Infinite Heat Order in 3+1 dimensions

Based on: 2604.01184

B. Bajc, G. Muco, F. Sannino, S. Wagner

New directions in the large-charge expansion - Giulia Muco, 22.06.26

In this talk:

- Some background
- Symmetry non-restoration in QFT: a (3+1)-d example?
- The finite-N story

The question we are dealing with:

Does spontaneous symmetry breaking survive at arbitrarily high temperatures?

- Thermodynamics naively suggests **no**:

  At finite T we minimise

$$F = E - ST$$

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- More concretely, a "cluster decomposition" property can be proven:

$$\exists \quad \beta_* \quad : < A(x)B(y) >_\beta \xrightarrow{|x-y| \gg \beta} < A(x) >_\beta < B(y) >_\beta \, , \qquad \beta < \beta_*$$

arXiv:2503.22789

… But:

There are examples (experimentally observed!) in which the systems acquire order when heated (for a short range of T):

- Rochelle salt becomes a little more ordered than at lower temperatures
- Liquid He_3 solidifies when heated

How?

… going back to:

$$\exists \quad \beta_* \ : < A(x)B(y) >_\beta \xrightarrow{|x-y| \gg \beta} < A(x) >_\beta < B(y) >_\beta \, , \qquad \beta < \beta_*$$

This statement is based on assumptions!

e.g.:

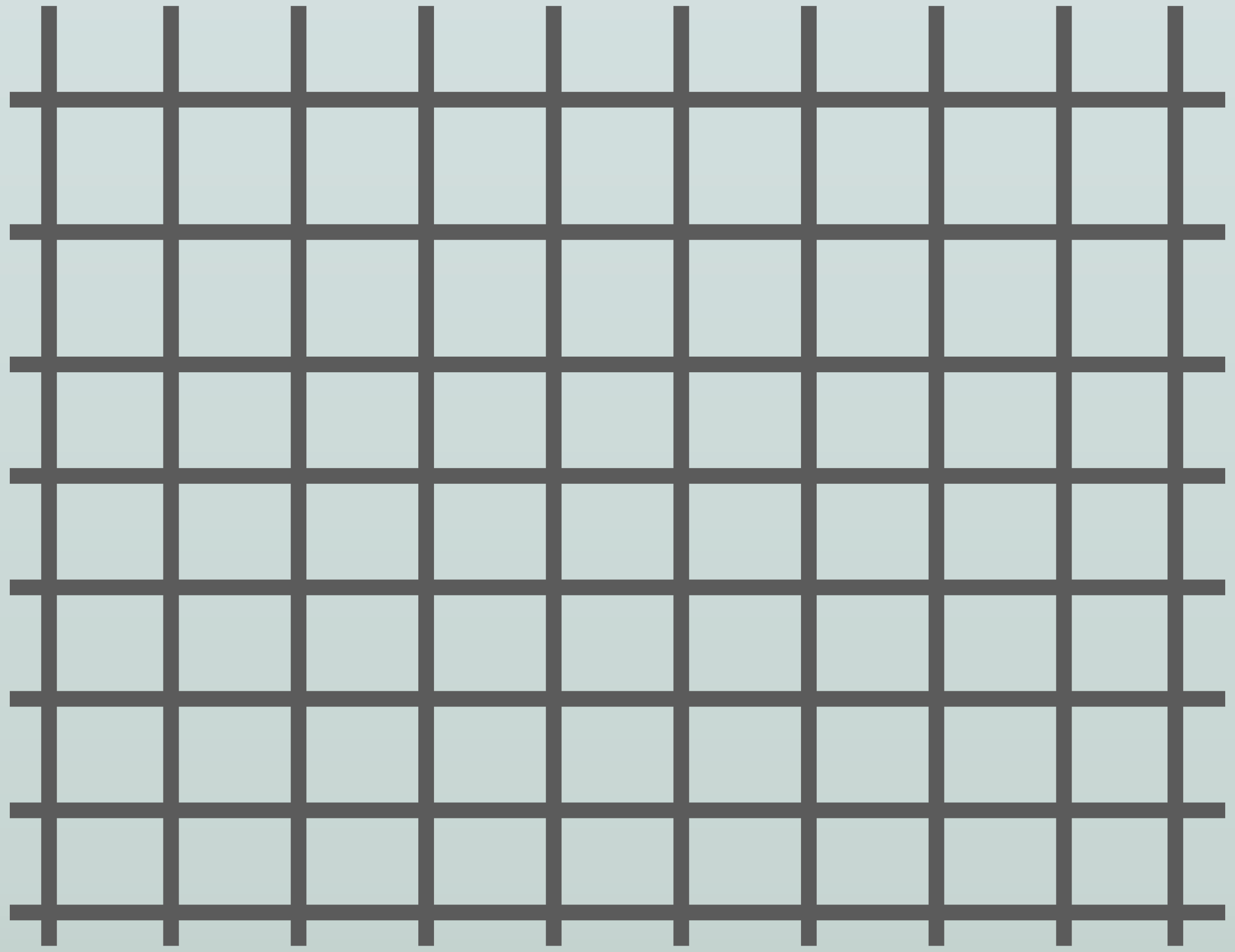
1. $\exists$ a uniform probability distribution over the space of events Ω
2. The space of events is a direct product: $\Omega = \Omega_1 \times \ldots \Omega_N$

It is possible to escape these in QFT!

A 2-d example on a lattice

Consider a system on an $L \times L$ square lattice with Hamiltonian

$$H = U \sum_{u \sim v} n_u^2 n_v^2 + \sum_v n_v$$



Han, Huang,
Komargodski, Lucas,
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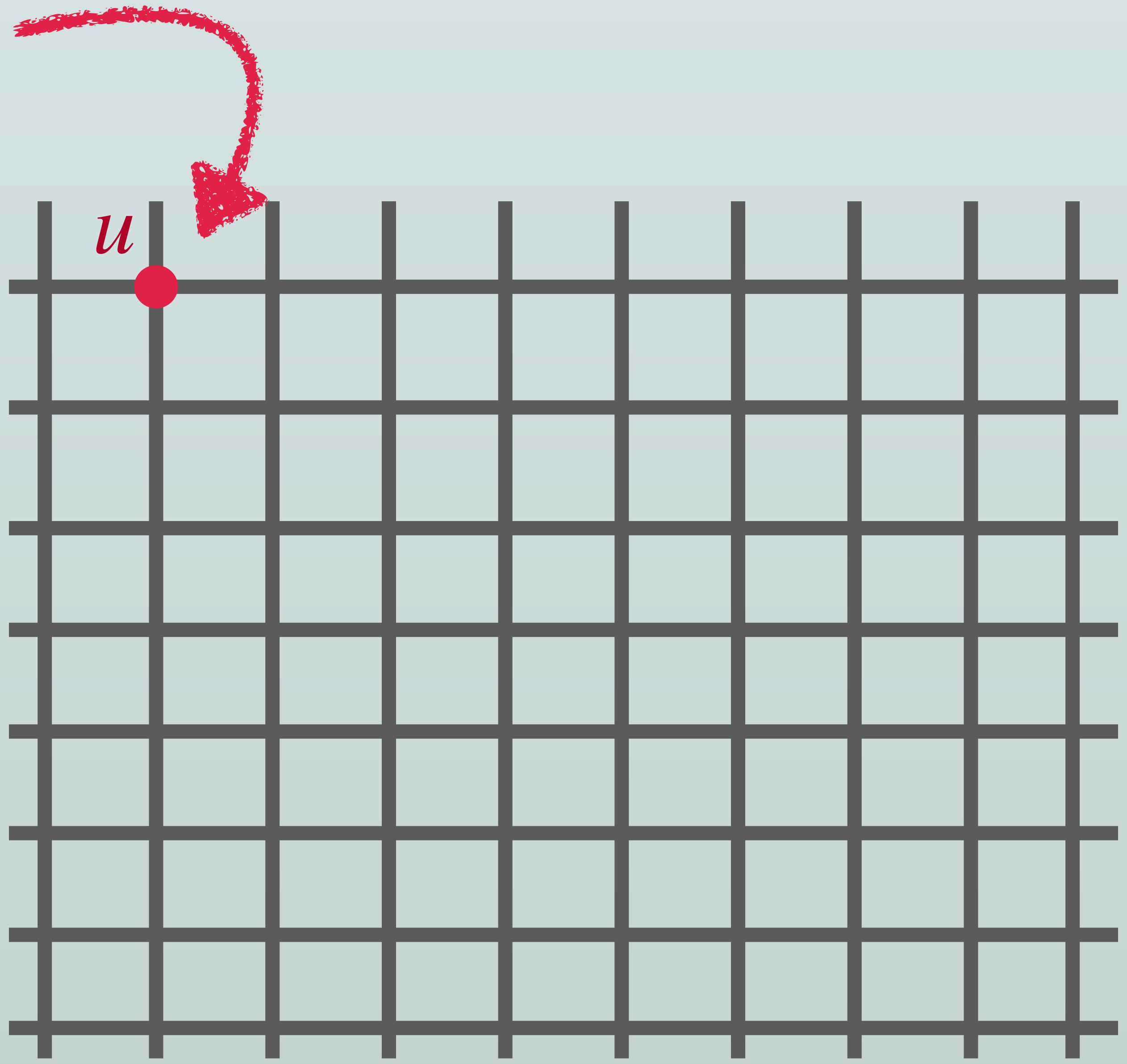

$n_u \in \{0,1,2,\ldots\}$

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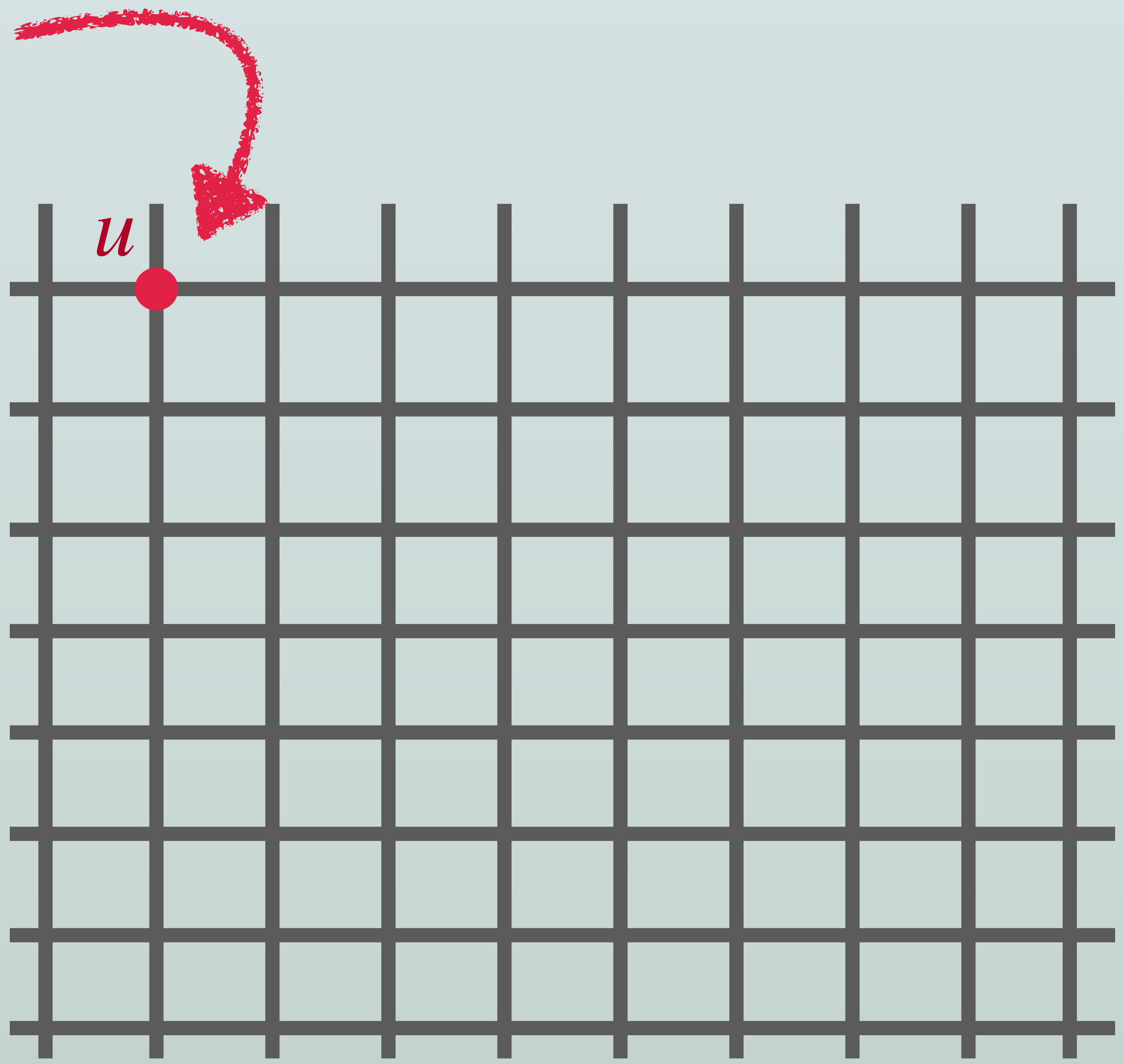

Infinite number of possible microstates!

→ Potential to evade the theorem!

Han, Huang,
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The free energy F of the solid is smaller than the one of the gas.

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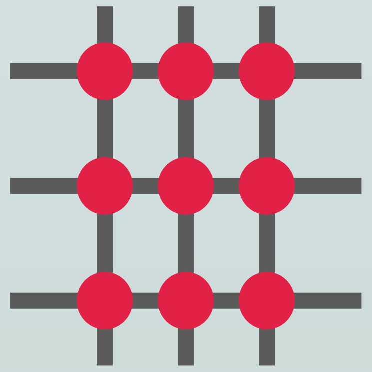

Gas:

All sites are occupied

$$\beta \bar{n}_g^4 \sim 1 \quad => \quad \bar{n}_g \sim T^{1/4}$$

$$Z_g \sim \bar{n}_g^{L^2} \sim T^{L^2/4}$$

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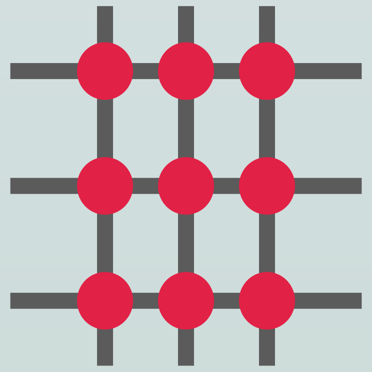

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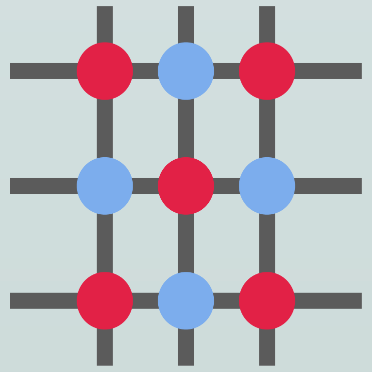

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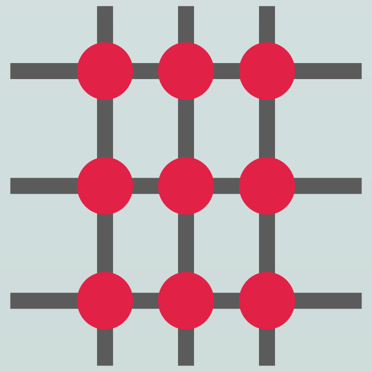

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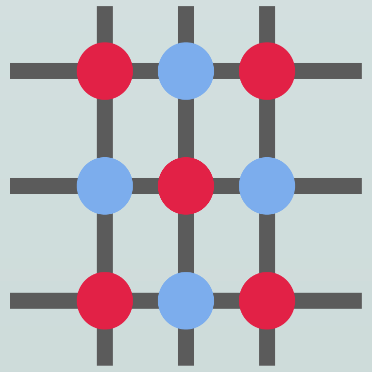

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The dominant contribution to $Z(\beta)$ comes from the checkerboard pattern for sufficiently high T!

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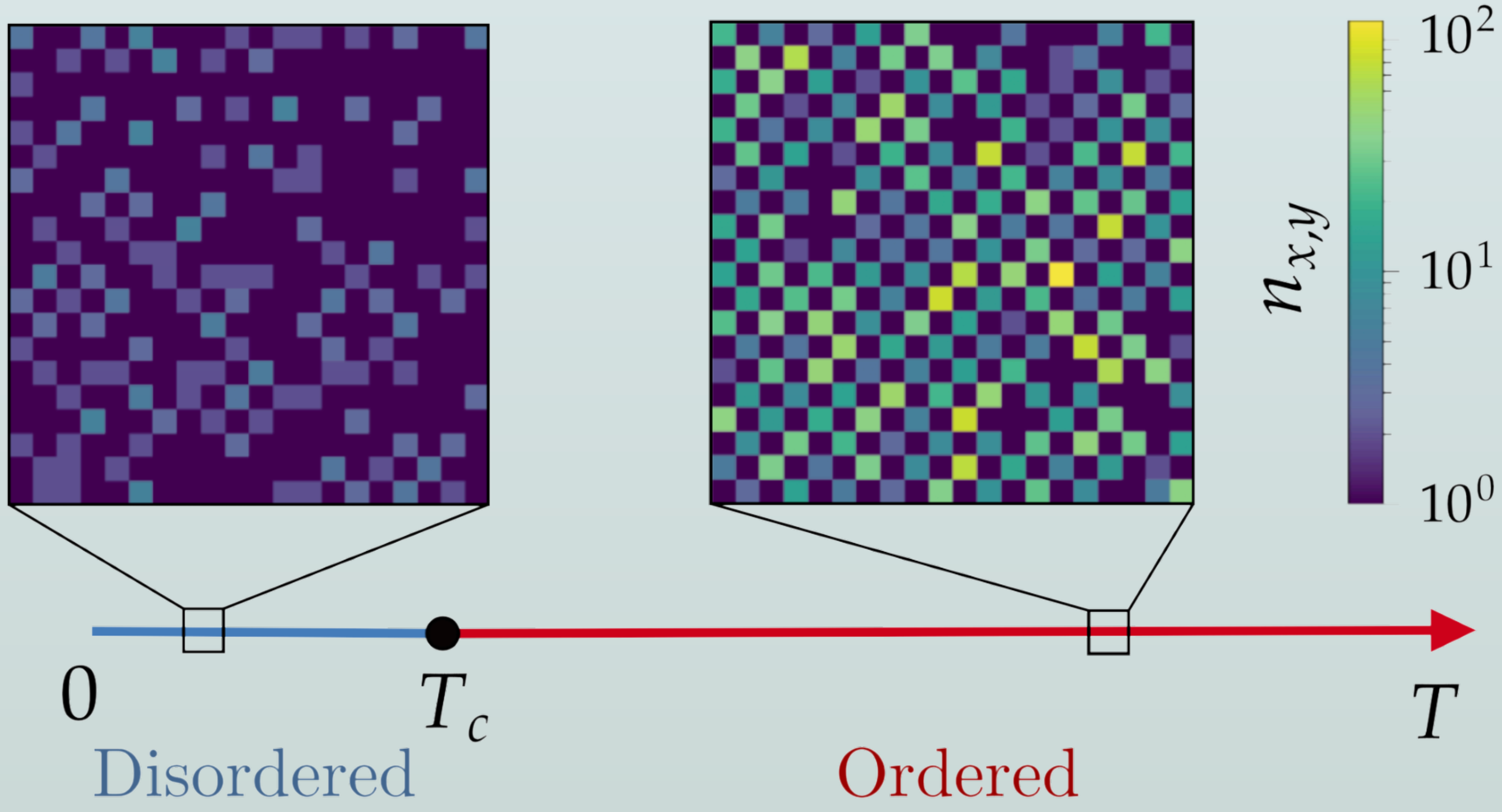
… in fact:

Result of a Monte Carlo simulation:



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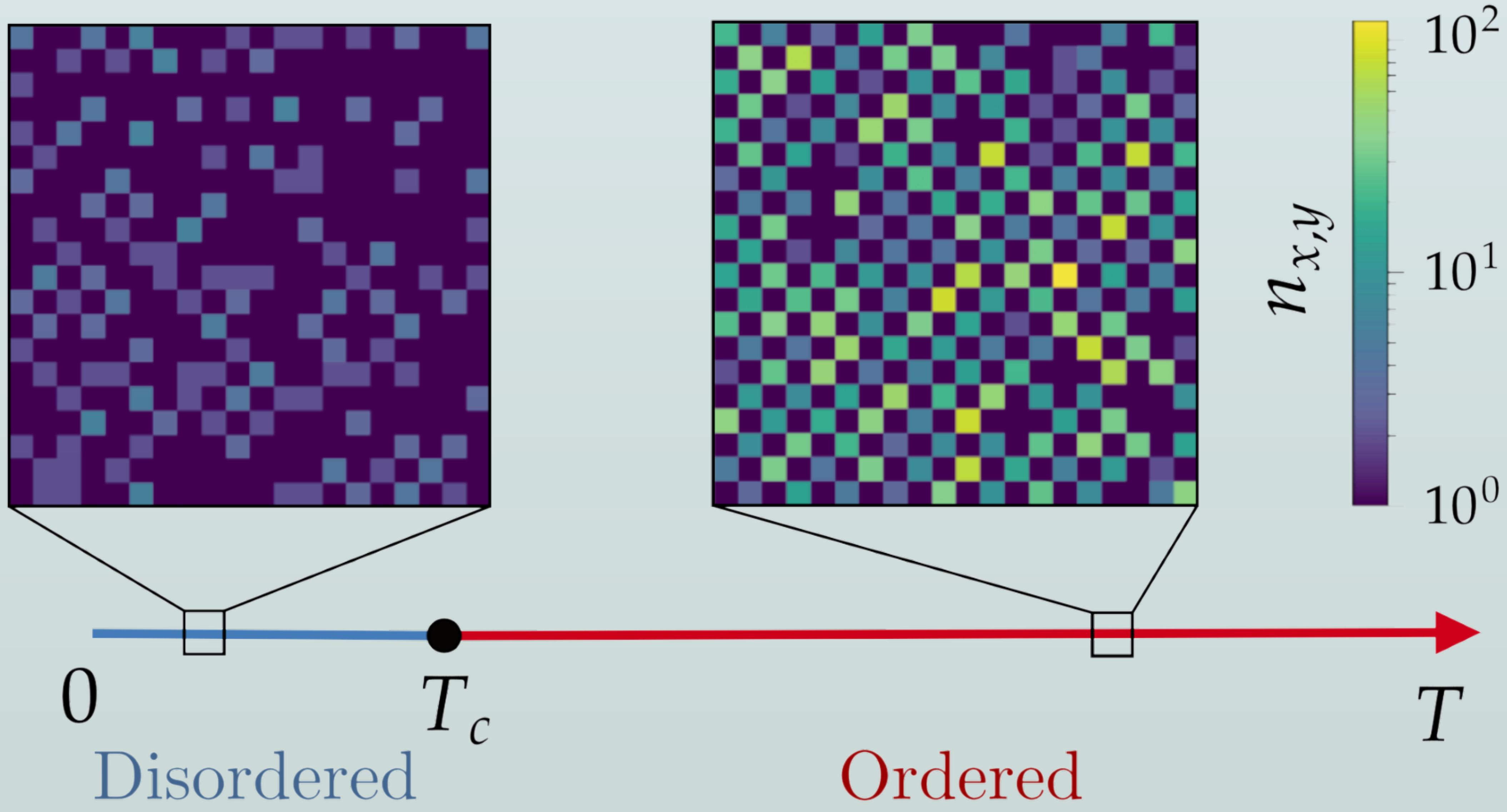

the high-temperature phase is a solid phase that spontaneously breaks the lattice translational symmetry!

The **underlying mechanism** that makes the solid phase overall entropically favourable is that sacrificing some entropy to have order opens up a large phase space of configurations through the fluctuations permitted at each site.

In other words: order in one degree of freedom enables other degrees of freedom to strongly fluctuate. The same idea applies in QFT.

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Symmetry non-restoration in (3+1)-d QFTs

- How do we investigate symmetry non-restoration in QFT?
  1. Put our models at finite T
  2. Compute the resulting **thermal masses** (from ΔV_T)
  3. If the mass matrix of the theory has negative eigenvalues, there is condensation

- This topic in the realm of QFT was first studied by Weinberg (1974)

$$V = \frac{\lambda_1}{4}\phi_1^4 + \frac{\lambda_2}{4}\phi_2^4 - \frac{\lambda}{2}\phi_1^2\phi_2^2 \qquad \lambda_{1,2} > 0 \quad , \quad \lambda^2 < \lambda_1\lambda_2$$

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... at high T: $$\Delta V_T = \frac{T^2}{24}\Big((3\lambda_1 - \lambda)\phi_1^2 + (3\lambda_2 - \lambda)\phi_2^2\Big)$$

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for $\lambda > 3\lambda_2$: $< \phi_2 > \neq 0, \quad \mathbb{Z}_2 \times \mathbb{Z}_2 \to \mathbb{Z}_2$

- But: this theory is not UV complete! The scalar couplings increase with the energy
- What we can actually say: assuming a physical cutoff, for temperatures below it, one observes the phenomenon of symmetry non-restoration
- Our task is then to find a **UV complete** theory in **(3+1)-d** in which symmetries remain broken at arbitrarily high temperatures

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- Our task is then to find a **UV complete** theory in **(3+1)-d** in which symmetries remain broken at arbitrarily high temperatures

Bajc, Lugo, Sannino,
arXiv:2012.08428

Strategy:

- Look for **complete asymptotically free** theories (these require the presence of gauge fields!) $=>$ ansatz for the RGEs and the couplings λ_I $\longrightarrow$ $m_T^2(\lambda_I) < 0$?
- Compute $\Delta V_T(\lambda_I)$

arXiv:2012.08428

What kind of theories are considered?

Complete asymptotically free

- Gauge singlet scalars
- Gauged scalars
  1. $SU(N)$ gauge group
  2. $SU(N_{c_1}) \times SU(N_{c_2})$
     - A. $\phi_i \sim \square$
     - B. $\phi_i \sim \mathbf{adj}$

Asymptotically safe

- Litim-Sannino (arXiv:1406.2337)
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The case of gauge singlet scalar fields

Consider a $SU(N_c)$ gauge theory with $N_f = N_{f_1} + N_{f_2}$ Dirac fermions ($\square$) coupled to the $\phi_{1,2}$ scalars:

$$\mathscr{L}_Y = \phi_1 \sum_{i=1}^{N_{f_1}} y_{1i} \bar{\psi}_{1i} \psi_{1i} + \phi_2 \sum_{i=1}^{N_{f_2}} y_{2i} \bar{\psi}_{2i} \psi_{2i} \qquad \mathbb{Z}_2 \times \mathbb{Z}_2 \quad : \quad \begin{cases} \phi_i \to -\phi_i \\ \psi \to i\gamma_5 \psi \end{cases}$$

$$\text{asymptotic freedom} \;\; => \;\; \alpha_g \propto 1/t \quad \text{for large } t = \log(\mu/\mu_0)$$

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complete asymptotic freedom $=>$ $\alpha_g \propto 1/t$ for large $t = \log(\mu/\mu_0)$

↪ all couplings λ_I must vanish in the UV at least as fast as α_g.

With the requirement of $m_T^2 < 0$ it is sufficient to study the fixed flow solution in which $\lambda_I \propto 1/t$.

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- This transforms the problem of solving the RGEs (coupled partial differential eq.s) to the problem of solving a system of *polynomial* equations!
- In practice, we define

$$\alpha_g = \frac{g^2}{(4\pi)^2}, \quad \alpha_{y_i} = \frac{y_i^2}{(4\pi)^2}, \quad \alpha_{\lambda_i} = \frac{\lambda_i}{(4\pi)^2}, \quad \alpha_\lambda = \frac{\lambda}{(4\pi)^2}$$

And look for solutions of asymptotic form $\alpha_a = \frac{\tilde{\alpha}_a}{t}, \;\; a = g,\, y_i,\, \lambda_i,\, \lambda \;\; (\tilde{\alpha}_a$ constant)

- ... turns out: it is **not** possible to find a solution of the RGEs $\tilde{\alpha}_a$ such that $m_T^2 < 0$
- We turn our attention to **gauged scalars**.

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$SU(N_{c_1}) \times SU(N_{c_2})$ with adjoint scalars

Infinite-N limit

$$V = \frac{\lambda_1'}{4}\,\mathrm{Tr}\big(\Sigma_1^4\big) + \frac{\lambda_2'}{4}\,\mathrm{Tr}\big(\Sigma_2^4\big) + \frac{\lambda_1}{4}\left[\mathrm{Tr}\big(\Sigma_1^2\big)\right]^2 + \frac{\lambda_2}{4}\left[\mathrm{Tr}\big(\Sigma_2^2\big)\right]^2 - \frac{\lambda}{2}\,\mathrm{Tr}\big(\Sigma_1^2\big)\,\mathrm{Tr}\big(\Sigma_2^2\big)\,.$$

Field	Spin	SU(N_{c1})	SU(N_{c2})	Multiplicity
$A_\mu^{(1)}$	1	**adj**	**1**	–
$A_\mu^{(2)}$	1	**1**	**adj**	–
Σ_1	0	**adj**	**1**	1
Σ_2	0	**1**	**adj**	1
Ψ_1	$\frac{1}{2}$	$\square$	**1**	N_{f1}
Ψ_2	$\frac{1}{2}$	**1**	$\square$	N_{f2}

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$$\Delta V_T = \frac{(4\pi)^2 T^2}{48\log T}\left(\left(\tilde{\lambda}_1 + 2\tilde{\lambda}_1' - \frac{N_{c_2}}{N_{c_1}}\tilde{\lambda} + 12\tilde{\alpha}_1\right)\mathrm{Tr}\Sigma_1^2 + \left(\tilde{\lambda}_2 + 2\tilde{\lambda}_2' - \frac{N_{c_1}}{N_{c_2}}\tilde{\lambda} + 12\tilde{\alpha}_2\right)\mathrm{Tr}\Sigma_2^2\right)$$

A solution for bounded potential with negative thermal mass square exists ✓

arXiv:2012.08428

$SU(N_{c_1}) \times SU(N_{c_2})$ with fundamental scalars

Infinite-N limit

$$V = \frac{\lambda_1}{2}\left(\vec{\varphi}_1^{\,*}\cdot\vec{\varphi}_1\right)^2 + \frac{\lambda_2}{2}\left(\vec{\varphi}_2^{\,*}\cdot\vec{\varphi}_2\right)^2 - \lambda\left(\vec{\varphi}_1^{\,*}\cdot\vec{\varphi}_1\right)\left(\vec{\varphi}_2^{\,*}\cdot\vec{\varphi}_2\right)$$

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we define:

$$g_i^2 = \frac{(4\pi)^2\tilde{\alpha}_i}{N_{c_i}t} \;,\quad \lambda_i = \frac{(4\pi)^2\tilde{\lambda}_i}{N_{c_i}t} \;,$$

$$\lambda = \frac{(4\pi)^2\tilde{\lambda}}{\sqrt{N_{c_1}N_{c_2}}\,t} \;,\quad i = 1,\, 2$$

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... at large N:

$$\Delta V_T = (4\pi)^2\frac{T^2}{24\log T}\left(\left(2\left(\tilde{\lambda}_1 - \sqrt{\frac{N_{c_2}}{N_{c_1}}}\tilde{\lambda}\right) + 3\tilde{\alpha}_1\right)(\vec{\varphi}_1^{\,*}\cdot\vec{\varphi}_1) + \left(2\left(\tilde{\lambda}_2 - \sqrt{\frac{N_{c_1}}{N_{c_2}}}\tilde{\lambda}\right) + 3\tilde{\alpha}_2\right)(\vec{\varphi}_2^{\,*}\cdot\vec{\varphi}_2)\right)$$

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... defining $\tilde{\lambda}_{\pm} = \frac{1}{2}(\tilde{\lambda}_1 \pm \tilde{\lambda}_2) \quad , \quad \tilde{\alpha}_{\pm} = \frac{1}{2}(\tilde{\alpha}_1 \pm \tilde{\alpha}_2)$

the RGEs become

$$-\tilde{\lambda}_1 = 2\tilde{\lambda}_1^2 + 2\tilde{\lambda}^2 - 6\tilde{\alpha}_1\tilde{\lambda}_1 + \frac{3}{2}\tilde{\alpha}_1^2$$

$$-\tilde{\lambda}_2 = 2\tilde{\lambda}_2^2 + 2\tilde{\lambda}^2 - 6\tilde{\alpha}_2\tilde{\lambda}_2 + \frac{3}{2}\tilde{\alpha}_2^2$$

$$-\tilde{\lambda} = 2\left(\tilde{\lambda}_1 + \tilde{\lambda}_2\right)\tilde{\lambda} - 3\left(\tilde{\alpha}_1 + \tilde{\alpha}_2\right)\tilde{\lambda}$$

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$$-\tilde{\lambda} = 2\left(\tilde{\lambda}_1 + \tilde{\lambda}_2\right)\tilde{\lambda} - 3\left(\tilde{\alpha}_1 + \tilde{\alpha}_2\right)\tilde{\lambda}$$

A solution is: $\tilde{\alpha}_- = 0 \quad , \quad \tilde{\lambda}_+ = \frac{6\tilde{\alpha}_+ - 1}{4} \quad , \quad \tilde{\lambda}_-^2 + \tilde{\lambda}^2 = \frac{1}{16}(24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1) \quad ,$

$$\tilde{\alpha}_+ \geq (3 + \sqrt{3})/12 \quad .$$

This solution supports symmetry non-restoration at arbitrary high temperatures ✓

arXiv:2012.08428

… But, is this a valid solution? One must check:

1. Our solution to the RGEs leads to a potential which is bounded from below

$$\lambda_1\lambda_2 - \lambda^2 > 0 \quad \Leftrightarrow \quad \tilde{\lambda}_+^2 - \tilde{\lambda}_-^2 - \tilde{\lambda}^2 > 0$$

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$$\lambda_1\lambda_2 - \lambda^2 > 0 \quad \Leftrightarrow \quad \underset{\substack{\shortparallel \\ \frac{12\tilde{\alpha}_+^2}{16}}}{\tilde{\lambda}_+^2 - \tilde{\lambda}_-^2} - \tilde{\lambda}^2 > 0$$

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… But, is this a valid solution? One must check:

1. Our solution to the RGEs leads to a potential which is bounded from below ✓

$$\lambda_1\lambda_2 - \lambda^2 > 0 \quad \Leftrightarrow \quad \tilde\lambda_+^2 - \underset{\displaystyle\frac{12\tilde\alpha_+^2}{16}}{\underset{\shortparallel}{\tilde\lambda_-^2}} - \tilde\lambda^2 > 0$$

2. What's the allowed range for N_{f_i}/N_{c_i}?

$$\tilde\alpha_- = 0 \quad \Leftrightarrow \quad \tilde\alpha_1 = \tilde\alpha_2$$

So: $$N_{c_1} g_1^2 = N_{c_2} g_2^2 \quad \Leftrightarrow \quad \frac{N_{f_1}}{N_{c_1}} = \frac{N_{f_2}}{N_{c_2}}$$

… and: $$\frac{1}{2}\left(2 + 3\sqrt{3}\right) \leq \frac{N_{f_1}}{N_{c_1}} < \frac{11}{2} \quad .$$

arXiv:2012.08428

The finite number degrees of freedom story

- We focus on the $SU(N_{c_1}) \times SU(N_{c_2})$ gauge theory with fundamental scalars
- At finite-N:

$$-\tilde{\lambda}_1 = \left(2 + \frac{8}{xN_{c2}}\right)\tilde{\lambda}_1^2 + 2\tilde{\lambda}^2 - 6\left(1 - \frac{1}{x^2N_{c2}^2}\right)\tilde{\alpha}_1\tilde{\lambda}_1$$
$$+\frac{3}{2}\tilde{\alpha}_1^2\left(1 + \frac{1}{xN_{c2}} - \frac{4}{x^2N_{c2}^2} + \frac{2}{x^3N_{c2}^3}\right)$$

$$-\tilde{\lambda}_2 = \left(2 + \frac{8}{N_{c2}}\right)\tilde{\lambda}_2^2 + 2\tilde{\lambda}^2 - 6\left(1 - \frac{1}{N_{c2}^2}\right)\tilde{\alpha}_2\tilde{\lambda}_2$$
$$+\frac{3}{2}\tilde{\alpha}_2^2\left(1 + \frac{1}{N_{c2}} - \frac{4}{N_{c2}^2} + \frac{2}{N_{c2}^3}\right)$$

$$-\tilde{\lambda} = \tilde{\lambda}\left[-\frac{4}{N_{c2}\sqrt{x}}\,\tilde{\lambda} + 2\left(1 + \frac{1}{xN_{c2}}\right)\tilde{\lambda}_1 + 2\left(1 + \frac{1}{N_{c2}}\right)\tilde{\lambda}_2\right.$$
$$\left.-3\left(\tilde{\alpha}_1\left(1 - \frac{1}{x^2N_{c2}^2}\right) + \tilde{\alpha}_2\left(1 - \frac{1}{N_{c2}^2}\right)\right)\right]$$

$$\mu_i^2 = 3\tilde{\alpha}_i\left(1 - \frac{1}{N_{ci}^2}\right) + 2\tilde{\lambda}_i\left(\frac{1}{N_{ci}} + 1\right) - 2\sqrt{\frac{N_{cj}}{N_{ci}}}\,\tilde{\lambda} \quad ,$$

$$(i, j) = (1,2), (2,1)$$

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We rewrite the couplings as:

$$\tilde{\lambda}_\pm = \tilde{\lambda}_\pm^{(0)} + \frac{1}{N_{c2}}\,\tilde{\lambda}_\pm^{(1)},$$

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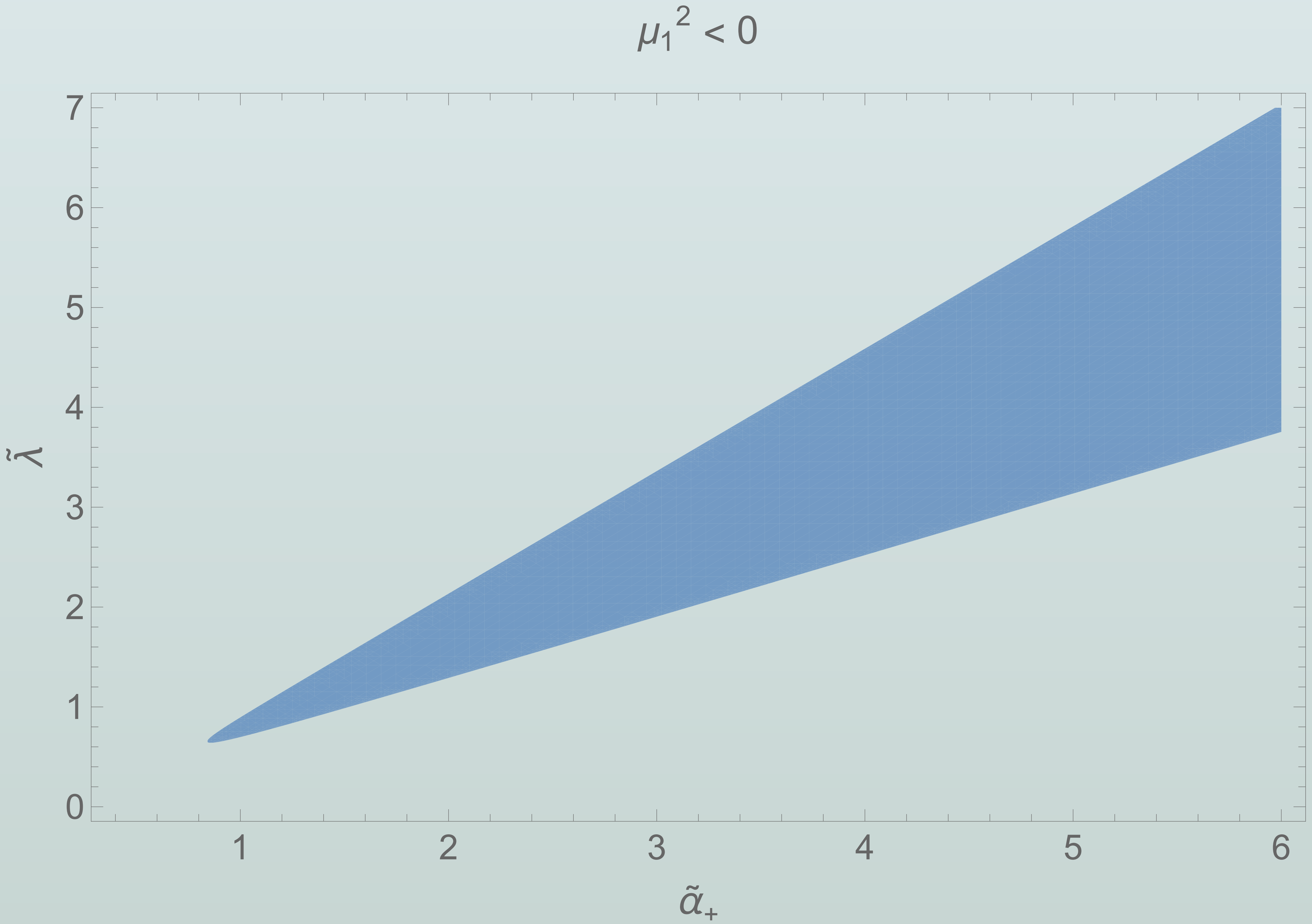
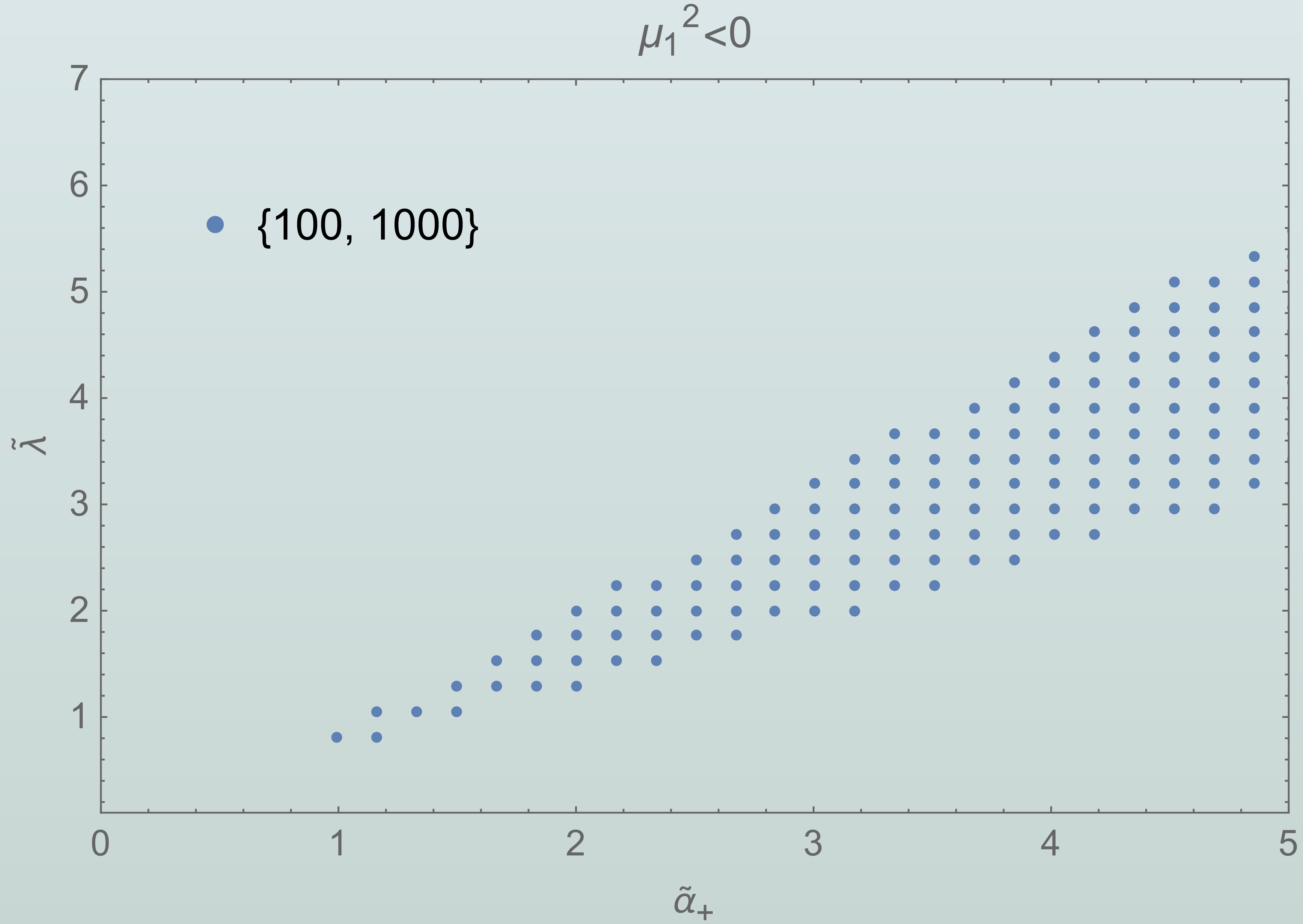
NLO: we now solve for them

LO: already solved!

$1/N$ result:



Numerical evaluation (no $1/N$ truncation)

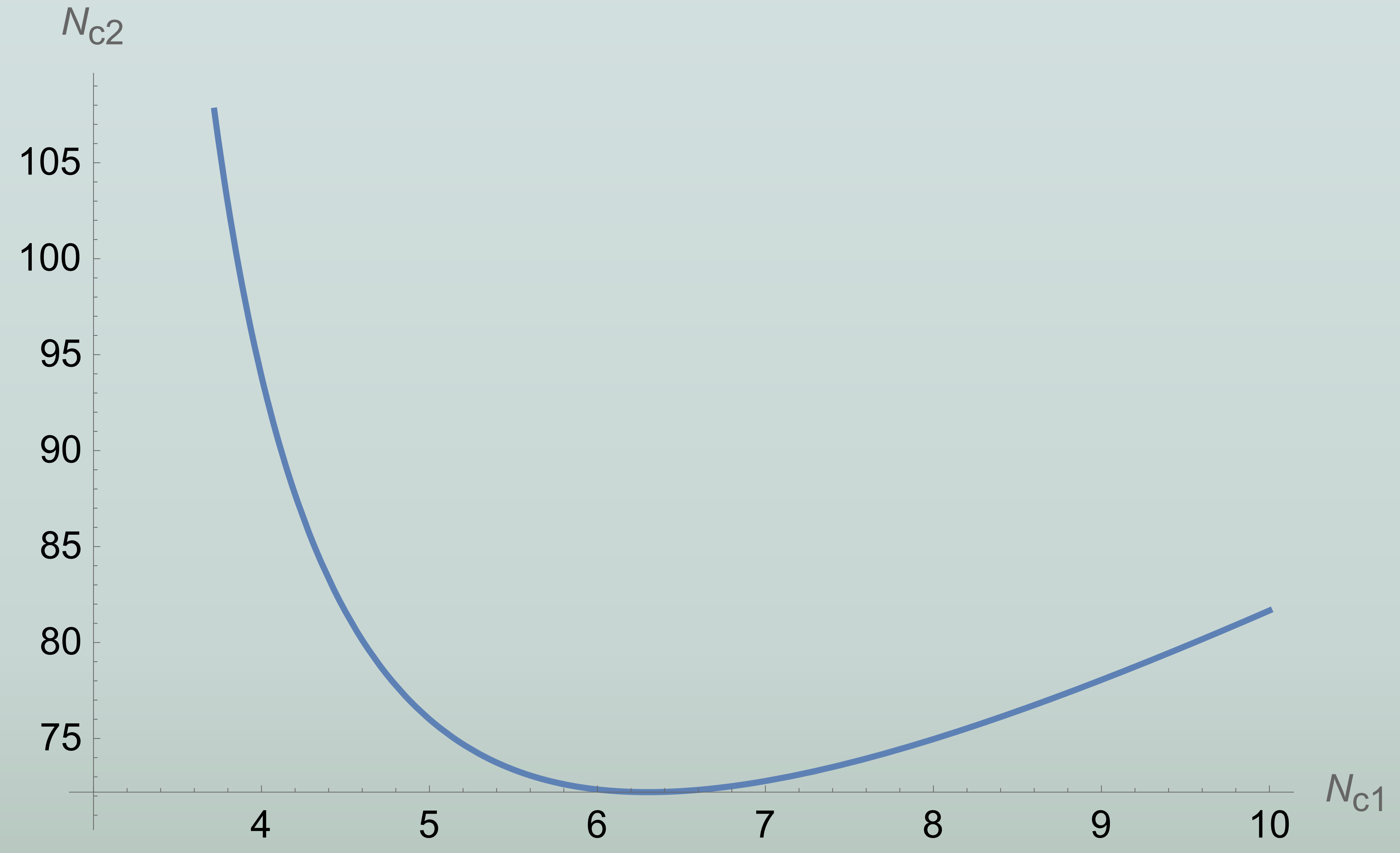


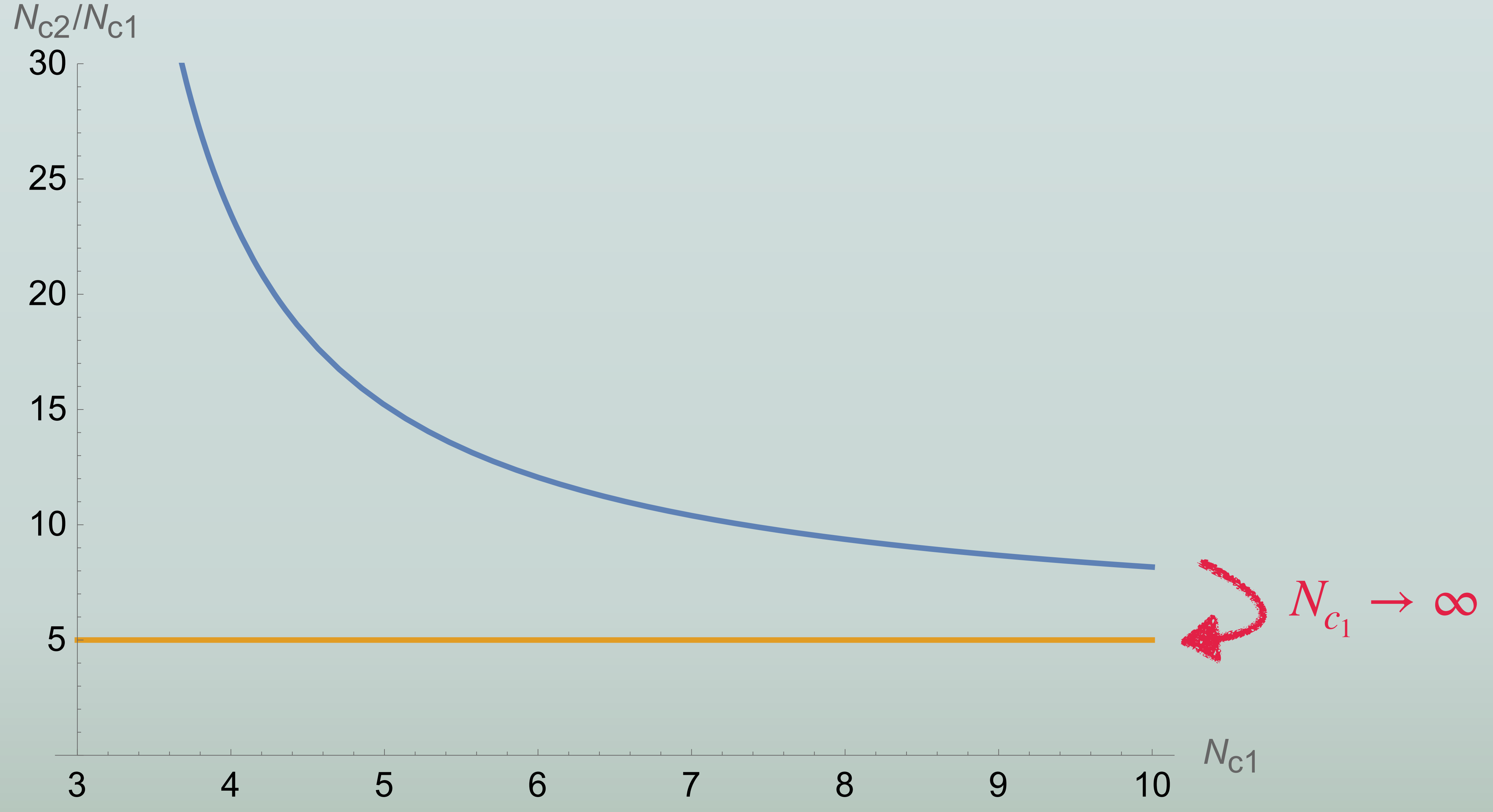
Bottom line: symmetry non-restoration survives with a finite number of degrees of freedom at arbitrarily high T!

On the lower bound on the colour degrees of freedom

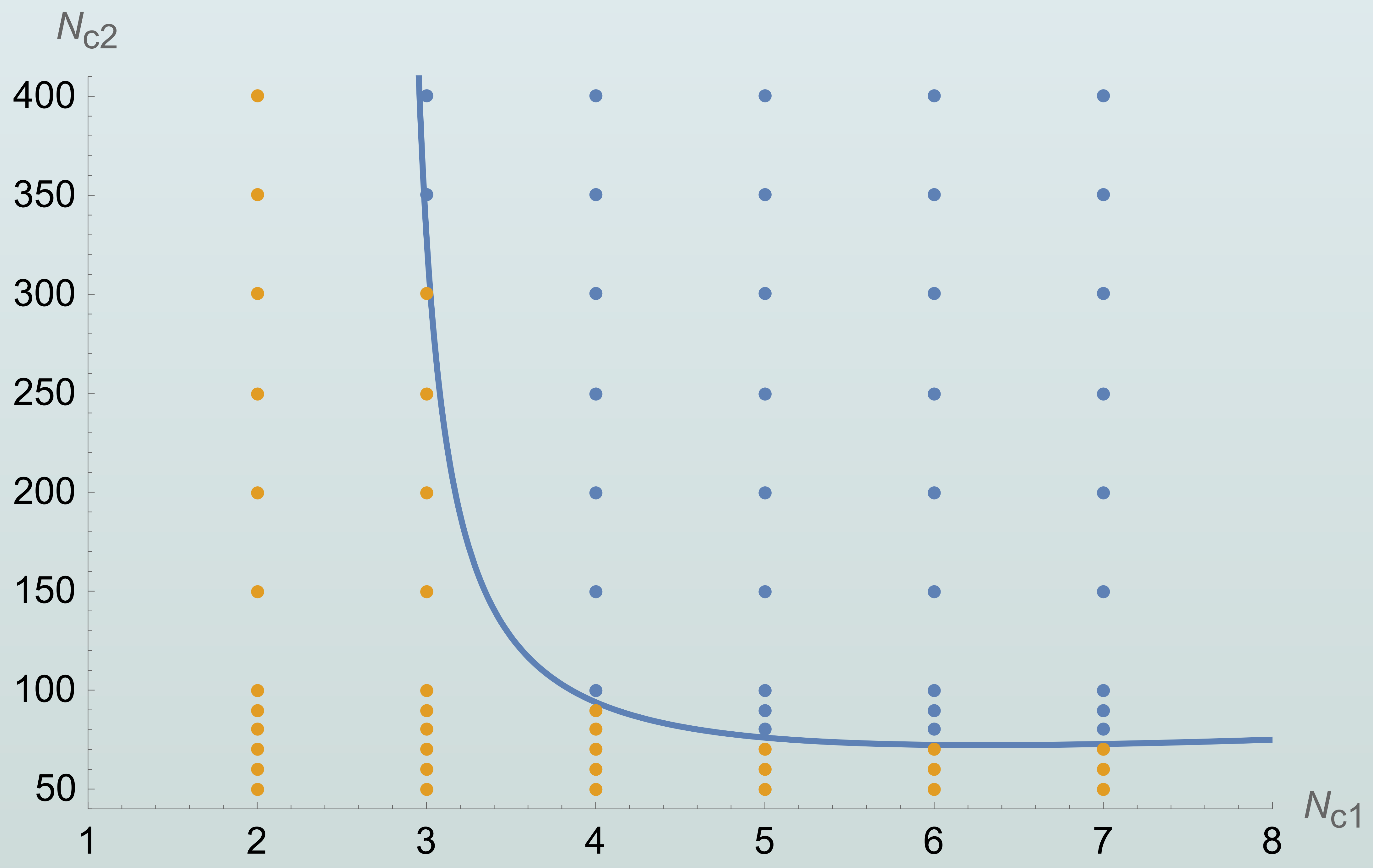
- What's the lowest possible bound on N_{c_i} $(i = 1,\ 2)$ below which symmetry must restore at high temperature?
- Within perturbation theory we find:

$$N_{c2} \geq \frac{10N_{c1}^4 + 31N_{c1}^3 + 12N_{c1}^2 - 29N_{c1} - 20}{2N_{c1}^3 - 3N_{c1}^2 - 9N_{c1} + 5} \quad .$$



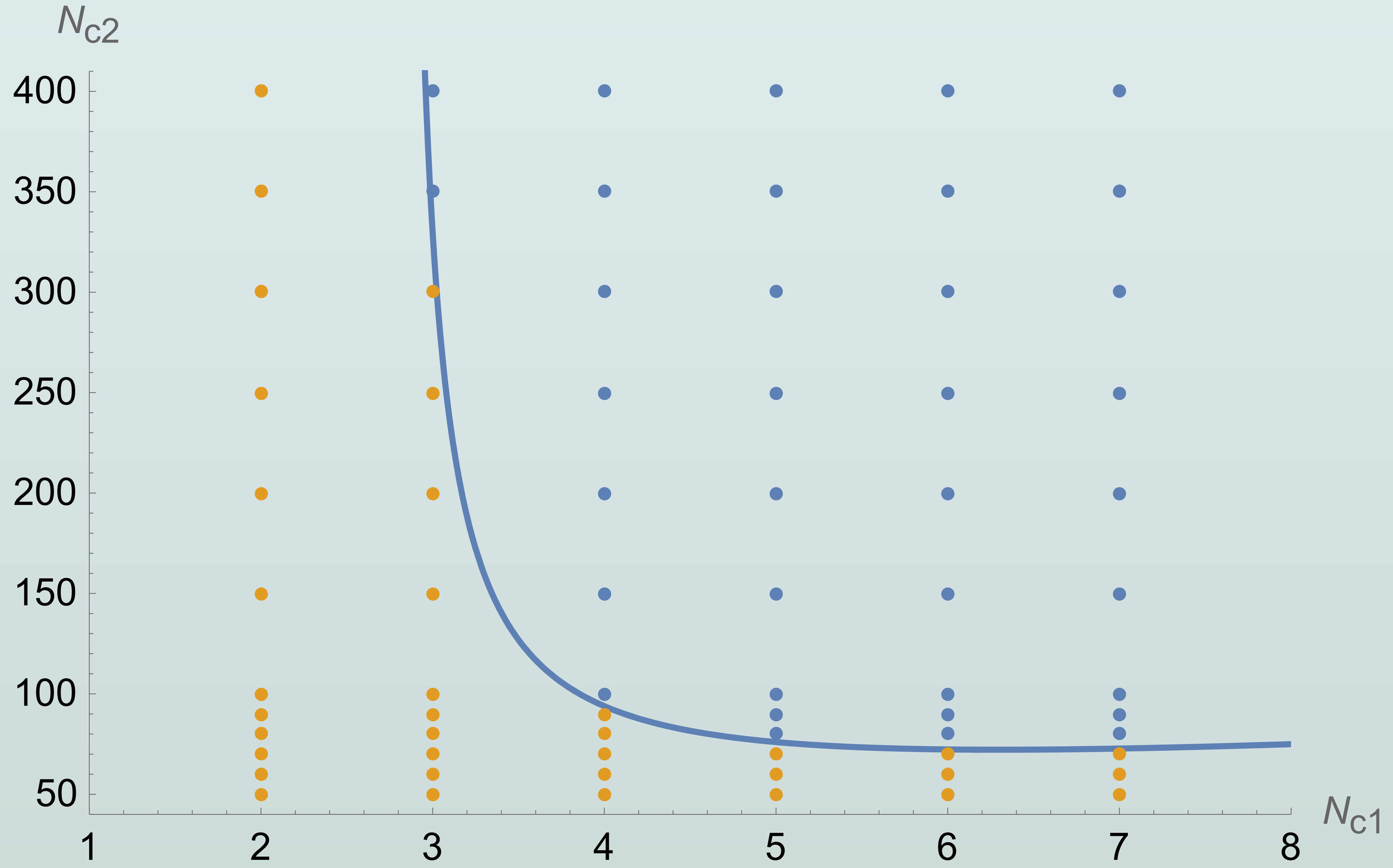


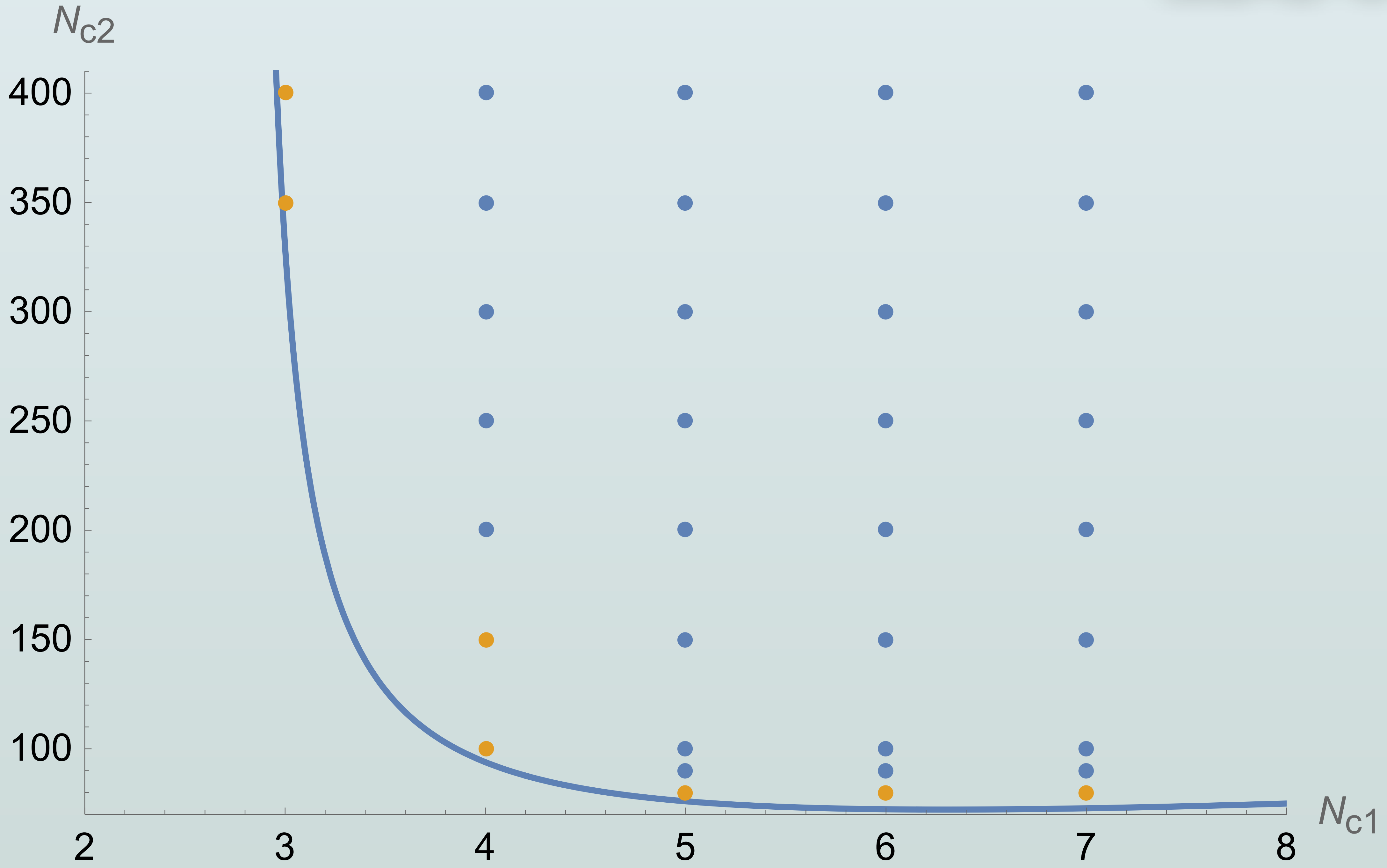
Numerical results (no $1/N$) truncation



Regions in the $N_{c_1} - N_{c_2}$ plane with (blue) and without (yellow) a solution with symmetry non-restoration. The boundary between them is well described by the perturbative constraint!

Numerical results (no $1/N$) truncation





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We lose some solutions when taking into account integer $N_{f_{1,2}}$

Conclusions & Summary

- We now know that there exists (at least) one example of a (3+1)-d theory with a finite number of degrees of freedom that realises symmetry non-restoration at arbitrarily high T
- Symmetry non-restoration might be the answer to some longstanding phenomenological issues, e.g. the problems of domain walls and magnetic monopoles simply wouldn’t arise in cosmology.

Thank you!