Looking at bulk points in general geometries

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Introduction

- AdS/CFT duality allows us to understand gravity in anti-de Sitter spacetime in terms of a non-gravitating CFT living on its asymptotic boundary.

$$\text{bulk gravity in AdS} \quad \leftrightarrow \quad \text{boundary CFT}$$

[Maldacena]

- Dictionary relates boundary correlators to bulk correlators.

[Gubser-Klebanov-Polyakov, Witten]

- **Our broad motivation:** How to self-consistently arrive at bulk geometries from holographic CFT correlators?
- Simple bulk experiments using conformal correlators seem useful towards this.

Introduction

- In empty AdS (CFT vacuum), sharp statement of bulk locality at sub-AdS length scales: correlator factorizes to flat-space amplitude.

  [Polchinski; Gary-Giddings-Penedones; Heemskerk-Penedones-Polchinski-Sully ...]

  [Maldacena-Simmons-Duffin-Zhiboedov, Minwalla et al ...]

- Does such a flat-space limit hold in arbitrary CFT states dual to a geometry?
  A **universal factorization formula** exists in states dual to bulk geometry.

  [**Simon Caron-Huot, JC, Keivan Namjou; Looking at bulk points in general geometries, 2502.14963**]

- We can also capture boundary imprint of bulk causality and read off the local metric.

  [**Simon Caron-Huot, JC, Keivan Namjou; Boundary imprint of bulk causality, 2501.13182**]

Metric ansatz

- **Notation:** x, p denote boundary labels, X, P denote bulk labels, z denotes bulk radial coordinate.

- Spacetime metric obeys time and transverse translations.

$$\mathrm{d}s^2 = \frac{1}{z^2}\left(-A(z)\,\mathrm{d}T^2 + \frac{\mathrm{d}z^2}{B(z)} + \delta_{ab}\,\mathrm{d}X^a\mathrm{d}X^b\right) \tag{1}$$

  Boundary at $z = 0$, where $A = B \to 1$.
  Empty AdS: $A = B = 1$ everywhere.

- Planar black holes are given by:

$$A(z) = B(z) = 1 - \frac{z^d}{z_{\mathrm{h}}^d}$$

Ingredients for bulk scattering

- **Questions:**
  1. How to sharply pick up a point in the bulk with four point correlator?
  2. Scattering data falling inside horizon lost! What amplitudes in black hole background?

- Directed wavepackets: Use them to introduce boundary momentum, which introduces a bulk momentum as wavepackets travel.

- General time-ordering: We can consider inclusive observables instead of in-out observables. Correlators defined on single or multiple time-folds.

Directed wavepackets

- To introduce momentum, we work with the smeared (boundary) operators:

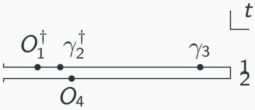
$$O_{x,p,\sigma} = \int \mathrm{d}^d(\delta x)\, \psi^*_{p,\sigma}(\delta x)\, O(x+\delta x), \qquad (2)$$

and its conjugate for $O^\dagger_{x,p,\sigma}$. Defined the smeared Fourier transform as:

$$\psi_{p,\sigma}(\delta x) = \exp\left(\mathrm{i} p_\mu \delta x^\mu - \frac{1}{2}\sigma^{-1}_{\mu\nu}\, \delta x^\mu \delta x^\nu\right), \qquad \det\sigma > 0, \qquad (3)$$

- These operators shoot wavepackets into the bulk. Our boundary correlators consist of such smeared operator insertions.

Radar observable



- We consider the expectation value $\langle\Psi|O_4(x_4)\ \gamma_3(x_3)\ \gamma_2^\dagger(x_2)\ O_1^\dagger(x_1)|\Psi\rangle$.

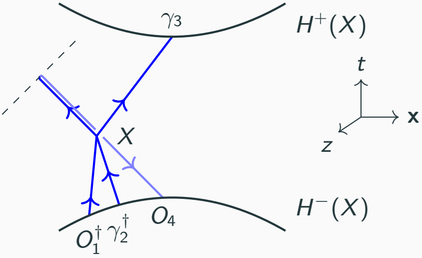
$$= \sum_{\text{out}} \langle\Psi|O_4(x_4)\ |\text{out}\rangle\ \langle\text{out}|\ \gamma_3(x_3)\ \gamma_2^\dagger(x_2)\ O_1^\dagger(x_1)|\Psi\rangle \tag{4}$$

- **Experiment**: Say we identify $O_1 = O_4$, then we basically measure the two point photon correlator

$$\langle\xi|\gamma_3(x_3)\ \gamma_2^\dagger(x_2)\ |\xi\rangle \tag{5}$$

over the state $|\xi\rangle = O_1|\Psi\rangle$. Kind of a radar!

Radar in the AdS bulk



- Sharp features when x_1, x_2, and x_4 approach the past lightcone and x_3 approaches the future lightcone of a common bulk point X.
- Dashed line is (arbitrary) late time at which the “out” states are summed over.
- By tuning shooting momenta, we can get a flat-space amplitude near X.

Boundary-bulk scattering dictionary

- GKPW dictionary: decompose correlator as product of bulk-to-boundary propagators.
- Bulk: massless scalar field theory. Use semiclassical WKB analysis to obtain these propagator at high energy (the geometric optics limit).
- High-energy WKB of propagator in plane wave limit takes the form:

$$\langle\Psi|\Phi(X+\delta X)\, O_p^\dagger|\Psi\rangle = \mathcal{C}^{\pm}_{\Delta,p}\sqrt{\mathcal{D}(p;X)}\exp\Big(\mathrm{i}S(p;X)+\mathrm{i}P_M\delta X^M\Big), \quad (6)$$

- Define a basis of early-time modes $a_{X,P}/a^\dagger_{X,P}$ and a basis of late-time modes $b_{X,P}/b^\dagger_{X,P}$. Roughly quantum field at X:

$$\langle\Psi|\Phi(X+\delta X)\ a^\dagger_{X,P}|\Psi\rangle = \mathrm{e}^{\mathrm{i}P\cdot\delta X} \quad (7)$$

- Relation between the bulk modes and the boundary smeared operators.

$$O_p^\dagger \ \simeq\ \sqrt{\mathcal{D}(p;X)}\, e^{\mathrm{i}S(p;X)} \times \begin{cases} \mathcal{C}^{+}_{\Delta,p}a^\dagger_{X,P}, & \text{if } O^\dagger \text{ is in the past of } X, \\ \mathcal{C}^{-}_{\Delta,p}b^\dagger_{X,P}, & \text{if } O^\dagger \text{ is in the future of } X. \end{cases} \quad (8)$$

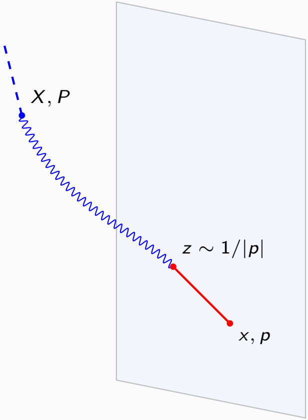


Figure 1: Evolution of a wavepacket into the bulk.

Flat-space amplitude near X

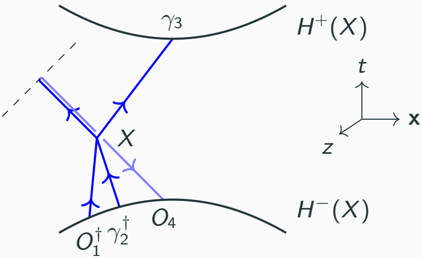
- Away from plane wave limit, general dictionary:

$$O^\dagger_{x,p,\sigma} \quad \text{on the boundary} \quad \leftrightarrow \quad \int \mathrm{d}(\delta P)\, \psi^{\text{bulk}}_{x,p,\sigma;X}(\delta P)\, a^\dagger_{X,P+\delta P} \quad \text{in the bulk.} \tag{9}$$

- Using this, we relate a n-point boundary correlator to another n-point bulk amplitude, schematically:

$$\begin{gathered} \langle\Psi|(\text{product of } O_{x,p,\sigma},\, O^\dagger_{x,p,\sigma}\text{'s})|\Psi\rangle \\ \updownarrow \\ \int \prod_{i=1}^{n} \mathrm{d}(\delta P_i)\, \psi^{\text{bulk}}_i(\delta P_i)\, \langle 0|(\text{product of } a,\ a^\dagger,\ b,\ b^\dagger\text{'s})|0\rangle^{\text{bulk}}_{\text{near X}}\,. \end{gathered} \tag{10}$$

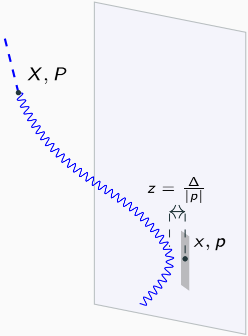
The bulk amplitude could be a standard in-out amplitude or it could be a generalized non-time-ordered amplitude.



$$\langle\Psi|O_{x_4^-,p_4,\sigma_4}O_{x_3^+,p_3,\sigma_3}O^\dagger_{x_2^-,p_2,\sigma_2}O^\dagger_{x_1^-,p_1,\sigma_1}|\Psi\rangle$$
$$\approx\int\left[\prod \mathrm{d}P_j\,\psi_j\right]\langle 0|a_{X,P_4}b_{X,P_3}a^\dagger_{X,P_2}a^\dagger_{X,P_1}|0\rangle^{\text{bulk}} \tag{11}$$
$$=\int\left[\prod \mathrm{d}P_j\,\psi_j\right]\sqrt{-g}\,(2\pi)^{d+1}\delta^{d+1}\left(\textstyle\sum_j\alpha_j P_j\right)\mathrm{i}\mathcal{M}(s,t),$$

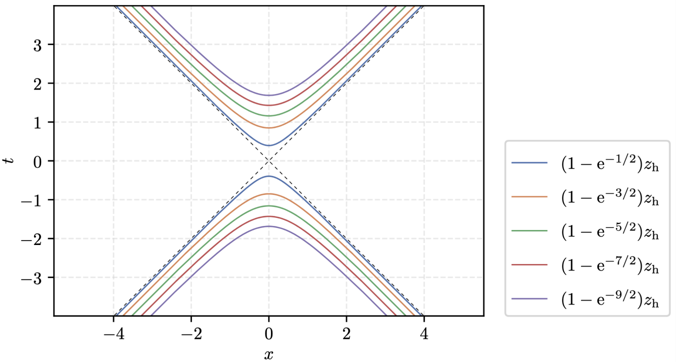
where $s=-(P_1+P_2)^2$ and $t=-(P_2-P_3)^2$.

Extensions:



- **Massive fields:** No timelike geodesics from boundary to $z \sim \Delta/|p|$. Particle tunnels into the bulk from boundary! Adjust WKB phase with "$-$" sign instead of "i" to account for tunneling. Complex trajectory.
- **Light spinning fields:** Eg. currents and stress tensor. Story similar to scalar but more polarizations, could be both transverse and longitudinal.

Boundary hyperboloids



Boundary hyperboloids for different radii outside a planar AdS_5 black hole.

The curves become actual hyperbolas as $z \to 0$.

Same as "lightcone cuts" of [Engelhardt, Horowitz]

Bulk depth from hyperboloids

- Geometric optics implies null geodesics are invariant under rescaling of boundary momentum p:

$$p_\mu \frac{\partial x^\nu}{\partial p_\mu} = 0 = p_\mu \frac{\partial x^\mu}{\partial p_\nu}. \tag{12}$$

  Powerful relations, allow us to measure bulk depth!

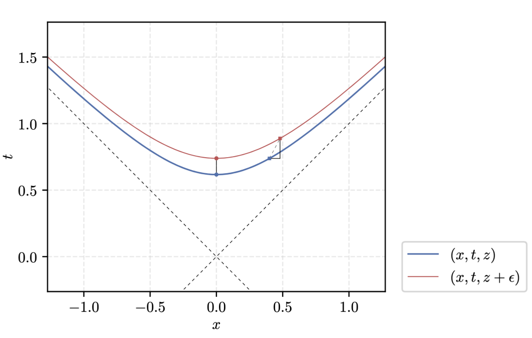
- **Eg:** Consider parallax (from two adjacent eyes at the boundary), which measures depth, $= \partial x^\mu / \partial p_\nu$.

- The exterior curvature of hyperboloid simplifies:

$$\tilde{K}^{\dot\mu\dot\nu} = \frac{1}{\sqrt{-p^2}} \frac{\partial x^{\dot\mu}(p; X)}{\partial p_{\dot\nu}} \qquad \text{(Minkowski boundary).} \tag{13}$$

- The parallax is same as the curvature radius of the hyperboloid!

Reading off the bulk metric: a simple example



- A bulk displacement δX is null *iff* two hyperboloids are tangent at a point.
- Assume metric form near X (with translation symmetry):

$$\mathrm{d}s^2 = -A(z)\mathrm{d}T^2 + B(z)\mathrm{d}z^2 + C(z)\delta_{ab}\mathrm{d}X^a\mathrm{d}X^b \tag{14}$$

- Strategy: find equal normal vectors first. At $x = 0$, hyperboloid variation fixes $\frac{A}{B} = \frac{(\delta z)^2}{(\delta t)^2}$. Any other point fixes $\frac{A}{C}$.
- Need only two evaluations! Scale factor from factorization formula.

What can black hole interior tell us about CFTs?

- Is there an exterior experiment whose answer is given by the interior geometry? Yes, with an **exponential cost** $e^{\frac{-\beta\omega}{2}}$!

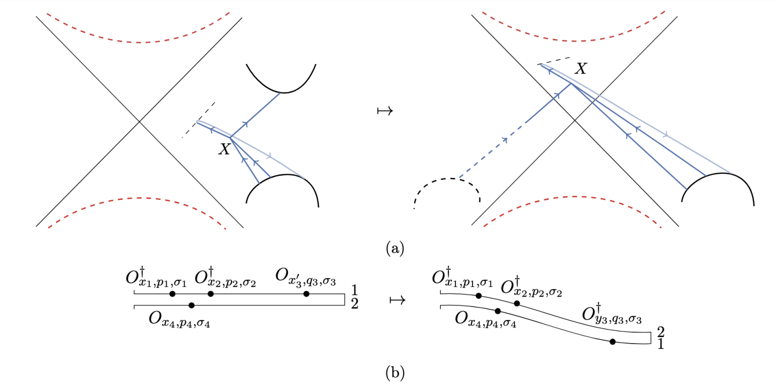
- We can choose a different smearing function which introduces "future" bulk modes near the bulk point X located within black hole interior?

- Use poles $(t' = -t \pm \mathrm{i}\beta/2)$ to mimic operators shot in from the left CFT, using right-side hyperboloids:

$$\frac{\beta}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}t'}{(t'+t)^2+\frac{\beta^2}{4}}\ e^{\pm\mathrm{i}\omega t'}\ O_R(t') = e^{\frac{-\beta\omega}{2}}\ e^{\mp\mathrm{i}\omega t}\ O_R\left(-t\pm\frac{\mathrm{i}\beta}{2}\right) \sim e^{\frac{-\beta\omega}{2}}\ e^{\mp\mathrm{i}\omega t}\ O_L(t) \tag{15}$$

[**Simon Caron-Huot, JC, to appear**]

Black hole interior?



Opposite Euclidean evolution violates path integral convergence rules, like Kontsevich Segal. Still, a valid answer to the four point function.

Black hole interior?

- Cross-checks: Works in Rindler space. Cross-ratios of the boundary 4 pt correlator go to the second sheet and hit the bulk point discontinuity.
- **Note:** Doesn't address physics of infalling observer (or firewalls)!
- To address questions about infalling observer, use bulk points from two sided correlations on TFD state.
- Possible geometric implications for Hayden-Preskill protocol?

Black hole singularity from two-point function?

Exact R-current spectral density in $\mathcal{N} = 4$ SYM ($\lambda = \infty$)

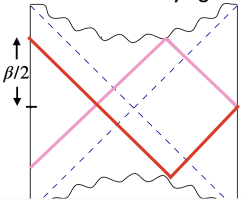
[Myers, Starinets& Thomson 0706.0162]

$$\mathrm{Im}\, G^{x,x}_{\mathrm{ret}}(\omega, k=0) = \frac{\pi\omega^2(1-e^{-\beta\omega})}{(1-e^{-\frac{\beta\omega}{2}(1-i)})(1-e^{-\frac{\beta\omega}{2}(1+i)})}$$

$$= \pi\omega^2 \left(1 + e^{-\frac{\beta\omega}{2}(1-i)} + e^{-\frac{\beta\omega}{2}(1+i)} + e^{-2\frac{\beta\omega}{2}(1-i)} + e^{-2\frac{\beta\omega}{2}(1+i)} + \dots\right)$$

$T=0$

Non-perturbatively decaying at large ω



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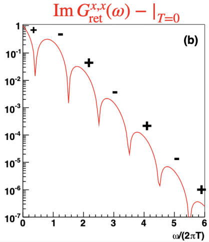

Geodesics bouncing off singularity at two-point (Fidkowski et al 03) match with vacuum-subtracted spectral density (Teaney 06)!

[**Nima Afkhami-Jeddi, Simon Caron-Huot, JC, Alex Maloney; in progress**]

Future questions

- Universality of the formula reflects equivalence principle: flat-space like amplitude is independent of the bulk point X.
  Why should bulk scattering amplitudes be the same around every bulk point?
- Do AdS-corrections to Virasoro Shapiro amplitudes provide corrections to subleading WKB?
- Can the bulk Raychaudhuri equations and focussing theorems be derived from properties of boundary correlators?
- Unrealistic assumption to derive factorization formula: that all bulk interactions are short-ranged. Multiplicative corrections for long-range?

Thank you!