Noncommutative Formulation of Classical Mechanics

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$$\partial_t \rho^L = \{H, \cdot\}\, \rho^L \ ;$$

$$\{H, \cdot\} = \partial_q\, H \partial_p - \partial_p H\, \partial_q$$

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$$i\hbar\,\partial_t \chi = \widetilde{H}\chi$$

The "auxiliary" variables

$$\widetilde{H} = \partial_q H\, \widetilde{q} + \partial_p H\, \widetilde{p} + \alpha_{_H}(q,p,t)$$

$$\widetilde{q} = i\hbar\, \partial_p \ , \quad \widetilde{p} = -i\hbar\, \partial_q$$

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$$[q, \widetilde{p}] = [\widetilde{q}, p] = i\hbar$$

Commutators

$$\frac{1}{i\hbar}[q, \widetilde{p}] = \frac{1}{i\hbar}[\widetilde{q}, p] = \{q, p\}$$

$$\frac{1}{i\hbar}[\widetilde{H}, G(q, p, t)] = \{H, G(q, p, t)\}$$

Tilde-images / General map

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$$\begin{aligned} \widetilde{u} &= i\hbar\,\{u,\,\cdot\} + \alpha_u \\ &= \partial_q u\,\widetilde{q} + \partial_p u\,\widetilde{p} + \alpha_u(q,p,t) \end{aligned}$$

Map of main operations

- $\widetilde{u+v} = \widetilde{u} + \widetilde{v} + \alpha_{u+v}$
- $\widetilde{uv} = \widetilde{u}v + u\widetilde{v} + \alpha_{uv}$
- $\widetilde{\{u,v\}} = \frac{1}{i\hbar}[\widetilde{u}, \widetilde{v}] + \alpha_{\{u,v\}}$

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- $\widetilde{u}$ is linear in $\widetilde{q}$ and $\widetilde{p}$
- $\widetilde{u}$ has an arbitrary part $\alpha_u(q, p, t)$

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  Note: The tilde-variables will surface in a Q-C system

But what does it mean?

- Auxiliary/unphysical?

But what does it mean?

- Auxiliary/unphysical?
- Non-classical?

But what does it mean?

- Auxiliary/unphysical?
- Non-classical?
- Related to statistical mechanics?

Thank you!

Come check out my talk in the CAP Congress!
Classical Hilbert space, statistical mechanics and gauge freedom

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