Regge's Inferno

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This presentation is based on

★ arXiv: 2603.10197 with **with Alessio Miscioscia, Fedor Popov**
★ wip **Alessio Miscioscia, Fedor Popov, and Sunjin Choi**

General problem

Determine the operator spectrum

$$\mathcal{O}_{\Delta, J_i, Q_a}$$

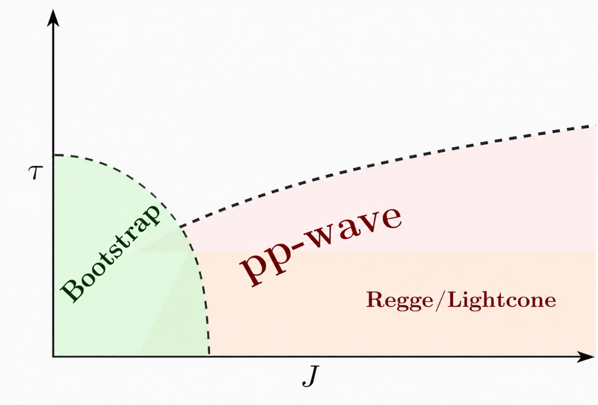
in an interacting CFT.

- Low-lying operators: numerical bootstrap.
- Large charge: EFT/superfluid descriptions.
- Large spin: today.

In 2+1 dimensional theories operators are classified by representations of $SO(3)$ and there is only one Cartan element J, and

$$\tau \equiv \Delta - J \geq \frac{1}{2} \ .$$

We will start from this example as a warm up.
Our goal is to understand operators with very large J.



Asymptotic Freedom at Large J.

Recall: $\tau = \Delta - J$.
At large J one considers multi-twist operators

$$\partial_+^{m_1}\mathcal{O}\cdots\partial_+^{m_n}\mathcal{O}.$$

Their total spin is

$$\sum_i m_i + nJ(\mathcal{O}) \ .$$

Approximate additivity

For $m_i \gg 1$,

$$\Delta \approx n\Delta_0 + \sum_i m_i$$

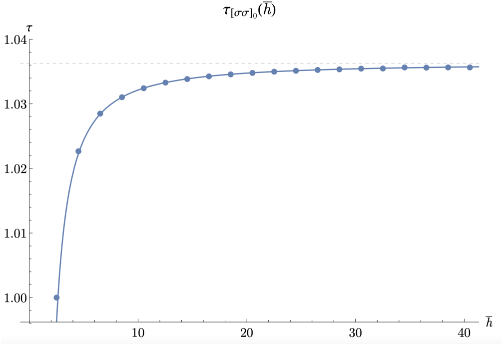
so twists add approximately:

$$\tau \approx n\tau_0.$$

At sufficiently large spin, composite operators behave as a *nearly-free Fock space of partons* .

[Alday, Maldacena; ZK, Zhiboedov; Fitzpatrick, Kaplan, Poland, Simmons-Duffin...]

The existence of such a Fock space even in strongly coupled theories is essentially confirmed.



Where the Fock-space picture breaks down

$$\partial_+^{m_1}\mathcal{O}\cdots\partial_+^{m_n}\mathcal{O}.$$

Interactions between partons grow with the number of constituents.

Breakdown scale

$$\tau_{\text{strong}} \sim \sqrt{J}$$

At this scale, one should replace the partonic picture by a different description.

[Cuomo,ZK; Fardelli, Fitzpatrick, Li; Kravchuk, Mann]

Inspired by recent work

Semi-universality of CFT$_d$ entropy at large spin

Harsh Anand[a,1] Nathan Benjamin,[b,2] Vipul Kumar,[a,3] Shiraz Minwalla,[a,4] Jyotirmoy Mukherjee,[a,5] Sridip Pal,[c,6] Asikur Rahaman[a,7]

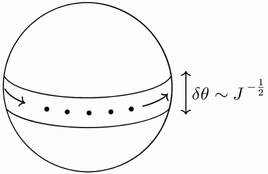
we will now describe a scaling limit with the properties

$$\boxed{\frac{\tau}{\sqrt{J}} = \text{finite} \;, \quad J \to \infty}$$

The Fock-space picture is only valid for $\frac{\tau}{\sqrt{J}} \ll 1$.

We found the following Brinkmann metric captures these states:

$$\boxed{ds^2 = -(1+y^2)dt^2 + dtdx + dy^2}$$



We obtain this metric as a Penrose limit around the equator of S^2. This metric admits a Heisenberg isometry group H_3 generated by

$$K_1 = \partial_x,$$

$$K_2 = \cos t\,\partial_y + y\sin t\,\partial_x, \quad K_3 = \sin t\,\partial_y - y\cos t\,\partial_x.$$

Heisenberg symmetry organizes the high-spin spectrum.

We do not have to view this geometry as a Penrose limit. We can also view it as time evolution by

$$\underbrace{\Delta + M_{+-}}_{=\tau, \geq 0} + \underbrace{P_-}_{\geq 0} \quad .$$

The P_- term is necessary to resolve infinite degeneracies and to obtain sensible thermodynamics. (It arises naturally from the Penrose limit, too.)

In general space dimension d with one large angular momentum we obtain the metric

$$\boxed{ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2} \ . \qquad (1)$$

with a Heisenberg isometry group $H_{2(d-1)+1}$.

An interesting application is to partition functions $\tau = \Delta - J$

$$Z(\beta, \varepsilon) = \sum e^{-\beta\tau - \beta\varepsilon J} = \text{Exp}\left[\frac{2\pi\beta}{\varepsilon} F(\beta) + O(\varepsilon^0)\right].$$

Alternatively,

$$e^S \sim \text{Exp}\left[\sqrt{J}\tilde{F}\left(\frac{\tau}{\sqrt{J}}\right) + O(\varepsilon^0)\right].$$

Interpretation

$$\frac{2\pi}{\varepsilon} \sim \sqrt{J}$$

acts as *emergent volume*. This is just the volume of the spatial slice of the pp wave. $F(\beta)$ is the pressure of the quantum fields on the pp wave.

Emergent Extensivity

The large-spin sector behaves like a thermodynamic system with entropy

$$S(E, V) = V\, s(E/V)$$

The macroscopic variable is *effective volume generated by large angular momentum*.

Locality on the ppwave implies

$$\log Z \propto \beta \times \underbrace{(\text{effective volume})}_{\frac{2\pi}{\epsilon}} \times F(\beta).$$

This is the geometric origin of the universal large-spin density of states.

Due to the large prefactor in the exponent

$$Z \sim \mathrm{Exp}\left[\frac{2\pi\beta}{\varepsilon}F(\beta) + O(\varepsilon^0)\right] \ ,$$

$$e^S \sim \mathrm{Exp}\left[\sqrt{J}\tilde{F}\left(\frac{\tau}{\sqrt{J}}\right) + O(\varepsilon^0)\right]$$

there may be phase transitions, i.e. non-analyticity in $F, \tilde{F}$.
This *does not* require large N.

For instance, free scalars in 2+1 dimensions give

$$F(\beta) = \frac{1}{4\pi\beta^2} \sum_{m=1}^{\infty} \frac{1}{m^2 \sinh(m\beta/2)} .$$

similar formulas hold in other free conformal theories. In this model, there are no phase transitions.

A more complicated model is the $O(N)$ model, which we solve in the large N limit in this high spin regime.
For each longitudinal momentum $-i\frac{\partial}{\partial x} = p$ the problem reduces to the diagonalization of the one-dimensional operator

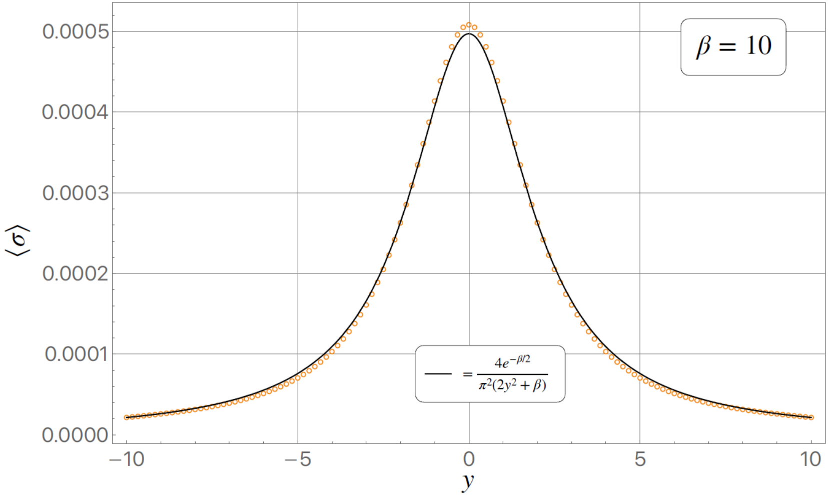
$$H_p[\sigma] = -\partial_y^2 + 4p^2y^2 + \sigma(y),$$

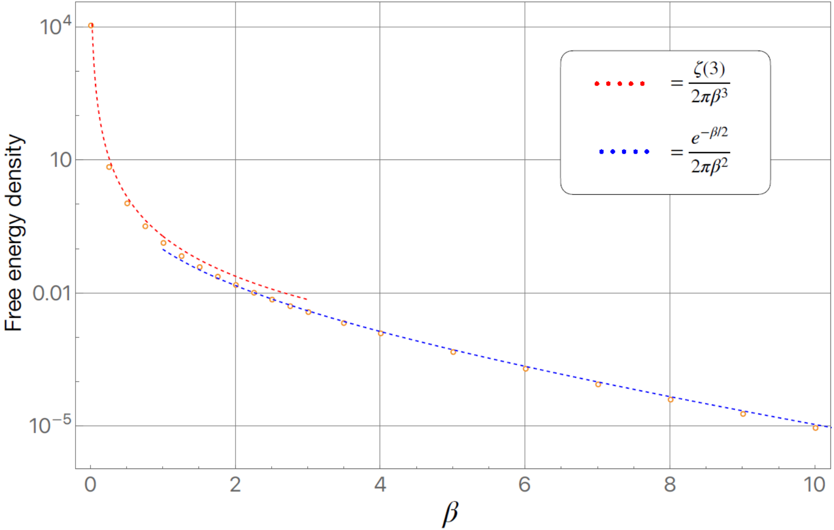
We then calculate the partition function

$$\frac{\log Z}{V_{pp-wave}} = -\int_0^\infty \frac{dp}{2\pi}\sum_a \log\left(1 - e^{-\beta E_a(p)}\right).$$

regualarize, and extremize over $\sigma(y)$.

We do not know how to do this analytically on the pp wave.





10^4
10
0.01
10^{-5}
Free energy density
0
2
4
6
8
10
β
$= \frac{\zeta(3)}{2\pi\beta^3}$
$= \frac{e^{-\beta/2}}{2\pi\beta^2}$

We see a crossover between operators with $\tau/\sqrt{J} \gg 1$ (fluid-like) vs small $\tau/\sqrt{J} \ll 1$ (Fock-space).

A richer example is $SU(2)$ $\mathcal{N} = 4$ SYM theory in 3+1 dimensions. Since $\Delta - J$ commutes with supersymmetry, we can preserve supersymmetry in the pp wave. We can calculate the usual partition function or a protected version thereof.

$$Z_{protected} = \mathrm{Tr}_{BPS}(-1)^F p^{\Delta - J + \frac{2}{3}r} \ .$$

The theory is placed in a pp wave with a constant chemical potential. We denote $p = e^{-\beta}$.

$$Z_{protected} = \mathrm{Tr}_{BPS}(-1)^F p^{\Delta - J + \frac{2}{3}r} \ .$$

This again diverges and needs to be regularized by an "effective volume" term, or alternatively, by adding $\epsilon(\Delta + J)$ the Hamiltonian.

While in flat space the theory is *deconfined* at any temperature, on the pp wave it is *confined* at low temperature and deconfined at high temperature. We will soon see that the confinement is "geometric".

The spectrum of spinning local operators encodes this phenomenon.

We can also interpret this phase transition as *monotonous* to *fortuitous* states. (Actually there are more phases.)

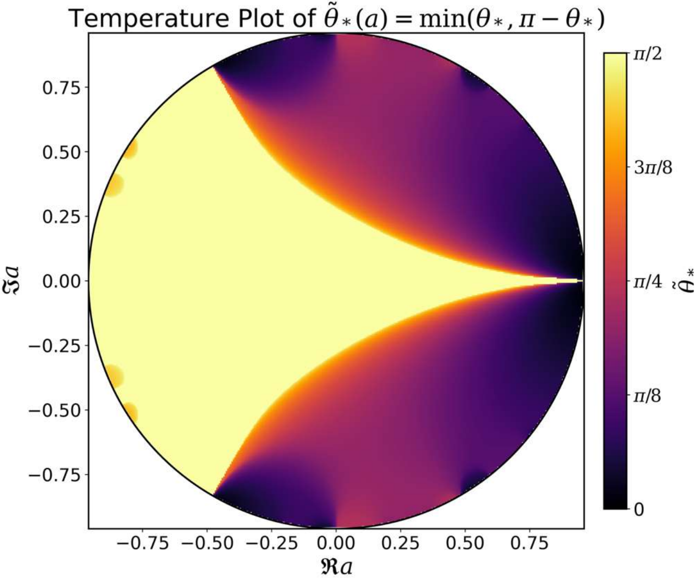
$$Z_{protected} \sim \int d\mu_{SU(2)}(\theta) \ \exp\left[\frac{1}{\epsilon} W_p(\theta)\right] ,$$

with

$$W_p(\theta) = \sum_{n=1}^{\infty} \frac{-1}{n^2} \frac{(1-p^{n/3})^2}{1+p^{n/3}+p^{2n/3}} \left[1 + 2\cos(2n\theta)\right] \ ,$$

and $p = e^{-\beta}$.

In the complex plane $a = p^3$ we find the following phase diagram



We see confinement at low temperatures and deconfined ("fortuitous") phases at high temperature on the pp wave.

Most of the literature is about the phases at infintie N, e.g. [...Cherman, Dhumunturao ...] here it is just $SU(2)$ SYM.

We would like to identify this transition in the highly spinning cohomology with the fortuituy of
[Chang, Lin; Choi, Kim, Lee, Park; ...]

Why do gauge fields confine on the pp wave?
The response to a static point charge at $x_1 = x_2 = 0$, $x = x_0$ is described by

$$\Phi(r, v) = q \int_{-\infty}^{\infty} \frac{dk}{2\pi} \, e^{ik(x - x_0)} \, g_k(r),$$

where

$$\left[-\frac{1}{r} \frac{d}{dr} \left(\frac{r}{1 + r^2} \frac{d}{dr} \right) + k^2 \right] g_k(r) = \frac{\delta(r)}{2\pi r}.$$

The electromagnetic field is

$$F = -\left(dt - \frac{dx}{1 + r^2} \right) \wedge d\Phi.$$

The stored Killing energy is

$$\mathcal{E} = \frac{1}{2} \int dx_1 dx_2 dx \left[\frac{|\nabla_\perp \Phi|^2}{1 + r^2} + (\partial_x \Phi)^2 \right].$$

For a point charge this energy is divergent both in the UV and in the IR:

$$\boxed{\mathcal{E} = \frac{q^2}{8\pi\epsilon} + \frac{q^2}{2\pi^2} \log L + O(1)}$$

This log confinement persists for arbitrary weakly coupled gauge fields. It should be related to
[Gross,Wilczek; Korchemsky, Radyushkin; Gubser, Klebanov, Polyakov...]

Consider again the metric

$$ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2 \ .$$

It describes time evolution by

$$\underbrace{\Delta + M_{+-}}_{=\tau, \geq 0} + \underbrace{P_-}_{\geq 0} \ .$$

A variant of it,

$$ds^2 = -\sum_{i=1}^{d-1} x_i^2 dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2 \ ,$$

describes time evolution by $\Delta + M_{+-}$.
In the absence of the $1+$, there is infinite degeneracy but the spectrum is bounded from below.

Now fix $d = 3$ and pass to a rotating frame in the x_1, x_2 plane. In flat space that means that for $\sqrt{x_1^2 + x_2^2} > 1$ we exceed the speed of light and the theory is sick. The metric in this rotating frame has a corresponding sickness.

Indeed, in flat space, $H - J$ is not bounded from below.

In our case the speed of light "grows" as $\sqrt{x_1^2 + x_2^2} > 1$ and hence the metric does not have any sickness in the rotating frame. Therefore, causality implies

$$\boxed{H - J = \Delta - J_1 - J_2 \geq 0\ .}$$

This is a new unitarity bound. Previously it was know that $\Delta - max(J_1, J_2) > 0$.
(In special cases stronger bounds can be found in [Cordova,Diab].)

Geometry in $3+1$ dimensions: a family of null geodesics

The physics of the situation with $J_{1,2} \to \infty$ with $J_1/J_2 = \mathrm{finite}$ is interesting.
With two large angular momenta, fast-spinning partons live at *different* latitudes on S^3.

So one cannot zoom around a single null geodesic. Instead one performs a generalized Penrose limit focusing on a family of null geodesics.

$$ds^2 = -(dt' + 2udy)^2 + (dv + 2udy)^2 + du^2 + dy^2 .$$

The isometries include

$$\times (H_3 \rtimes SO(2)),$$

with H_3 preserved by time translations.

$$ds^2 = -(dt' + 2udy)^2 + (dv + 2udy)^2 + du^2 + dy^2 .$$

This metric indeed has a global timelike Killing vector and no ergosphere.

If we begin with

$$ds^2 = -\left(1 + \sum_{i=1}^{d-1} x_i^2\right) dt^2 + dxdt + \sum_{i=1}^{d-1} dx_i^2 \ .$$

and transform to a rotating frame we obtain the same result.

Locality implies

$$\log Z = \beta \, \mathrm{Vol}(H_3) \, F(\beta).$$

with the replacement

$$\mathrm{Vol}(H_3) \longleftrightarrow \frac{\pi^2}{2\varepsilon_1\varepsilon_2}.$$

Therefore, In $3+1$ dimensions, with, $\tau = \Delta - J_1 - J_2$

$$Z(\beta, \varepsilon_{1,2}) = \sum_{\mathcal{O}} e^{-\beta\tau - \beta\varepsilon_1 J_1 - \beta\varepsilon_2 J_2} = \mathrm{Exp}\left[\frac{\beta\pi^2}{2\varepsilon_1\varepsilon_2} F(\beta) + O(\varepsilon^0)\right].$$

A subtlety is that since we only know that

$$\boxed{H - J = \Delta - J_1 - J_2 \geq 0}$$

but we do not know how to prove that there is a twist gap, the Fock space picture at low temperature may or may not be universal. Either way the Fock space picture breaks down for

$$\tau_{\text{strong}} \sim (J_1 J_2)^{1/3} \qquad (3+1 \text{ d with two large spins}).$$

The appropriate limit that we have been describing in 3+1 dimensions is

$$J_{1,2} \to \infty \ , \quad J_1/J_2 = \text{finite} \ , \quad \frac{\tau}{(J_1 J_2)^{1/3}} = \text{finite} \ .$$

The geometry

$$ds^2 = -(dt' + 2udy)^2 + (dv + 2udy)^2 + du^2 + dy^2 .$$

captures precisely these operators.

- Large spin with fixed $\tau/\sqrt{J}$ and $\tau/(J_1 J_2)^{1/3}$ is captured by pp waves.
- Large spin creates an *effective thermodynamic limit*. This allows phase transitions and we have discussed an example of a phase transition between *monotonous* and *fortuitous* states.
- Locality on pp waves provides a universal explanation of the large-spin free-energy scalings.
- In $3+1$ dimensions, causality leads to a new unitarity bound

$$\Delta - J_1 - J_2 \geq 0 \ .$$