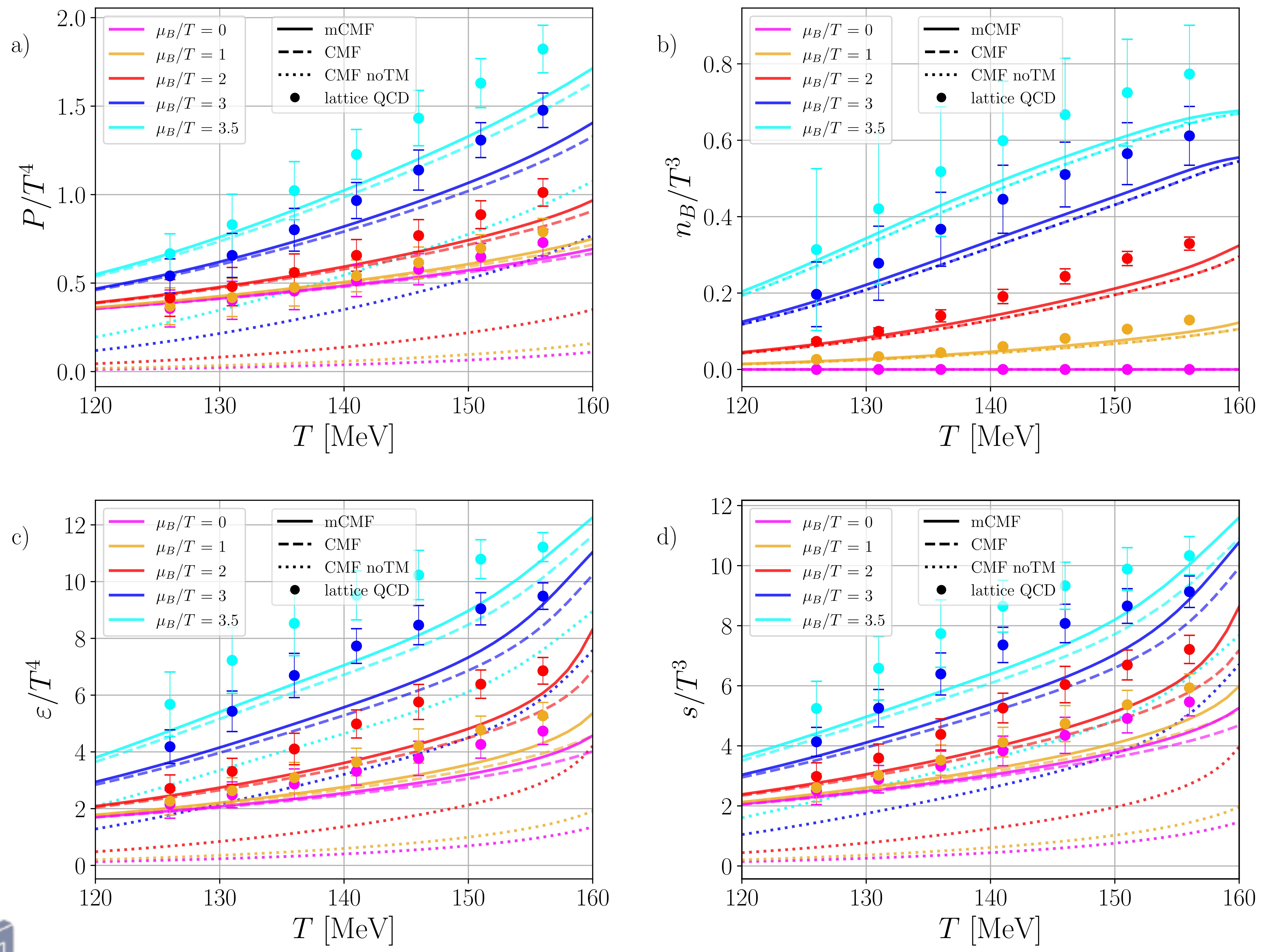
$$m^{*2}_{\pi^0/\pi^+/\pi^-} = m_\pi^2 \frac{\sigma}{\sigma_0}$$

- Modified masses change particle populations.

R. Kumar *et al.*, PRD 111, 074029 (2025)

Comparison with lattice QCD thermodynamics

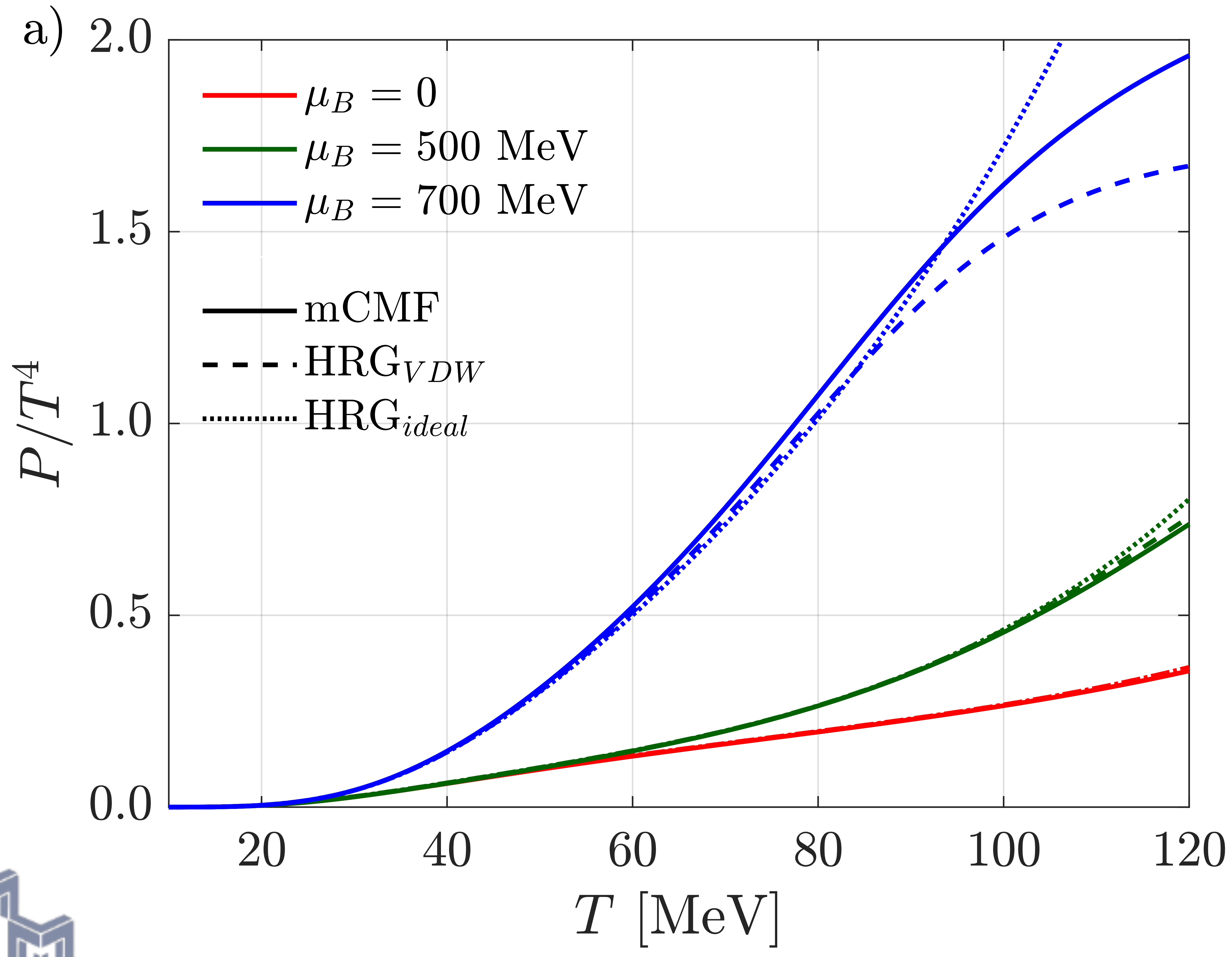


- Improved agreement with lattice data for:
  - Pressure
  - Baryon density
  - Energy density
  - Entropy density
- Valid up to T ≈ 160 MeV

S. Borsányi, et al. PRL 126 (2021)

R. Kumar *et al.*, PRD 111, 074029 (2025)

Comparison with HRG thermodynamics



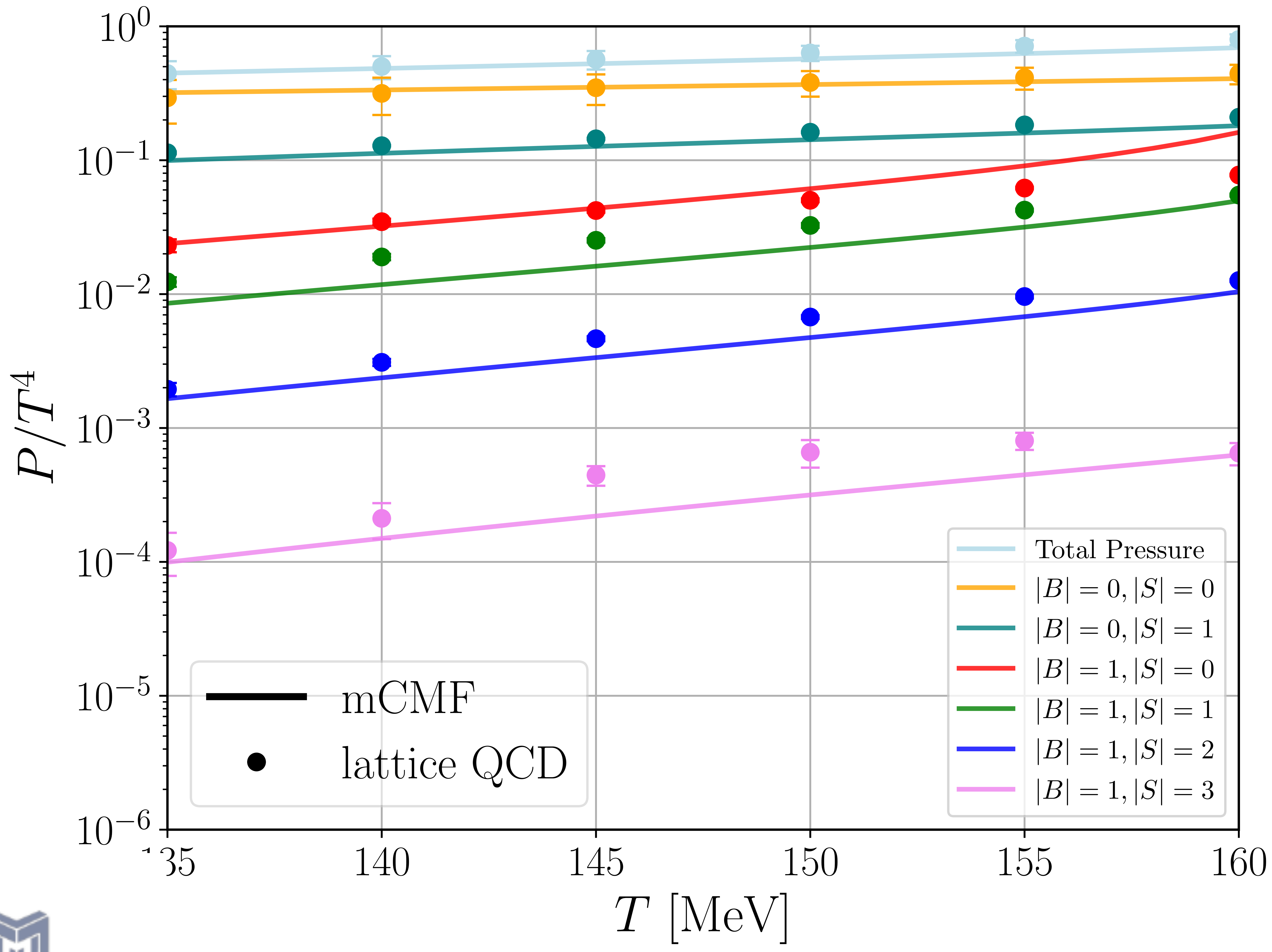
HRG model

- Reproduces HRG behavior at low to intermediate temperature T
- The agreement can be further improved by including quark degree of freedom

V. Vovchenko, *et al*. Comp. Phys. Com. 244, 295 (2019)

R. Kumar *et al*., PRD 111, 074029 (2025)

Comparison with Partial Pressures



Partial Pressures

- Validates hadronic content of CMF
- Good agreement across sectors:
  - Baryonic
  - Strange

P. Alba, et al. PRD 96 (2017)

R. Kumar *et al.*, PRD 111, 074029 (2025)

Summary

- Extended the CMF model by including pseudoscalar and vector meson interactions.
- We achieved an unprecedented level of agreement with extrapolated lattice QCD and HRG thermodynamics that could be improved by adding quark degrees of freedom
- Our goal is to obtain a realistic equation of state for dense matter at finite temperature, which will be useful in neutron stars and neutron star merger simulations.

Outlook:

- Obtain the EoS at finite isospin fraction including neutron star matter. *Talk by J. Grefa*
- Include quarks and a deconfinement potential *On going work, Poster by M K*
- Improve the mean-field approximation or go beyond mean field approximation *On going work by J. Grefa*

Thank you for your attention!

APPENDIX

Multiplets

- Baryon Octet

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda_0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

- Scalar Matrix: Mean-Fields

$$X = \begin{pmatrix} \frac{\delta^0+\sigma}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{-\delta^0+\sigma}{\sqrt{2}} & 0 \\ 0 & 0 & \zeta \end{pmatrix}$$

- A_p matrix

$$A_p = \frac{1}{\sqrt{2}} \begin{pmatrix} m_\pi^2 f_\pi & 0 & 0 \\ 0 & m_\pi^2 f_\pi & 0 \\ 0 & 0 & 2m_K^2 f_K - m_\pi^2 f_\pi \end{pmatrix}$$

Multiplets

- Pseudoscalar Nonet

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- Vector Meson Nonet : Mean Fields (ω,ρ and ϕ)

$$V = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \phi \end{pmatrix}$$

- Pseudoscalar Singlet

$$Y = \sqrt{\frac{1}{3}}\eta_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The CMF Lagrangian

$$\mathcal{L}_{\mathrm{CMF}} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{int}} + \mathcal{L}_{\mathrm{scal}} + \mathcal{L}_{\mathrm{vec}} + \mathcal{L}_{\mathrm{esb}}$$

$$\mathcal{L}_{\mathrm{kin}} = i\,\mathrm{Tr}\left(\bar{B}\gamma_\mu D^\mu B\right) = i\sum_{i\in B}\left(\bar{\psi}_i\gamma_\mu\partial^\mu\psi_i\right)$$

$$\mathcal{L}_{\mathrm{scal}} = -\frac{1}{2}k_0\chi_0^2\left(\sigma^2+\zeta^2+\delta^2\right) + k_1\left(\sigma^2+\zeta^2+\delta^2\right)^2 + k_2\left[\frac{\sigma^4+\delta^4}{2}+\zeta^4+3(\sigma\delta)^2\right]$$
$$+ k_3\chi_0\left(\sigma^2-\delta^2\right)\zeta + k_{3N}\chi_0\left(\frac{\sigma^3}{\sqrt{2}}+\frac{3}{\sqrt{2}}\sigma\delta^2+\zeta^3\right) - k_4\chi_0^4 + \frac{\epsilon}{3}\chi_0^4\ln\left[\frac{\left(\sigma^2-\delta^2\right)\zeta}{\sigma_0^2\zeta_0}\right]$$

$$\mathcal{L}_{\mathrm{vec}} = \frac{1}{2}\left(m_\omega^2\omega^2+m_\phi^2\phi^2+m_\rho^2\rho^2\right) + \mathcal{L}_{\mathrm{vec}}^{\mathrm{SI}}$$

$$\mathcal{L}_{\mathrm{esb}}^{u} = -\left[m_\pi^2 f_\pi\sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta\right]$$

$$\mathcal{L}_{\mathrm{int}} = -\sum_{i\in B}\bar{\psi}_i\left[\gamma_0\left(g_{i\omega}\omega+g_{i\rho}\rho+g_{i\phi}\phi\right)+g_{i\sigma}\sigma+g_{i\zeta}\zeta+g_{i\delta}\delta\right]\psi_i$$

Formalism: CMF thermodynamic potential

$$\begin{aligned}\frac{\Omega^H}{V} &= \frac{\Omega}{V} + \frac{\Omega^M_{\text{th}}}{V}\,, \\ &= U_M + \frac{\Omega^B_{\text{th}}}{V} + \frac{\Omega^M_{\text{th}}}{V}\,,\end{aligned} \tag{1}$$

$$U_M = \mathcal{L}_{\text{vec}} - \mathcal{L}_{\text{scal}} - \mathcal{L}_{\text{esb}} + \mathcal{L}_{\text{vac}}\,, \tag{2}$$

$$\frac{\Omega^B_{\text{th}}}{V} = -T \sum_{i\in\text{baryons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \left(\ln\left[1 + e^{-\frac{1}{T}\left(E^*_i(k) - \mu^*_i\right)}\right] + \ln\left[1 + e^{-\frac{1}{T}\left(E^*_i(k) + \mu^*_i\right)}\right]\right)\,, \tag{3}$$

$$\frac{\Omega^M_{\text{th}}}{V} = T \sum_{i\in\text{mesons}} \frac{\gamma_i}{2\pi^2} \int dk k^2 \ln\left[1 - e^{-\frac{1}{T}\left(E^*_i(k) - \mu^*_i\right)}\right]\,. \tag{4}$$

In-medium meson mass

$$m^{*2}_{\varphi_{ij}} = \lim_{\varphi \to \langle \varphi \rangle} \frac{\partial^2}{\partial \varphi_i \partial \varphi_j} U \,,$$

$$m^{*^2}_{\pi^0/\pi^+/\pi^-} = m_\pi^2 \frac{\sigma}{\sigma_0}$$

$$m^{*^2}_{K^+/K^-} = \frac{0.5 m_K^2 \left(2\zeta + \sqrt{2}\,(\delta + \sigma)\right)\left(\sqrt{2}\sigma_0 + 2\zeta_0\right)}{\left(\sigma_0 + \sqrt{2}\zeta_0\right)^2}$$

$$m^{*2}_{\omega} = m_\omega^2 + 6 g_4 \left(\frac{Z_\phi}{Z_\omega}\right)\phi^2$$

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$$m^{*^2}_{\eta^8} = \frac{m_\pi^2 \sigma \sigma_0 + \sqrt{2}\zeta \left(\sqrt{2} m_K^2 \left(\sqrt{2}\sigma_0 + 2\zeta_0\right) - 2 m_\pi^2 \sigma_0\right)}{\sigma_0^2 + 4\zeta_0^2}$$