

# **NR** conformal collider physics

**Subham Dutta Chowdhury**

**with Cyuan-Han Chang, Ian Mout & Dam Thanh Son**



**Funded by  
the European Union**

**Les Diablerets, 2026**

"Exotic High Energy Phenomenology" (X-HEP), project funded by the European Union - Grant Agreement n. 101039756

Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the ERC Executive Agency (ERCEA). Neither the European Union nor the granting authority can be held responsible for them

# What is NRCFT?

Consider the symmetries of the Schrödinger equation in  $d$  space dimensions,

$$\left( i\partial_t + \frac{\nabla^2}{2M} \right) \Psi(t, \vec{x}) = 0$$

Spatial and time translations, galilean boosts, spatial rotations and  $U(1)$  rotations (Particle conservation)

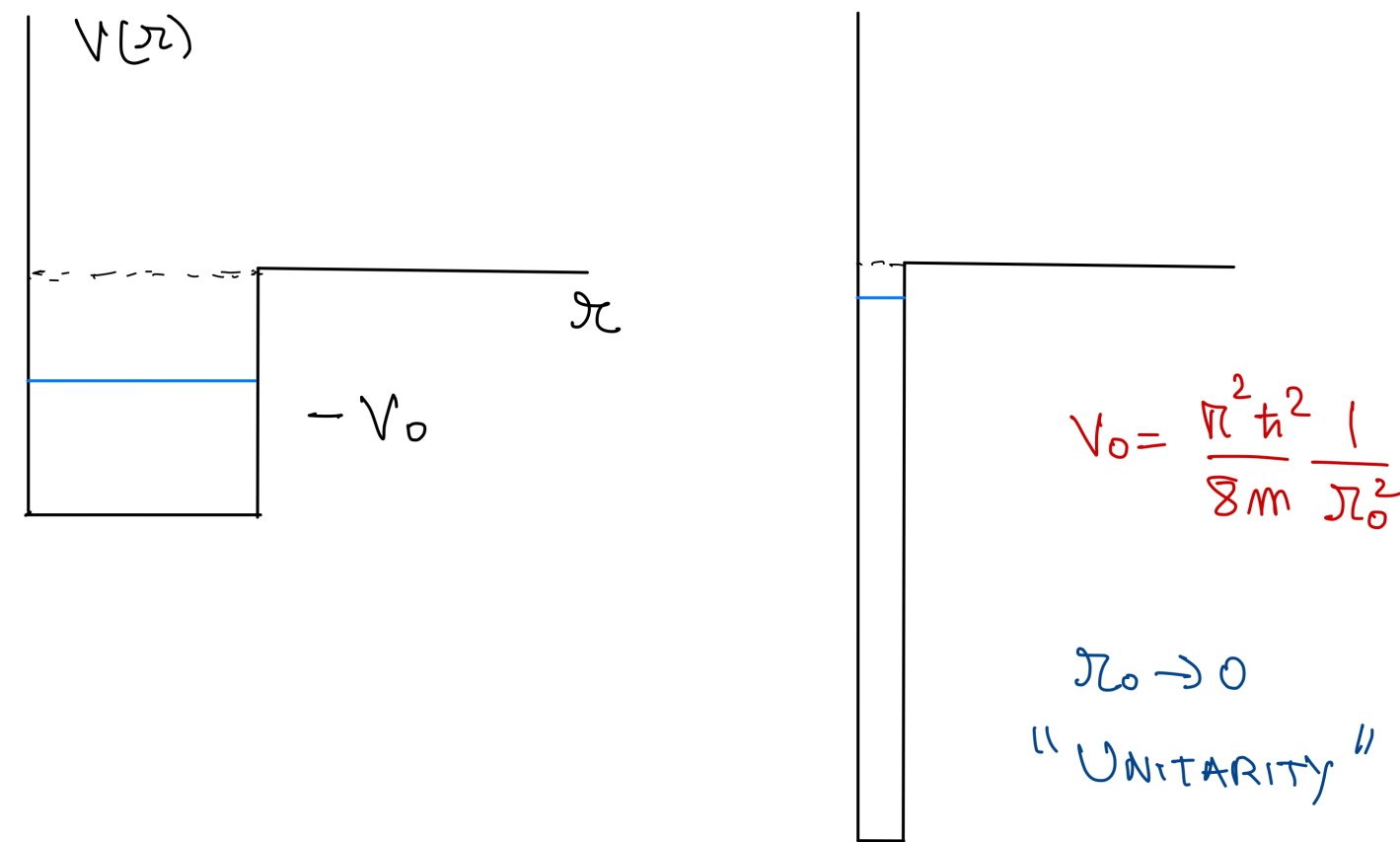
Also, scale transformation  $D$  and conformal transformation  $C$

$$\Psi(t, \vec{x}) \rightarrow e^{\lambda d/2} \Psi(e^{-2\lambda} t, e^{-\lambda} \vec{x})$$

$$\Psi(t, \vec{x}) \rightarrow (1 - ct)^{-d/2} e^{\frac{-iM\vec{x}^2 c}{1-ct}} \Psi\left(\frac{t}{1-ct}, \frac{\vec{x}}{1-ct}\right)$$

# What is NRCFT?

## Quantum mechanics



Fermions with short range interaction, with large scattering length.

**Strongly interacting system!**

This system enjoys **scale** and **conformal** symmetries (akin to free Schrödinger equation):

**Fermions at unitarity**

## Second quantised version

$$\mathcal{L} = \sum_{\sigma=\uparrow\downarrow} \psi_{\sigma}^{\dagger} \left( i\partial_t + \frac{\nabla^2}{2} \right) \psi_{\sigma} + c \psi_{\downarrow}^{\dagger} \psi_{\uparrow}^{\dagger} \psi_{\uparrow} \psi_{\downarrow}.$$

[Nishida & Son 2007]

$c$  is irrelevant: needs renormalisation + tuning

↓  
**Fixed point**

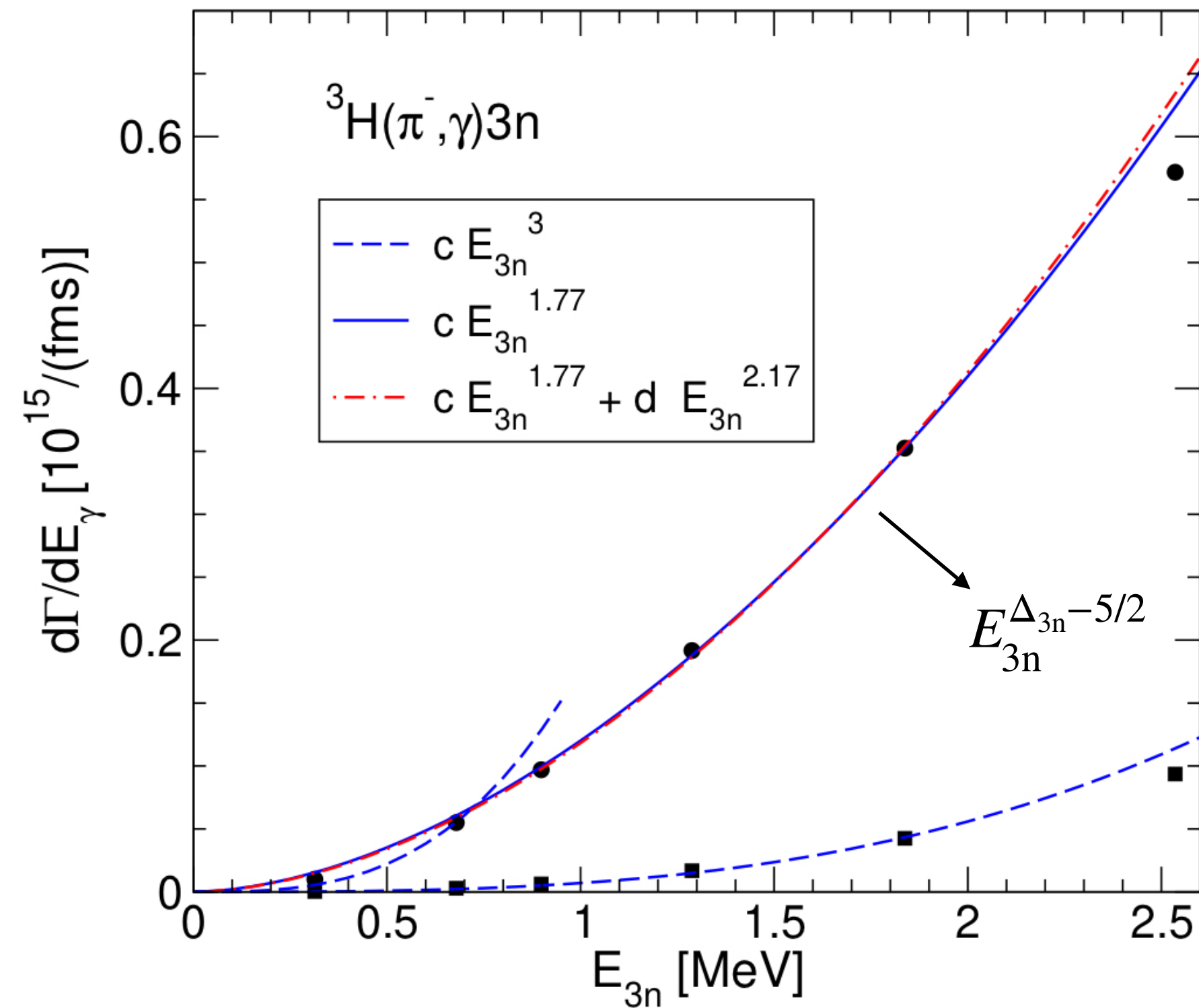
Can define primary operators  $\Phi(t, \vec{x})$  with dimensions  $\Delta_{\Phi}$  and particle number  $N_{\Phi}$

$$\begin{aligned} \Phi(t, \vec{x}) &\rightarrow e^{\lambda \Delta_{\Phi}} \Phi(e^{-2\lambda} t, e^{-\lambda} \vec{x}) \\ \Phi(t, \vec{x}) &\rightarrow (1 - ct)^{-\Delta_{\Phi}} e^{\frac{i N_{\Phi} \vec{x}^2 c}{1 - ct}} \Phi\left(\frac{t}{1 - ct}, \frac{\vec{x}}{1 - ct}\right) \end{aligned}$$

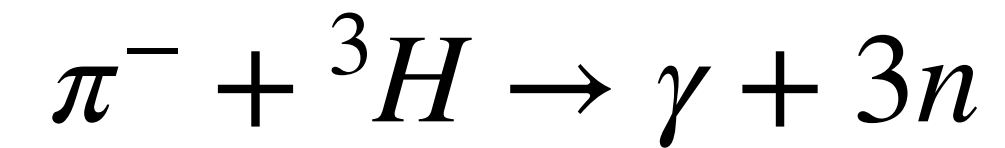
convention:  $N_{\psi} = -1$

# Where is NRCFT?

ref: HWH's talk



[Golok et al 2018]



$$a_n \sim -19 \text{ fm}, r_n \sim 2.8 \text{ fm}$$

$$\frac{\hbar^2}{2ma_n^2} \sim 0.1 \text{ Mev} \leq E_{3n} \leq 5 \text{ Mev} \sim \frac{\hbar^2}{2mr_n^2}$$

Within this energy range, the three-neutron state can be effectively described by a local operator in the NRCFT of fermions at unitarity.

$$\Delta_{3n} = 4.2727$$

[Son & Hammer 2021,  
SDC, Mishra & Son 2023,  
Beane, Orlando & Reffert 2025]

# Where is NRCFT?

Droplets of  ${}^3\text{He}$  atoms are energetically unstable unless number of atoms  $> 20 - 40$

[Pandharipande et al 1986, .....]

The decay rate of a near-threshold boson droplet (into free atoms) has **universal scaling behaviour**.

$$\Gamma(E) \sim \underbrace{E^{\Delta_o - 5/2}}$$

[Son, Stephanov & Yee 2021]

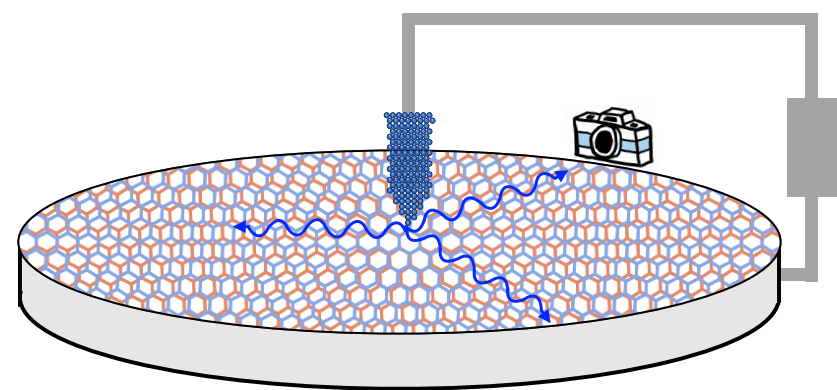
“effectively” described by

$$\mathcal{L} = \Psi^\dagger \left( i\partial_t + \frac{\nabla^2}{2m_\Psi} \right) \Psi + L_{\text{NRCFT}}[\psi_a] + g(O^\dagger \Psi + \Psi^\dagger O), \quad O = \psi_a^N \quad \text{Lowest dimensional N-boson operator.}$$

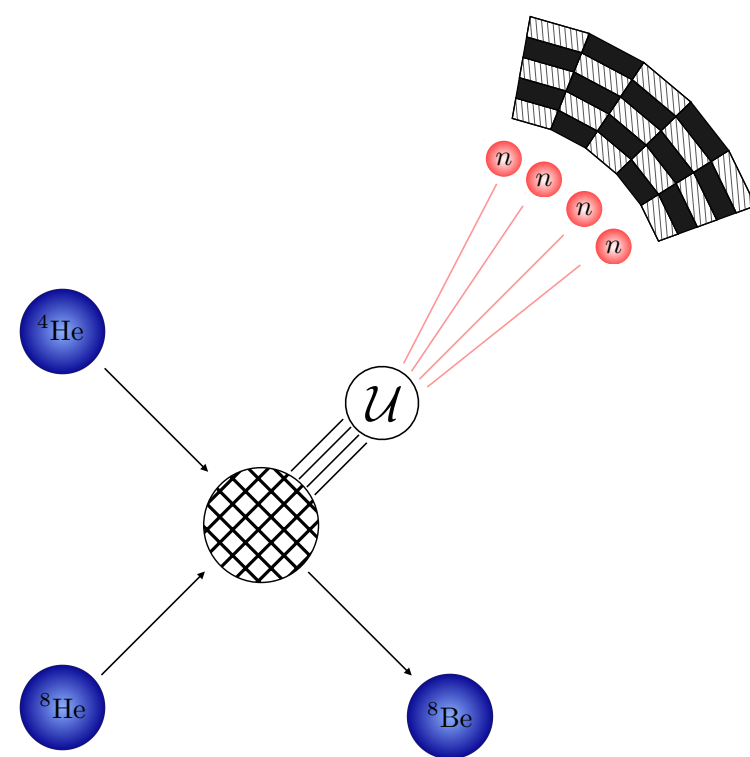
$\Psi$  describes the resonance and  $\psi_a$  describes the particles that constitute the resonance.

$$\Gamma(E) \sim \text{Im } \Sigma_\Psi$$

# Why is NRCFT?



Can we measure the momenta/energy distribution of neutrons/decay of droplet of *He* atoms in such experiments?



$$\sim \frac{\langle O(q) \mathcal{D}(\hat{n}_1) \cdots \mathcal{D}(\hat{n}_k) O^\dagger(q) \rangle}{\langle O(q) O^\dagger(q) \rangle}$$

Is there an operator definition of these observables analogous to relativistic CFTs?

Constraints due to galilean, conformal symmetry?

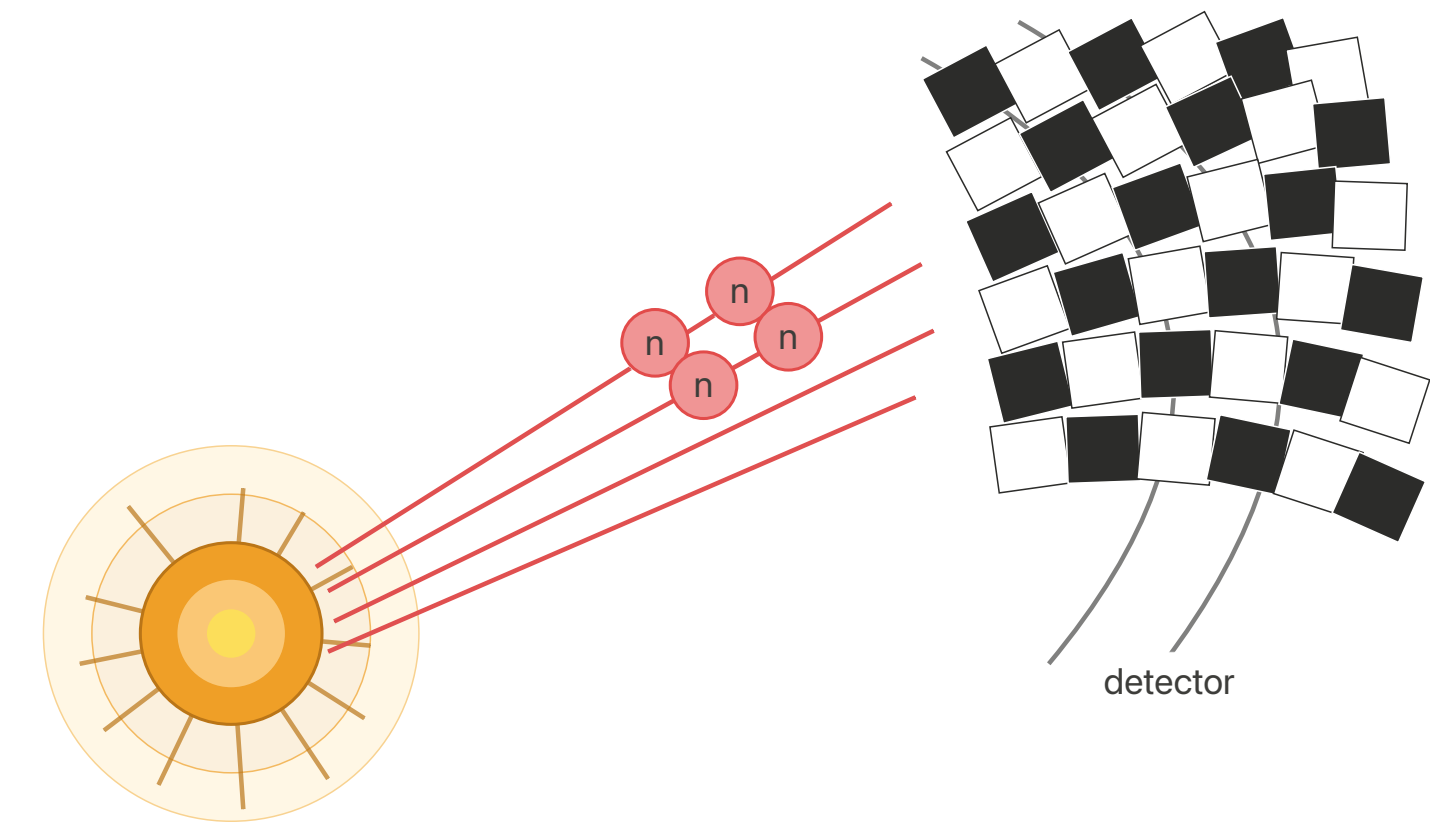
Potentially useful for high precision low energy experiments proposed in T. Aumann et al (2020)

.....

# Warmup: free theory

How does one define detectors in free theories, e.g., non-relativistic fermions ?

$$\mathcal{L} = \sum_{\sigma=\uparrow\downarrow} \psi_{\sigma}^{\dagger} \left( i\partial_t + \frac{\nabla^2}{2m} \right) \psi_{\sigma}.$$



Want to measure energy deposited at large distance from the collision event.

$$\mathcal{E}(\hat{n}) = \sum_{\sigma=\uparrow,\downarrow} \int_{\vec{k}} \frac{k^2}{2m} a_{\sigma}^{\dagger}(k) a_{\sigma}(k) \delta^{(2)}(\hat{k} - \hat{n}) \quad \psi(t, \vec{x}) = \int_{\vec{k}} e^{i(\vec{k} \cdot \vec{x} - \frac{\vec{k}^2}{2m}t)} a_{\sigma}(k),$$

The detector literally measures energy per particle that is deposited at  $\hat{n}$

Want a more theory independent / operator version!

# Warmup: free theory

Rewriting the energy detector as flux

$$\begin{aligned} \lim_{r \rightarrow \infty} \psi \left( \frac{m}{k_0} r, r \hat{n} \right) &= \lim_{r \rightarrow \infty} \int_{\vec{k}} e^{i \left( k_n r - \frac{(k_n^2 + k_\perp^2)}{2m} \frac{m}{k_0} r \right)} a_\sigma(k) \\ &\simeq \left( \frac{-ik_0}{2\pi r} \right)^{\frac{3}{2}} a_\sigma(k_0 \hat{n}) + O \left( \frac{1}{r^{\frac{5}{2}}} \right) \end{aligned}$$

$$\vec{k}_n = \hat{n} \cdot \vec{k} \hat{n}, \quad \vec{k}_\perp = \vec{k} - \vec{k}_n$$

A more theory-independent definition

$$\mathcal{E}(\hat{n}) = \frac{1}{2m} \int dk_0 k_0 \lim_{r \rightarrow \infty} r^3 n \left( \frac{m}{k_0} r, r \hat{n} \right) = \frac{1}{2} \int dk_0 \lim_{r \rightarrow \infty} r^3 \hat{n} \cdot \vec{j} \left( \frac{m}{k_0} r, r \hat{n} \right).$$

$$n(t, \vec{x}) = \sum_{\sigma=\uparrow, \downarrow} \psi_\sigma^\dagger(t, \vec{x}) \psi_\sigma(t, \vec{x}),$$

$$j^i(t, \vec{x}) = \frac{-i}{2m} \sum_{\sigma=\uparrow, \downarrow} \left( \psi_\sigma^\dagger(t, \vec{x}) \partial_{\vec{x}}^i \psi(t, \vec{x}) - \partial_{\vec{x}}^i \psi_\sigma^\dagger(t, \vec{x}) \psi_\sigma(t, \vec{x}) \right).$$

Valid beyond free theory, as we will show.

# Warmup: free theory

graded by “velocity”  $v = \frac{k_0}{m}$

$$\mathcal{E}(\hat{n}) = \int_0^\infty dv \mathcal{E}_v(\hat{n}), \quad \mathcal{E}_v(\hat{n}) = \frac{m}{2} \lim_{r \rightarrow \infty} r^3 \hat{n} \cdot \vec{j} \left( \frac{r}{v}, r\hat{n} \right) = \frac{mv}{2} \lim_{r \rightarrow \infty} r^3 \vec{n} \left( \frac{r}{v}, r\hat{n} \right)$$

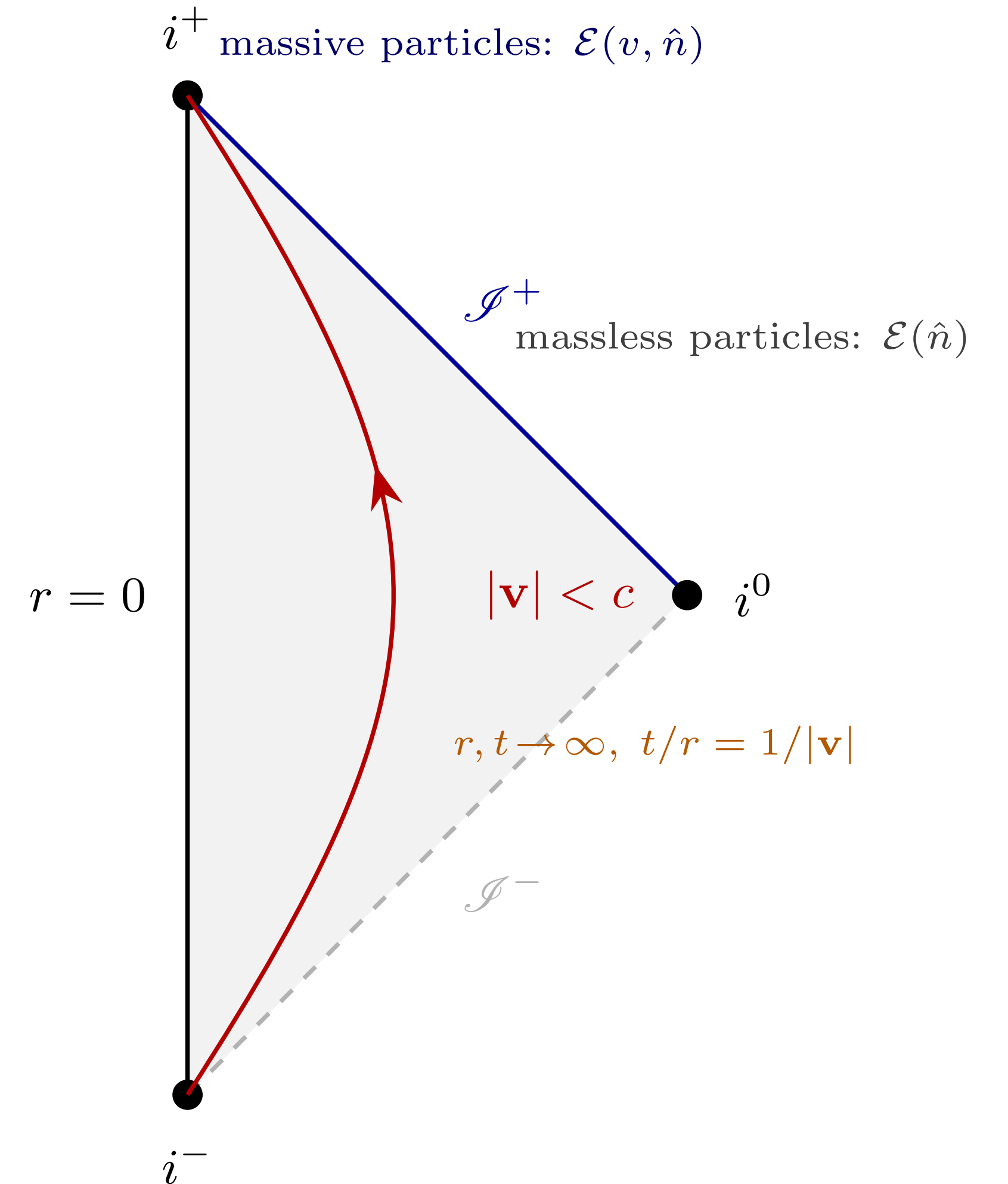
we will be primarily interested in one-point functions of the detector:

$$\langle \mathcal{E}_v(\hat{n}) \rangle_{O(\omega, \vec{k})} \equiv \frac{\frac{1}{2} \lim_{r \rightarrow \infty} r^3 \langle 0 | O(\omega, \vec{k}) \hat{n} \cdot \vec{j} \left( \frac{r}{v}, r\hat{n} \right) O^\dagger(\omega, \vec{k}) | 0 \rangle}{\langle 0 | O(\omega, \vec{k}) O^\dagger(\omega, \vec{k}) | 0 \rangle}.$$

In free theory, particles travel in straight lines to spatial infinity,

$$\int d\Omega_{\hat{n}} \int dv \langle \mathcal{E}_v(\hat{n}) \rangle_{O(\omega, \vec{k})} = \omega.$$

We set  $m = 1$  for remainder of the discussion.



Analogy with detectors for massive particles in relativistic setting

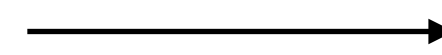
[Mateu, Stewart & Thaler 2012]

# Detectors in general NRCFTs

Not so obvious in interacting theory

$$\int d\Omega_{\hat{n}} \int dv \langle \mathcal{E}_v(\hat{n}) \rangle_{O(\omega, \vec{k})} \stackrel{?}{=} \omega.$$

- Existence of  $j^i, n$
- Schrödinger symmetry



$$\mathcal{E}(\hat{n}) = \int_0^\infty dv \mathcal{E}_v(\hat{n}),$$
$$\mathcal{E}_v(\hat{n}) = \frac{1}{2} \lim_{r \rightarrow \infty} r^3 \hat{n} \cdot \vec{j} \left( \frac{r}{v}, r\hat{n} \right) = \frac{v}{2} \lim_{r \rightarrow \infty} r^3 \vec{n} \left( \frac{r}{v}, r\hat{n} \right)$$

continues to be a good detector definition even in interacting theory!

# NRCFT basics

Rotations, translations, galilean boosts, dilatations and special conformal transformations, generated by the set

$$\{M_{ij}, P_i, K_i, D, C\}$$

$A \backslash B$	$P_j$	$K_j$	$D$	$C$	$H$
$P_i$	0	$-i\delta_{ij}N$	$-iP_i$	$-iK_i$	0
$K_i$	$i\delta_{ij}N$	0	$iK_i$	0	$iP_i$
$D$	$iP_j$	$-iK_j$	0	$-2iC$	$2iH$
$C$	$iK_j$	0	$2iC$	0	$iD$
$H$	0	$-iP_j$	$-2iH$	$-iD$	0

$\{M_{ij}, P_i, K_i, D, C, N\}$  admit a representation in terms of  $n, j^i$

$$[n(\vec{x}), n(\vec{y})] = 0, \quad [n(\vec{x}), j_i(\vec{y})] = -in(\vec{y})\partial_i^x \delta^{(3)}(\vec{x} - \vec{y}),$$

$$[j_i(\vec{x}), j_j(\vec{y})] = -i \left( j_j(\vec{x})\partial_i^x + j_i(\vec{y})\partial_j^x \right) \delta^{(3)}(\vec{x} - \vec{y}).$$

[Nishida & Son 2007]

$$[H, N] = [H, P_i] = [H, M_{ij}] = 0, \quad [N, \text{any}] = 0$$

# NRCFT basics

Local operators, are labelled as primary if,

$$[K_i, O(0,0)] = 0, \quad [C, O(0,0)] = 0$$

They are also usually labelled by two quantum numbers,

$$[D, O(0,0)] = \Delta_O O(0,0), \quad [N, O(0,0)] = N_O$$

Descendants which are generated by  $P_i$  and  $H$

$$[H, O] = -i\partial_t O, \quad [P_i, O] = -i\partial_i O$$

While  $n$  transforms like a primary operator  $j^i$  **does not**,

“Alien operator”: neither primary nor descendants

[Golkar & Son 2014]

$$[K^i, j^j(t, \vec{x})] = -it\partial_x^i j^j(t, \vec{x}) + i\delta^{ij}n(t, \vec{x}), \quad [C, j^i(t, \vec{x})] = -i\left(t^2\partial_t + t\vec{x} \cdot \partial_{\vec{x}} + t\Delta_j\right)j^i(t, \vec{x}) + i\vec{x}^i n(t, \vec{x})$$

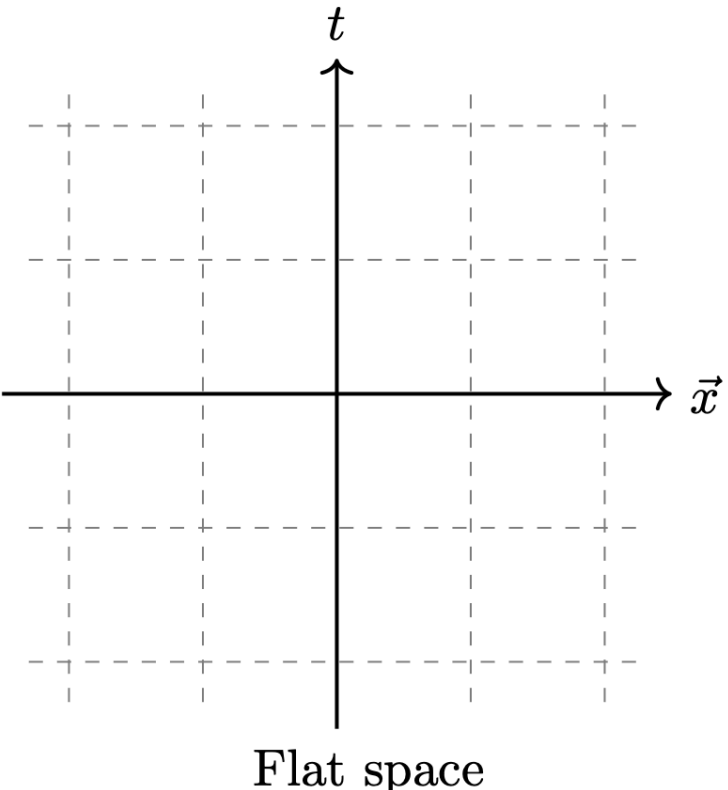
# State-operator correspondence

Operators are in one-to-one correspondence with eigenstates of the system in harmonic potential.

$$H_\omega = H + \omega^2 C, \quad |O\rangle = e^{\frac{-H}{\omega}} O^\dagger(0,0) |0\rangle$$

$$H_\omega |O\rangle = \omega \Delta_O |O\rangle$$

(we set  $\omega = 1$  (not to be confused with frequency/energy) for the rest of the talk)

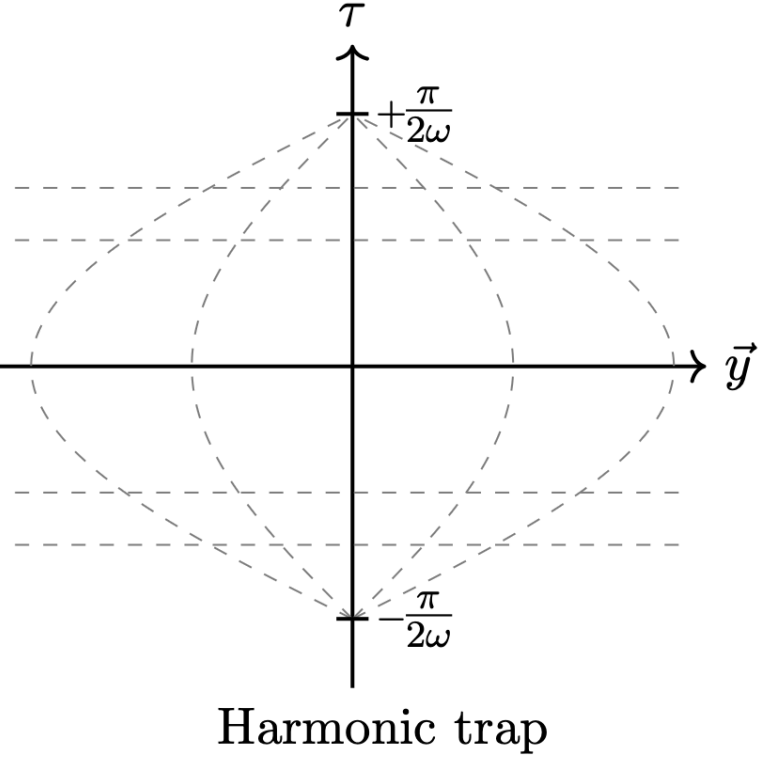


$$\omega t = \tan(\omega\tau)$$

$$\vec{x} = \vec{y} \sec(\omega\tau)$$

$$\omega\tau = \arctan(\omega t)$$

$$\vec{y} = \frac{\vec{x}}{\sqrt{1 + \omega^2 t^2}}$$



Primary operators transform in the following way:

$$O(t, \vec{x}) = (\cos \tau)^{\Delta_O} \exp\left(-\frac{i}{2} N_O \vec{y}^2 \tan \tau\right) \tilde{O}(\tau, \vec{y})$$

Operator in the oscillator frame.

[Nishida & Son 2007, Goldberger, Khandker & Prabhu 2014, Boisvert, Fadda, Kulp & Yazdi 2025]

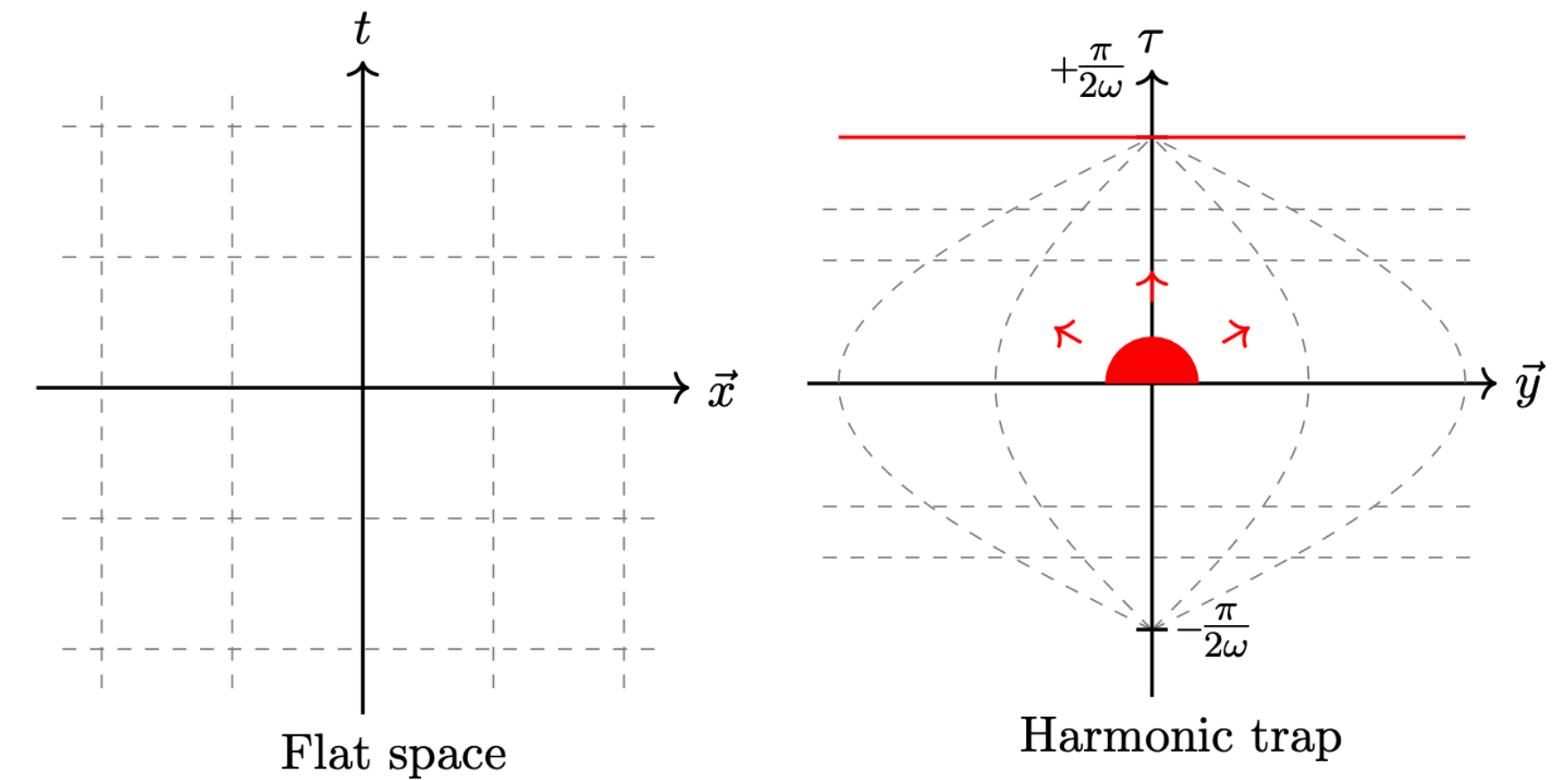
# Detector in harmonic trap

The current and charge density transform as:

$$j^i(t, \vec{x}) = (\cos \tau)^4 [j^i(\tau, \vec{y}) + \tan \tau \vec{y}^i \tilde{n}(\tau, \vec{y})], \quad n(t, \vec{x}) = (\cos \tau)^3 \tilde{n}(\tau, \vec{y}),$$

Detector limit:  $t = \frac{r}{v}, \vec{x} = r\hat{n}, r \rightarrow \infty \implies \tau = \frac{\pi}{2}, \vec{y} = v\hat{n}$

$$\mathcal{E}_v(\hat{n}) = \frac{v^4}{2} \tilde{n} \left( \frac{\pi}{2}, v\hat{n} \right).$$



Pictorial representation of  $\int d\Omega_{\hat{n}} \int dv \mathcal{E}_v(\hat{n})$  in harmonic trap geometry

# Ward identities and symmetries

Harmonic trap/oscillator frame immediately makes the **symmetries manifest!**

$$\int d\Omega_{\hat{n}} \int dv \mathcal{E}_v(\hat{n}) = \int d^3\vec{v} \frac{\vec{v}^2}{2} \tilde{n} \left( \frac{\pi}{2}, \vec{v} \right).$$

$$\int d\Omega_{\hat{n}} \int dv \mathcal{E}_v(\hat{n}) = e^{i(H+C)\frac{\pi}{2}} \left( \int d^3\vec{v} \frac{\vec{v}^2}{2} \tilde{n} (0, \vec{v}) \right) e^{-i(H+C)\frac{\pi}{2}}.$$

since  $\tilde{n}(0, \vec{y}) = n(0, \vec{y})$ , this is just the **flat-space conformal generator!**

$$\int d\Omega_{\hat{n}} \int dv \mathcal{E}_v(\hat{n}) = H.$$

Using  $SO(2,1)$  sub algebra of schrödinger symmetry:

$$[H, C] = -iD, \quad [D, C] = -2iC, \quad [D, H] = 2iH$$

Our detector therefore measures total energy injected into the system.

# Ward identities and symmetries

Harmonic trap geometry motivates us to also propose new detector observables measuring charge and total spatial momentum,

$$\int d^3\vec{v} \tilde{n} \left( \frac{\pi}{2}, \vec{v} \right) = N, \quad \int d^3\vec{v} \vec{v}^i \tilde{n} \left( \frac{\pi}{2}, \vec{v} \right) = P^i$$

With the final set of ward identities,

$$\frac{1}{2} \int d^3\vec{v} \vec{v}^2 \langle \tilde{n} \left( \frac{\pi}{2}, \vec{v} \right) \rangle_{O(\omega, \vec{k})} = \omega, \quad \int d^3\vec{v} \langle \tilde{n} \left( \frac{\pi}{2}, \vec{v} \right) \rangle_{O(\omega, \vec{k})} = -N_O, \quad \int d^3\vec{v} \vec{v}^i \langle \tilde{n} \left( \frac{\pi}{2}, \vec{v} \right) \rangle_{O(\omega, \vec{k})} = \vec{k}^i.$$

Valid for any (interacting) NRCFT.

# Ward identities and symmetries

We are interested in the one point function of the detectors in states created by primary operators.

$$\langle O(\omega, \vec{k}) \tilde{n} \left( \frac{\pi}{2}, v\hat{n} \right) O^\dagger(\omega', \vec{k}') \rangle \xrightarrow{\text{time and spatial translation invariance}} \delta(\omega - \omega') \delta^{(3)}(\vec{k} - \vec{k}') F(v\hat{n}, \vec{k}, \omega)$$

$$F(v\hat{n}, \vec{k}, \omega) \xrightarrow{\text{galilean and scale invariance}} \left( \omega + \frac{k^2}{2N_o} \right)^{\Delta_o - 4} f \left( \frac{v\hat{n} + \frac{\vec{k}}{N_o}}{\sqrt{\omega + \frac{k^2}{2N_o}}} \right)$$

f is an unconstrained function

- determine  $f$  for  $\vec{k} = 0$ : sufficient due to Galilean invariance.
- verifying ward identities for  $\vec{k} = 0 \implies$  ward identity for  $\vec{k} \neq 0$ .

# Examples: Free fermions

Hilbert space characterised by fixed number of spin up and spin down particles.

$$\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_{1,0} \otimes \mathcal{H}_{0,1} \otimes \mathcal{H}_{1,1} \otimes \mathcal{H}_{2,0} \otimes \dots$$

$$\mathcal{L} = \sum_{\sigma=\uparrow\downarrow} \psi_\sigma^\dagger \left( i\partial_t + \frac{\nabla^2}{2} \right) \psi_\sigma.$$

with multi-particle wave functions

$$\Psi(\{\vec{x}_i; \vec{y}_j\}) \equiv \Psi(\underbrace{\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m}_{\text{anti-symmetry due to Pauli exclusion principle}}; \vec{y}_1, \vec{y}_2, \dots, \vec{y}_n)$$

anti-symmetry due to Pauli exclusion principle

Simple for free fermions

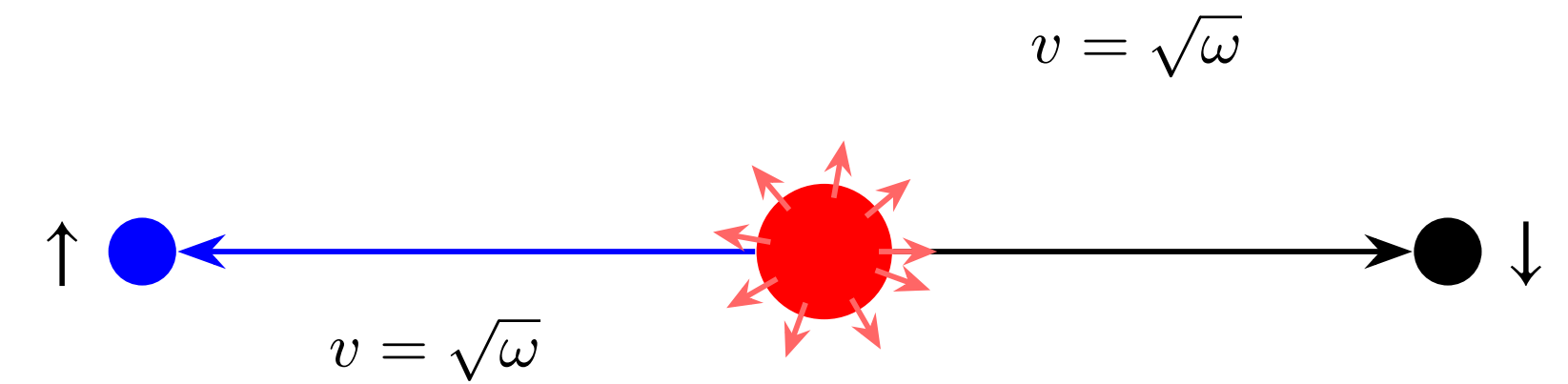
$$\frac{1}{\sqrt{p!q!}} \begin{vmatrix} \phi_1(\vec{x}_1) & \dots & \phi_1(\vec{x}_p) \\ \vdots & \ddots & \vdots \\ \phi_p(\vec{x}_1) & \dots & \phi_p(\vec{x}_p) \end{vmatrix} \times \begin{vmatrix} \chi_1(\vec{y}_1) & \dots & \chi_1(\vec{y}_q) \\ \vdots & \ddots & \vdots \\ \chi_q(\vec{y}_1) & \dots & \chi_q(\vec{y}_q) \end{vmatrix}$$

Also need local operators, that create the states from vacuum:  $O_Q$   $\longrightarrow$  Degree  $Q$  polynomial of  $\psi_\uparrow, \psi_\downarrow$  and arbitrary number of derivatives to account for Pauli exclusion principle

Focus on primary  $O_Q$   $\longrightarrow$   $\langle O_Q(t, \vec{x}) O_Q^\dagger(0,0) \rangle = \frac{C e^{-\frac{iQ\vec{x}^2}{2t}}}{t^{\Delta_{O_n}}}$

# Examples: Free fermions

Warm up with two particle states: **outcome is kinematically constrained.**



states created by the local operator:  $O_2^{\text{free}}(t, \vec{x}) = \psi_{\downarrow} \psi_{\uparrow}(t, \vec{x})$

which overlap with the two particle sector of the hilbert space,

$$|\vec{k}_1, \vec{k}_2\rangle = \int_{\vec{x}, \vec{y}} \Psi_{\vec{k}_1, \vec{k}_2}(\vec{x}, \vec{y}) \psi_{\downarrow}^{\dagger}(\vec{y}) \psi_{\uparrow}^{\dagger}(\vec{x}) |0\rangle, \quad \Psi_{\vec{k}_1, \vec{k}_2}^{\text{free}}(\vec{x}, \vec{y}) = \frac{1}{V} e^{i(\vec{k}_1 \cdot \vec{x} + \vec{k}_2 \cdot \vec{y})}.$$

(We normalise our wave functions in a box of volume  $V$  and take continuum limit in the end.)

Insert  $\sum_{k_i} |\vec{k}_1, \vec{k}_2\rangle \langle \vec{k}_1, \vec{k}_2|$  due to **particle number conservation**,

$$\langle O_2(\omega, \vec{k}) \mathcal{E}(\hat{n}) O_2^{\dagger}(\omega', \vec{k}') \rangle \implies \langle \mathcal{E}_v(\hat{n}) \rangle_{O_2(\omega, 0)}^{\text{free}} = \frac{1}{2\pi\sqrt{\omega}} v^4 \delta(\omega - v^2)$$

- consistent with physical picture.
- verified using **two different definitions** of  $\mathcal{E}(\hat{n})$

# Examples: Free fermions

Three particle states: **Non-trivial**

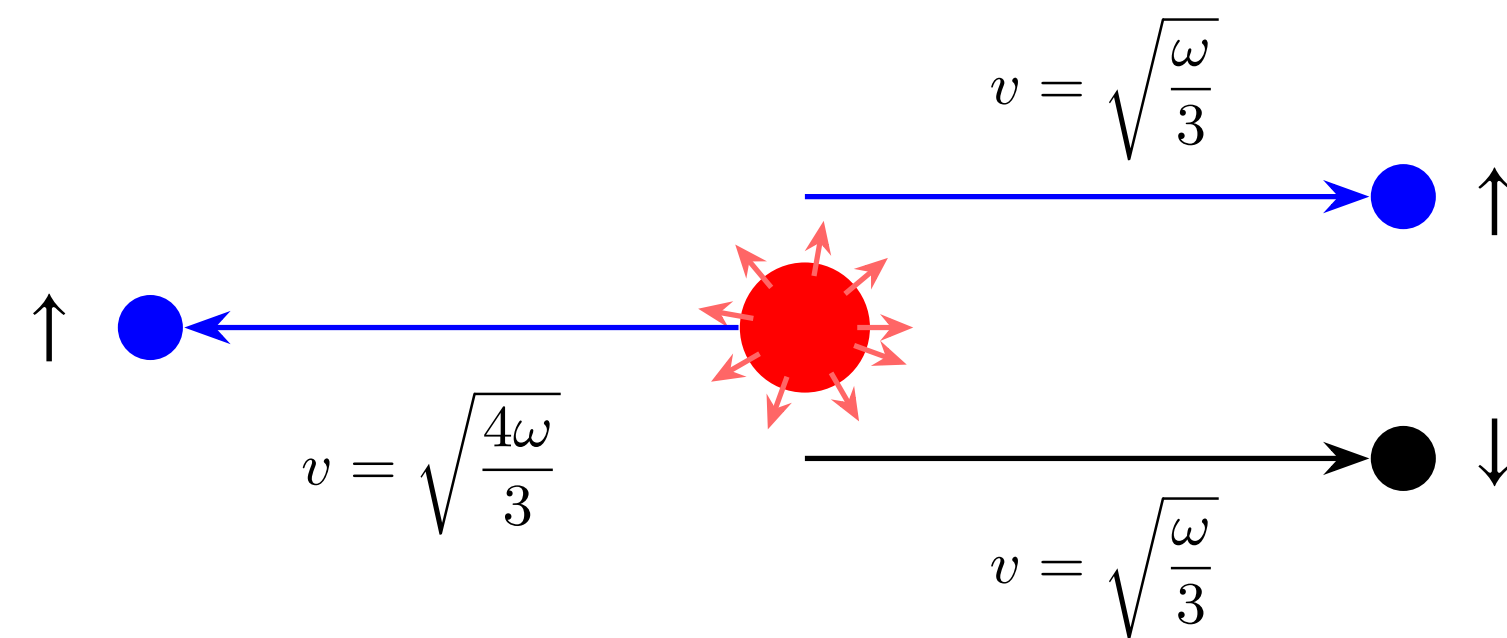
states created by the local operator:  $\epsilon \cdot O_3$ ,  $O_3^i(t, \vec{x}) = \psi_\uparrow \psi_\downarrow \partial^i \psi_\uparrow(t, \vec{x})$  a spin one operator

with the corresponding three particle Hilbert space

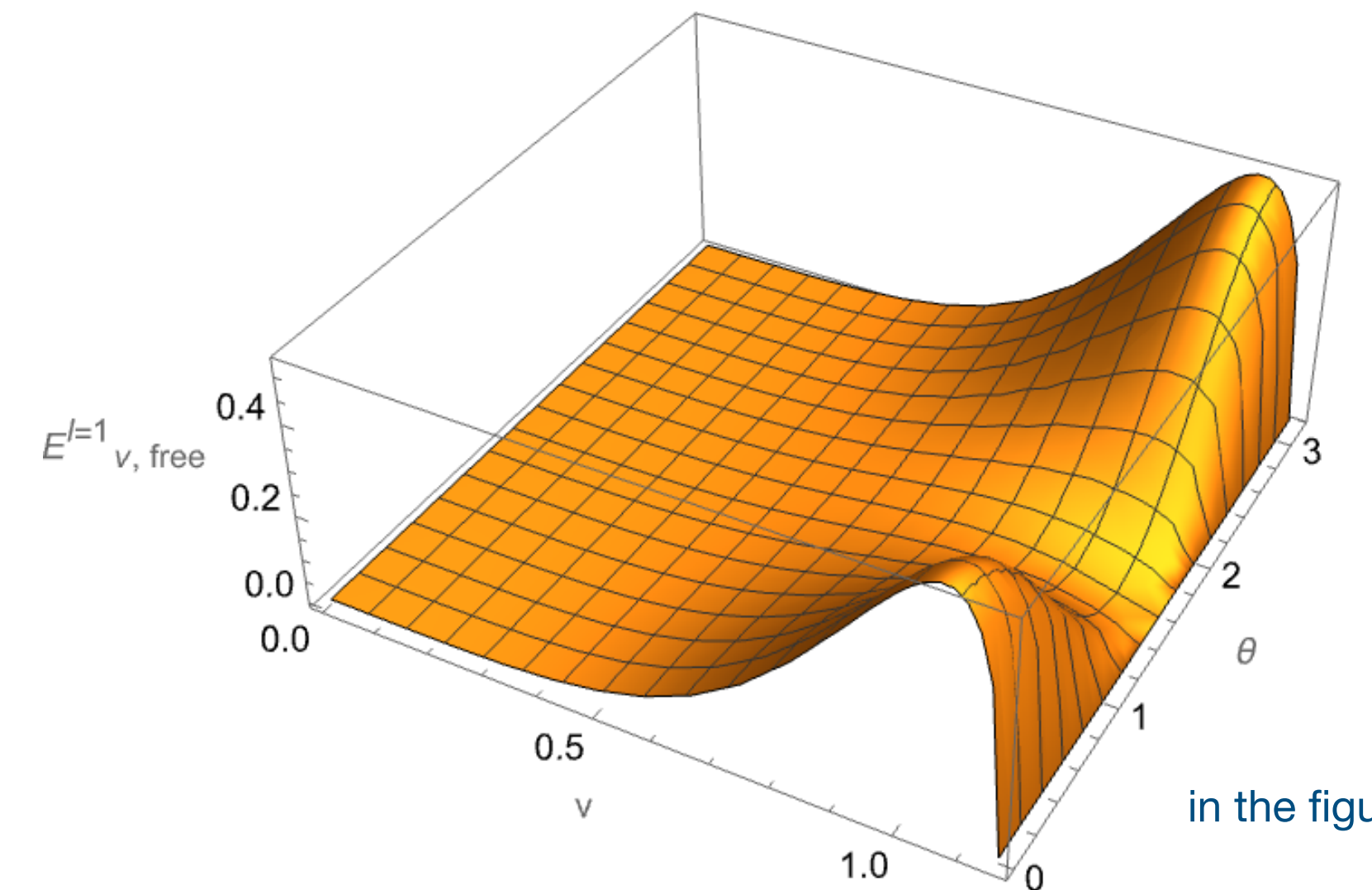
$$|\vec{k}_1, \vec{k}_2, \vec{k}_3\rangle = \int_{\vec{x}_i} \Psi_{\vec{k}_1, \vec{k}_2, \vec{k}_3}^{(3), \text{free}}(\vec{x}_1, \vec{x}_2, \vec{x}_3) \psi_\uparrow^\dagger(\vec{x}_1) \psi_\downarrow^\dagger(\vec{x}_2) \psi_\uparrow^\dagger(\vec{x}_3) |0\rangle, \quad \Psi_{\vec{k}_1, \vec{k}_2, \vec{k}_3}^{(3), \text{free}}(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \frac{1}{\sqrt{2}V^{3/2}} \left( e^{i(\vec{k}_1 \cdot \vec{x}_1 + \vec{k}_3 \cdot \vec{x}_3)} - e^{i(\vec{k}_1 \cdot \vec{x}_3 + \vec{k}_3 \cdot \vec{x}_1)} \right) e^{i\vec{k}_2 \cdot \vec{x}_2}$$

Appears naturally due to energy-momentum conservation!

$$\int_0^\infty dv \langle \mathcal{E}_v(\hat{n}) \rangle_{\epsilon \cdot O_3^{\ell=1}}^{\text{free}} = \int_0^{\frac{4\omega}{3}} dv \frac{9v^4 \sqrt{12\omega - 9v^2} \left( (4\omega - 3v^2) + 9v^2 \cos^2 \theta \right)}{32\pi^2 \omega^3}, \quad \epsilon \cdot \hat{n} = \cos \theta$$



Kinematic configuration for the maximal velocity



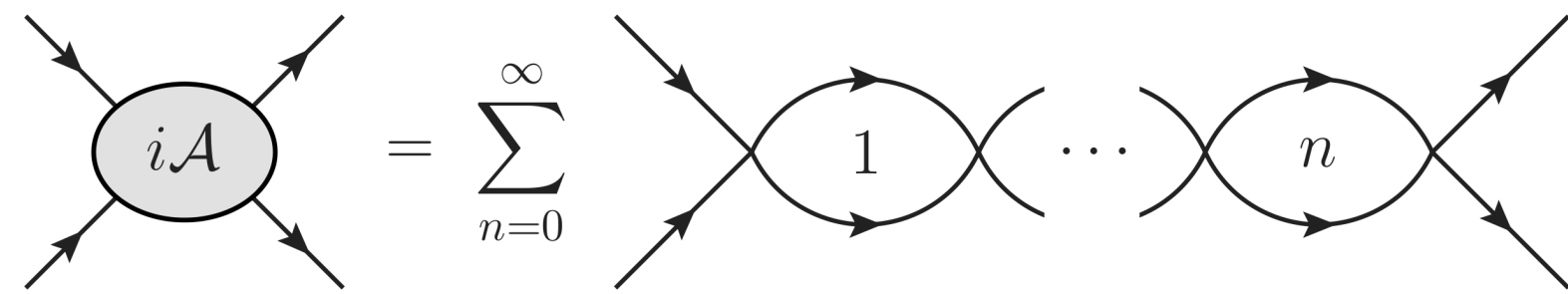
in the figures,  $\omega = 1$

- Also for scalar primary operators
- Free bosons

# Examples: Fermions at unitarity

$$\mathcal{L} = \sum_{\sigma=\uparrow\downarrow} \psi_{\sigma}^{\dagger} \left( i\partial_t + \frac{\nabla^2}{2} \right) \psi_{\sigma} + c\psi_{\downarrow}^{\dagger}\psi_{\uparrow}^{\dagger}\psi_{\uparrow}\psi_{\downarrow}.$$

$c$  is irrelevant: needs renormalisation + tuning



$$\frac{1}{4\pi a} = -\frac{1}{c} + \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2}, \quad a \rightarrow \infty$$

$$-iT(p)^{-1} \rightarrow -\frac{1}{4\pi} \sqrt{\frac{p^2}{4} - p_0 - i\epsilon},$$

A non-perturbative definition

$$\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_{1,0} \otimes \mathcal{H}_{0,1} \otimes \mathcal{H}_{1,1} \otimes \mathcal{H}_{2,1} \otimes \dots$$

$$\Psi(\{\vec{x}_i; \vec{y}_j\}) \equiv \Psi(\vec{x}_1, \vec{x}_2, \dots, \underbrace{\vec{x}_m; \vec{y}_1, \vec{y}_2, \dots, \vec{y}_n})$$

anti-symmetry due to Pauli exclusion principle

Unitarity condition: solve the free many-body Schrödinger equation with **Bethe-Peierls** boundary condition.

$$\Psi(\vec{x}_i, \vec{y}_j) \rightarrow \frac{C_0 \left( \frac{\vec{x}_i + \vec{y}_j}{2} \right)}{|\vec{x}_i - \vec{y}_j|} + O(|\vec{x}_i - \vec{y}_j|)$$

$$n(t, \vec{x}) = \sum_{\sigma=\uparrow,\downarrow} \psi_{\sigma}^{\dagger}(t, \vec{x}) \psi_{\sigma}(t, \vec{x}),$$

$$j^i(t, \vec{x}) = \frac{-i}{2} \sum_{\sigma=\uparrow,\downarrow} \left( \psi_{\sigma}^{\dagger}(t, \vec{x}) \partial_x^i \psi(t, \vec{x}) - \partial_x^i \psi_{\sigma}^{\dagger}(t, \vec{x}) \psi_{\sigma}(t, \vec{x}) \right).$$

# Examples: Fermions at unitarity

## Two-particle sector

Wavefunction:

$$\Psi_{\vec{P}_{\text{cm}},k}^{(2)}(\vec{x}_1, \vec{x}_2) = \frac{1}{\sqrt{2\pi R_{\text{max}} V}} \frac{\cos k |\vec{x}_1 - \vec{x}_2|}{|\vec{x}_1 - \vec{x}_2|} e^{i\vec{P}_{\text{cm}} \cdot \frac{\vec{x}_1 + \vec{x}_2}{2}}, \quad E_{\vec{P}_{\text{cm}},k} = \frac{P_{\text{cm}}^2}{4} + k^2,$$

relative coordinate  $r$  confined in a sphere with radius  $R_{\text{max}}$

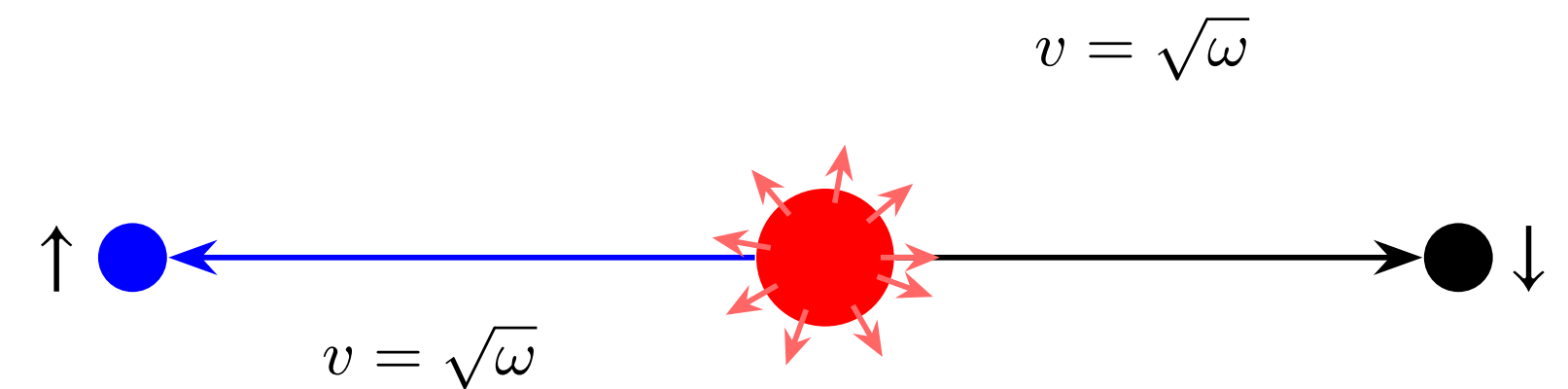
Local operator:

$$O_2(t, \vec{x}) = \lim_{|\vec{y}| \rightarrow 0} |\vec{y}| \psi_{\uparrow}(t, \vec{x} + \frac{\vec{y}}{2}) \psi_{\downarrow}(t, \vec{x} - \frac{\vec{y}}{2}), \quad \Delta = 2$$

[Nishida & Son 2007]

Kinematically trivial and (almost) same as free theory!

$$\langle \mathcal{E}_v(\hat{n}) \rangle_{O_2(\omega,0)}^{\text{interacting}} = \frac{\sqrt{\omega}}{2\pi} v^2 \delta(\omega - v^2)$$



The function  $f$  is different

# Examples: Fermions at unitarity

## Three-particle sector

Wavefunction:

$$|\Psi^{(3)}\rangle = \frac{1}{\sqrt{2}} \int d^3\vec{x}_1 d^3\vec{x}_2 d^3\vec{x}_3 \Psi^{(3)}(\vec{x}_1, \vec{x}_2, \vec{x}_3) \psi_{\uparrow}^{\dagger}(\vec{x}_1) \psi_{\downarrow}^{\dagger}(\vec{x}_2) \psi_{\uparrow}^{\dagger}(\vec{x}_3) |0\rangle.$$

$\downarrow \quad \downarrow$   
 anti-symmetry due to Pauli exclusion principle

Unitarity condition:

$$\Psi^{(3)}(\vec{x}_1, \vec{x}_2, \vec{x}_3) \rightarrow \frac{C_0 \left( \frac{\vec{x}_1 + \vec{x}_2}{2} \right)}{|\vec{x}_1 - \vec{x}_2|} + O(|\vec{x}_2 - \vec{x}_3|), \quad \Psi^{(3)}(\vec{x}_1, \vec{x}_2, \vec{x}_3) \rightarrow \frac{-C_0 \left( \frac{\vec{x}_3 + \vec{x}_2}{2} \right)}{|\vec{x}_3 - \vec{x}_2|} + O(|\vec{x}_3 - \vec{x}_2|),$$

Local operators:  $O_3^{l,m,s}(t, \vec{R}_{\text{cm}}), \quad \Delta(\ell) = s + \frac{5}{2}$

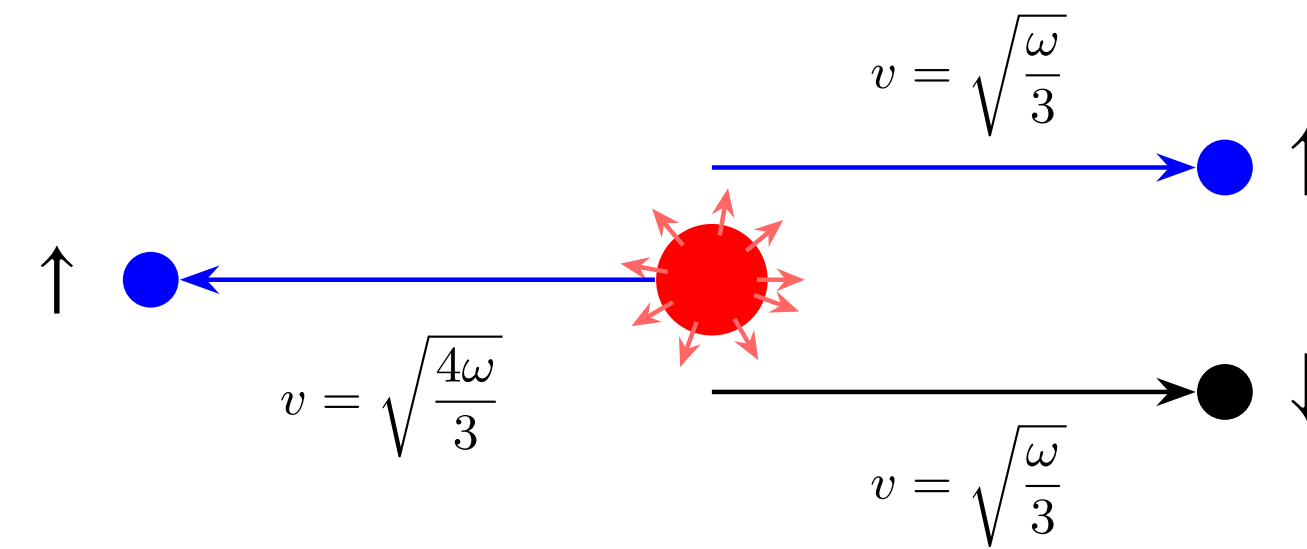
Infinite number of charge three operators labelled by  $s, \ell, m$  (determined by the properties of  $\Psi^{(3)}$ )

$$\begin{aligned} \ell = 0 : \quad \Delta &= 4.666, 7.627, 9.614, \dots 2n + \frac{7}{2}, \\ \ell = 1 : \quad \Delta &= 4.273, 6.878, 8.216, \dots 2n + \frac{5}{2}. \end{aligned}$$

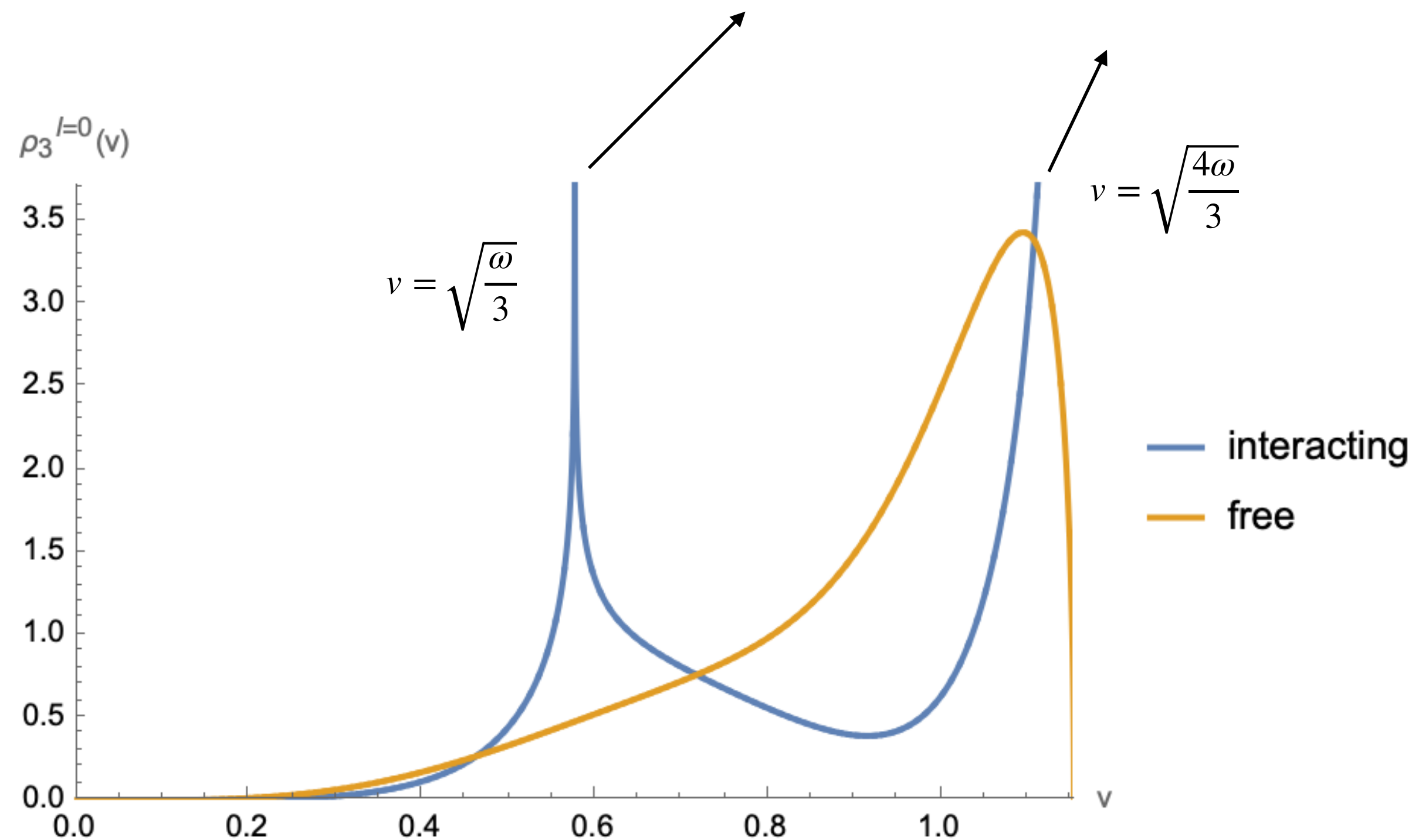
[Efimov 1971,  
Felix Werner thesis,  
SDC, Mishra & Son 2023]

# Examples: Fermions at unitarity

Scalar operator:  $O_3^{\ell=0, \text{interacting}}, \Delta_{O_3^{\ell=0}}^{\text{int}} = 4.666$   $O_3^{\ell=0, \text{free}} = \psi_{\downarrow} \psi_{\uparrow} \nabla^2 \psi_{\uparrow} - 2 \nabla_i \psi_{\downarrow} \psi_{\uparrow} \nabla^i \psi_{\uparrow}, \Delta_{O_3^{\ell=0}}^{\text{free}} = \frac{13}{2}$



(Integrable) divergences in the interacting wavefunction in these configurations due to the **unitarity** condition.



$$\rho_3^{\ell=0}(v) = \frac{4\pi}{\omega} \langle \mathcal{E}_v(\hat{n}) \rangle_{O_3^{\ell=0}(\omega, \vec{k})}$$

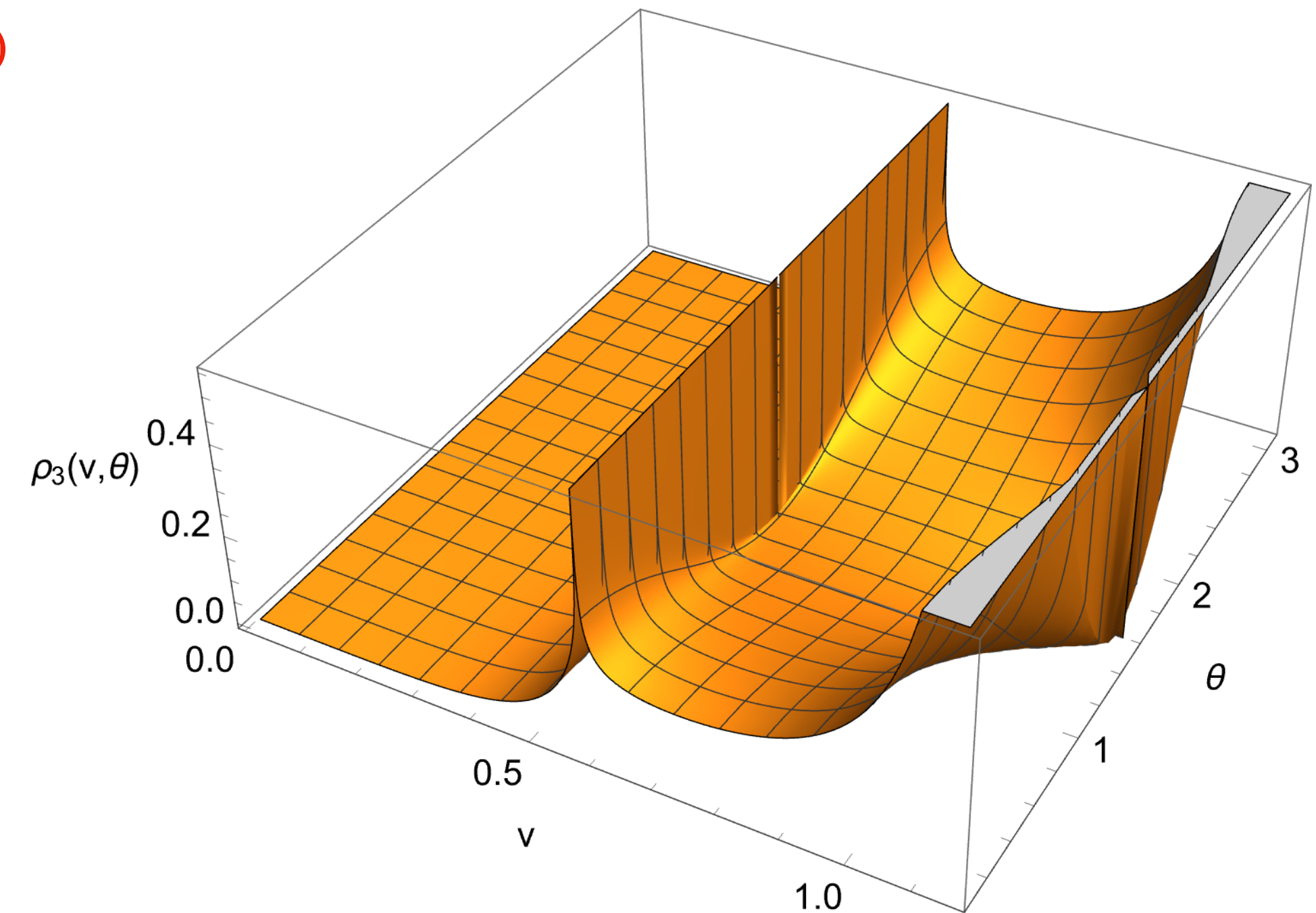
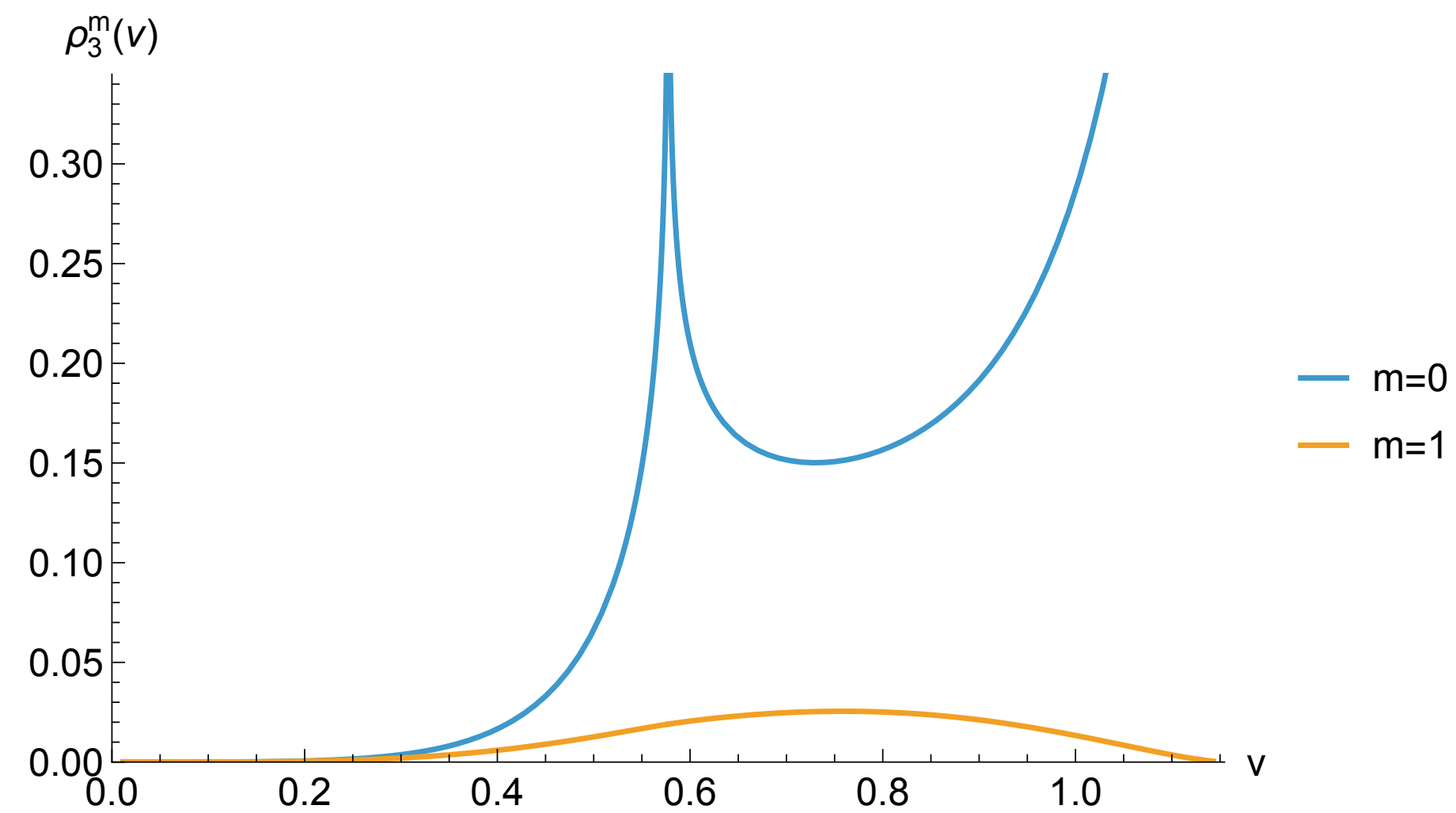
# Examples: Fermions at unitarity

Spin one operator:

$$\epsilon \cdot O_3^{\ell=1}, \quad \Delta_{\epsilon \cdot O_3^{\ell=1}}^{\text{int}} = 4.273$$

$$\rho_3(v, \theta) = \frac{2\pi}{\omega} \left( \cos \theta_\epsilon^2 \underbrace{\langle \mathcal{E}_v(\hat{z}) \rangle_{O_3^{m=0}(\omega, \vec{k})}}_{\rho_3^0(v)} + \sin \theta_\epsilon^2 \underbrace{\langle \mathcal{E}_v(\hat{z}) \rangle_{O_3^{m=1}(\omega, \vec{k})}}_{\rho_3^1(v)} \right)$$

$$\epsilon \cdot \vec{O}_3 = \sum_m (-1)^m \epsilon_m O_3^{\ell=1, -m}, \quad \epsilon_z = \hat{z}, \epsilon_\pm = \frac{1}{\sqrt{2}} (\mp \hat{y} - i \hat{x})$$



# Examples: Large charge

IR of Unitary fermions described by superfluid EFT

$$L = c_0 M^{\frac{3}{2}} X^{\frac{5}{2}} + \dots, \quad X = \partial_t \theta - \frac{(\nabla \theta)^2}{2M}$$

goldstone boson of the  $U(1)$  symmetry breaking

[Son & Wingate 2006.....]

using saddle point approximations of the insertions,

$$\langle \mathcal{E}_v(\hat{n}) \rangle_{O_{\Delta_Q}(\omega, 0)} \simeq \langle \mathcal{E}_v(\hat{n}) \rangle_{O_{\Delta_Q}(t_{12} = \frac{-i\Delta_Q}{\omega}, \vec{x}_{12} = 0)}, \quad \Delta_Q \sim Q^{\frac{4}{3}}$$

[Cuomo, Firat, Nardi & Ricci 2025]

$$j^i = \frac{-\partial_i \theta \rho}{M}, \quad \rho = \frac{5}{2} c_0 M^{\frac{3}{2}} X^{\frac{3}{2}}$$

using “master solution”  $\theta_s(t, \vec{x})$

[Beane, Orlando & Reffert 2024]

Interpret  $v_{\max} = \sqrt{\frac{16\omega}{3MQ}}$ , beyond which  $\tilde{n} \left( \frac{\pi}{2}, v\hat{n} \right)$  is imaginary, possibly signalling breakdown of leading order EFT.

$$\frac{27M^4 Q^4 v^4 \left( \frac{16\omega}{3MQ} - v^2 \right)^{3/2}}{1024\pi^2 r^3 \omega^3}$$

Satisfies the ward identities

Agrees with [Son, Stephanov & Yee 2021]

# Positivity ?

$$\mathcal{E}_\nu(\hat{n}) = \frac{\nu^4}{2} \tilde{n} \left( \frac{\pi}{2}, \nu \hat{n} \right)$$

$$\tilde{n}(\tau, \vec{y}) \equiv (\cos \tau)^{-3} n(t, \vec{x}), \quad \nu \geq 0$$

$$\mathcal{E}_\nu(\hat{n}) \geq 0$$

Non-relativistic analogue of ANEC

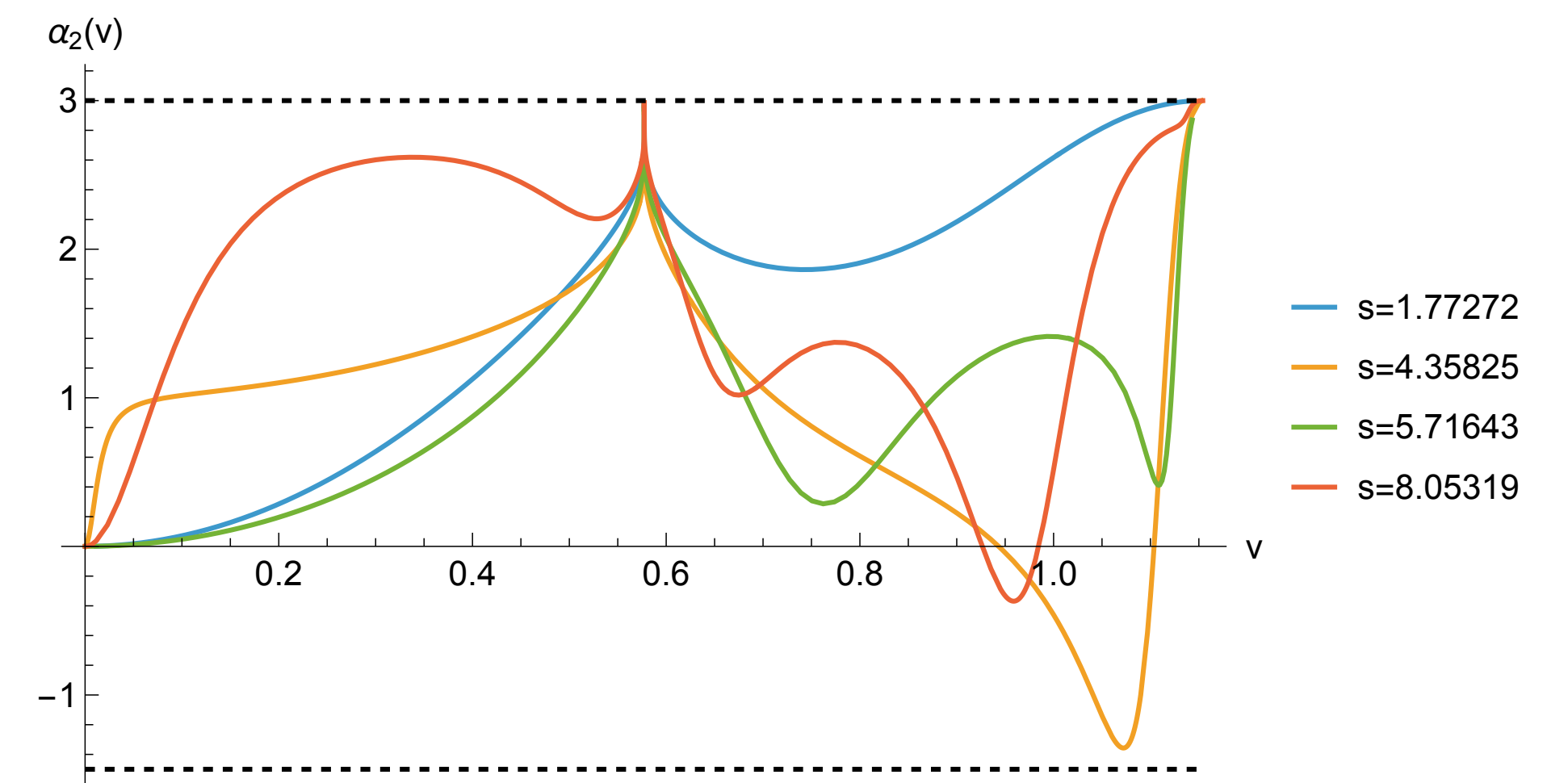
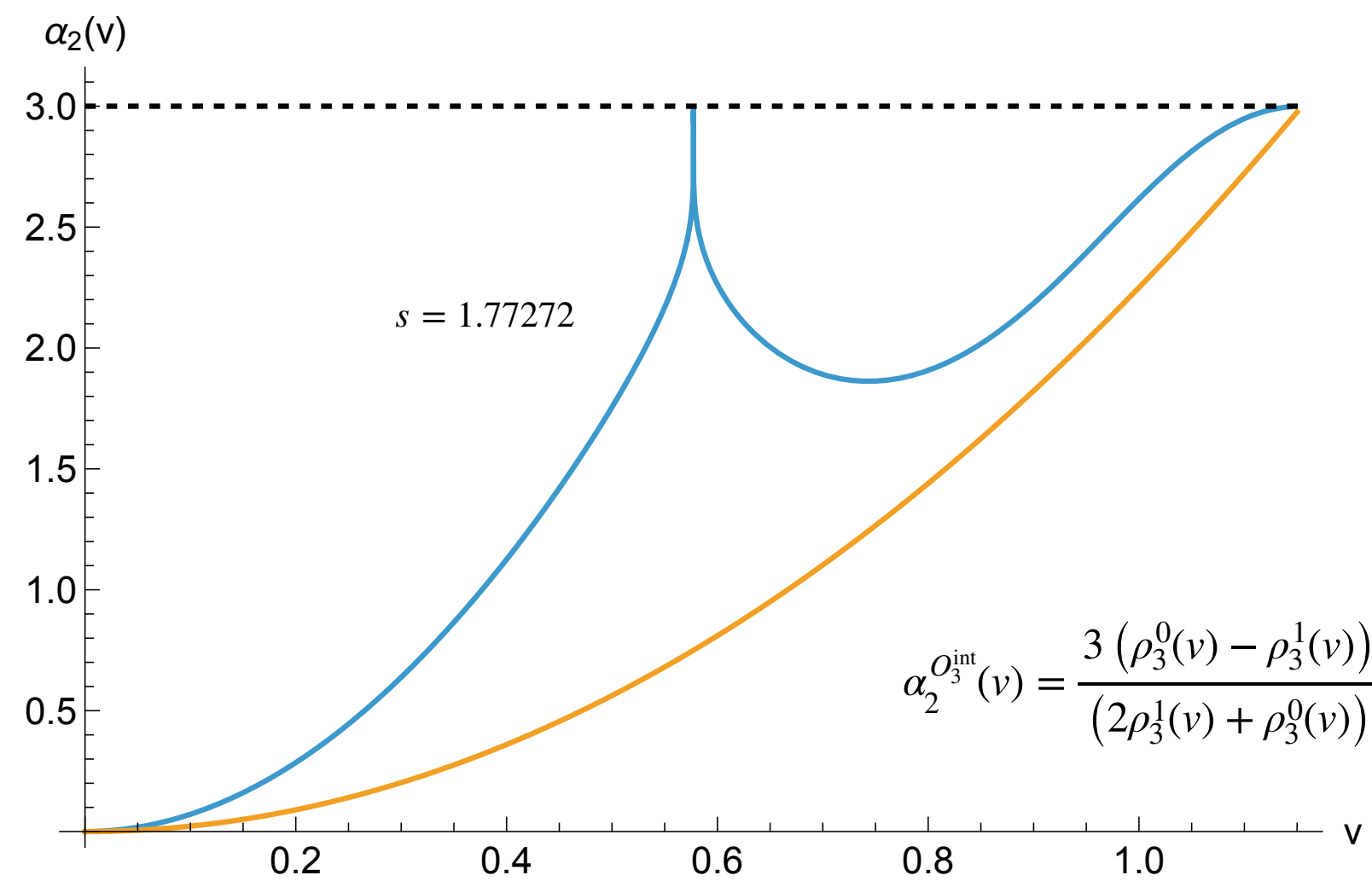
[Hofman & Maldacena 2008]

$$\langle \mathcal{E}_\nu(\hat{z}) \rangle_{\epsilon, O_3^{\ell=1}(\omega, 0)} \sim f(\nu) \left( 1 + \alpha_2(\nu) \left( \cos^2 \theta - \frac{1}{3} \right) \right)$$

$f(\nu) \geq 0$

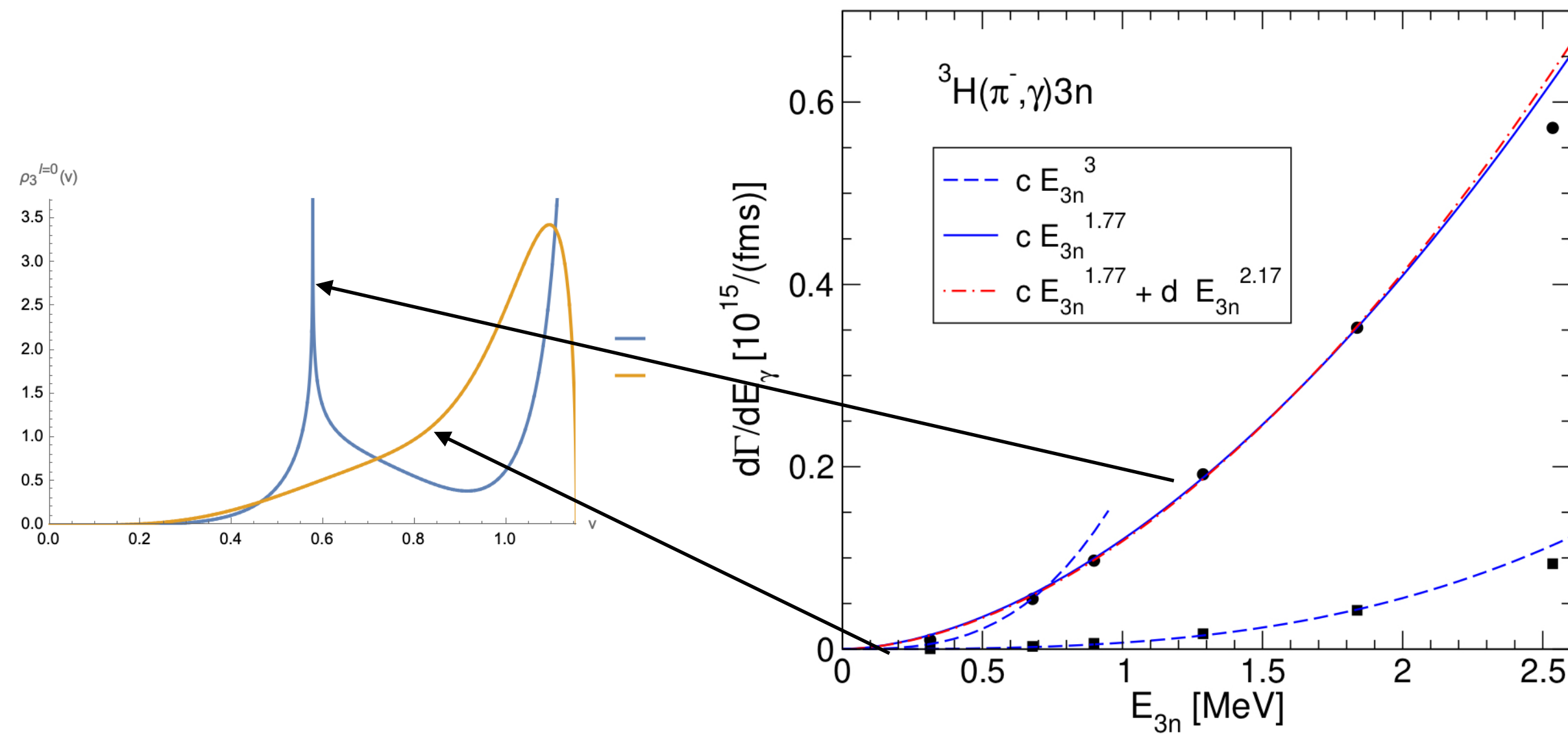
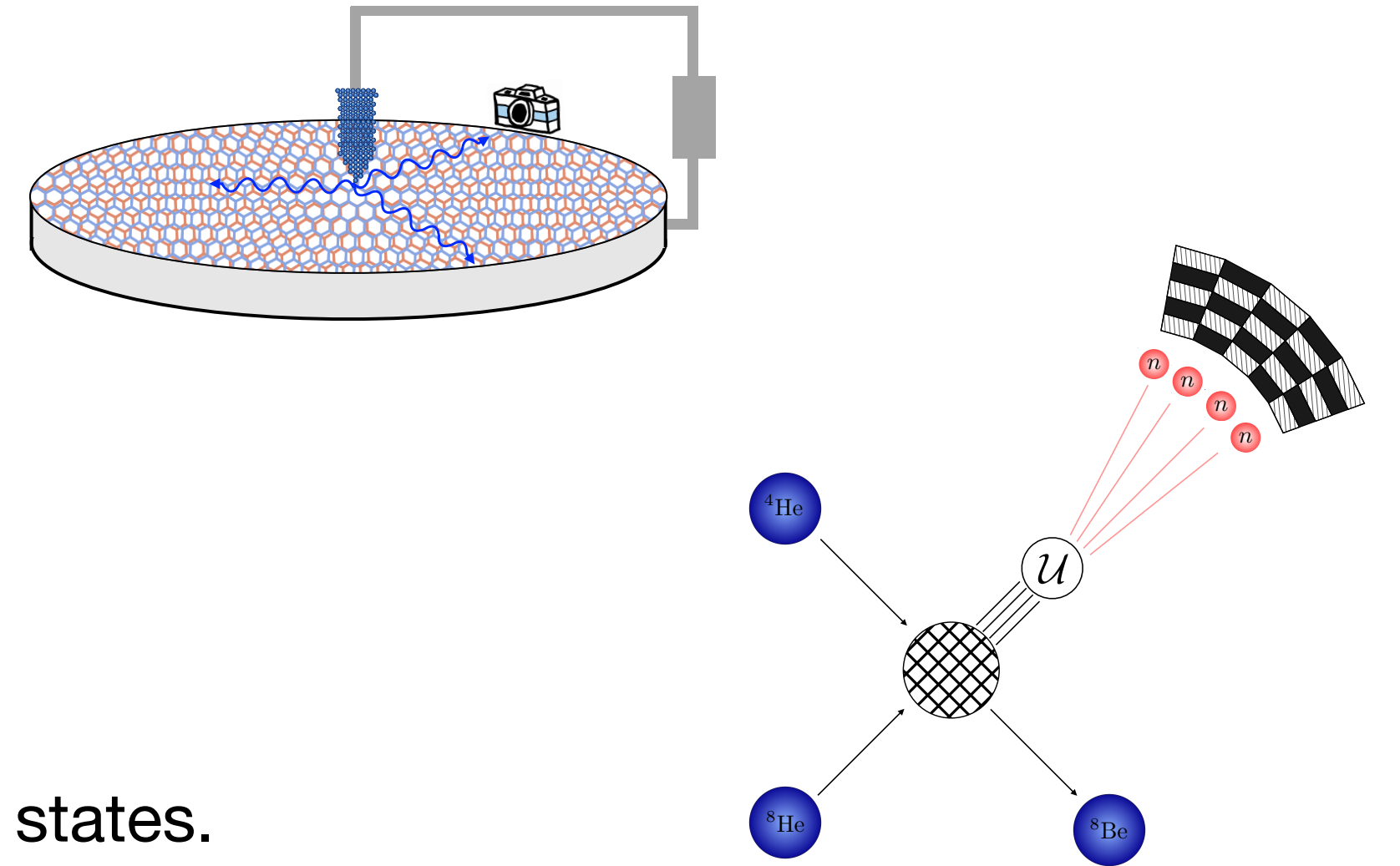
NR ANEC

$$-\frac{3}{2} \leq \alpha_2(\nu) \leq 3$$



# Summary

- Proposal for event-shapes in non-relativistic setting with Schrödinger invariance.
- Using symmetries to study ward identities and constrain correlations in non-trivial states.
- Explicit examples.
- Proposal for positivity.



# Future Directions

- Higher point detectors? Their OPE?
- Scattering length & effective range corrections?
- Proof of positivity?
- Anyons? Efimov states?..
- .....