



Neutrons as non-relativistic unparticles

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New directions in the large charge expansion, Les Diablerets, June 21-26, 2026

- Universality and the unitary limit
- Schrödinger symmetry
- Nuclear reactions with neutrons
- Short-distance production of hadrons
- Summary and Outlook

References:

HWH, D.T. Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021)

T. Backert, S. Dietz, HWH, S. König, D.T. Son, Phys. Rev. C **113**, 064001 (2026)

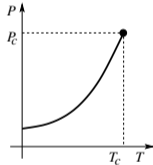
E. Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022)

E. Braaten, HWH, Phys. Rev. D **107**, 034017 (2023)

Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

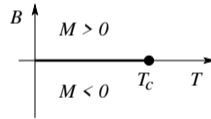
Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

■ Condensed matter systems near critical point



$$\rho_{\text{liq/gas}}(T) - \rho_c \longrightarrow \pm A(T_c - T)^\beta$$

Liquid-gas system



$$M_0(T) \longrightarrow A'(T_c - T)^\beta$$

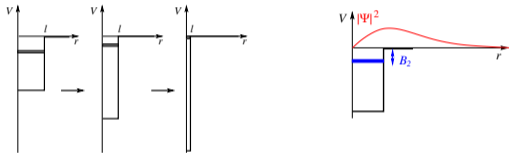
Ferromagnet (one easy axis)

$$\beta = 0.325$$

- Universality class determines critical exponents:
- Scale invariance (often conformal invariance)

- Consider short-ranged, resonant S-wave interactions \implies halo state
- Unitary limit: $a \rightarrow \infty, \ell \sim r_e \rightarrow 0$

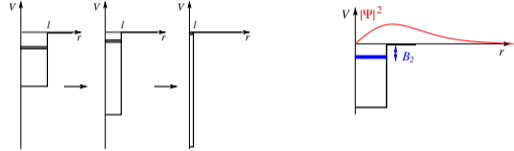
$$\mathcal{T}_2(k, k) \propto \left[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \dots} \quad -ik \right]^{-1} \sim i/k$$



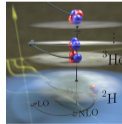
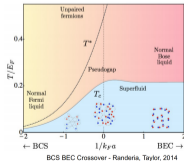
- Scattering amplitude scale invariant, saturates unitarity bound

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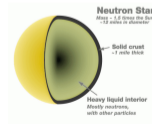
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- Scattering amplitude scale invariant, saturates unitarity bound
- BEC/BCS crossover, neutron matter, ... (cf. Schäfer, Baym, PNAS **118**, e2113775118 (2021))



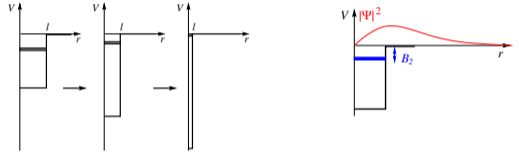
Sebastian König



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- Unitary limit: $a \rightarrow \infty, \ell \sim r_e \rightarrow 0$

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- Scattering amplitude scale invariant, saturates unitarity bound
- System has also (non-relativistic) conformal symmetry/Schrödinger symmetry
Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...
- Exploit approximate conformal symmetry for nuclear reactions with neutrons

$$a \approx -18.6 \text{ fm}, \quad r_e \approx 2.8 \text{ fm} \quad \implies \quad \underbrace{1/(ma^2)}_{0.1 \text{ MeV}} \ll E_n^{\text{cms}} \ll \underbrace{1/(mr_e^2)}_{5 \text{ MeV}}$$

- Non-relativistic conformal symmetry: **Schrödinger symmetry**

- Galilei symmetry

- space + time translations

- rotations

- Galilei boosts

- Scale transformations

$$\mathbf{x} \rightarrow e^\lambda \mathbf{x}, \quad t \rightarrow e^{2\lambda} t, \quad \psi \rightarrow e^{-\lambda \Delta} \psi$$

- Special conformal transformations

$$\mathbf{x} \rightarrow \frac{\mathbf{x}}{1 + \xi t}, \quad t \rightarrow \frac{t}{1 + \xi t}, \quad \psi \rightarrow \psi' = \dots$$

⇒ **preserves angles**

- 12 Parameters

- Generators: $H, \mathbf{P}, \mathbf{L}, \mathbf{K}, D, C$, satisfy **Schrödinger algebra**

- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e \sim \ell$)



- Renormalization group equation: $\Lambda \frac{d}{d\Lambda} \tilde{g}_2 = \tilde{g}_2(1 + \tilde{g}_2)$

- Two fixed points:

– $\tilde{g}_2 = 0 \Leftrightarrow a = 0 \Rightarrow$ no interaction, free particles

– $\tilde{g}_2 = -1 \Leftrightarrow 1/a = 0 \Rightarrow$ unitary limit \Rightarrow conformal/Schrödinger symmetry

- Neutrons: $a \approx -18.6$ fm, $r_e \approx 2.8$ fm \Rightarrow close to unitary limit \Rightarrow approximately conformal
 \Rightarrow multineutron system can be described by conformal field

- Two-point function of (primary) conformal field operator \mathcal{U}

$$G_{\mathcal{U}}(t, \mathbf{x}) = -i \langle T \mathcal{U}(t, \mathbf{x}) \mathcal{U}^\dagger(0, \mathbf{0}) \rangle = \mathcal{C} \frac{\theta(t)}{(it)^\Delta} \exp\left(\frac{iM\mathbf{x}^2}{2t}\right)$$

- Determined by symmetry up to overall constant \mathcal{C}
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega, \mathbf{p}) = -\mathcal{C} \left(\frac{2\pi}{M}\right)^{3/2} \Gamma\left(\frac{5}{2} - \Delta\right) \left(\frac{\mathbf{p}^2}{2M} - \omega\right)^{\Delta - \frac{5}{2}}$$

- pole only for $\Delta = 3/2$ (free field)
 - branch cut for $\Delta > 3/2$
- Non-relativistic analog of Georgi's unparticle (cf. Georgi, Phys. Rev. Lett. **98**, 221601 (2007))

■ Imaginary part of propagator

$$\text{Im } G_{\mathcal{U}}(\omega, \mathbf{p}) \sim \begin{cases} \delta\left(\omega - \frac{\mathbf{p}^2}{2M}\right), & \Delta = \frac{3}{2}, \\ \left(\omega - \frac{\mathbf{p}^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{\mathbf{p}^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

■ General \mathcal{U} does not behave like a particle (“unparticle/unucleus”) \Rightarrow continuous energy spectrum

■ Examples of unnuclei

- N free fields: $\mathcal{U} = \psi_1 \dots \psi_N$, $M = Nm_{\psi}$, $\Delta = 3N/2$
- N interacting fields: $\mathcal{U} = \psi_1 \dots \psi_N$, $M = Nm_{\psi}$, $\Delta > 3/2$

■ Corrections from finite a and r_0 (Dutta, Mishra, Son, Phys. Rev. D **109**, 016001 (2024); Beane, Orlando, Reffert, Phys. Rev. D **112**, 014028 (2025))

$$\text{Im } G_{\mathcal{U}}(\omega, 0) \sim \omega^{\Delta - \frac{5}{2}} \theta(\omega) \left(1 + \frac{c_1}{a\sqrt{m\omega}} + c_2 r_0 \sqrt{m\omega} + \dots \right), \quad c_2 = 0$$

- How to calculate scaling dimension Δ ? not given by symmetry \Rightarrow non-perturbative problem
 - (1) Δ can be obtained from field theory calculation
 - (2) Δ can be obtained from operator state correspondence

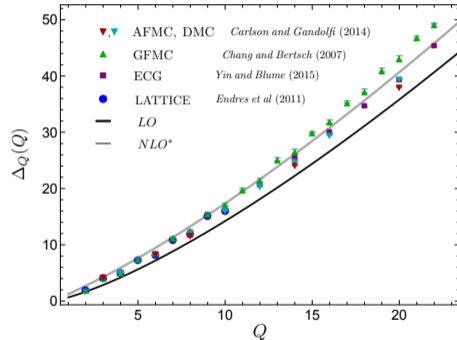
Δ of primary operator = (Energy of state in HO) / $\hbar\omega$

(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

N	S	L	\mathcal{O}	Δ	Δ_{free}
2	0	0	$\psi_1\psi_2$	2	3
3	1/2	1	$\psi_1\psi_2\nabla_j\psi_2$	4.27272	5.5
3	1/2	0	$\psi_1\nabla_j\psi_2\nabla_j\psi_2$	4.66622	6.5
4	0	0	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$	5.07(1)	8
5	1/2	1	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2\nabla_i\psi_1$	7.6(1)	10.5

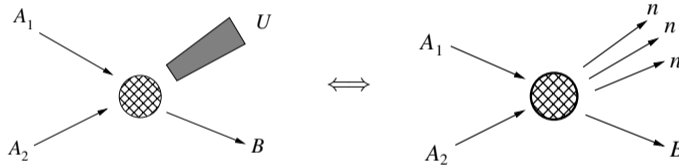
- Connection between Δ and energy of particles in a trap
- Predictions for $N \leq 70$ available (Endres et al., PRA **84**, 043644 (2011))

- Δ can be calculated in large charge expansion (\Rightarrow large N) expansion
- NLO* describes general trend well even at low N (one-parameter fit)
- Pairing effect?



S.R. Beane, D. Orlando, S. Reffert, Phys. Rev. D **112**, 014028 (2025)

- High-energy nuclear reaction with multi-neutron final state
(HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))



$$\Delta Mc^2 + E_{A_1} + E_{A_2} = E_B + E_U$$

- Assumption:** energy scale of primary reaction $\gg E_U - \frac{\mathbf{p}^2}{2M_U} = E_n^{cms}$
- Factorization:** $\frac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \text{Im } G_U(E_U, \mathbf{p})$ (cf. Briceno, Costa, HWH, Phillips, arXiv:2605.22805)
- Reproduces Watson-Migdal treatment of FSI for $2n$

■ Two ways to do experiments

(a) detect recoil particle B $\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta-5/2}, \quad E_0 = (1 + M_B/M_U)^{-1} E_{\text{kin}}$

(b) detect all final state particles **including neutrons**: $\frac{d\sigma}{dE} \sim (E_{xn}^{\text{cms}})^{\Delta-5/2}$

- Two ways to do experiments

(a) detect recoil particle B $\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta-5/2}, \quad E_0 = (1 + M_B/M_U)^{-1} E_{\text{kin}}$

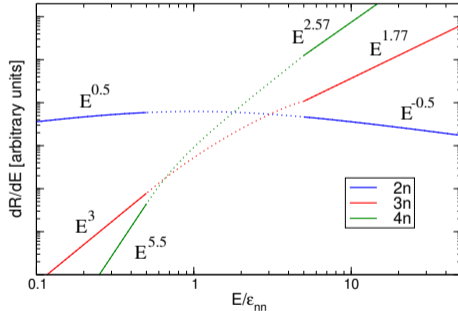
(b) detect all final state particles **including neutrons**: $\frac{d\sigma}{dE} \sim (E_{xn}^{\text{cms}})^{\Delta-5/2}$

- $2n$ case can be understood from dimer propagator in halo/pionless EFT ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \Rightarrow \text{Im } G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

- See also (Göbel et al., Phys. Rev. C **104**, 024001 (2021)) for ${}^6\text{He}(p, p\alpha)2n$

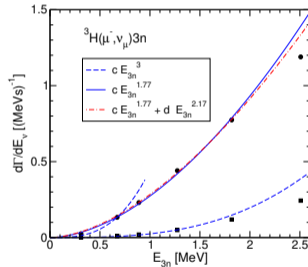
■ Power law behavior at low energies



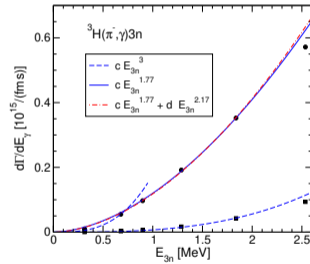
- $N = 5$: $E^8 \rightarrow E^{5.1}$
- $N = 6$: $E^{10.5} \rightarrow E^{6.2}$
- ...
- Predictions for $N \leq 70$ available
(M.G. Endres et al., PRA **84**, 043644 (2011))
- No peak except for $2n$

Braaten, HWH, Phys. Rev. D **107**, 034017 (2023)

- Radiative muon/pion capture on the triton (AV18 + UIX)



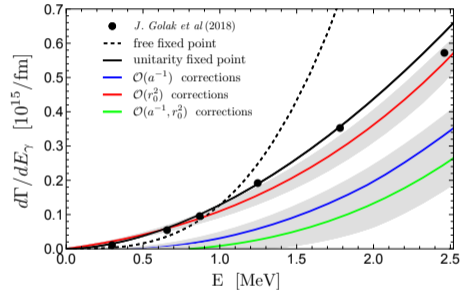
Golak et al., PRC **98**, 054001 (2018)



Golak et al., PRC **94**, 054001 (2016)

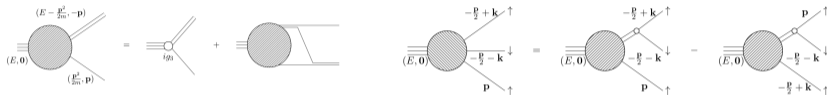
- Conformal prediction: $d\Gamma/dE \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}$, $0.1 \text{ MeV} \ll E_{3n} \ll 5 \text{ MeV}$

- Radiative pion capture on the triton (AV18 + UIX)
- Can be calculated in large charge (\Rightarrow large N) expansion
- Large $1/a$ corrections
- Can be calculated in pionless EFT

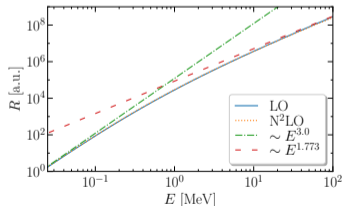
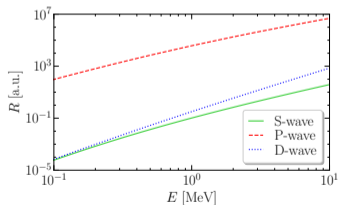


S.R. Beane, D. Orlando, S. Reffert, Phys. Rev. D **112**, 014028 (2025)

Point-production amplitude in pionless EFT



Reproduces conformal behavior (Backert, Dietz, HWH, König, Son, Phys. Rev. C **113**, 064001 (2026))

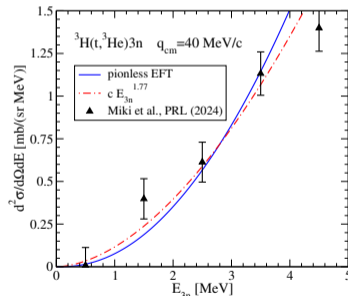
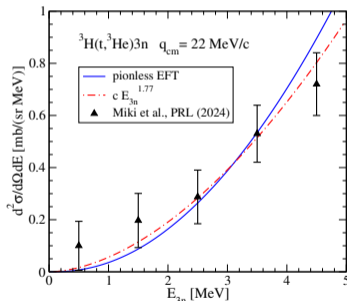


P-Wave

Free and unitary limit exponents reproduced, range corrections very small

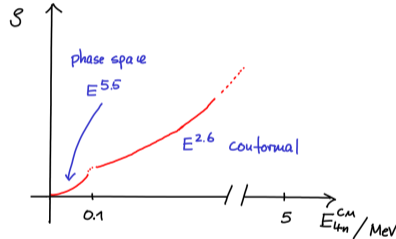
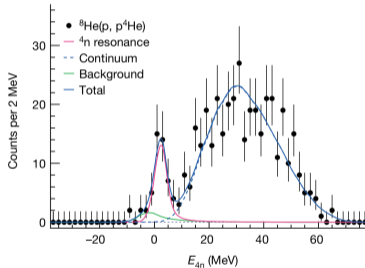
$3n$ distributions from experiment

- $3n$ case consistent with previous experiments for ${}^3\text{H}(\pi^-, \gamma)3n$ (Miller et al., Nucl. Phys. A **343**, 347 (1980))
- $3n$ distributions from ${}^3\text{H}({}^3\text{H}, {}^3\text{He})3n$ (Miki et al., Phys. Rev. Lett. **133**, 012501 (2024))



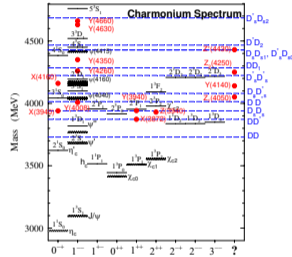
- Small q_{cm} data consistent

- Search for tetra-neutron resonances in ${}^8\text{He}(p, p\alpha)4n$ (M. Duer et al., Nature **606**, 678 (2022))



- Structure from initial state?
- Dineutron correlations can produce peak (Lazauskas, Hiyama, Carbonell, Phys. Rev. Lett. **130**, 102501 (2023))
(cf. Zhang, Elhatisari, Meißner, arXiv:2512.18849)

- **New $c\bar{c}$ states at B factories: X, Y, Z**
(cf. Godfrey, arXiv:0910.3409)
 - ▣ Challenge for understanding of QCD
 - ▣ Unitary limit relevant?
- **X(3872)** (Belle, CDF, BaBar, D0, LHCb)
- **Nature of X(3872) ?**
 - ▣ $\bar{D}^0 D^{0*}$ -molecule, tetraquark, charmonium hybrid, ...



$$m_X = (3871.65 \pm 0.06) \text{ MeV}, \quad \Gamma = (1.19 \pm 0.21) \text{ MeV}, \quad J^{PC} = 1^{++} \quad (\text{PDG 2023})$$

- **Assumption: X(3872) is weakly-bound $D^0\bar{D}^{0*}$ -molecule**
 - $\Rightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle) / \sqrt{2}, \quad B_X = (0.07 \pm 0.12) \text{ MeV} \approx 1 / (2\mu_{D\bar{D}^*} a^2)$
 - \Rightarrow **universal properties** (Braaten et al., 2003-2008; ...)

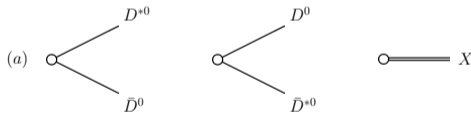
Neutral charm mesons and $X(3872)$

- Approximate unparticles of three D^0/D^{0*} mesons
- Interaction of $X(3872)$ with $D^0, \bar{D}^0, D^{0*}, \bar{D}^{0*}$ determined by large a
(Canham, HWH, Springer, PRD **80**, 014009 (2009))

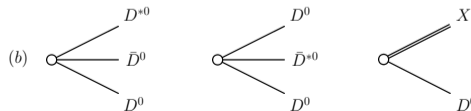
$$a_{D^0 X} = -9.7a \quad a_{D^{*0} X} = -16.6a$$

- Richer structure because of $X(3872)$ (bound state)

two charm mesons



three charm mesons



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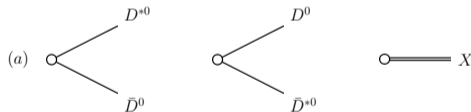
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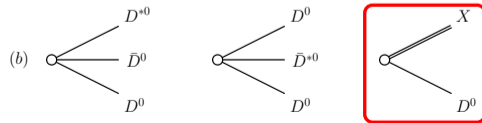
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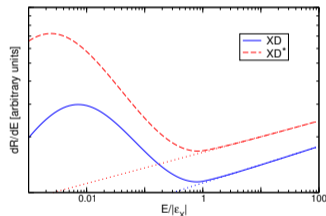


three charm mesons

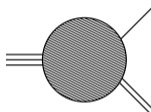


■ Universal scaling for unparticles of three neutral charm mesons

(Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022))



XD point production



Henkel, Unterberger, Nucl. Phys. B **660**, 407 (2003)

Fuertes, Moroz, Phys. Rev. D **79**, 106004 (2009)

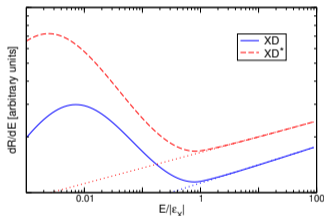
Volovich, Wen, JHEP **05**, 087 (2009)

$$\frac{dR}{dE} \sim (E^{-(\Delta_1+\Delta_2-\Delta_3)/2})^2 \sqrt{E} \approx E^{0.1} \quad \text{for } E/|\varepsilon_x| \gg 1 \quad \text{from conformal 3-pt. function}$$

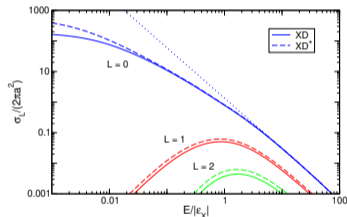
$$\Delta_1=3/2, \quad \Delta_2=2, \quad \Delta_3 \approx 3.10119/3.08697 \quad (\text{Braaten, HWH, Phys. Rev. D } \mathbf{107}, 034017 (2023))$$

■ Universal scaling for unparticles of three neutral charm mesons

(Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022))



XD point production



XD elastic scattering

$$\frac{dR}{dE} \sim (E^{-(\Delta_1+\Delta_2-\Delta_3)/2})^2 \sqrt{E} \approx E^{0.1}, \quad \sigma \sim E^{-1.6} \quad \text{for } E/|\epsilon_X| \gg 1$$

$$\Delta_1=3/2, \quad \Delta_2=2, \quad \Delta_3 \approx 3.10119/3.08697$$

- Universality in strongly interacting quantum systems
- High-energy nuclear reactions with final state neutrons
 - ⇒ (approximate) **conformal symmetry**
 - ⇒ **power law behavior of observables** determined by scaling dimension Δ
- Model-independent constraints on (certain) nuclear reactions
- Connection between reactions & properties of trapped particles

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- **Understand difference between $3n$ and $4n$, initial state dependence**
- Other applications & extensions
 - Neutral charm mesons, bottom mesons (Lin, HWH, Meißner, JHEP **04**, 092 (2026))
 - Two-component fermions in ultracold atom physics
 - Systems with the Efimov effect?
 - ⇒ bosonic atoms, nucleons, α particles
 - ⇒ **only discrete scale symmetry**, complex scaling dimensions