Semiclassics

at the Cusp

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Outline

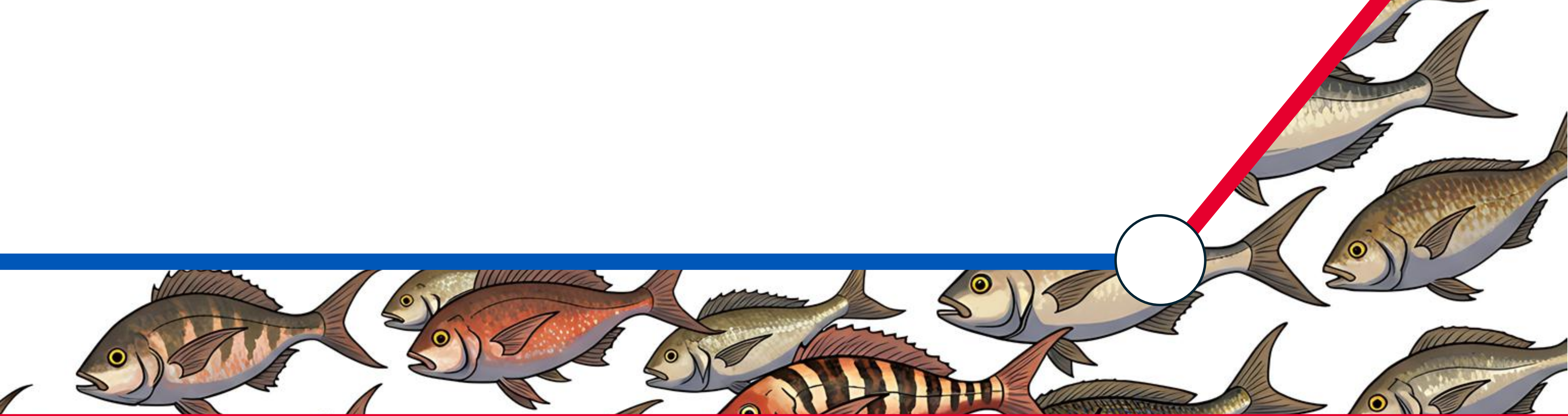
Motivation For Non-local Observables

Semiclassical Methods

Results and Directions

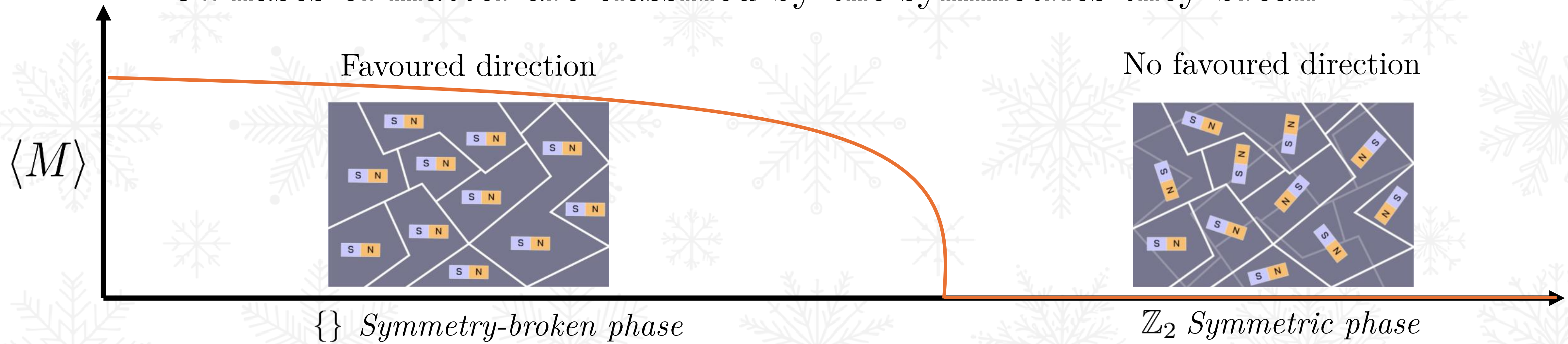
Upshot

- Large charge works for extended operators in gauge theory



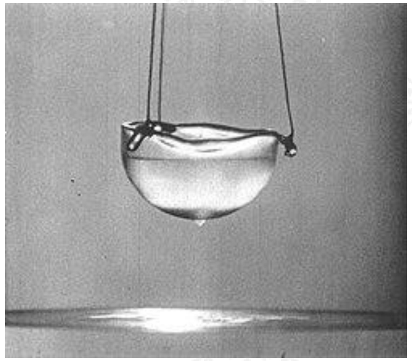
Order Parameter

- Landau Paradigm:
  - Phases of matter are classified by the symmetries they break



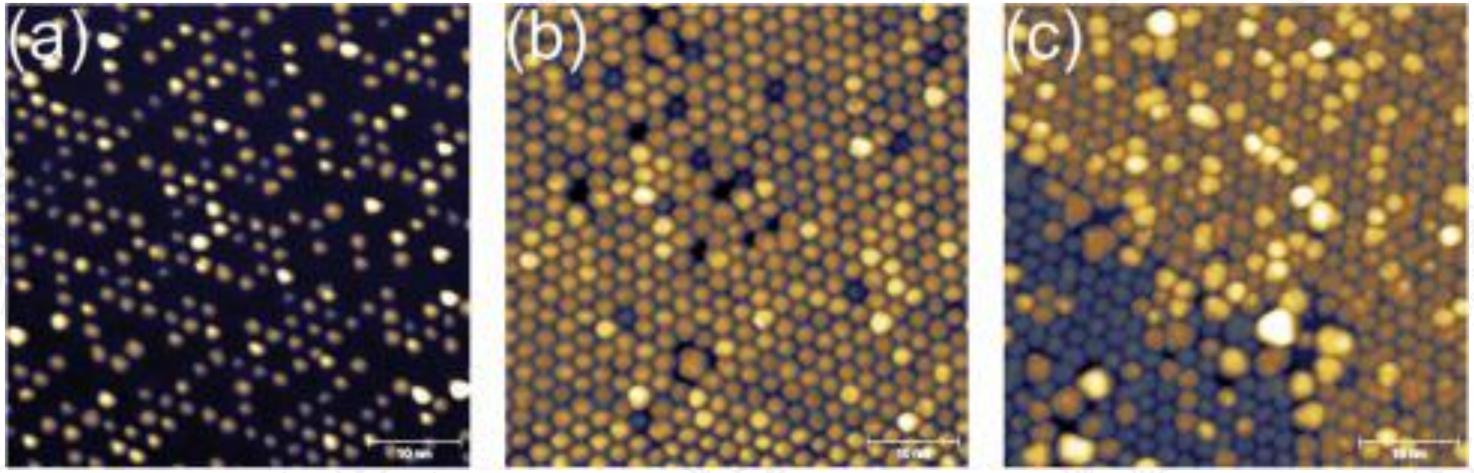
- Superfluid phase
- *U(1)* symmetry
  - *Conserved charge: particle number*

$$\langle \phi \rangle$$



Long range order

- How strongly do correlations persist over long distances?



Single-layer graphene sheets with long range order, Vesselli et al. (2017).

$$\langle \phi^*(x_2)\phi(x_1)\rangle \sim \frac{1}{|x_1 - x_2|^{(D-2)+\gamma/2}}$$

Anomalous dimension:
deviation from classical scaling dimension

Problem:

In superconductors, we expect the symmetry to be gauged

- Even the humble 2-pt function is not gauge invariant

$$\langle\phi^*(x_2)\phi(x_1)\rangle \to e^{i\left(\alpha(x_1)-\alpha(x_2)\right)}\langle\phi^*(x_2)\phi(x_1)\rangle$$

- Elitzur's Theorem:

$$\langle\phi^*(x_2)\phi(x_1)\rangle = 0$$

- Even if we insist, the result explicitly depends on the choice of gauge

$$\langle \phi^*(x_2)\phi(x_1)\rangle|_\xi \sim \frac{1}{|x_1 - x_2|^{(D-2)+\gamma(\xi)/2}}$$

- Feynmann diagram arguments say there is dependence only at 1-loop

$$\gamma(\xi) = \gamma_0 + \gamma_1(\xi)G + \mathcal{O}(G^2)$$

Dirac's Solution:

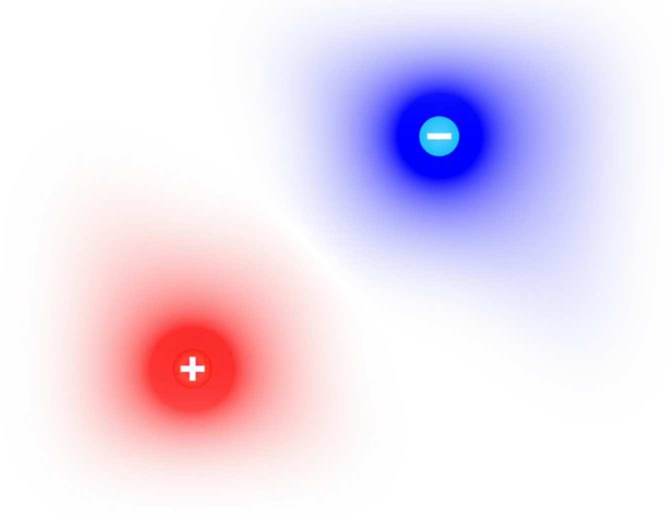
- Dress the fields with some nonlocal functional of the gauge field

$$\phi_J = \exp\left(\int dz\, J^\mu(z;x) A_\mu(z)\right) \phi(x)$$

$$\partial_\mu J^\mu = \delta(z - x)$$

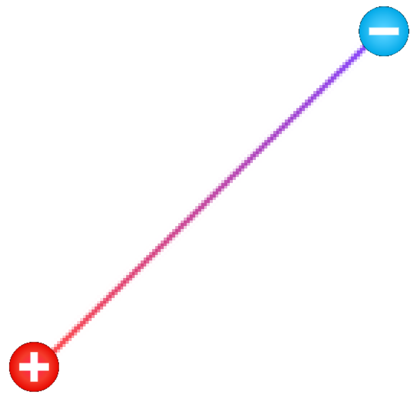
Two interesting cases of dressings:

Dirac dressing
(Coulomb field)



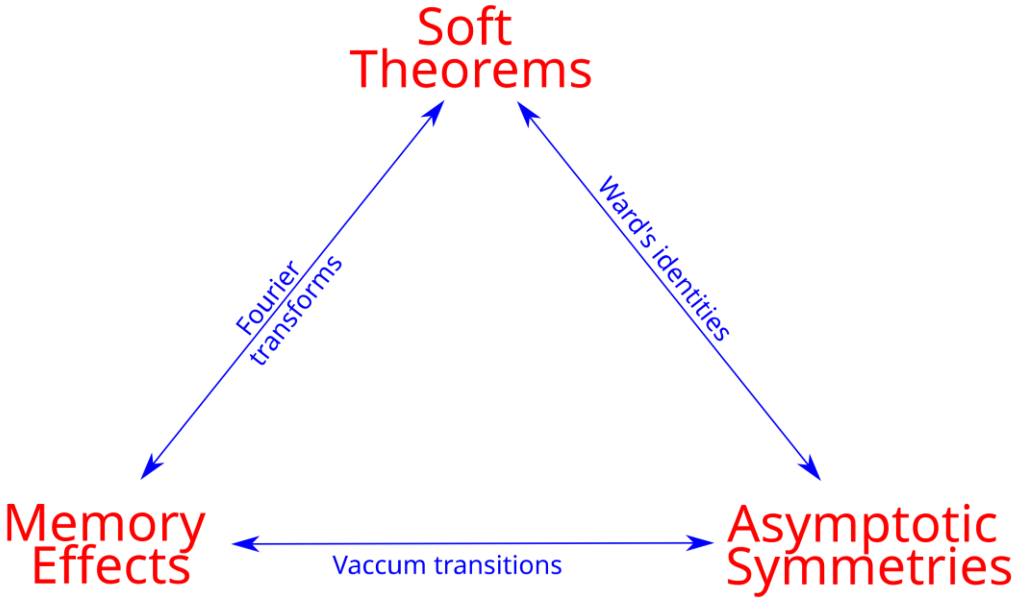
$$J^{\mu}_{D}(x;x_1,x_2) = \partial^{\mu}\left[G_0(y-x_2) - G_0(y-x_1)\right]$$

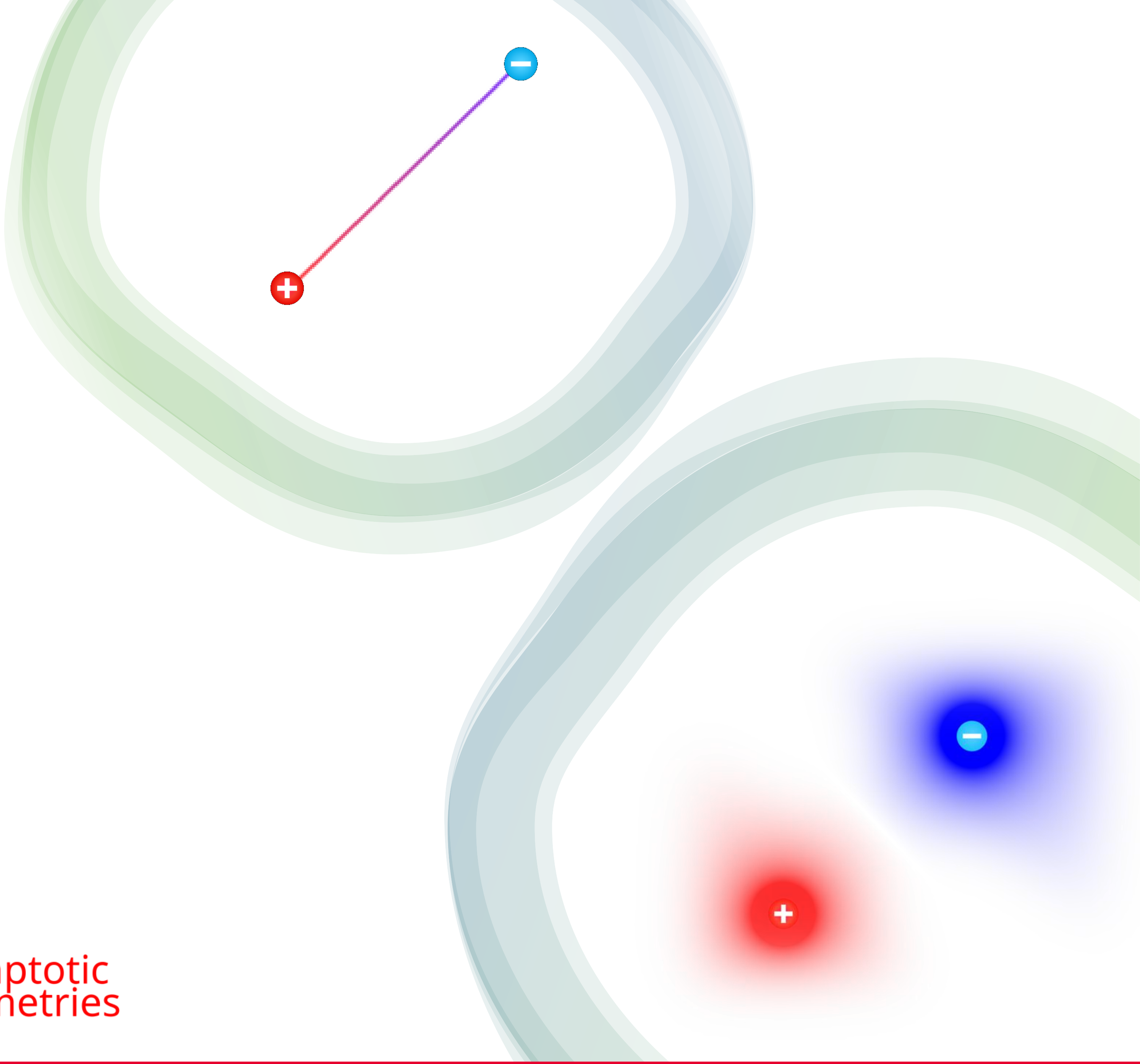
Mandelstam-Schwinger
dressing (Wilson line)



$$J^{\mu}_{\text{MS}}(x;\gamma) = \int_0^1 ds\delta\left(x-\gamma(s)\right)\frac{d\gamma^{\mu}(s)}{ds}$$

- Gauge-invariant
- Highly symmetric
- Interesting links to asymptotic symmetries and memory effect





Literature Claim:

- Dressing corresponds to a gauge choice.

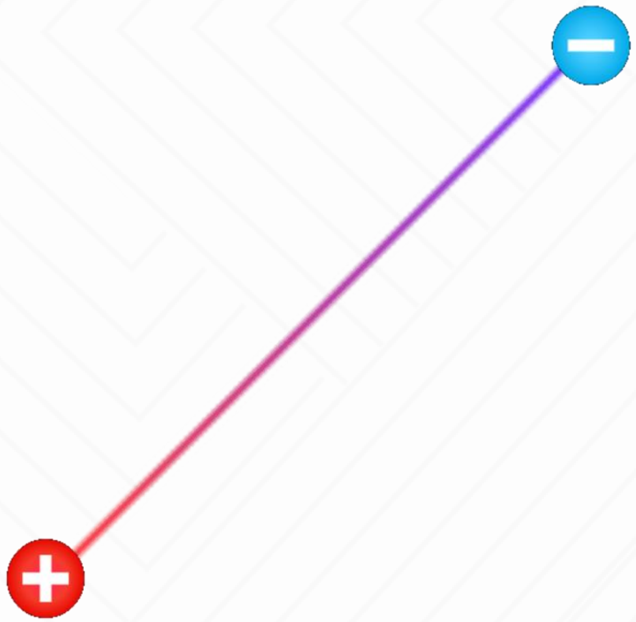
$$\gamma_{\xi=0} = \gamma_D$$

$$\gamma_{\xi=1-d} = \gamma_{MS}$$

[Kleinert, Schakel (2003)]

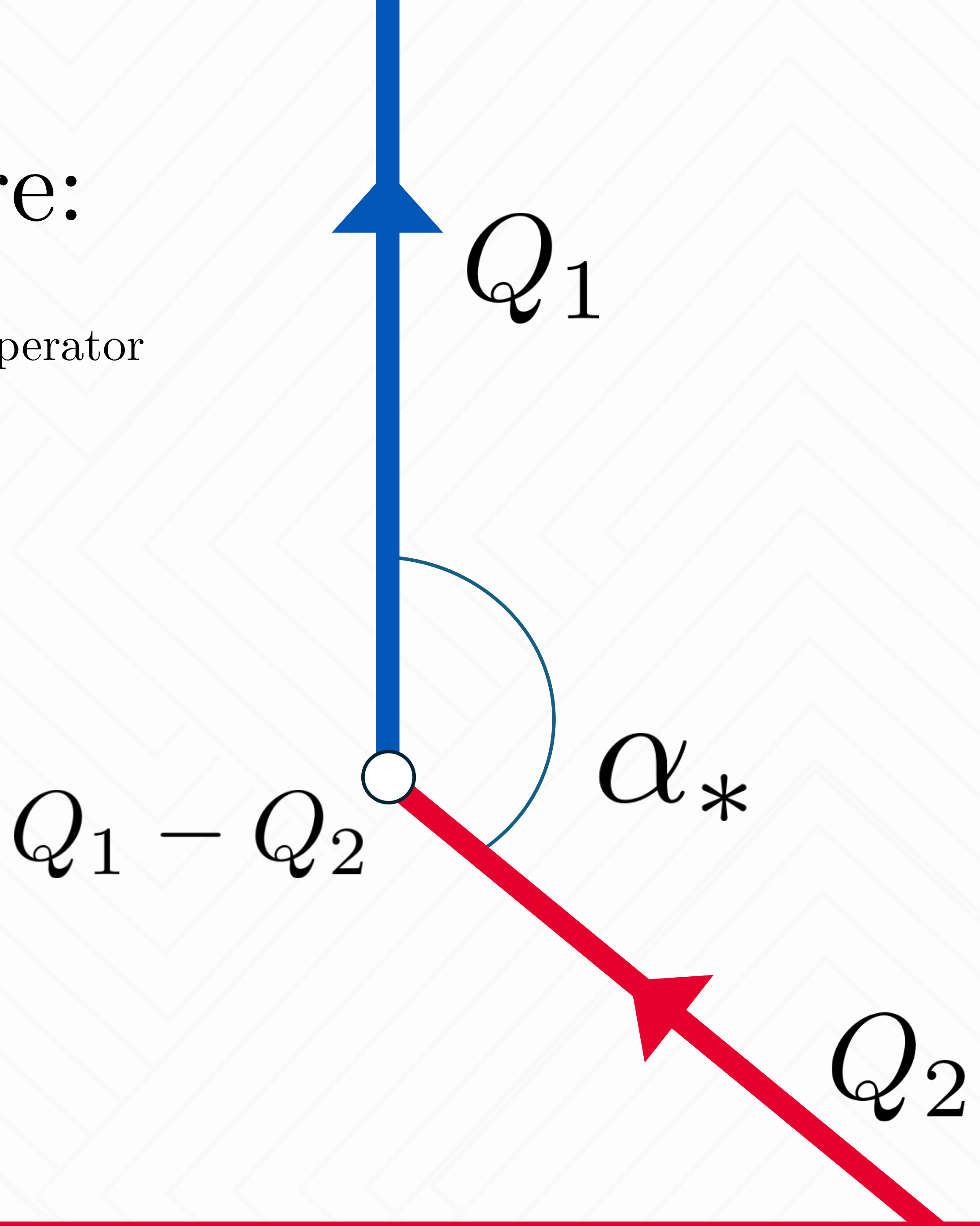
Beyond dressings

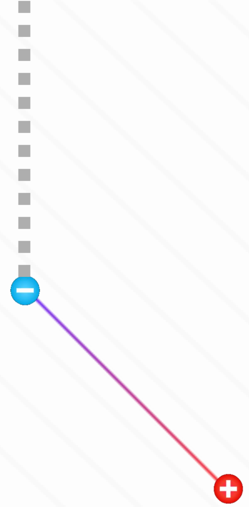
Lines are objects in their own right

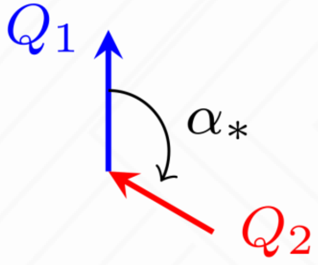


General picture:

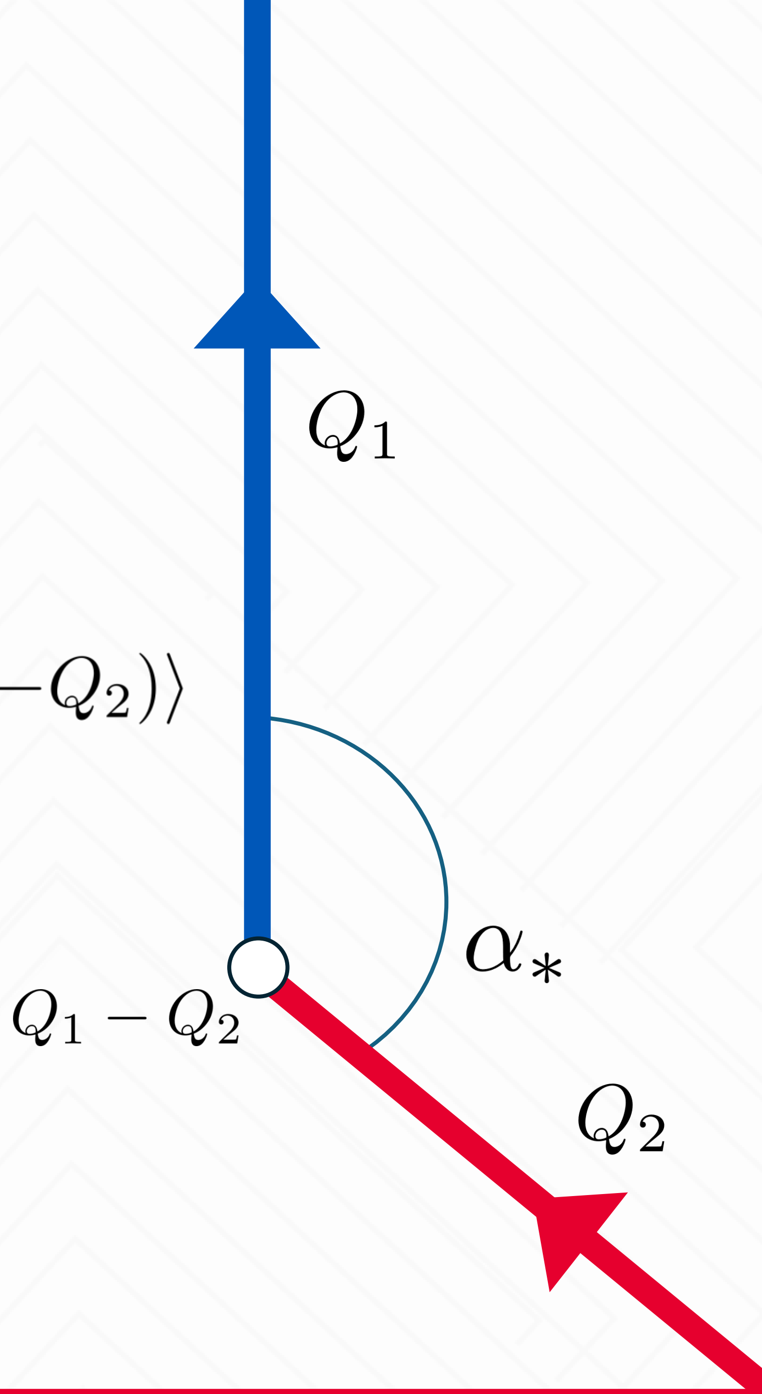
“Cusp” with defect-changing operator







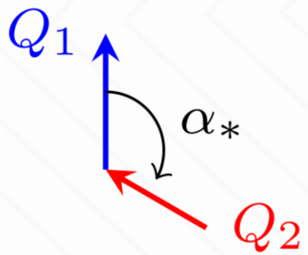
$$\Big\langle \quad \Big\rangle = \langle \phi^*(x_\infty)^{Q_1 - Q_2} W_{\gamma_1}(Q_1)\phi(0)^{Q_1 - Q_2} W_{\gamma_2}(-Q_2)\rangle$$



Cusp anomalous dimension

- Natural observable is the coefficient that describes the divergence

$$\log \Big\langle$$



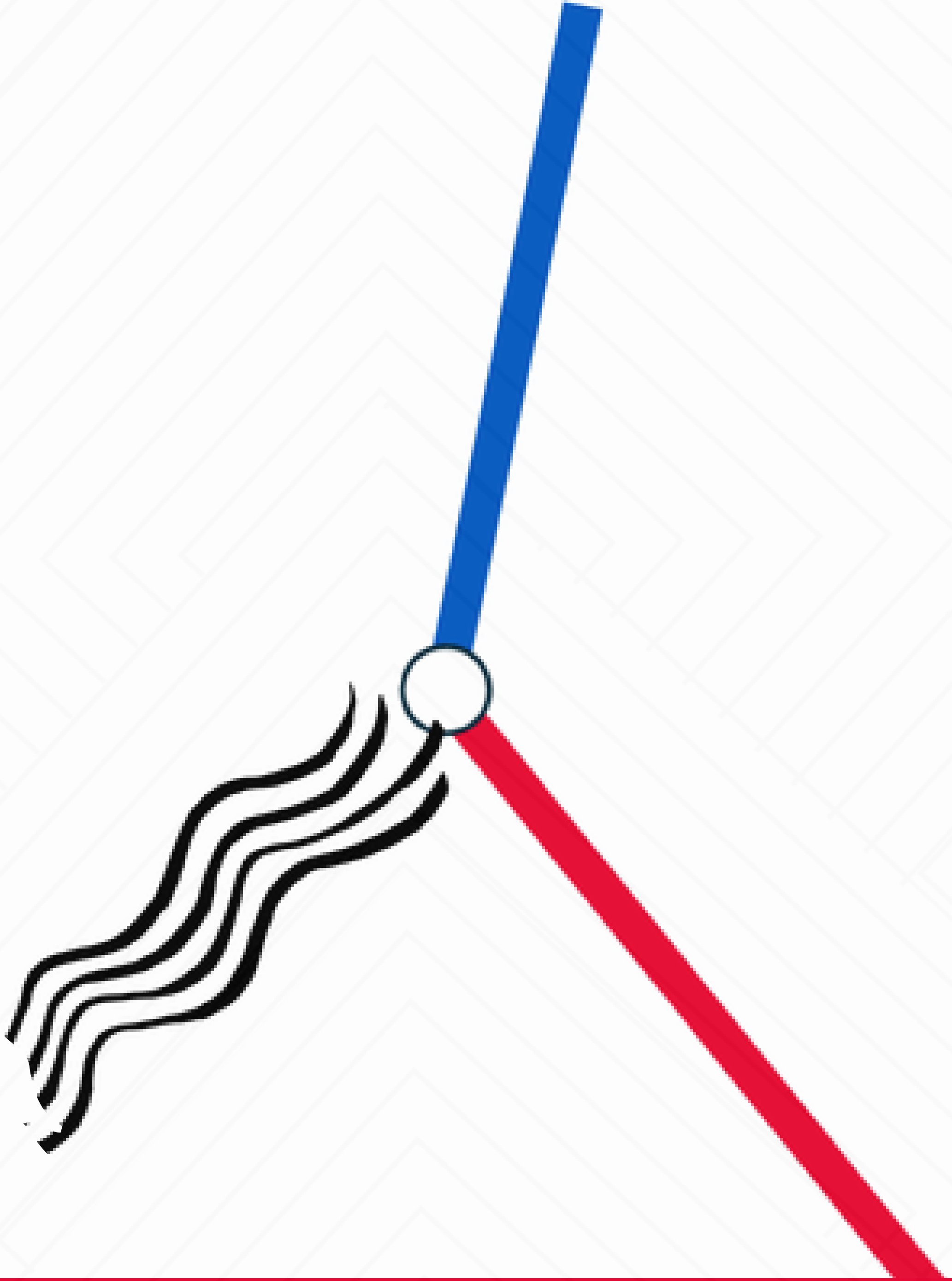
$$\Big\rangle = -\Gamma_{Q_1 Q_2}(\alpha_*) \log \frac{L_{IR}}{a_{UV}} + \dots ,$$

- What does this tell us?

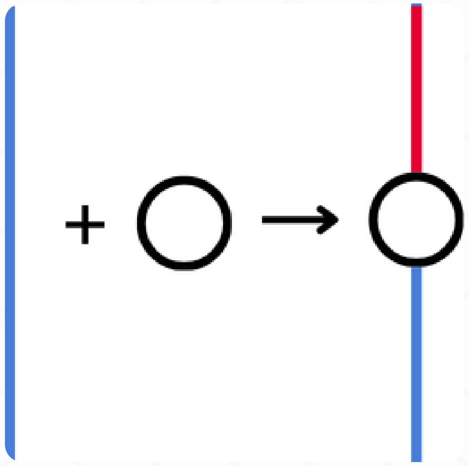
Bremsstrahlung

Change of velocity -> radiation emitted

$$\frac{1}{2}\beta_{Q_1Q_2}(\alpha_* - \pi)^2 \subset \Gamma_{Q_1Q_2}(\alpha_*)$$

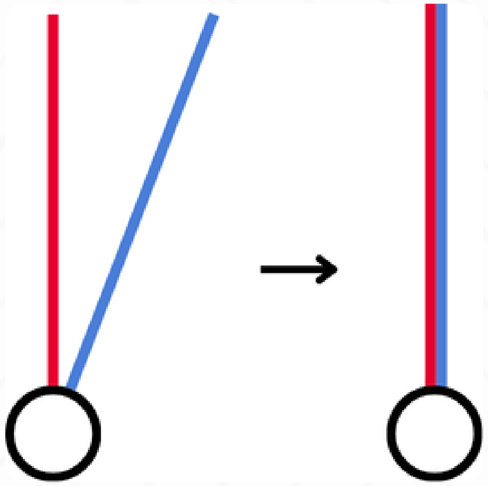


Defects



- Information about defect-changing operators (impurities)

$$\Delta_{Q_1Q_2}(\alpha_* - \pi)^0 \subset \Gamma_{Q_1Q_2}(\alpha_*)$$



- Encodes the self-fusion behaviour of the line

$$\frac{C_{Q_1Q_2}}{\alpha_*} \subset \Gamma_{Q_1Q_2}(\alpha_*)$$

Problem

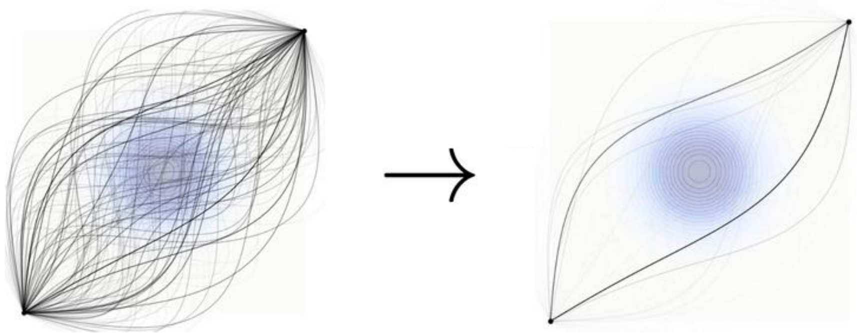
Interesting systems are strongly coupled

Solution

Semiclassical methods

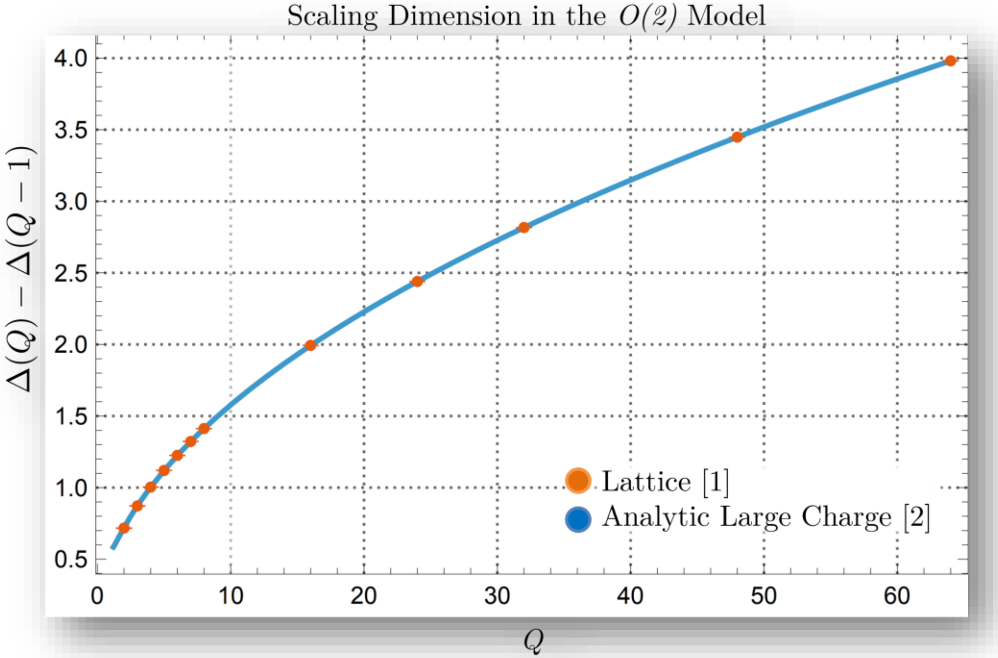
Semiclassical Methods

Idea: introduce a large parameter x to localise the path integral



$$Z = \int \mathcal{D}\phi e^{-S} \to \int \mathcal{D}\phi e^{-x\bar{S}} \approx e^{-xS_0}$$

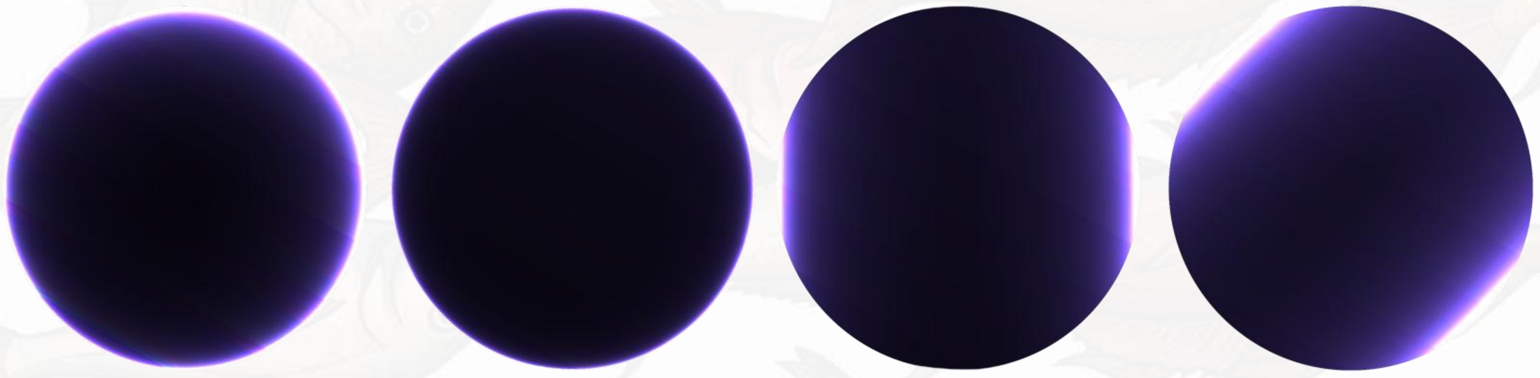
- Restrict to a sector of large quantum number: spin, charge, etc
- Works very well...



[1] Hasenbusch, M. (2026). [2] Banerjee, D., Chandrasekharan, S., & Orlando, D. (2018).

What about in Gauge Theory?

- Quantum numbers for global symmetries
- Gauge invariance?
- EFT picture: SSB vs Higgs Mechanism?



Abelian Higgs model

- Superconductivity

$$S_{\mathrm{AH}}[A,\phi] = \int d^d x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D^\mu \phi_a)^* D_\mu \phi_a + \frac{(4\pi)^2}{6} \lambda (\phi_a^* \phi_a)^2 \right]$$

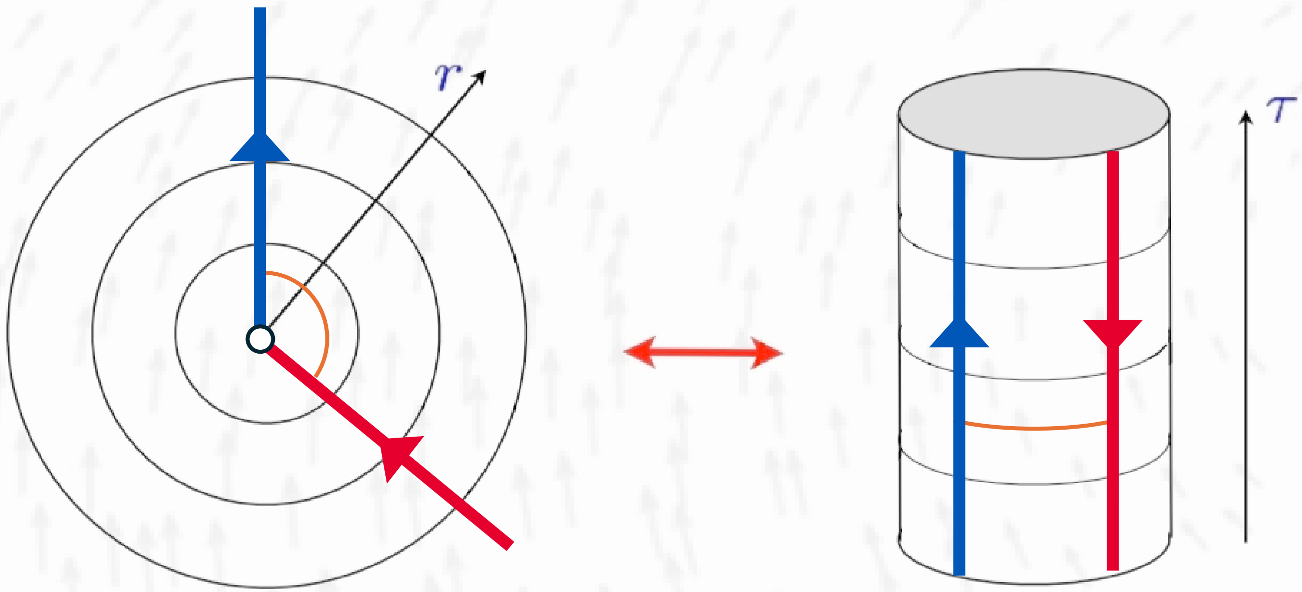
- λ controls superconductor rigidity and Cooper pair repulsion

Abelian Higgs model

- Can tune to a CFT!
- For sufficiently large (finite) N, in $d = 4 - \epsilon$:

$$\lambda^* = \frac{3\left(N+18\pm\sqrt{(N-180)N-540}\right)}{4N(N+4)}\epsilon + \mathcal{O}(\epsilon^2)\,, \qquad e^{*2} = \frac{24\pi^2}{N}\epsilon + \mathcal{O}(\epsilon^2)\,.$$

State Operator Correspondence



$$\Gamma_{Q_1 Q_2}(\alpha_*) = R \cdot E_{Q_1 Q_2, \alpha_*}$$

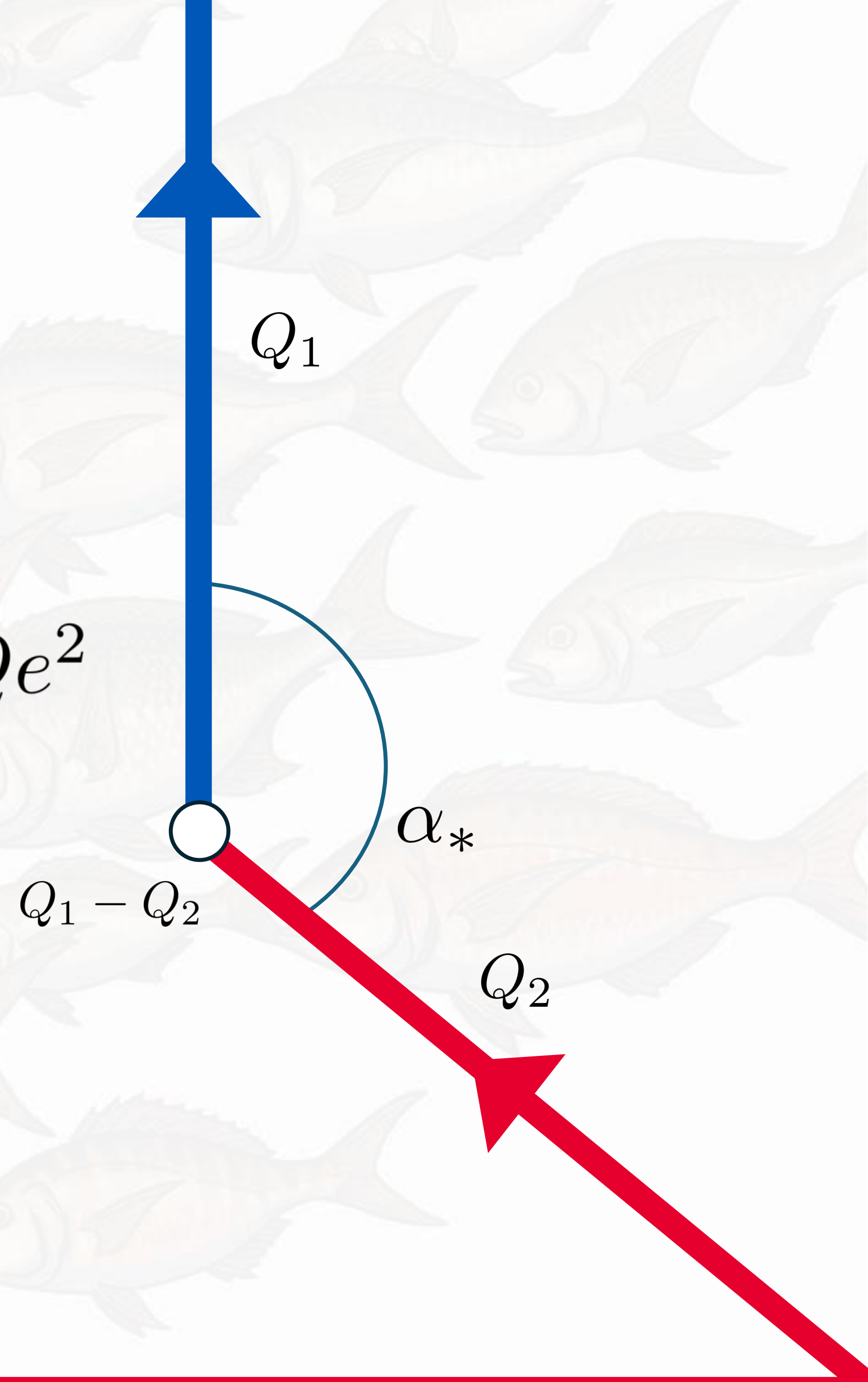
Semiclassical limit:

We take

$$Q_1 = q_1 Q, \quad Q_2 = q_2 Q, \quad \kappa = Q\lambda, \quad G = Qe^2$$

And the limit

$$Q \to \infty, \quad e \to 0, \quad G = Qe^2 \text{ fixed.}$$



Abelian Higgs model

- Write the insertions as a term in the action

$$S_{\text{ins}} = (Q_2 - Q_1) \int dx \, (\log(\phi)\delta(x) + \log(\phi^*)\delta(x - x_\infty)) \\ - ieQ_1 \int_{\gamma_1} A + ieQ_2 \int_{\gamma_2} A$$

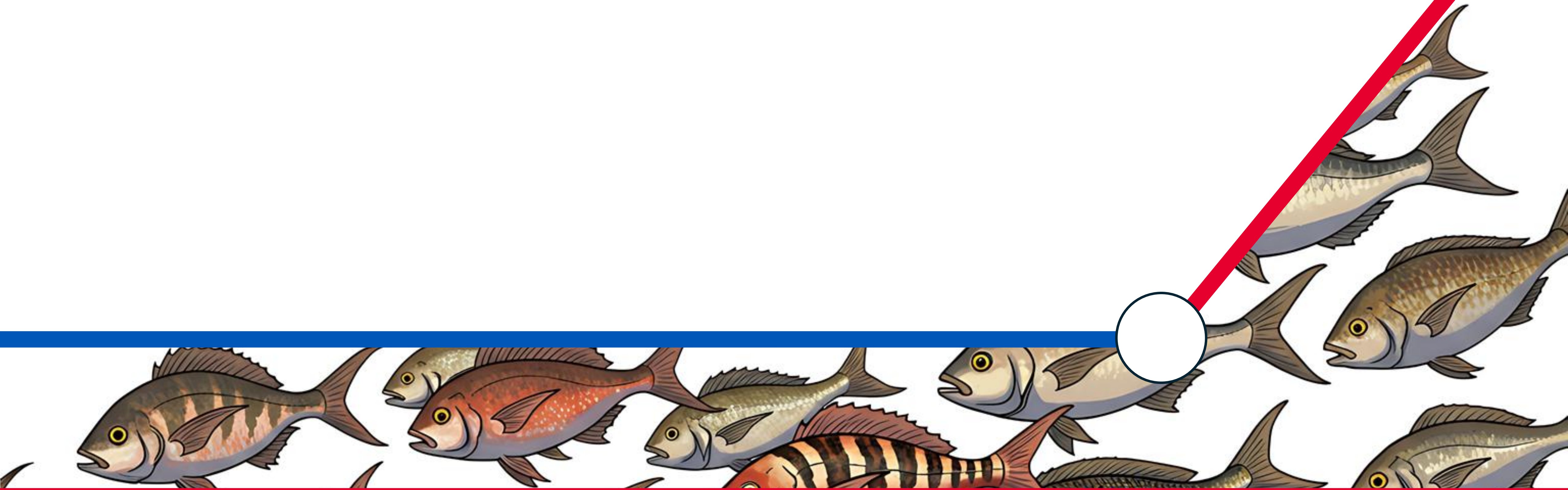
- Rescaling fields gives the total action

$$S + S_{\text{ins}} \equiv Q\bar{S} = Q\Bigg[\int dx \Big(\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + (\partial_\mu + i\sqrt{G}A_\mu)\phi^*(\partial_\mu - i\sqrt{G}A_\mu)\phi + \kappa(\phi^*\phi)^2 \\ + (q_2 - q_1)\,(\log(\phi_1)\delta(x) + \log(\phi_1^*)\delta(x - x_\infty))\Big) - iq_1\sqrt{G}\int_{\gamma_1} A + iq_2\sqrt{G}\int_{\gamma_2} A \Bigg]$$

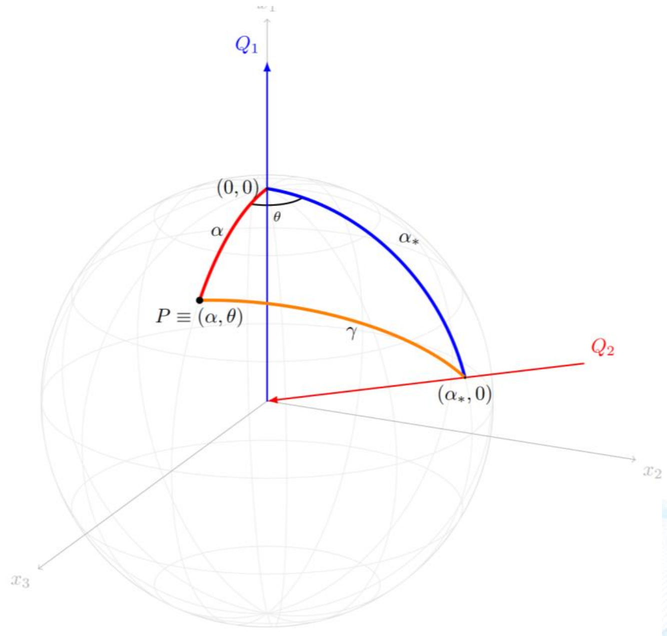
- Controlled by semiclassics!

Semiclassical limit exists!

- Can compute quantities via large charge



Degrees of Freedom



$$\phi = \frac{\rho}{\sqrt{2}} e^{i\chi}$$

$$\rho = \frac{\rho_c(\theta, \alpha)}{r} \qquad B(\theta, \alpha) = i\left(\sqrt{G}A + d\chi\right)$$

Manifestly gauge invariant

Saddle Point Equations

Matter

$$\triangle_{S^3}\, \rho_c(\alpha,\theta) = \left(1 - (RB(\alpha,\theta))^2 + \kappa(R\rho_c(\alpha,\theta))^2\right) \rho_c(\alpha,\theta)$$

Maxwell

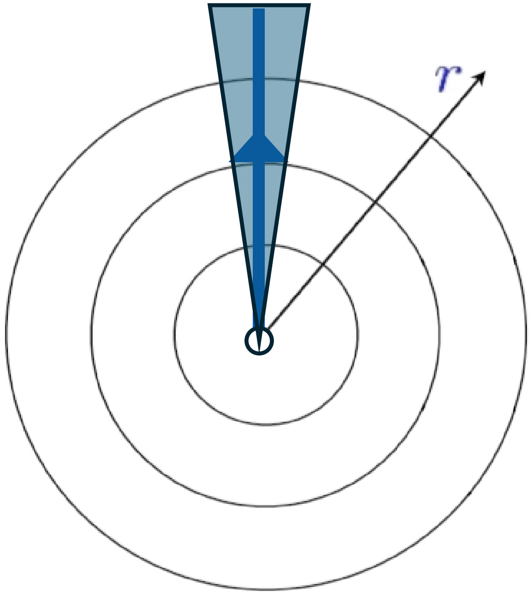
$$\triangle_{S^3}\, B(\alpha,\theta) = G\left((R\rho_c(\alpha,\theta))^2 B(\alpha,\theta) - \frac{q_1\delta(\alpha) - q_2\delta(\alpha-\alpha_*)\delta(\theta)}{2\pi R \sin^2\alpha \sin\theta}\right)$$

Gauss

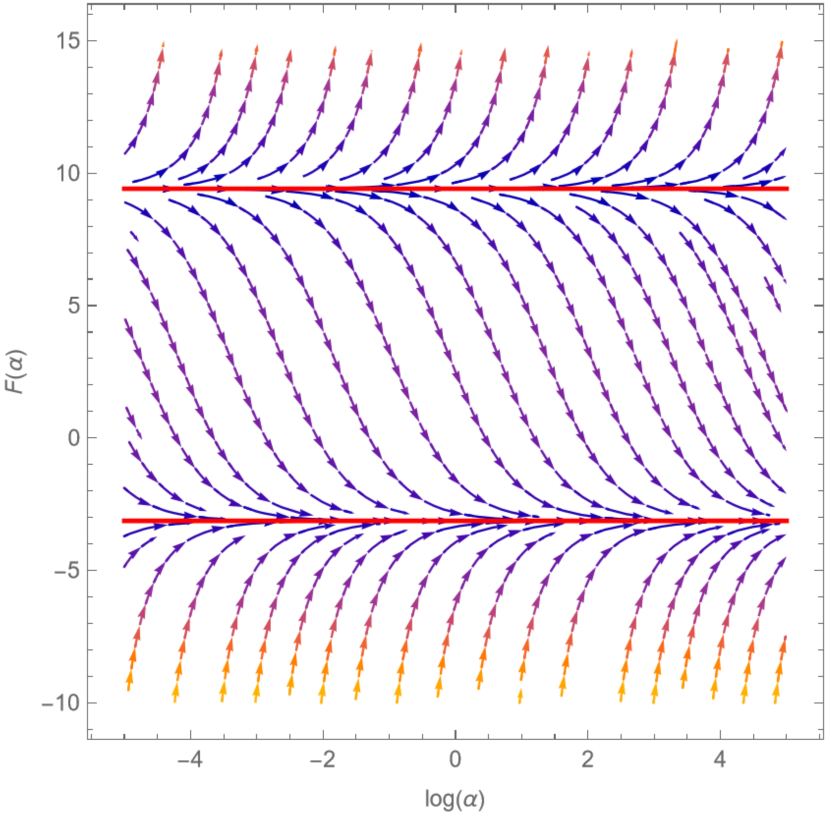
$$\int d\Omega_2 \rho_c^2(\alpha,\theta) B(\alpha,\theta) = \frac{q_1 - q_2}{2\pi R^3}$$

Boundary Conditions?

- If we treat the line as a genuine defect, potential non-trivial boundary conditions



$$F(\alpha)\int \alpha \rho_c^2 d\tau\Big|_{\alpha=\alpha_0}$$



$$\rho \sim \frac{1}{\alpha} + \frac{G^2}{16\pi^2}\frac{\log\alpha}{\alpha} + \ldots$$

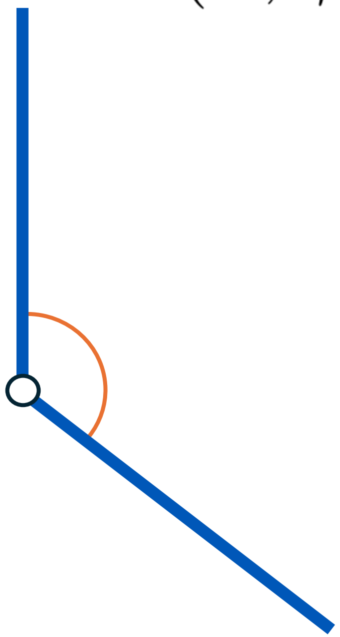
$$\rho \sim 1 - \frac{G^2}{16\pi^2}\log\alpha + \ldots$$

See also [Aharony, Cuomo, Komargodski, Mezei, Raviv-Moshe (2023)]

Trivial Defect-Change ($Q_1 = Q_2$)

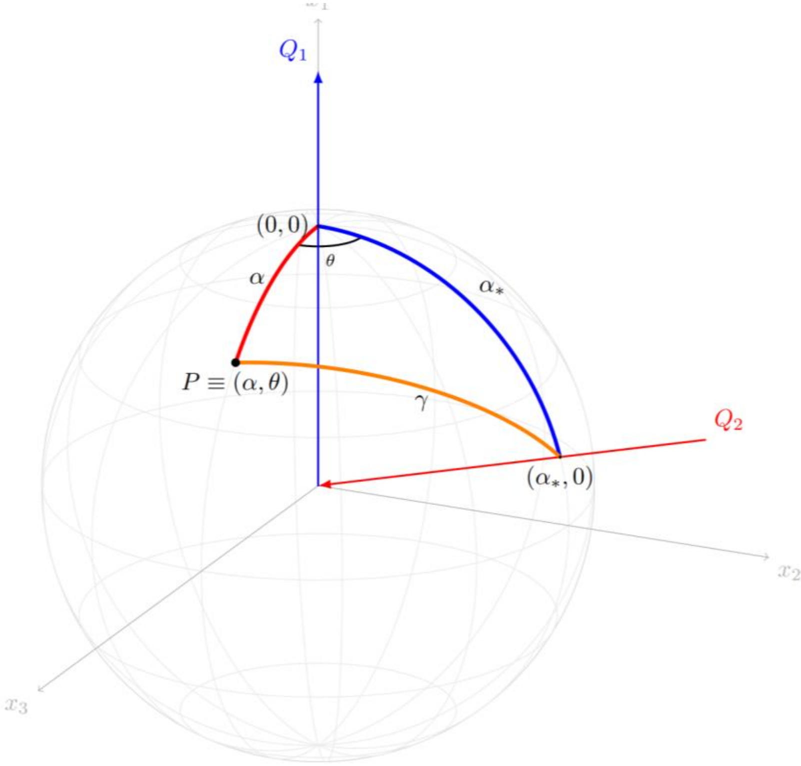
- Can actually solve exactly

$$B(\alpha,\gamma) = \frac{G}{4\pi^2 R}\left((\pi-\alpha)\cot(\alpha) - (\pi-\gamma)\cot(\gamma)\right)$$



$$\Gamma_{qq}^{-1,1}(\alpha_*) = -Q\,\frac{1+(\pi-\alpha_*)\cot(\alpha_*)}{4\pi^2}$$

$$\Gamma_{qq}^{-1,i} = 0 \quad i > 1.$$



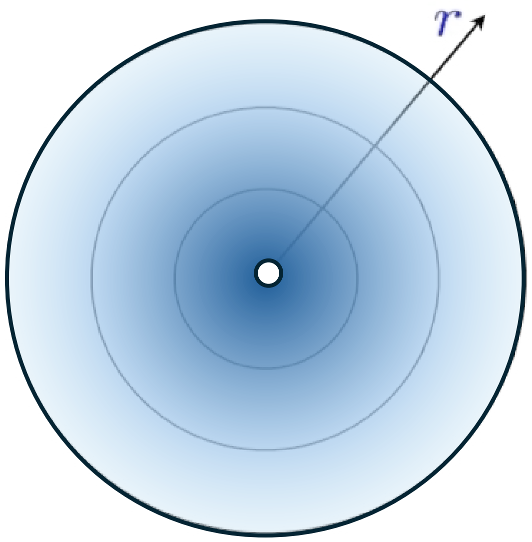
Perturbation theory in G

$$\rho \to r = r_0 + r_1 G + r_2 G^2 + \dots$$

$$B \to \mu = \mu_0 + \mu_1 G + \mu_2 G^2 + \dots$$

$$\Gamma_{Q_1 Q_2}(\alpha_*) = \sum_{i=-1}^{\infty} \sum_{j=0}^{\infty} \frac{G^j}{Q^i} \Gamma^{(i,j)}_{Q_1 Q_2}(\alpha_*)$$

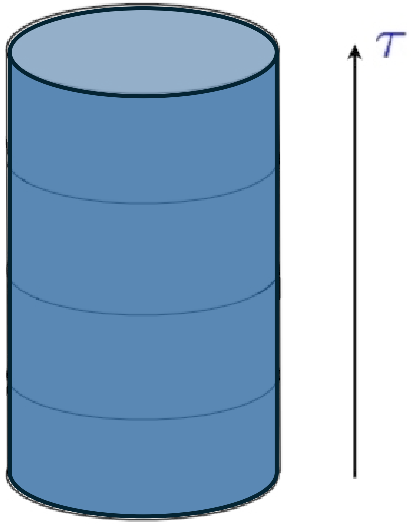
Leading Order Solution



- Reproduces the result of the ungauged model [Badel, Cuomo, Monin, Rattazzi (2019)], and the Dirac dressed case [Antipin, Bednyakov, Bersini, Panopoulos, Pikelner (2022)]

$$\mu_0 = \frac{2(6\pi^4)^{1/3} + \left(\sqrt{81\kappa^2(q_1-q_2)^2 - 48\pi^4} + 9\kappa(q_1-q_2)\right)^{2/3}}{(6\pi)^{2/3}\left(\sqrt{81\kappa^2(q_1-q_2)^2 - 48\pi^4} + 9\kappa(q_1-q_2)\right)^{1/3}} .$$

$$\Gamma^{(-1,0)}_{q_1 q_2} = \frac{q_1 - q_2}{4}\left(3R\mu_0 + \frac{1}{R\mu_0}\right) .$$

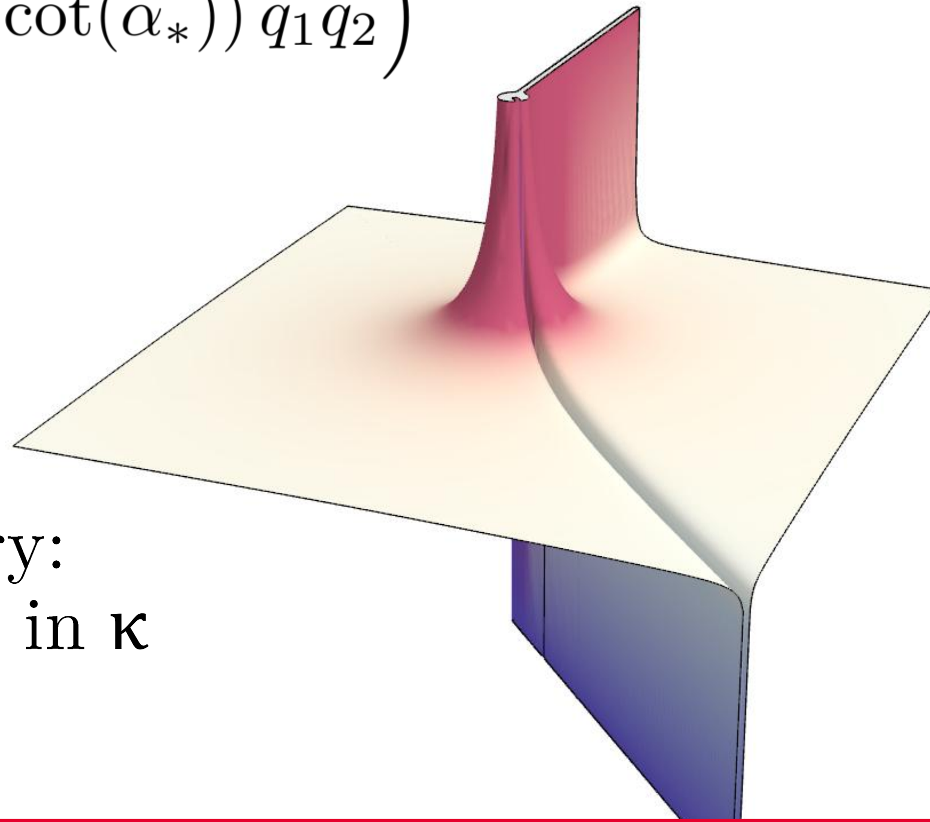


- Resummation of perturbation theory in κ!

Next-to-Leading Order Solution

- Independent of κ!

$$\Gamma^{(-1,1)}_{q_1 q_2} = -\frac{1}{16\pi^2}\left(3\,(q_1 - q_2)^2 + 4\,(1 + (\pi - \alpha_*)\cot(\alpha_*))\,q_1 q_2\right)$$



- All orders statement in perturbation theory: contributions from diagrams at each order in κ vanish

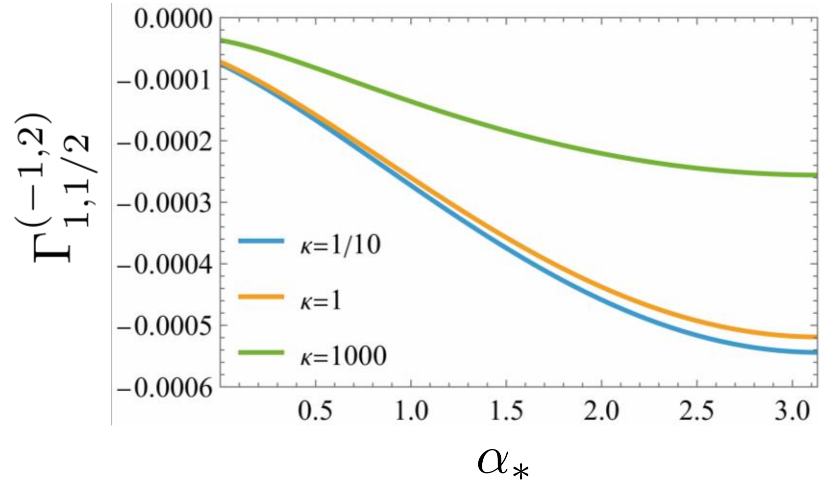
Next-to-Next-to-Leading Order Solution

- Non-trivial... and non-zero

$$\Gamma^{(-1,2)}_{q_1 q_2} = \frac{q_1 - q_2}{384\pi^4 R\mu_0 (1-(R\mu_0)^2)^3} \left(H_1(R\mu_0)\left(q_1 - q_2\right)^2 + q_1 q_2 H_2(R\mu_0, \alpha_*)\right),$$

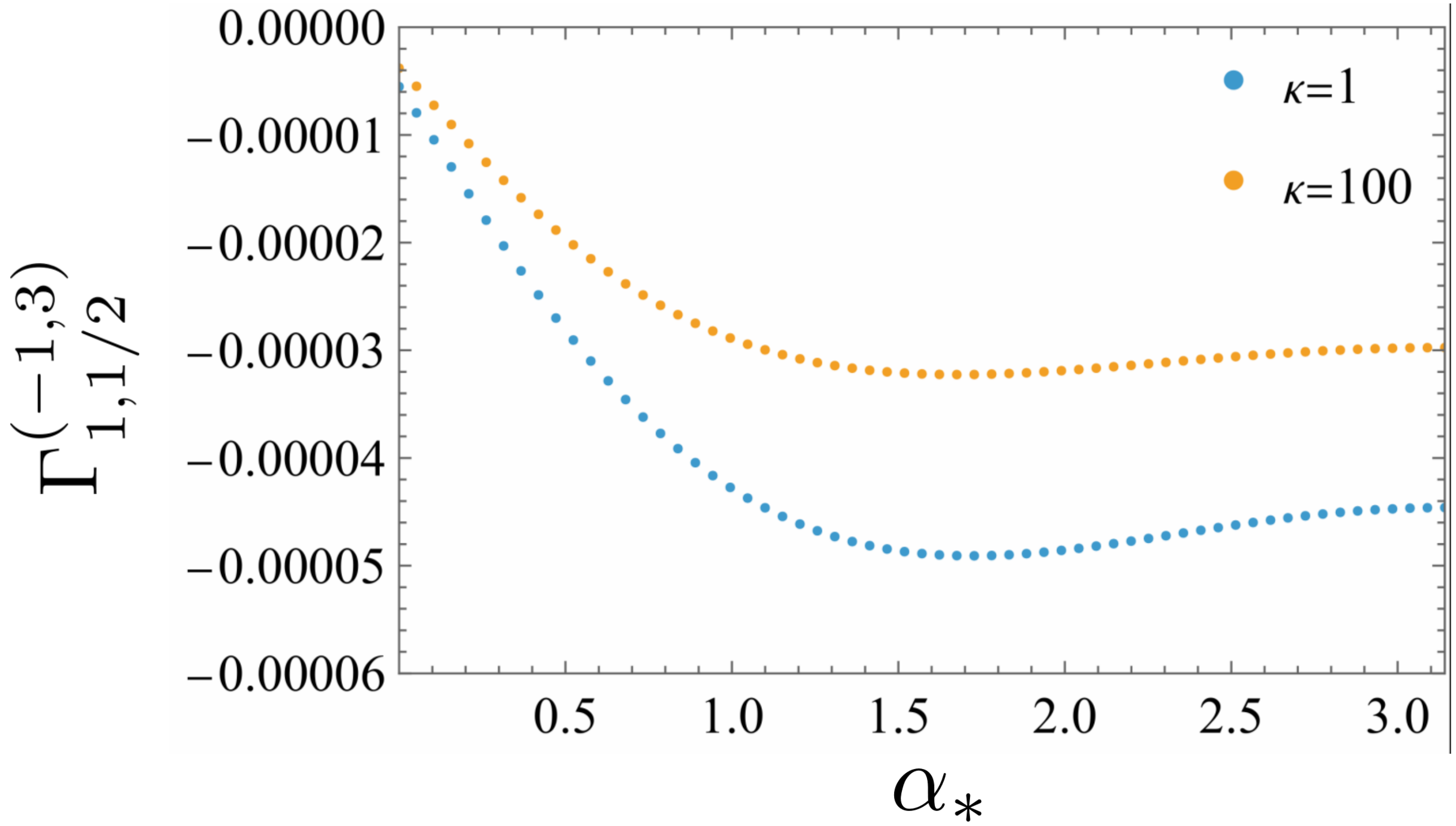
$$H_1 = -9(R\mu_0)^6 - 15(R\mu_0)^4 + 45(R\mu_0)^2 + 3 + 4\pi^2\left((R\mu_0)^2 - 1\right)^2\left(1 - 3(R\mu_0)^2\right) + 24\pi\left((R\mu_0)^2 - 1\right)\sqrt{2(R\mu_0)^2 - 3}(R\mu_0)^2 \coth\left(\pi\sqrt{2(R\mu_0)^2 - 3}\right),$$

$$H_2 = 12\left(1 - (R\mu_0)^2\right)\Bigg(4(R\mu_0)^2(1 + (\pi - \alpha_*)\cot(\alpha_*)) + (2\pi - \alpha_*)\alpha_*\left(3(R\mu_0)^4 - 4(R\mu_0)^2 + 1\right) - 4\pi(R\mu_0)^2\left(\csc(\alpha_*)\operatorname{csch}\left(\pi\sqrt{2(R\mu_0)^2 - 3}\right)\sinh\left((\pi - \alpha_*)\sqrt{2(R\mu_0)^2 - 3}\right) + \sqrt{2(R\mu_0)^2 - 3}\coth\left(\pi\sqrt{2(R\mu_0)^2 - 3}\right)\right)\Bigg),$$



NNNLO Numerics

- Energy integral too hard to evaluate analytically



Order Q Expansion $(\kappa \ll 1)$

$$Q[\Gamma^{(-1,0)}_{Q_1Q_2} + \Gamma^{(-1,1)}_{Q_1Q_2} G + \Gamma^{(-1,0)}_{Q_1Q_2} G^2 + \ldots] + \ldots =$$

$$Q\Bigg[\left((q_1 - q_2) + \frac{(q_1-q_2)^2\kappa}{8\pi^2} + \mathcal{O}(\kappa^2)\right) G^0 - \frac{3(q_1-q_2)^2 + 4q_1q_2(1+(\pi-\alpha_*)\cot(\alpha_*))}{16\pi^2} G$$
$$+ \Bigg(\frac{q_1-q_2}{384\pi^4}\left(8q_1q_2\left(\alpha_*(\alpha_* - 2\pi)(\alpha_* - \pi)\cot(\alpha_*) - 2\pi^2\right) + \left(3 - 8\pi^2\right)(q_1 - q_2)^2 + \mathcal{O}(\kappa)\right)\Bigg) G^2 + \mathcal{O}(G^3)\Bigg] + \mathcal{O}(Q^0)$$

- Reproduces perturbative Feynman diagram expansion at each order

Order Q Expansion ($\kappa \gg 1$)

$$Q[\Gamma^{(-1,0)}_{Q_1Q_2} + \Gamma^{(-1,1)}_{Q_1Q_2} G + \Gamma^{(-1,0)}_{Q_1Q_2} G^2 + \ldots] + \ldots =$$

$$Q\Bigg[\left(\frac{3(q_1-q_2)^{4/3}\kappa^{1/3}}{2^{7/3}\pi^{2/3}} + \left(\frac{\pi}{2}\right)^{2/3}(q_1-q_2)^{2/3}\kappa^{-1/3} + \mathcal{O}(\kappa^{-1})\right)G^0$$

$$- \frac{3(q_1-q_2)^2+4q_1q_2(1+(\pi-\alpha_*)\cot(\alpha_*))}{16\pi^2}G$$

$$-\left(\frac{1}{64}\left((3+4\pi^2)(q_1-q_2)^2+12q_1q_2(2\pi-\alpha_*)\alpha_*\right)\left(\frac{(q_1-q_2)^2}{4\pi^{10}\kappa}\right)^{1/3}\right.$$

$$\left.+\mathcal{O}(\kappa^{-1})\right)G^2 + \mathcal{O}(G^3)\Bigg]\Bigg] + \mathcal{O}(Q^0)$$

Characteristic large-charge scaling behaviour

- New results beyond perturbation theory!
- Full formula interpolates between both regimes

Fluctuations

- Want $\mathcal{O}(Q^0)$ contributions to the anomalous dimension

$$Q\Gamma^{(-1)}_{q_1 q_2} + Q^0\Gamma^{(0)}_{q_1 q_2} + \ldots$$

- Obtained from spectrum of fluctuations around classical solution

Dispersion Relations

- Need eigenvalues of each block of inverse propagator matrix, eg:

$$\mathcal{D}_S^{-1} = \begin{pmatrix} -\omega^2 + \Delta_{S^{d-1}} + m_d^2 - \mu^2 + 3\kappa r^2 & -2i\mu\omega & -2i\sqrt{G}\mu r \\ 2i\mu\omega & -\omega^2 + \Delta_{S^{d-1}} + Gr^2 & 0 \\ -2i\sqrt{G}\mu r & 0 & -\omega^2 + \Delta_{S^{d-1}} + Gr^2 \end{pmatrix}$$

$$= V_0 + \sqrt{G}V_1 + GV_2 + \mathcal{O}\left(G^{3/2}\right),$$

- Function of position-dependant fields. Use matrix element pert. Theory:

$$\lambda_{S,i} = \lambda_{S,i}^{(0)} + G\left(\langle v_{S,i}|V_2|v_{S,i}\rangle + \sum_{j\neq i} \frac{|\langle v_{S,i}|V_1|v_{S,j}\rangle|^2}{\lambda_{S,i}^{(0)} - \lambda_{S,j}^{(0)}}\right) + \mathcal{O}(G^2)$$

Mass Spectrum

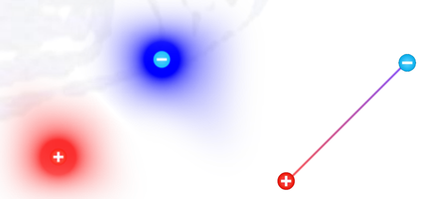
$$m^2_{S,-} = G\frac{q_1 - q_2}{2\pi^2 R^3 \mu_0} + \mathcal{O}(G^2) \qquad m^2_{S,+} = G\frac{q_1 - q_2}{2\pi^2 R^3 \mu_0} + \mathcal{O}(G^2)$$

$$m^2_{S,3} = 6\mu_0^2 - 2m_d^2 + \mathcal{O}(G^2) \qquad m^2_{T} = (d-2) + G\frac{q_1 - q_2}{2\pi^2 R^3 \mu_0} + \mathcal{O}(G^2)$$

$$m^2_{h} = 6\mu_0^2 - 2m_d^2 + \mathcal{O}(G^2) \qquad m^2_{\mathrm{gh}} = G\frac{q_1 - q_2}{2\pi^2 R^3 \mu_0} + \mathcal{O}(G^2)$$

$$m^2_{II,1} = \mathcal{O}(G^2) \qquad m^2_{II,2} = \mu_0^2 + \mathcal{O}(G^2)\,.$$

- Expectation: line breaks $SO(d,1) \times \frac{SU(N)}{\mathbb{Z}_N} \to SO(d-2) \times \mathcal{D} \times \frac{SU(N-1)}{\mathbb{Z}_{N-1}}$
- No Type-I Goldstone (Higgs), S- and gauge field massive



Contribution to Anom. Dim.

$$\Gamma_{q_1 q_2}^{(0,0)} = \frac{8\sqrt{6(R\mu_0)^2-2}-3(R\mu_0)^4(N+4)-6(R\mu_0)^2 N+16R\mu_0(N-1)-7N+12}{16} + \frac{1}{2}\sum_{\ell=1}^{\infty}\sigma(\ell)$$

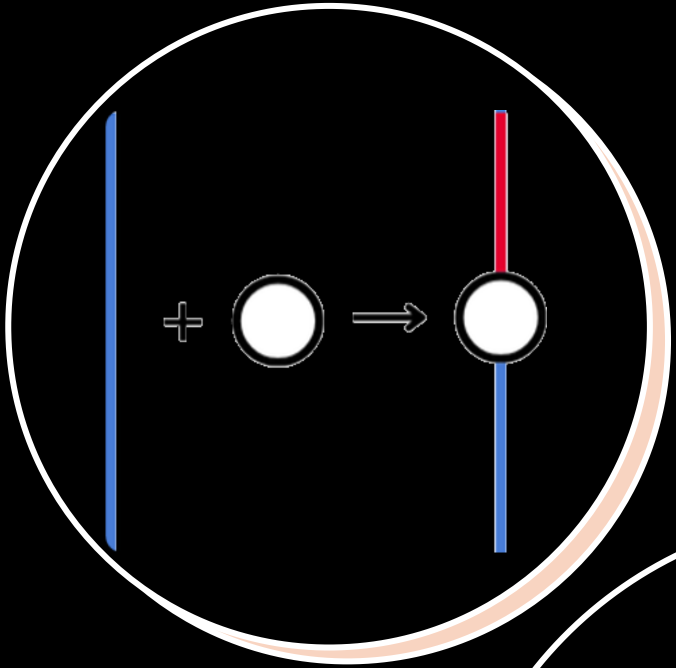
- Complicated but known
- Reproduces usual LO fluctuations
- Can sum eigenvalues for NLO corrections

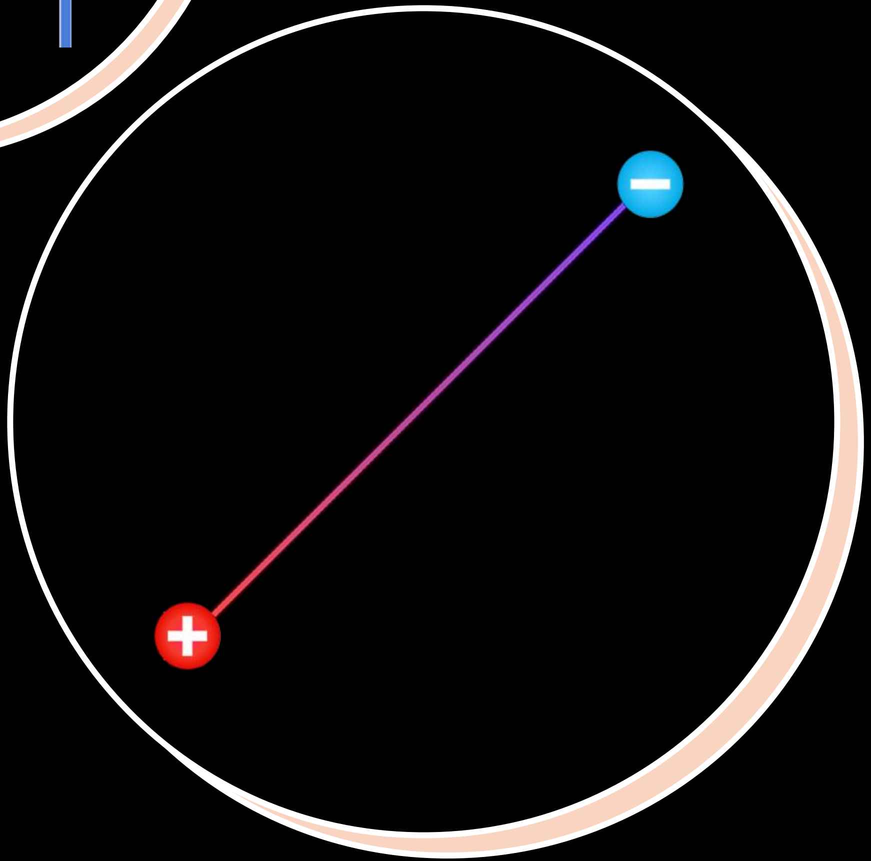
Contribution to Anom. Dim.

$$\Gamma^{(0,0)}_{q_1 q_2} = -\tfrac{N+5}{3}\tfrac{6\kappa}{(4\pi)^2}(q_1 - q_2) + \tfrac{3-N}{9}\Big(\tfrac{6\kappa}{(4\pi)^2}(q_1 - q_2)\Big)^2$$
$$+ \tfrac{2}{27}(2(N+7)\zeta(3) + N - 18)\Big(\tfrac{6\kappa}{(4\pi)^2}(q_1 - q_2)\Big)^3$$
$$- \tfrac{2}{81}((12N + 65)\zeta(3) + 5(N+15)\zeta(5) + 4N - 146)\Big(\tfrac{6\kappa}{(4\pi)^2}(q_1 - q_2)\Big)^4 + \ldots$$

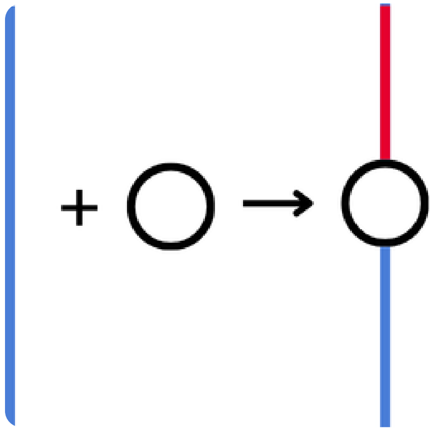
- Each order is again a resummation of perturbation theory in κ

Observables

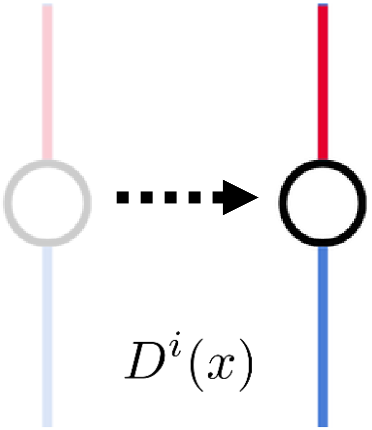




Observables: Defect Changing Operator



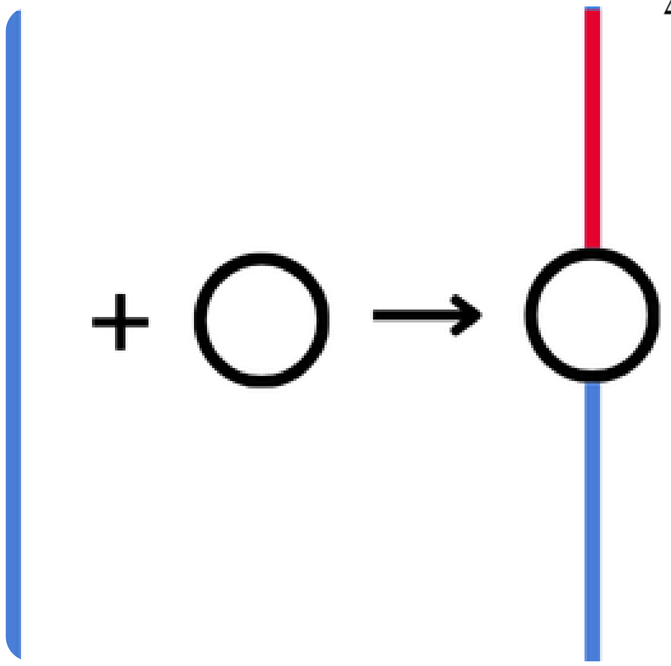
$$\Gamma_{q_1 q_2}(\alpha_*) = \Delta_{q_1 q_2} + \frac{1}{2}\beta_{q_1 q_2}(\alpha_* - \pi)^2 + \mathcal{O}\big((\alpha_* - \pi)^3\big)$$



β related to displacement operator:

$$\partial_\mu T^{\mu i} = \delta(x_\perp) D^i(x_{||})$$

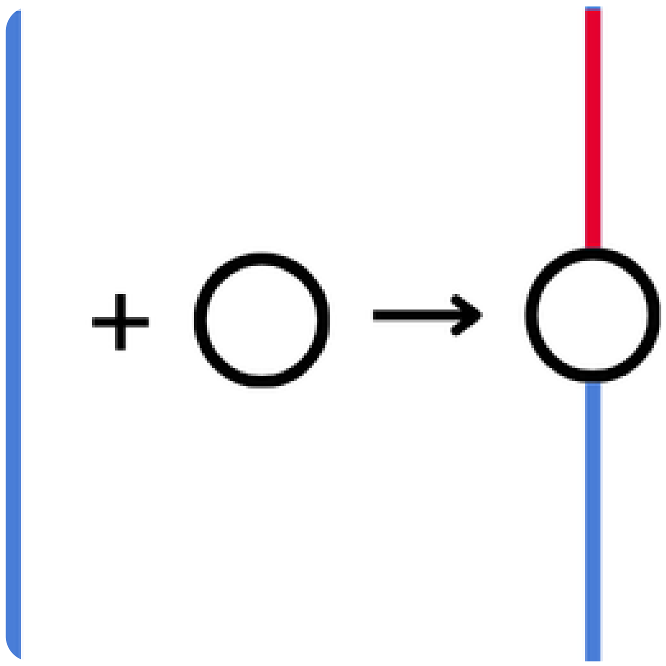
Defect Changing Operator: Small Limit



$$\Delta_{q_1q_2} = Q\Bigg[\left((q_1-q_2) + \frac{(q_1-q_2)^2\kappa}{8\pi^2} - \frac{(q_1-q_2)^3\kappa^2}{32\pi^4} + \dots\right) - \frac{3(q_1-q_2)^2}{16\pi^2}G$$
$$+ \Bigg(\frac{(3-8\pi^2)(q_1-q_2)^3 - 24\pi^2 q_1q_2(q_1-q_2)}{384\pi^4} + \frac{\kappa\left(15\pi^2(\pi^2-6)q_1q_2(q_1-q_2)^2 + 2(45-15\pi^2+2\pi^4)(q_1-q_2)^4\right)}{5760\pi^6}$$
$$+ \frac{\kappa^2\left(120\pi^2(\pi^2-18)q_1q_2(q_1-q_2)^3 + (765-720\pi^2+32\pi^4)(q_1-q_2)^5\right)}{184320\pi^8} + \dots\Bigg)G^2 + \dots\Bigg]$$
$$- \left[\frac{(N+5)(q_1-q_2)\kappa}{8\pi^2} + \frac{(N-3)(q_1-q_2)^2\kappa^2}{64\pi^4} + \dots\right] + \mathcal{O}(Q^0G)\,,$$

$$\beta_{q_1q_2} = -\frac{QG}{6\pi^2} + \left(\frac{(3+\pi^2)(q_1-q_2)}{72\pi^4} - \frac{(q_1-q_2)^2\kappa}{32\pi^6} + \frac{(180-105\pi^2+14\pi^4)(q_1-q_2)^3\kappa^2}{34560\pi^8} + \dots\right)q_1q_2QG^2$$
$$+ \mathcal{O}(QG^3, Q^0G)\,,$$

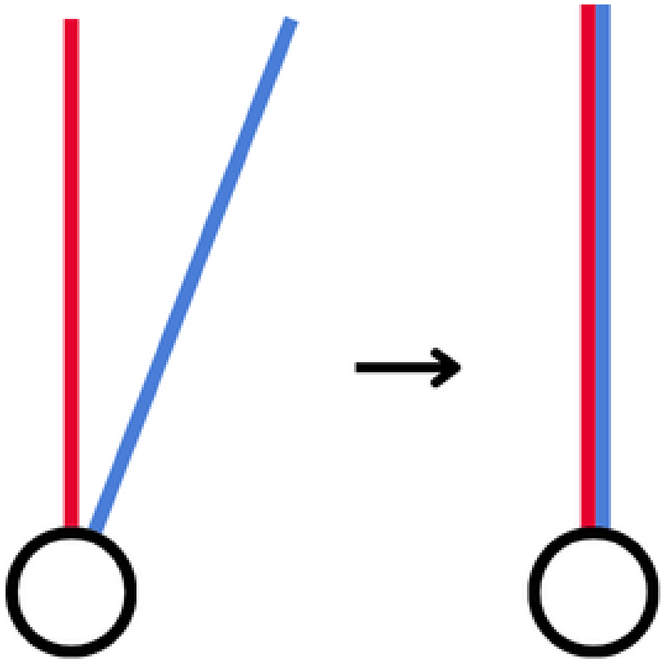
Defect Changing Operator: Large Limit



$$
\begin{aligned}
\Delta_{q_1 q_2} = Q\Bigg[& \left(\frac{3}{4}\left(\frac{\kappa(q_1-q_2)^4}{2\pi^2}\right)^{1/3} + \ldots\right) - \frac{3(q_1-q_2)^2}{16\pi^2} G \\
& - \left(\frac{1}{64}\left(\left(3+4\pi^2\right)(q_1-q_2)^2 + 12\pi^2 q_1 q_2\right)\left(\frac{(q_1-q_2)^2}{4\pi^{10}\kappa}\right)^{1/3} + \ldots\right) G^2 + \ldots \Bigg] \\
& + \Bigg[\frac{1}{192}\Big(4(N+4)\log((q_1-q_2)\kappa) - 4(3\sqrt{3} + 13 + 8\log(4\pi) - 15\coth^{-1}\sqrt{3}) \\
& - N(15 + 8\log(4\pi)) + 12\gamma_E(N+4)\Big)\left(\frac{\kappa^4(q_1-q_2)^4}{2\pi^8}\right)^{1/3} + \ldots\Bigg] + \mathcal{O}(Q^0 G)\,,
\end{aligned}
$$

$$
\beta_{q_1 q_2} = -\frac{QG}{6\pi^2} + \left(\frac{3}{16}\left(\frac{(q_1-q_2)^2}{4\pi^{10}\kappa}\right)^{1/3} + \ldots\right) q_1 q_2 Q G^2 + \mathcal{O}(QG^3, Q^0 G)\,.
$$

Observables: Defect Fusion

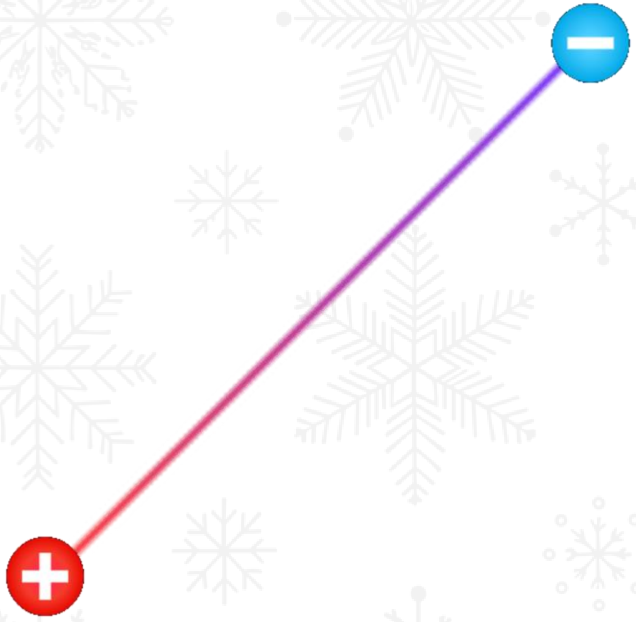


$$\Gamma_{q_1 q_2} = \frac{C_{q_1(-q_2)(\overline{q_1-q_2})}}{\alpha_*} + \Delta_{(q_1-q_2)0} + \alpha_{q_1 q_2}\,\alpha_*^{\Delta_{\mathrm{irr}}-1} + \dots$$

$$C_{q_1(-q_2)(\overline{q_1-q_2})} = -\frac{GQ}{4\pi} q_1 q_2 = -\frac{e^2}{4\pi} Q_1 Q_2$$

- Casimir energy is independent of κ!

Observables: Order Parameter



$$\langle \phi^*(x_2) \exp\left(\int dz J^{\mu}_{MS}(z;x) A_{\mu}(z)\right) \phi(x_1) \rangle$$

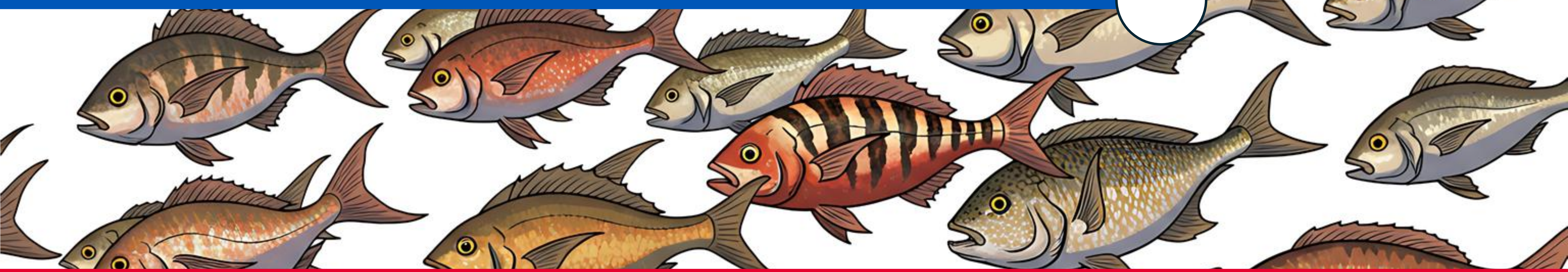
$$\gamma_{\xi=0} = \gamma_D$$

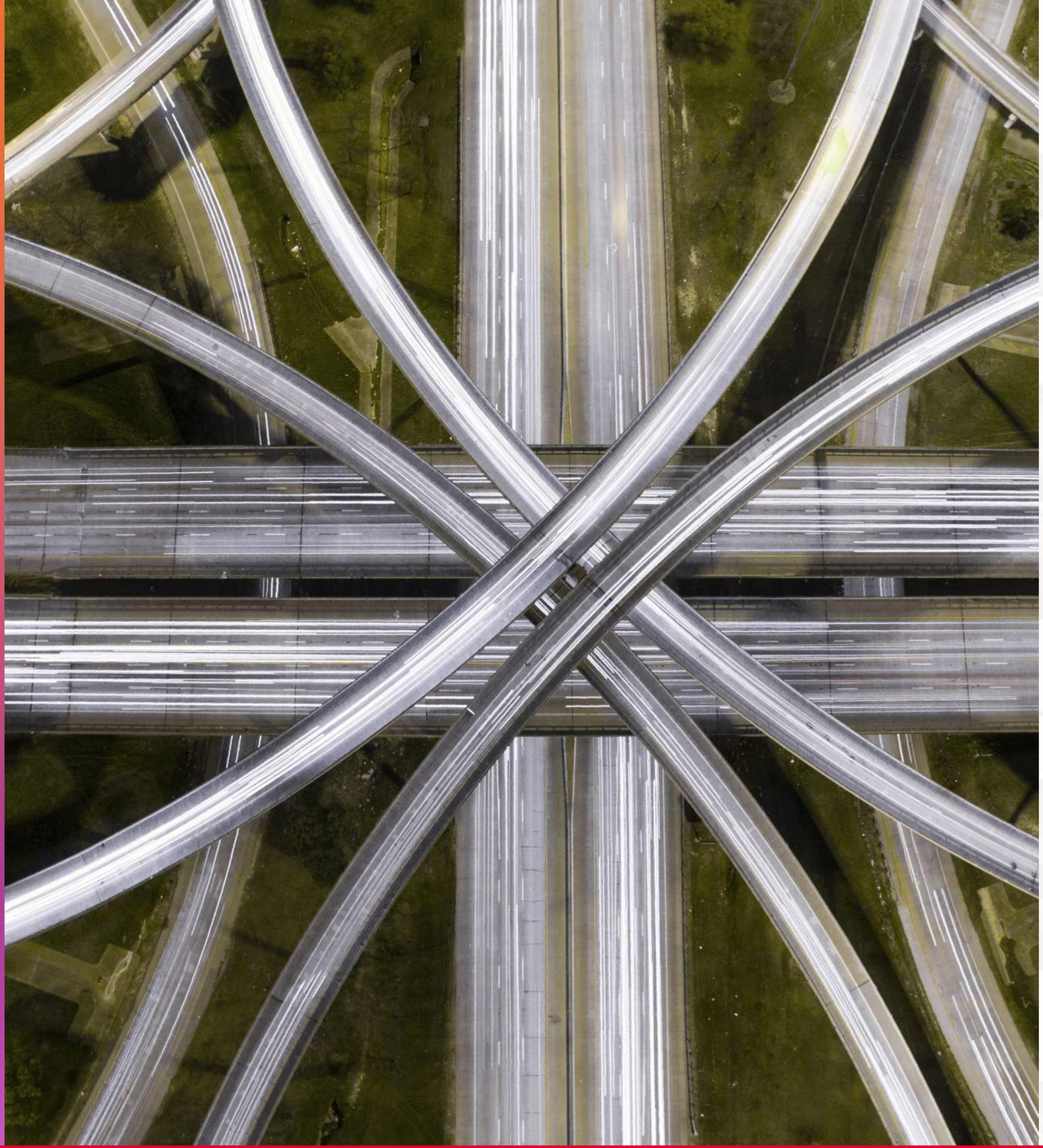
$$\gamma_{\xi=1-d} \neq \gamma_{MS}!$$

- Dirac dressing differs from MS dressing at all loops, not just 1-loop

To summarise:

- <u>Large charge works in gauge theory</u>
- Semiclassical expansion in Q
- Elucidation of SC order parameter
- Gauge redundancy ⇒ no I-Goldstone
- Non-perturbative results





Further directions

- Large G regime
- Nonabelian lines
- Supersymmetric Extensions
- Junctions
- Multiple defect changes

Thankyou!