No Shift, Sherlock

Shota Komatsu

Based on an upcoming paper with
José Calderón Infante (Caltech), Lucía Córdova (Amsterdam), Irene Valenzuela (CERN/IFT Madrid)

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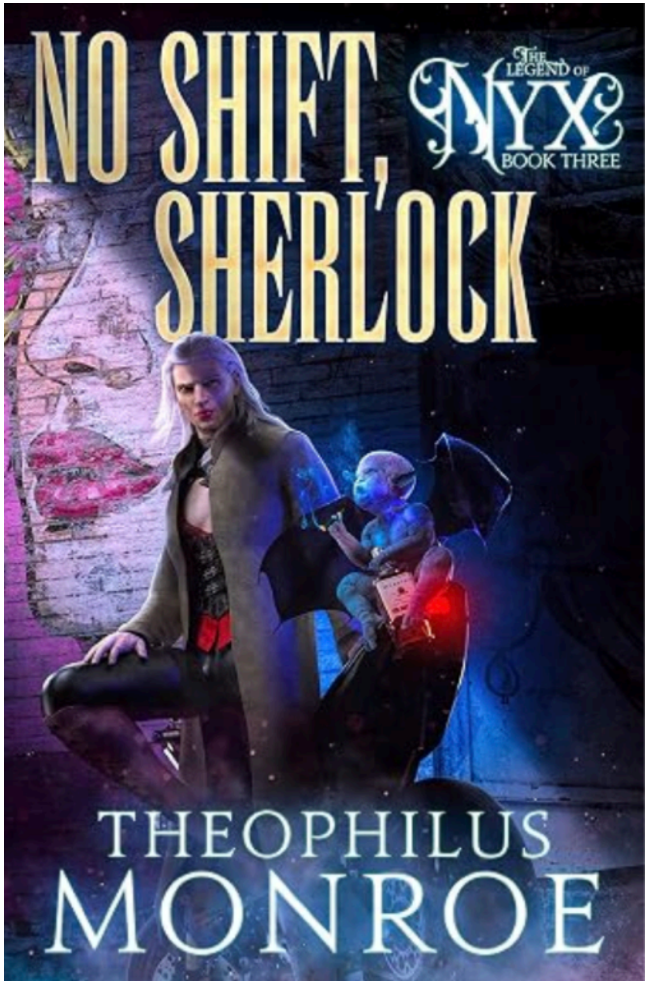

AI summary:
An urban-fantasy mystery in which Nyx, a badass vampire hunter, investigates soul-draining murders tied to a missing grimoire and a supernatural threat against unnatural beings.

No global symmetry in quantum gravity

- Swampland Program: Quantum gravity has imprints on IR EFT

  "In quantum gravity, not every low-energy EFT is allowed!"

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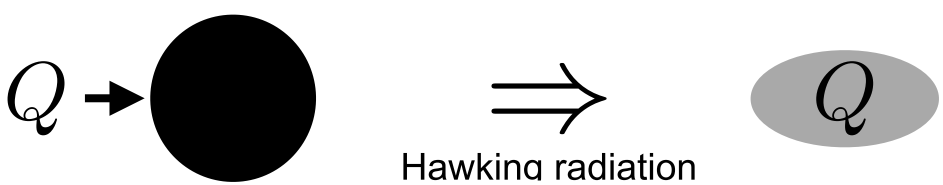
- One of the most famous examples:

  No global symmetry conjecture : Coleman, Gidding, Strominger, Kallosh, Linde, Linde, Susskind, …. Banks, Seiberg,…+ many others

  No global symmetry in quantum gravity: either gauged or broken in UV.

- Various indirect arguments:
  perturbative string theory, black hole entropy, etc….



States with $E < E_0$ with arbitrary charge Q

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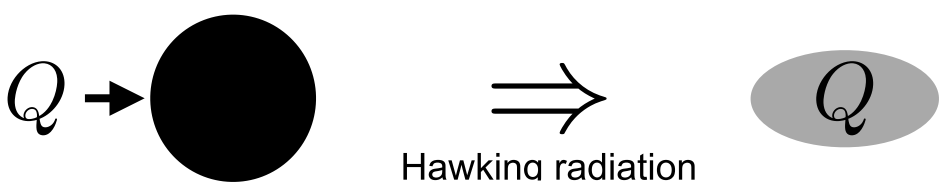
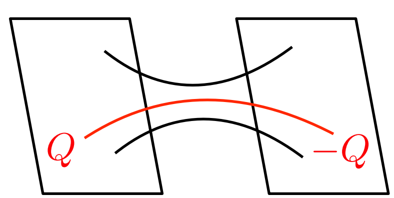


States with $E < E_0$ with arbitrary charge Q

- Symmetry violation from wormholes.

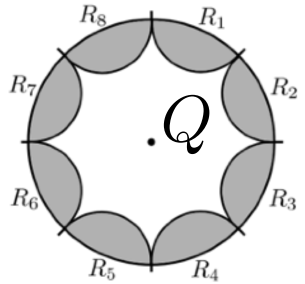
  ….Chen Lin, Hsin, Iliesiu, Yang, Milekhin, Tajdini, Belin, de Boer, Nayak, Sonner, Bah, Chen, Maldacena….

$\overline{AA^*} \supset$



No global symmetry in quantum gravity

- An argument based on AdS/CFT [Harlow, Ooguri 19]



Global symmetry in AdS QG is incompatible with entanglement wedge reconstruction (EWR).

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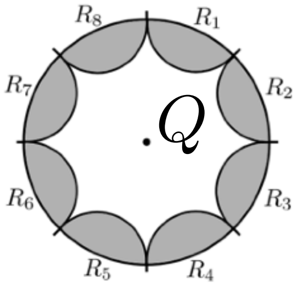

Global symmetry in AdS QG is incompatible with entanglement wedge reconstruction (EWR).

- EWR works only in the semiclassical gravity regime, expected to be modified by stringy / QG corrections.

- But no-global-symmetry conjecture is believed to be valid beyond semiclassical GR.

- Can we prove no global symmetry conjecture in full QG regime by translating it to a CFT statement using AdS/CFT?

No global symmetry in AdS = ?? in CFT

- Well-known fact about AdS/CFT:

  Gauge symmetry in AdS QG $\leftrightarrow$ Conserved current on CFT

$$A_\mu \leftrightarrow J_\mu$$

- Global symmetry in AdS QG $\leftrightarrow$

  Global symmetry in CFT without conserved current

$$\mathcal{O}_Q \quad \to \quad e^{i\alpha Q}\mathcal{O}_Q \qquad \text{but no } J_\mu$$

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- **CFT counterpart of the no-global symmetry conjecture:**

CFTs with stress tensor $T_{\mu\nu}$ and global symmetry G but no conserved current J_μ must be inconsistent.

No global symmetry in AdS = ?? in CFT

CFTs with stress tensor $T_{\mu\nu}$ and global symmetry G but no conserved current J_μ must be inconsistent.

- Clear CFT statements, but not necessarily easy to prove.
  - How do we characterize CFT with global symmetry without using conserved current?
  - As long as we look at finite number of correlators, it is difficult to distinguish "accidental" selection rules from genuine global symmetry.
  - Precisely speaking, there are counterexamples (e.g. multiple copies of Maxwell theories). $J_\mu = A^{\nu,(i)}\partial_\mu A_\nu^{(j)} - A^{\nu,(j)}\partial_\mu A_\nu^{(i)}$

- We don't know (yet) how to prove this in general. But there is actually one setup in which we can make progress.

Shift symmetry is easier

- Shift symmetry: theory is invariant under $\phi \rightarrow \phi + c$
- Clear distinction between gauged and global symmetries.
  Global shift symmetry: massless scalar
  Gauged shift symmetry: Higgs mechanism, massive scalar, no symmetry

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CFT counterpart of shift symmetry in AdS

- Shift symmetry also changes the boundary value of ϕ: $\phi_{\rm bdy} \to \phi_{\rm bdy} + c$
- In AdS/CFT, $\phi_{\rm bdy}$ is exactly marginal coupling constant in CFT.

$$Z_{\phi_{\rm bdy}} = \left\langle e^{-\int \phi_{\rm bdy} \mathcal{O}} \right\rangle$$

$$\phi : \text{massless} \quad \mapsto \quad \mathcal{O} : \text{marginal } (\Delta = d)$$

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- So shift symmetry in AdS corresponds to
  CFT which is invariant under the change of exactly marginal coupling constant $\phi_{\rm bdy} \to \phi_{\rm bdy} + c$

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- So shift symmetry in AdS corresponds to
  CFT with a "trivial conformal manifold" (parametrized by $\phi_{\rm bdy}$), on which no CFT data changes

  or equivalently,

  CFT with an exactly marginal operator $\mathcal{O}$ and the deformation by $\mathcal{O}$ leaves all the CFT data invariant.

CFT counterpart of no shift in AdS

CFT "No Shift" conjecture

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* One technical assumption (*non-derivative condition*):

$$\int_D \mathcal{O} \quad \neq \quad U_{\partial D}$$

- Integral of $\mathcal{O}$ in domain D cannot be expressed as an operator supported only on ∂D
- Excludes descendant $\mathcal{O}$ and other pathological cases (e.g. non-compact boson in 2d, Maxwell with non-compact gauge group)

** It is important to consider CFT with stress tensor. Without stress tensor, there **are** examples of trivial conformal manifold.

(More on next slide)

Examples of trivial conformal mfd without $T_{\mu\nu}$

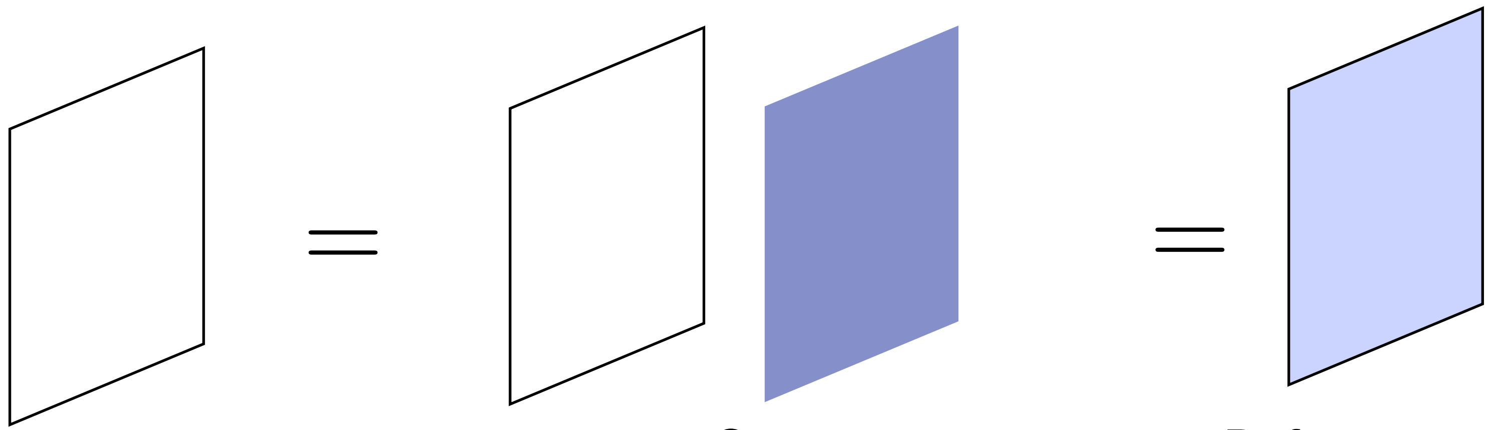
- Example 1: CFT with symmetry G and conformal defect breaks it down to H.

Drukker, Kong, Sakkas, ….

→ Defect conformal manifold G/H , every point on the manifold is equivalent.

$$\partial_\mu J^\mu = \underline{\tau(x_\parallel)}\delta^{d-p}(x_\perp)$$

Tilt operator, $\Delta = p$, marginal



Defect = Defect, Symmetry operator ($\phi \to \phi + c$) = Defect + $\exp\left(+c\int\tau\right)$

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Example: BPS Wilson loop in $\mathcal{N} = 4$ SYM

$$\mathrm{Tr}\left[\mathrm{P}\exp\left(\int dt\, iA_\mu \dot{x}^\mu + \Phi_6|\dot{x}|\right)\right]$$

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- Example 2: Free shift symmetric scalar in AdS without gravity.

→ obviously UV complete, generalized free field CFT, no stress tensor.

Plan

1. CFT argument for no shift symmetry
2. Generalization to higher form
3. Modular bootstrap
4. Conclusion

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General strategy

- A proof by contradiction in two steps:

- **Step 0**: Assume ∃CFT with a trivial conformal manifold

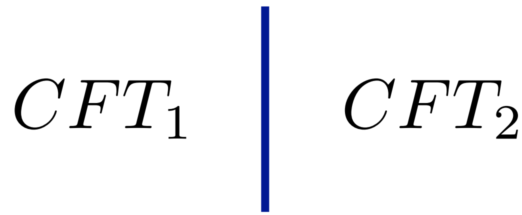
- **Step 1**: Use a CFT bootstrap argument to show one can construct "identity conformal interface" for any two points on a trivial conformal manifold.

- **Step 2**: Show that the existence of the identity conformal interface between nearby CFTs contradicts conformal perturbation theory

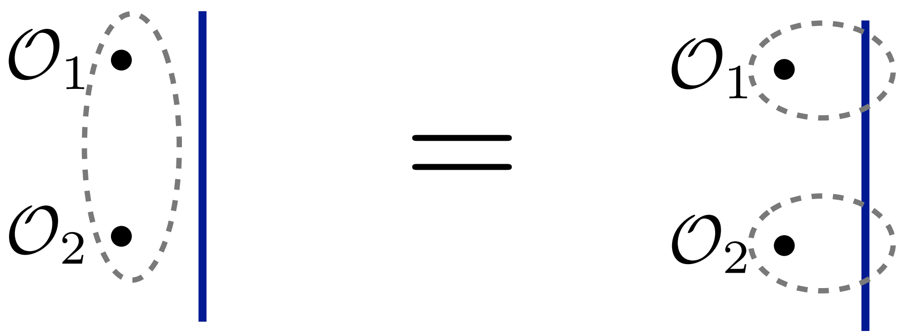
Step 1: **Trivial conformal mfd implies identity interface**

Conformal interfaces

- Conformal interface $SO(d+1,1) \rightarrow SO(d,1)$



- Needs to satisfy various bulk-interface crossing eq



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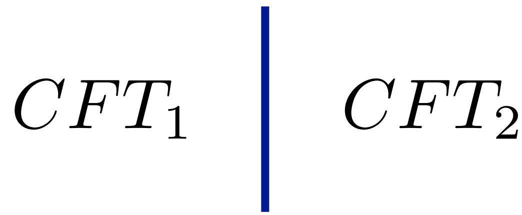

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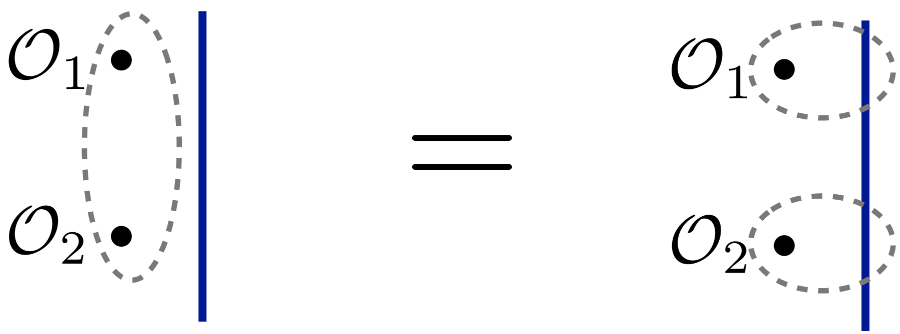

- Interface crossing eq: Bulk CFT data $\Rightarrow$ Interface CFT data
- Microscopic construction of interfaces: imposing bc to bulk fields, coupling to localized dof….
- In the modern bootstrap, sol's to crossing = conformal interface

Trivial conformal mfd implies identity interface

- On trivial conformal manifold, CFT_1 and CFT_2 share the same bulk CFT data by assumption.
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- On trivial conformal manifold, CFT_1 and CFT_2 share the same bulk CFT data by assumption.
- Interface crossing eq: Bulk CFT data $\Rightarrow$ Interface CFT data
- 1-to-1 correspondence between sol's for interface bootstrap: interfaces within CFT_1 $\leftrightarrow$ interfaces connecting $CFT_{1,2}$

$$CFT_1 \;\Big|\; CFT_1 \quad \longleftrightarrow \quad CFT_1 \;\Big|\; CFT_2$$

- In particular, it implies the "identity interface" between $CFT_{1,2}$ that corresponds to inserting nothing in CFT_1

$$CFT_1 \;\vdots\; CFT_1 \quad \longleftrightarrow \quad CFT_1 \;\Big|\; CFT_2$$

Comment

- I have shown that, if no CFT data changes, one can construct “identity interface” between $CFT_{1,2}$.
- This sounds obvious, but it is actually a key point of the proof.
- **Contrapositive:** If there is no identity interface, CFT data changes.

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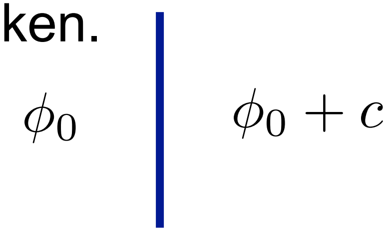
- I have shown that, if no CFT data changes, one can construct "identity interface" between $CFT_{1,2}$.

- This sounds obvious, but it is actually a key point of the proof.

- **Contrapositive:** If there is no identity interface, CFT data changes.

- **Gravity counterpart:** If there is no tensionless domain wall, shift symmetry is dynamically broken.



- There are arguments pointing out difficulty of constructing such domain walls within gravitational EFT due to large back reaction.

Bah, Jefferson, Roumpedakis, Waddleton,…

- But such arguments do not tell anything about shift-symmetry breaking. So the claim is highly non-trivial from gravity viewpoint.

- In AdS_3/CFT_2, non-topologicalness $\leftrightarrow$ symmetry breaking (Later)

modular bootstrap

Step 2: **Identity interface contradicts conformal perturbation theory**

Identity interface for nearby CFTs

- Let's analyze the "identity interface" between nearby CFTs.

$$= CFT_1 + \delta\phi \int_{x\perp \geq 0} \mathcal{O}$$

- Used a bootstrap argument to show its existence without knowing microscopic constructions.
- But we do know
  - In the limit $CFT_{1,2}$ coincide, it reduces to "inserting nothing".
  - Infinitesimally, $CFT_2 = CFT_1 + \delta\phi \int \mathcal{O}$

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- So infinitesimally,

"Identity interface" = $CFT_1 + \delta\phi \int_{x_\perp \geq 0} \mathcal{O} + \delta\rho \int_{x_\perp = 0} \mathcal{O}'$

Additional deformation localized at interface

- $\mathcal{O}'$ can be a local op. on the interface or some non-local expression on the interface.

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  Additional deformation localized at interface

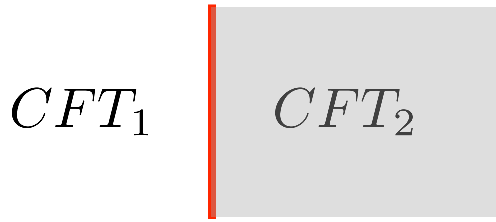
- $\mathcal{O}'$ can be a local op. on the interface or some non-local expression on the interface.

- No deformation away from the interface is allowed since it will violate the stress tensor conservation in the bulk. $\partial_\mu T^{\mu\nu} \neq 0$

- (In CFT without stress tensor, such deformations are allowed)

  [Herzog, Shamir]

Identity interface is inconsistent



"Identity interface" =

$$CFT_1 + \delta\phi \int_{x_\perp \geq 0} \mathcal{O} + \delta\rho \int_{x_\perp = 0} \mathcal{O}'$$

- Consider "Displacement operator" D.

$$\partial_\mu T^{\mu\perp} = D(x_{||})\delta^{d-1}(x_\perp)$$

- For the deformation above, one can show [SK, Kusuki, Meineri, Ooguri '25]

$$D = \delta\phi\, \mathcal{O} + \delta\rho\, \partial_\perp \mathcal{O}' + (\text{higher order})$$

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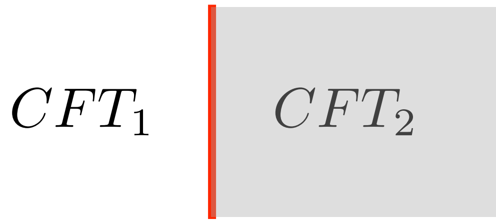

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$$D = \delta\phi\, \mathcal{O} + \delta\rho\, \partial_\perp \mathcal{O}' + (\text{higher order})$$

- But for the identity interface, $D = 0$.

- The only way to get $D = 0$ is to set $\mathcal{O} \propto \partial_\perp \mathcal{O}'$. But this violates the *non-derivative assumption*. **Contradiction.**

- Thus, the identity interface contradicts with conformal perturbation. Hence, the shift symmetry must be broken.

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Generalization to higher form symmetry

- 1-form symmetries of Maxwell theory.

$$dF = 0\,, \quad d * F = 0$$

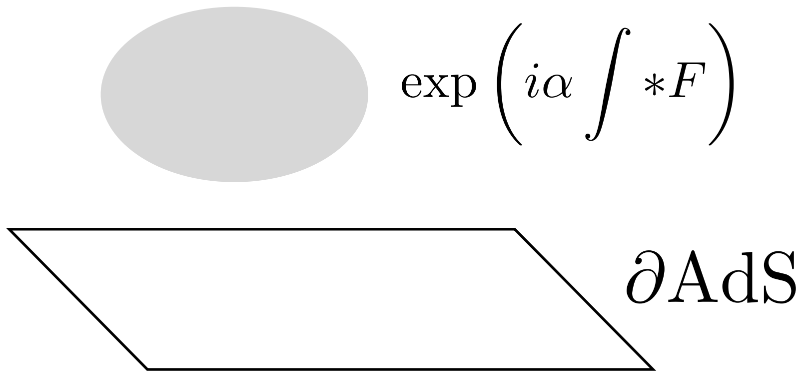
- In QG, they are expected to be violated by the presence of charged particles.

Related to charge completeness conjecture

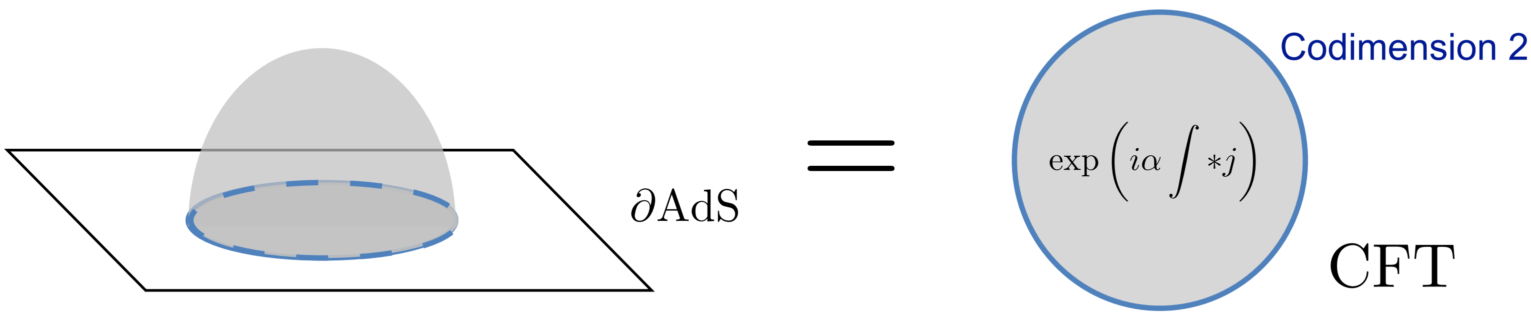
- Can we show the absence of 1-form symmetries of Maxwell theory in AdS QG?

Generalization to higher form symmetry

- If there was (electric) 1-form symmetry, one can construct the codim-2 topological interface in AdS.



- By pushing them to boundary, one gets a monodromy defect (co-dimension 2 + codimension 1)

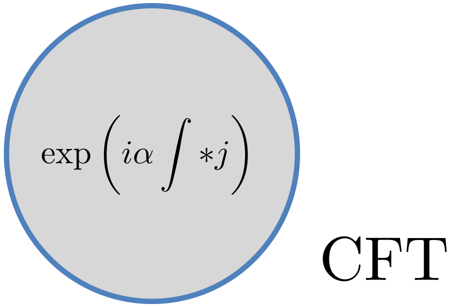


- 1-form symmetry in the bulk predicts that the monodromy defect is topological.

CFT counterpart of no 1-form symmetry in AdS

CFT "No 1-form" conjecture

In CFT with stress tensor, monodromy defects cannot be topological.



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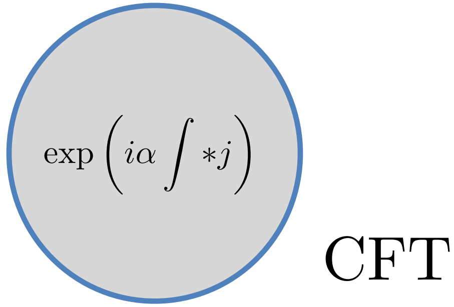

- Working in $\alpha \ll 1$ regime, one can show that the displacement operator is nonzero.

$$D \propto j$$

Contradiction to topologicalness.

- One can also argue that any local deformation at the defect (edge of the circle) cannot remedy this.

Thus, the topological monodromy defect cannot arise in CFT.

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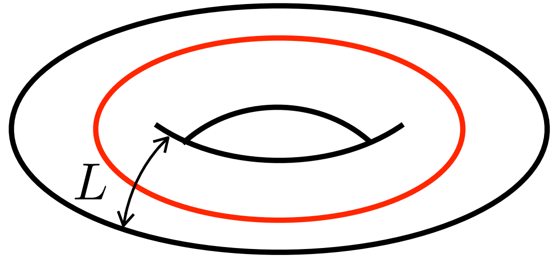
Bootstrap estimate on 1-form symmetry violation



- Consider a torus partition function with $e^{i\alpha\int *j}$ inserted.
- Untwisted channel (L is time)

$$I(\alpha) = \sum_a e^{-E_a L + i\alpha Q_a}$$

- Twisted channel (L is space)

$$I(\alpha) \overset{L\to 0}{\sim} e^{-E_0(\alpha)/L}$$

$E_0(\alpha)$: Energy of ground state in twisted sector

- Equating both sides and taking derivatives:

$$\partial_\alpha^2 E_0(\alpha)\big|_{\alpha=0} = \lim_{L\to 0} L e^{\frac{c}{12L}} \sum_a Q_a^2 e^{-E_a L}$$

Non-topologicalness $\leftrightarrow$ violation of 1-form symmetry (existence of charged op)

Bootstrap estimate on 1-form symmetry violation

$$\partial_\alpha^2 E_0(\alpha)\Big|_{\alpha=0} = \lim_{L\to 0} L e^{\frac{c}{12L}} \sum_a Q_a^2 e^{-E_a L}$$

- $E_0(\alpha)$ can be computed by perturbation in α .

$$\int_{x_1\le x,y\le x_2} dxdy\langle j(x)j(y)\rangle = C_J\int_{x_1\le x,y\le x_2} dxdy\frac{1}{|x-y|^2} = C_J\log|x_1-x_2|$$

$$\Rightarrow E_0(\alpha) = C_J\alpha^2 + \cdots$$

- Quantitative estimate on the violation of 1-form symmetry

$$\langle Q^2\rangle \overset{\Delta\gg 1}{\sim} C_J\sqrt{\frac{\Delta}{c}}$$

- Analysis seems generalizable for bulk 0-form symmetry.

[In progress]

Conclusion

- CFT-based argument for no shift symmetry in AdS QG.
  - Works beyond the semi-classical gravity regime.
  - But works only for shift symmetry + spontaneously broken symmetries.

- Conformal interfaces seem useful for analyzing swampland-related question. [SK, Kusuki, Meineri, Ooguri]

- Monodromy defects also seem important.

  ⇒ A systematic bootstrap analysis of monodromy defect?
  Application to weak gravity conjecture?

- Quantitative bootstrap bound in higher dimensions?

- Other related swampland conjectures? Infinite distance conjecture?

- Non-spontaneously broken symmetries?