Semiclassical Canovaccio

FOR

Quantum Field Theory

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SSM
Scuola Superiore Meridionale

SEMPER ARDENS
ADVANCE

The Canovaccio

From Commedia dell'arte to Quantum Physics

Skeletal script, a basic plot

Essential acts and staging

Semiclassics:

Mathematical skeleton to solve QFT

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Letter

Exact results for scaling dimensions of neutral operators in scalar conformal field theories

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We determine the scaling dimension Δ_n for the class of composite operators ϕ^n in the $\lambda\phi^4$ theory in $d = 4 - \epsilon$ taking the double scaling limit $n \to \infty$ and $\lambda \to 0$ with fixed λn via a semiclassical approach. Our results resum the leading power of n at any loop order. In the small λn regime we reproduce the known diagrammatic results and predict the infinite series of higher-order terms. For intermediate values of λn we find that Δ_n/n increases monotonically approaching a $(\lambda n)^{1/3}$ behavior in the $\lambda n \to \infty$ limit. We further generalize our results to neutral operators in the ϕ^4 in $d = 4 - \epsilon$, ϕ^3 in $d = 6 - \epsilon$, and ϕ^6 in $d = 3 - \epsilon$ theories with $O(N)$ symmetry.

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Semiclassical canovaccio for composite operators

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Abstract: We present a novel semiclassical framework tailored to determine the scaling dimensions of heavy neutral composite operators in conformal field theories (CFTs) which are inaccessible with other current methodologies. It utilizes the state-operator correspondence to map the desired scaling dimensions to the semiclassical energy spectrum of periodic homogeneous field configurations on a cylinder. As concrete applications, we provide detailed analyses for the ϕ^4 theory near four dimensions and ϕ^6 near three dimensions, semiclassically determining the full spectrum of neutral operators transforming according to different Lorentz representations. Our methodology is presented pedagogically and is readily applicable to a vast class of CFTs.

Lectures on Semiclassical Methods

for

Composite Operators

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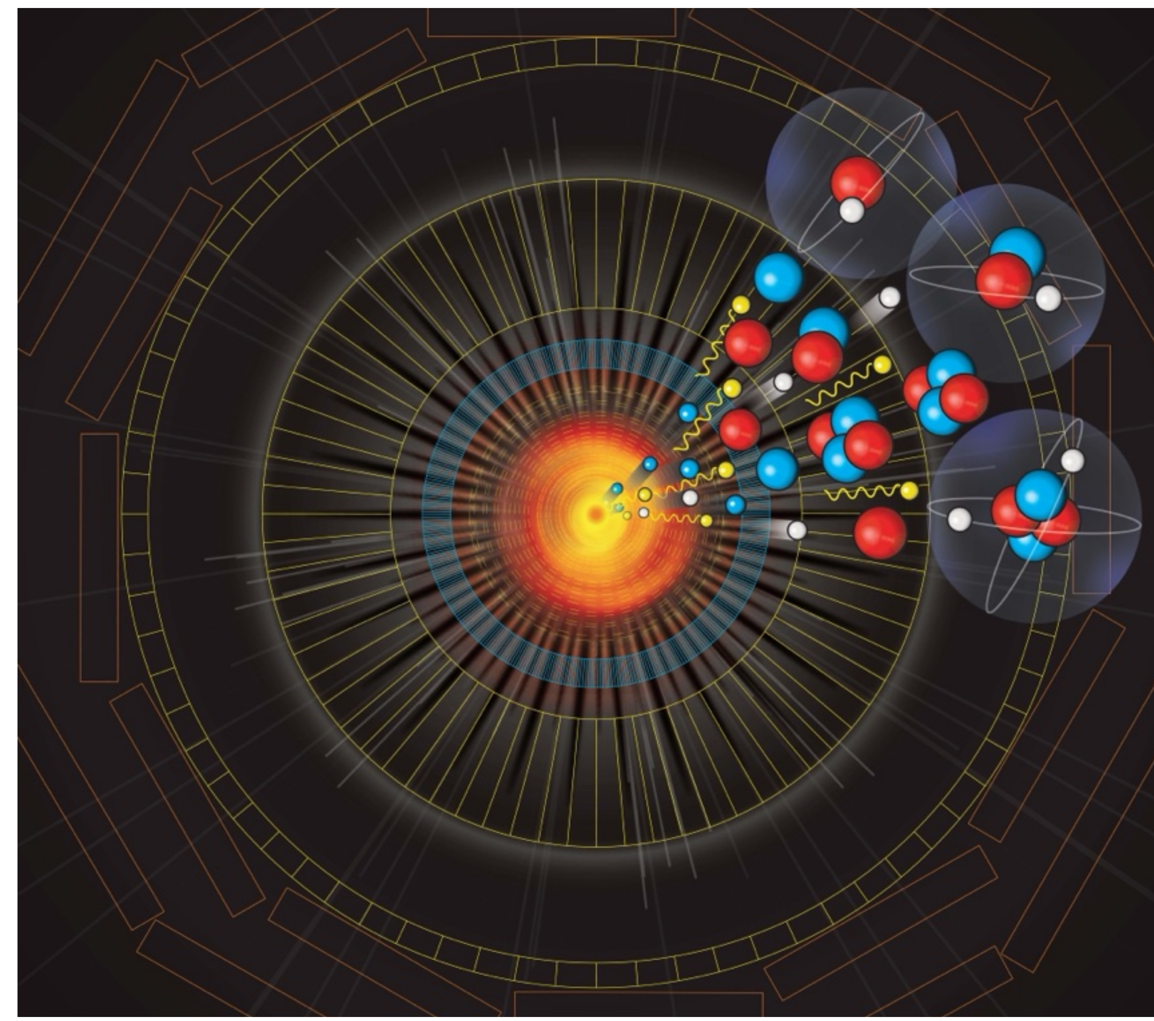
Abstract

These lecture notes are intended as a coherent introduction to conformal field theory in general, and composite operators in particular, through a semiclassical framework for computing scaling dimensions, with emphasis on operators of the form ϕ^n. In doing so, they aim to fill a gap in the literature and to help decode some of the relevant concepts. The physical idea is that at large n an (heavy) operator creates a highly occupied state. Through the state–operator correspondence, this state lives on the cylinder $\mathbb{R} \times S^{d-1}$, and its scaling dimension is the corresponding energy of the theory on the cylinder. The notes are organized as a self-contained route from conformal symmetry to semiclassical dynamics. Part I reviews the conformal group, primary operators, radial quantization, the state–operator correspondence, and operator mixing. Part II builds the semiclassical framework, first in the free scalar theory, where the dimension of ϕ^n is recovered in three independent ways, and then through the double-scaling limit, the action variable, and Bohr–Sommerfeld quantization. Part III develops the general machinery of periodic saddles, Floquet theory, fluctuation determinants, the Gel'fand–Yaglom method, and the Gutzwiller trace formula. Part IV applies the framework to the $O(N)$ ϕ^4 theory in $d = 4 - \epsilon$ at the Wilson–Fisher fixed point, deriving

Quantum Field Theory

1920- Born, Heisenberg, Jordan, Dirac (1927)....

Reconciles SR and QM

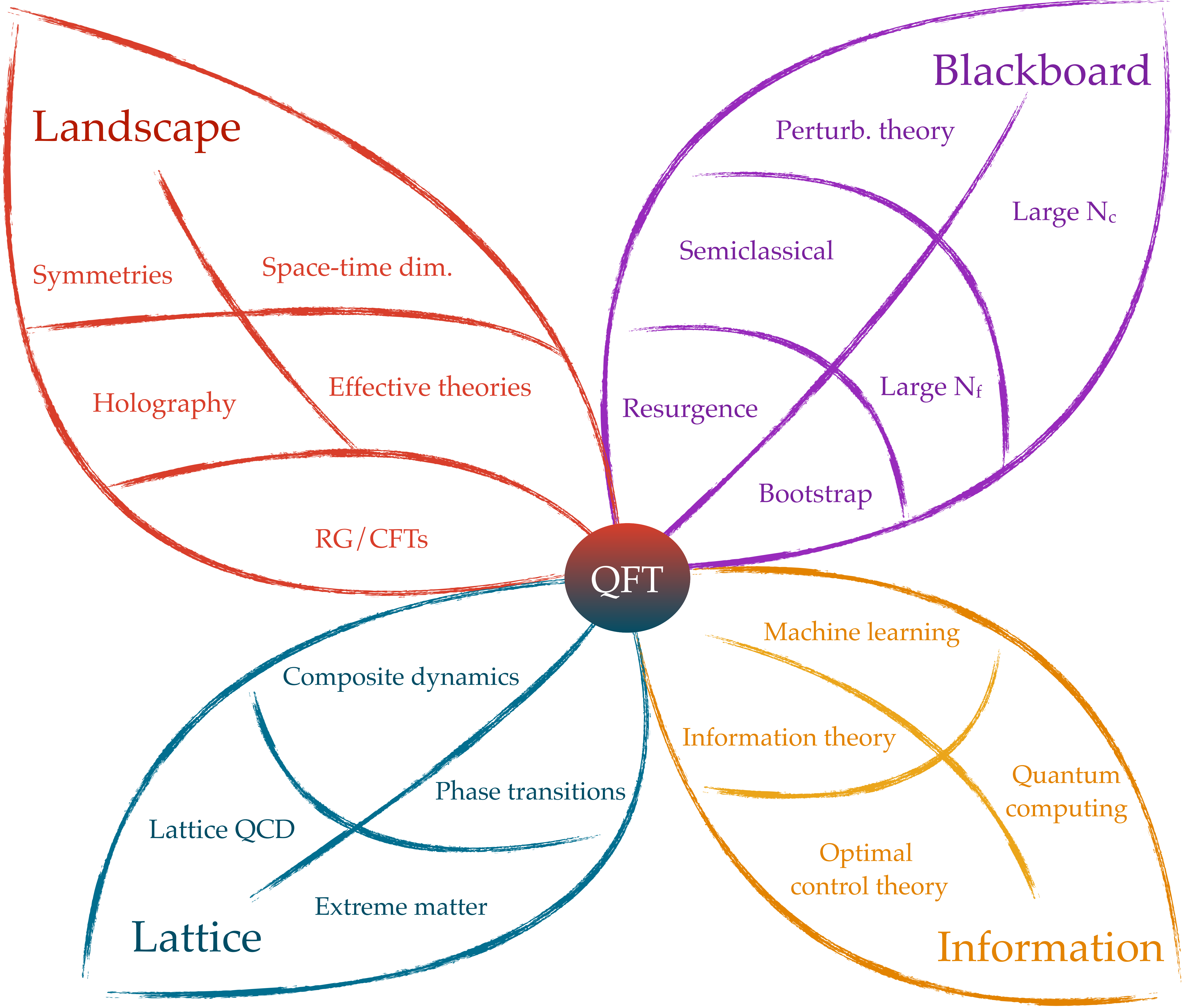


Underpins:

- Cosmology and Gravity
- Astro, nuclear and particle physics
- Condensed matter physics
- Advances in mathematics

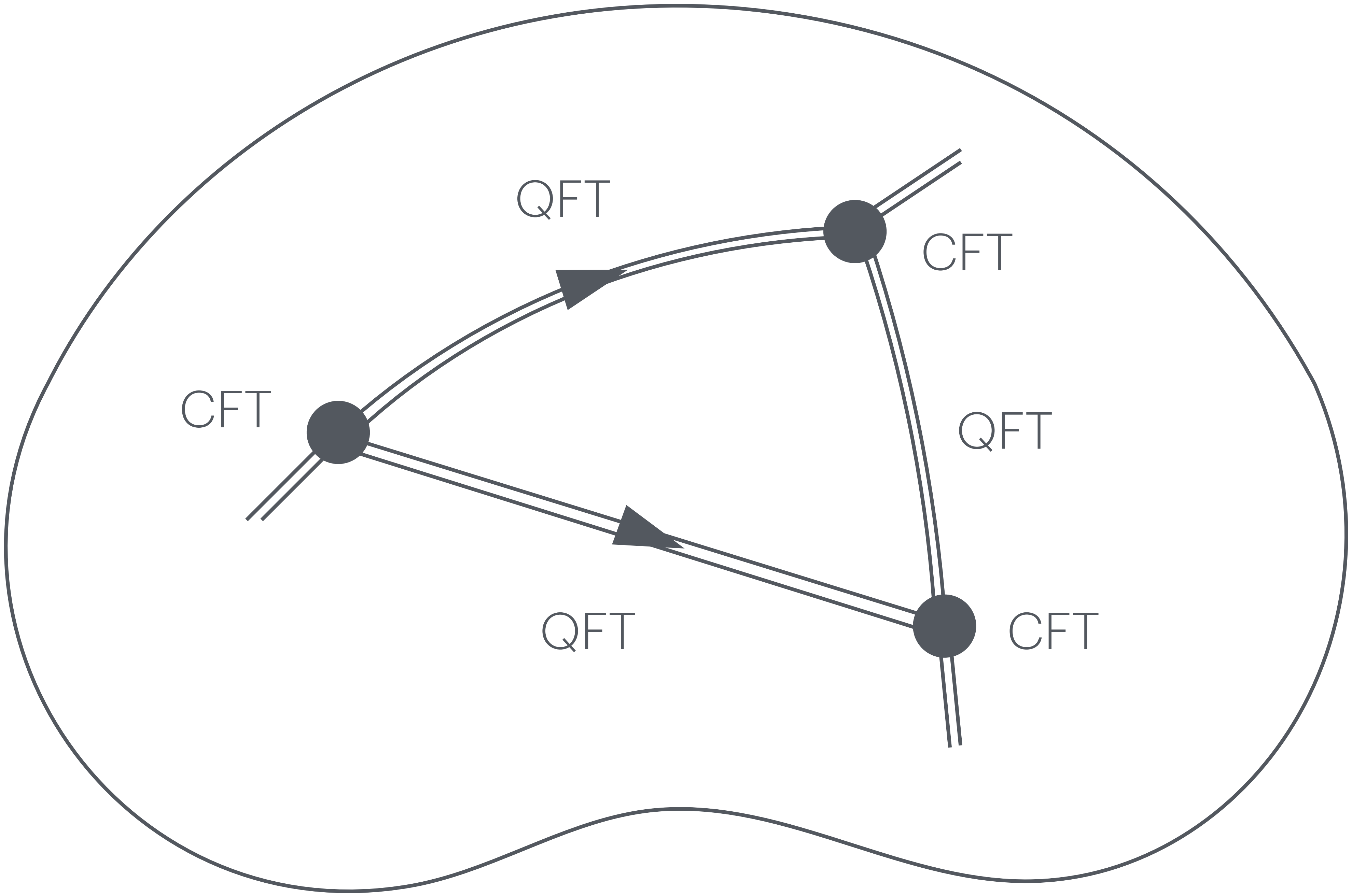
All fundamental forces of Nature are QFTs

Universal language across scales and dimensions



QFT
Landscape
Symmetries
Space-time dim.
Holography
Effective theories
RG/CFTs
Blackboard
Perturb. theory
Large N_c
Semiclassical
Resurgence
Large N_f
Bootstrap
Machine learning
Information theory
Quantum computing
Optimal control theory
Information
Composite dynamics
Phase transitions
Lattice QCD
Extreme matter
Lattice

QFT Space



Solving CFT

CFT data

Operators: Primary or descendants

Primary operator: Highest-weight states of the conformal group representations

Descendants: Obtained acting on primary with P_μ

Interactions: controlled by OPE coefficients

$$\mathcal{O}_i(x)\,\mathcal{O}_j(y) = \sum_k \lambda_{ijk}\,\frac{1}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}}\,\mathcal{O}_k(y)$$

Scaling dimension spectrum & OPE coefficients completely and uniquely solves the CFT, enabling computations of any arbitrary n-point correlator.

The Ising Model

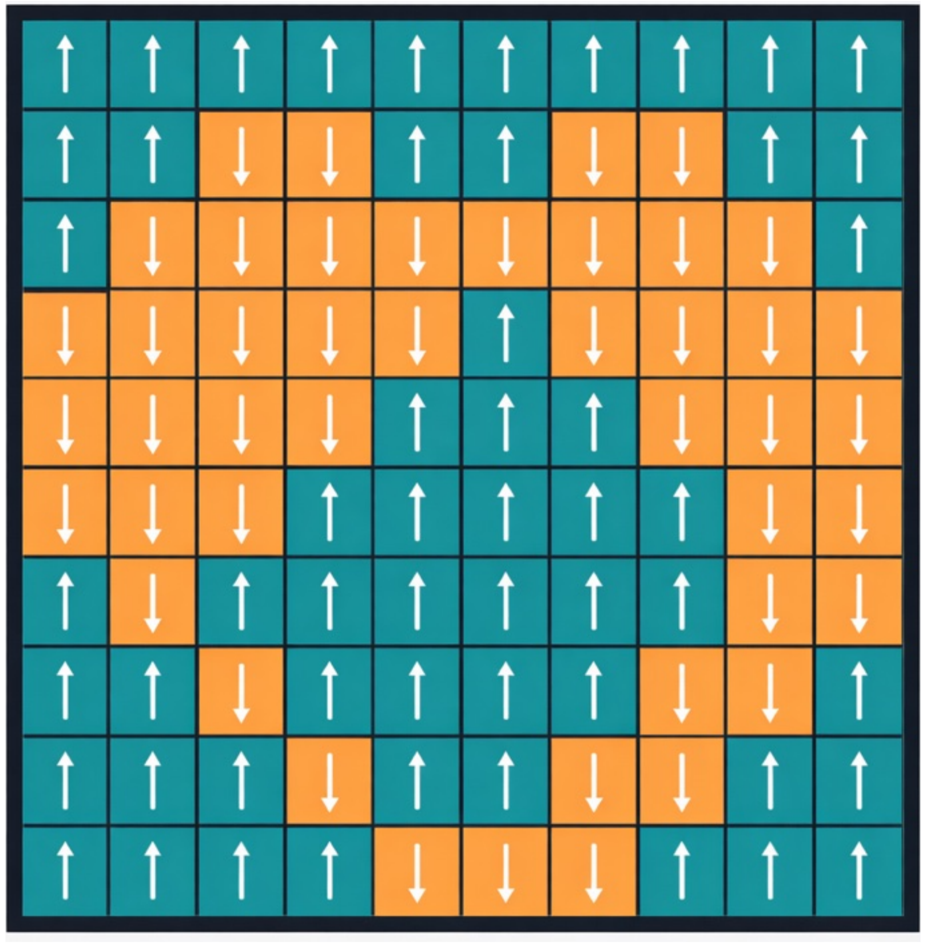
Quintessential example

Lattice spins: $s_i = \pm 1$; coupling J; field h

$$H = -J \sum_{\langle ij \rangle} s_i s_j - h \sum_i s_i$$

Control parameters: Temperature & dim.

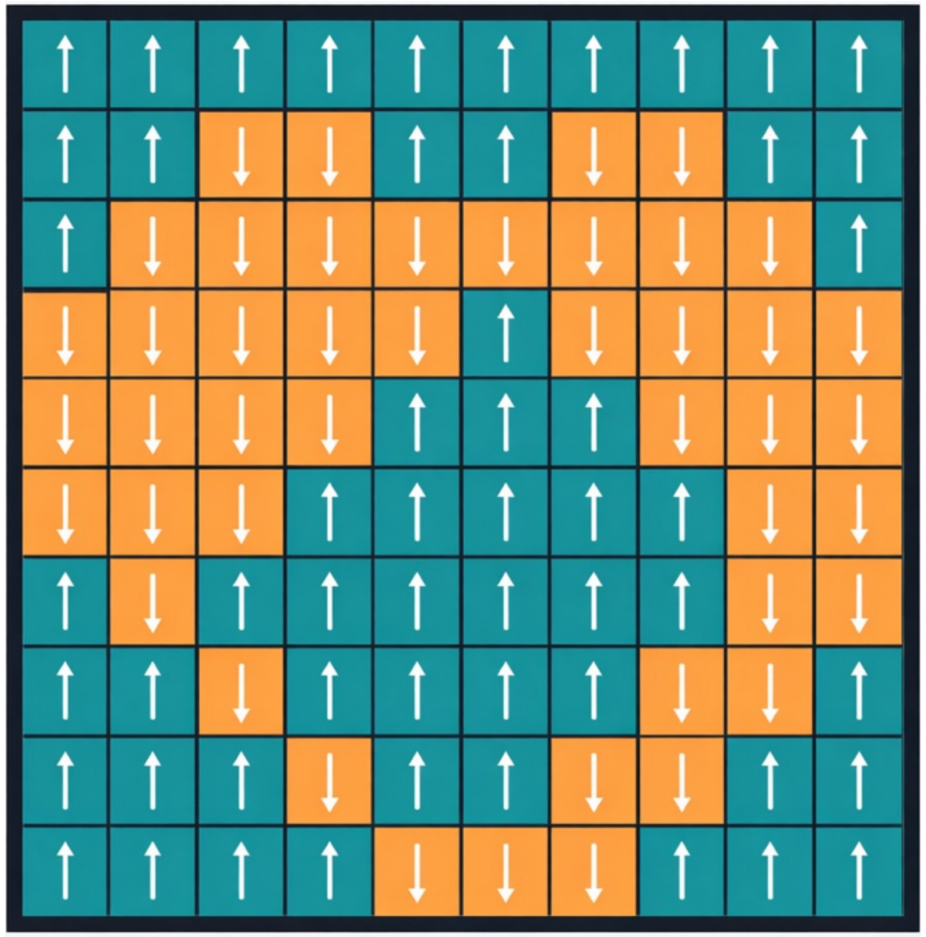
Magnetization $m = \frac{1}{N} \sum_i \langle s_i \rangle$



The Ising Model

1920, Wihelm Lenz

Lattice model of ferromagnetism.



1924

Lenz's student, Ernst Ising showed:

Phase transition does not occur in one dim.

1944

2-dim Ising model solved by Lars Onsager

Ordered phase transition

2000

3D Ising: Nondeterministic Polynomial (NP) complete problem, Sorin Istrail

Emergence of Conformal Symmetry

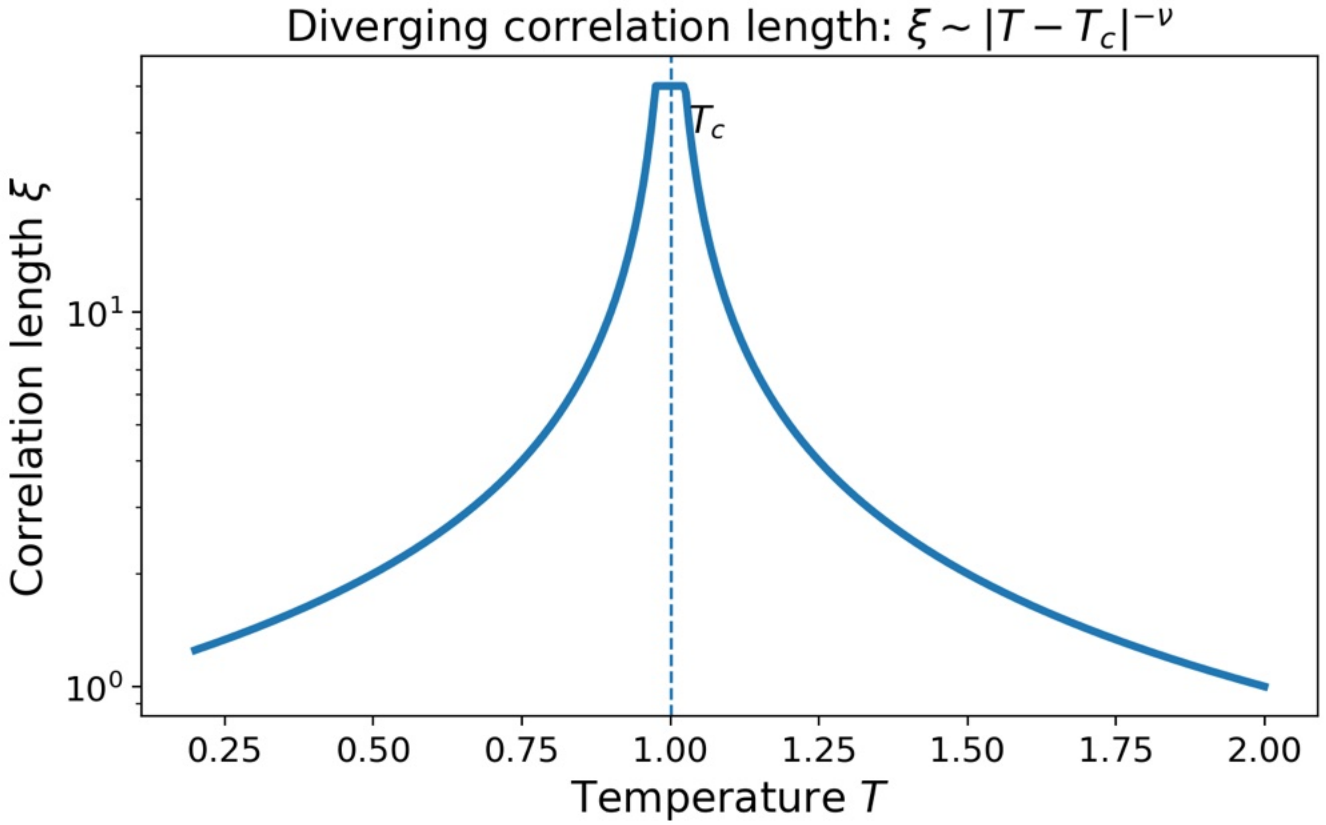
Why is it a CFT at T_c?

As $\xi \to \infty$ the system is **scale invariant**: $x \to \lambda x$!

Enhancement to Conformal Symmetry:

Local QFT: Scale invariance, combined with rotational and translation invariance generally enhances to conformal invariance!

Macrophysics at T_c is a CFT!



3D Ising Model CFT - Challenges

A biased list

The full theory is encoded in the spectrum of operators

Composite operators *proliferate fast & mix* under coarse-graning

Systematic, high-precision map of higher composites is needed

Needed for precision corrections to scaling and detailed dynamics

Composite operators are the frontier where the theory becomes rich, and hard!

Universality is the first layer

Composite operators $\Rightarrow$ Ising CFT becomes a quantitive QFT

Composite operators

What do we want compute?

$$\langle \phi^n(x_f)\,\phi^n(x_i)\rangle = \frac{1}{Z}\int \mathcal{D}\phi \;\; \phi^n(x_f)\,\phi^n(x_i) \;\; e^{i\,S[\phi]}$$

$$Z = \int \mathcal{D}\phi \; e^{i\,S[\phi]}$$

Dynamics in d-dimensions controlled by $S[\phi]$

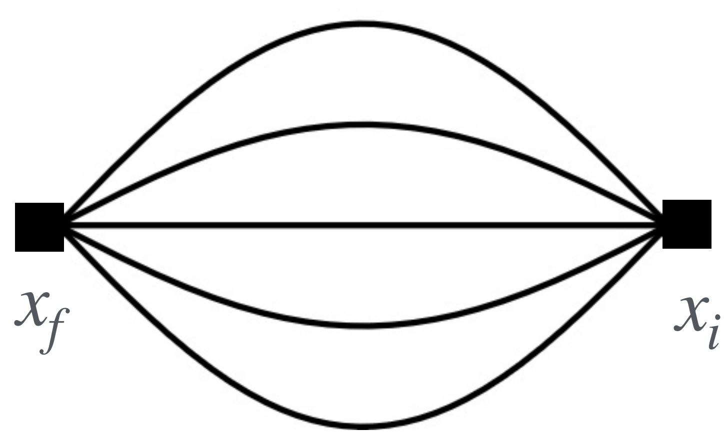
Simplest CFT

The *boring* free theory

$$S[\phi] = \frac{1}{2}\int d^d x\, (\partial\phi)^2$$

Diagrammatically

$$\langle \phi^n(x_f)\phi^n(x_i)\rangle = n!$$



$$= n!\,[G(x_f - x_i)]^n$$

$$n! \simeq e^{n(\log n - 1)}$$

Why semiclassics?

Free theory CFT

$$\phi^n(x) = e^{n \log \phi(x)}$$

$$\langle \phi^n(x_f)\phi^n(x_i)\rangle = \frac{1}{Z}\int \mathscr{D}\phi \;\; \exp\left[-\frac{1}{2}\int d^dx\,(\partial\phi)^2 + n\log\phi(x_f) + n\log\phi(x_i)\right]$$

Let's rescale the fields

$$\phi(x) = \sqrt{n}\,\varphi(x)$$

Quantum Mechanics Analogue

Free theory CFT

Kinetic rescaling: $$(\partial\phi)^2 = n\,(\partial\varphi)^2$$

Insertion rescaling: $$n\log\phi(x) = n\left(\frac{1}{2}\log n + \log\varphi(x)\right)$$

$$\langle\phi^n(x_f)\phi^n(x_i)\rangle = n^n\ \frac{1}{Z}\int\mathscr{D}\varphi\ e^{-n\,S_{\text{eff}}[\varphi]}$$

$$S_{\text{eff}}[\varphi] = \int d^dx\,\frac{1}{2}(\partial\varphi)^2 - \log\varphi(x_f) - \log\varphi(x_i)$$

$$n \sim \frac{1}{\hbar}$$

Controls corrections like $\hbar$ in semiclassical QM

Saddle-point evaluation

Free theory CFT

Solve for the classical E.o.M for $S_{\text{eff}}[\varphi]$

$$\varphi_{\text{cl}}(x) \equiv v(x) = \frac{G(x-x_f)}{v(x_f)} + \frac{G(x-x_i)}{v(x_i)}$$

Evaluate action on EoM $S_{eff}[v] = 1 - \log G(x_f - x_i)$

Compute large n correlator

$$\langle \phi^n(x_f)\phi^n(x_i)\rangle \simeq n^n \exp\Big(-n\, S_{eff}[v]\Big) = e^{n(\log n - 1)}\left[G(x_f - x_i)\right]^n$$

$n! \simeq e^{n(\log n-1)}$ Large n imprint!

Scaling dimensions

Free theory CFT

In d-dimensions is n times the dimension of the free scalar

$$\langle \phi^n(x)\phi^n(0)\rangle \propto \frac{1}{|x|^{2\Delta_n}} \qquad \Delta_n = \frac{d-2}{2}n$$

Recovered recalling $G(x) \propto |x|^{-(d-2)}$

$$\langle \phi^n(x_f)\phi^n(x_i)\rangle = e^{n(\log n - 1)}\left[G(x_f - x_i)\right]^n \propto |x|^{-n(d-2)}$$

So far

Free Theory CFT

Semiclassics *emerges as* $n \sim 1/\hbar$

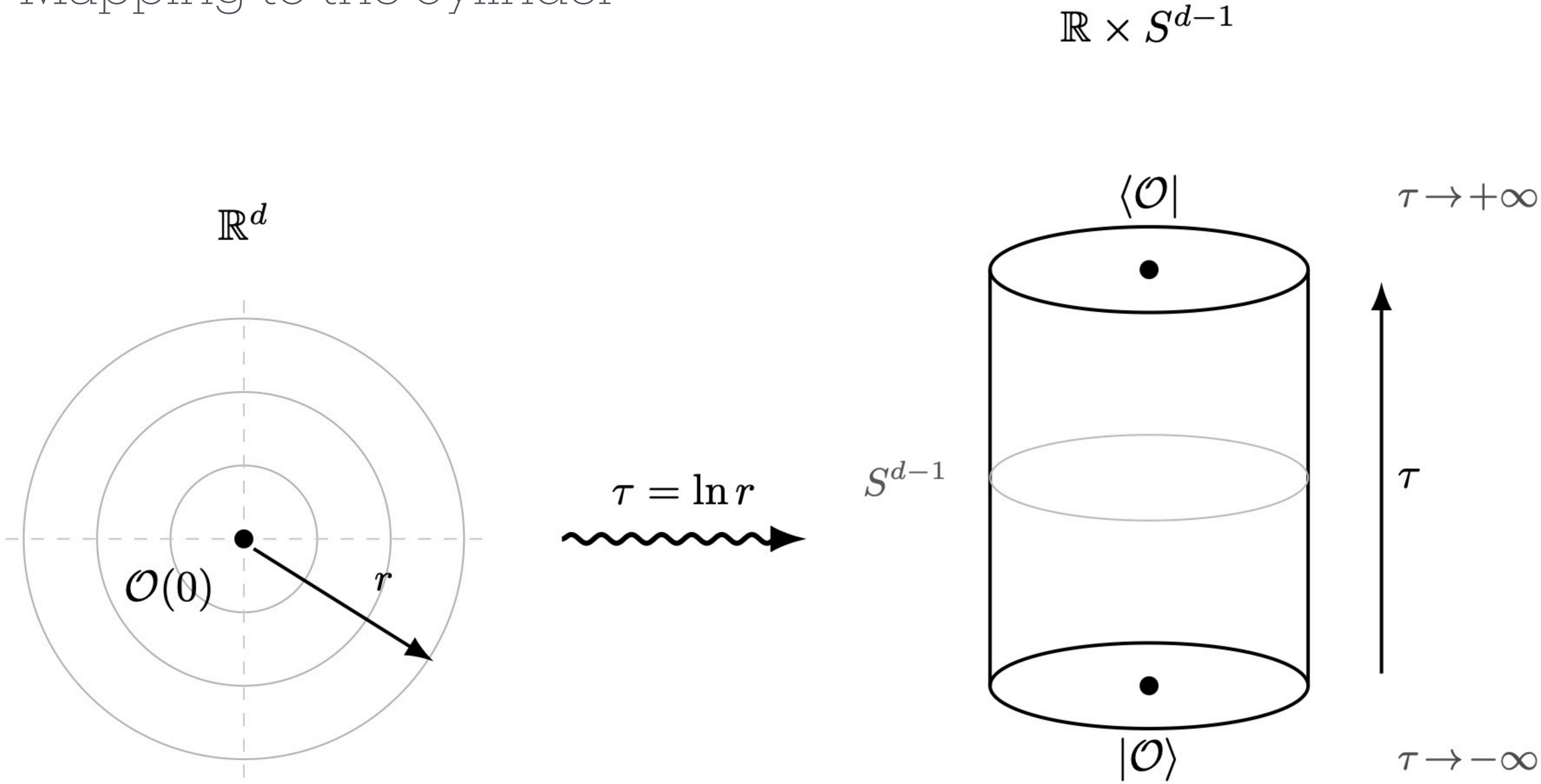
Composite operators *dimensions via saddle-point evaluation*

Still too involved for interacting theory. One more step!

Energy spectra are “typically” easier than computing correlators!

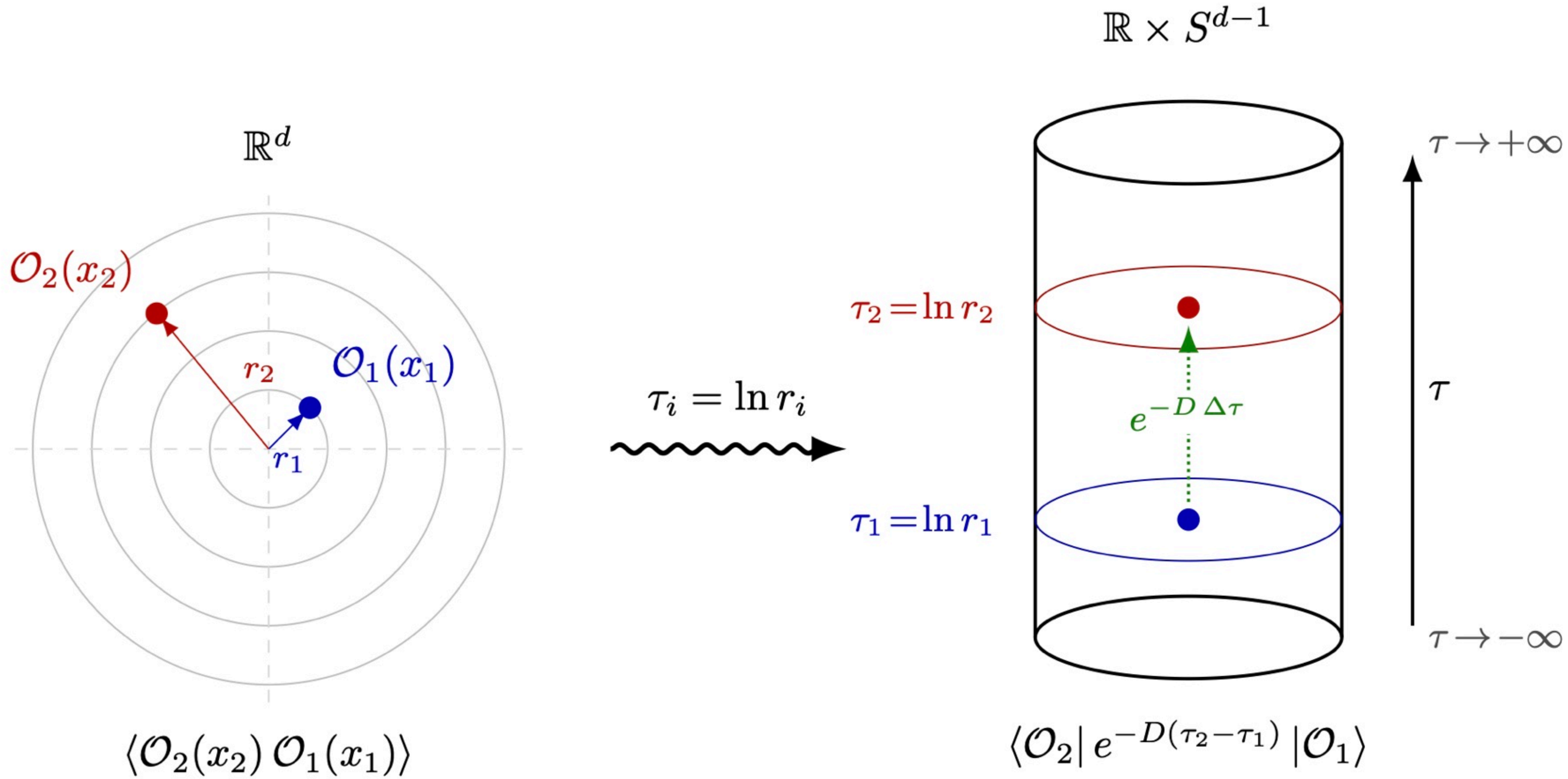
State - Operator Correspondence

Mapping to the cylinder



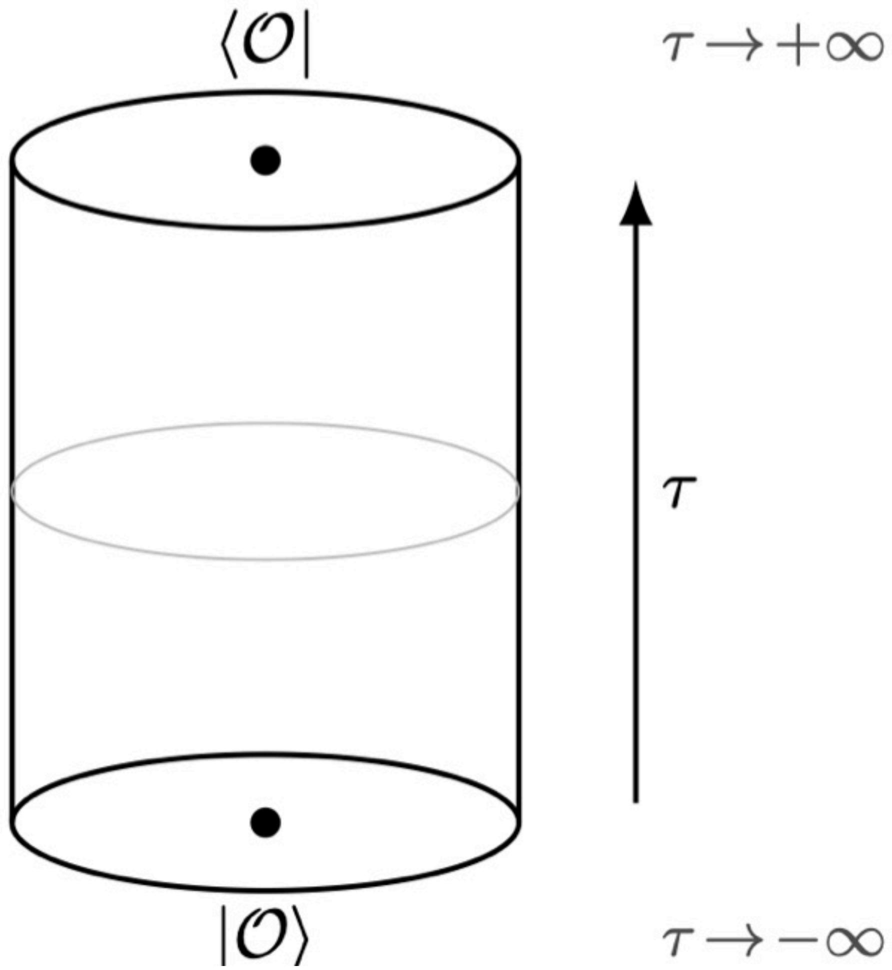
State - Operator Correspondence

Mapping to the cylinder



State - Operator Correspondence

Mapping to the cylinder



$$\mathbb{R}^d \quad \Rightarrow \quad \mathbb{R} \times S^{d-1}$$

Lorentzian time, $\tau = it$,

$\mathcal{O}$ insertions @ asympt. past & future

$$\Delta = r\,E$$

E: Energy states living on the d-1 spheres of radius r

From CFT to Energy States

Classical dynamics on the cylinder

A conformally coupled scalar field has action

$$S = \frac{1}{2}\int d^d x \sqrt{g} \left(g^{\mu\nu}\partial_\mu\phi\, \partial_\nu\phi + \xi\, \mathscr{R}\, \phi^2 \right), \qquad \xi = \frac{d-2}{4(d-1)} \quad \text{on} \quad \mathbb{R}\times S^{d-1}$$

Induced scalar mass

$$\mu^2 = \xi\mathscr{R} = \frac{(d-2)^2}{4r^2}, \qquad \mu = \frac{d-2}{2r}$$

Scalar curvature

$$\mathscr{R} = \frac{(d-1)(d-2)}{r^2}$$

Equations of motion on Cylinder

Classical dynamics on the cylinder

$$\phi_{class}(t, \Omega) = v(t)$$

E.o.M and solution

$$\ddot{v} + \mu^2 v = 0 \quad \Rightarrow \quad v(t) = \boxed{A}\cos(\mu t + t_0)$$

Yet unknown

Orbit period $$T = \frac{2\pi}{\mu}$$

t_0 - translational invariance

Action variable

Bohr-Sommerfeld quantization

Canonical momentum conjugate

$$\Pi = \frac{\partial L_{\text{Cyl}}}{\partial \dot{v}} = \Omega_{d-1} r^{d-1} \dot{v}$$

Action variable

$$I = \oint \Pi \, dv = \Omega_{d-1} r^{d-1} \int_0^T \dot{v}^2$$

On the solution

$$\dot{v} = -A\mu \sin(\mu t + t_0)$$

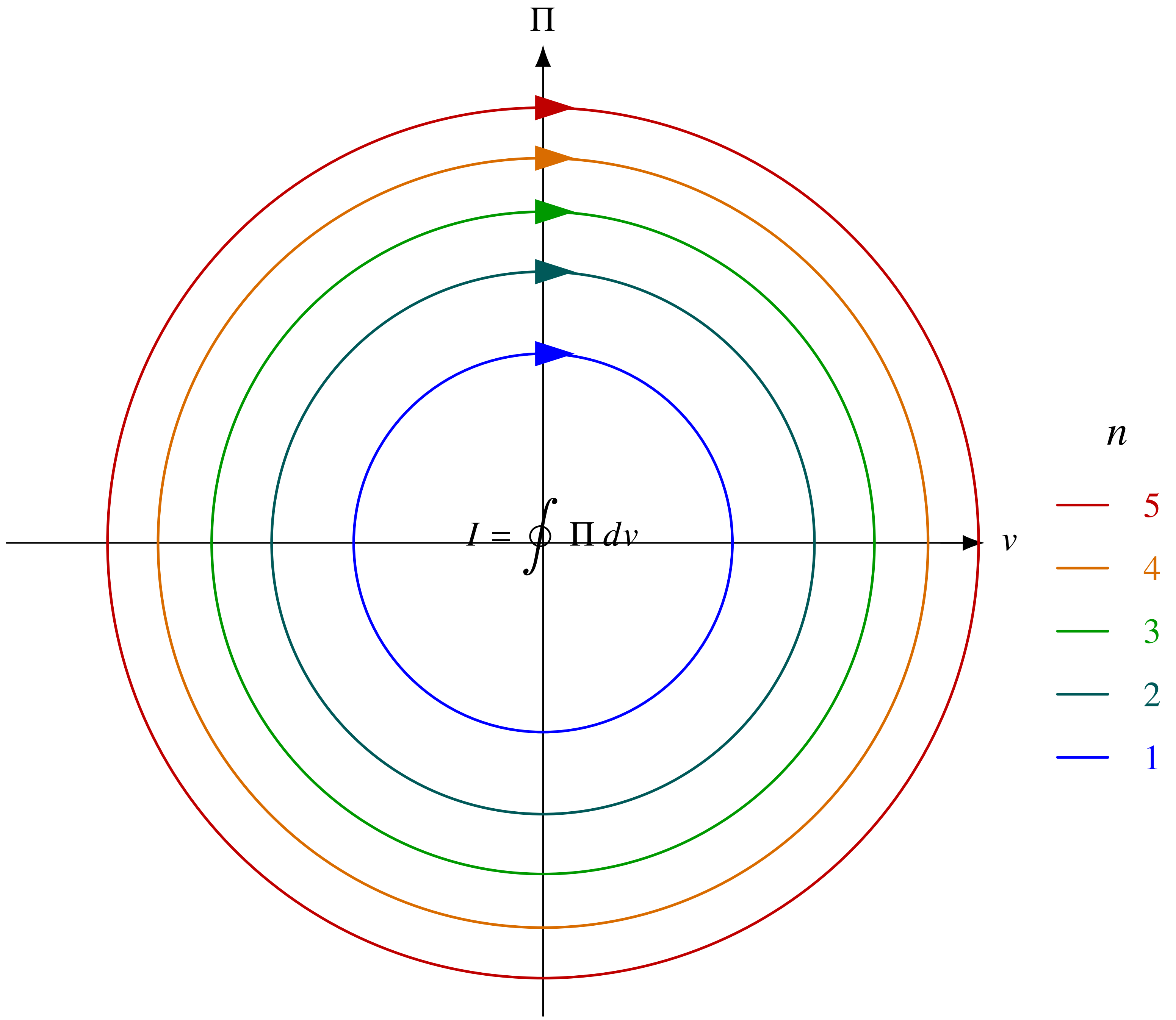
$$I = \pi \, \Omega_{d-1} \, r^{d-1} \boxed{A^2} \mu$$

Action variable

Bohr-Sommerfeld quantization

Action variable

$$I = \oint \Pi \, dv$$



Quantized orbits

Bohr-Sommerfeld quantization

Quantization $$I = 2\pi n$$ n-emergence

Final quantized orbit
$$v(t) = \frac{2\sqrt{n}}{\sqrt{(d-2)\Omega_{d-1} r^{d-2}}} \cos(\mu t + t_0)$$

A now fixed !

Spectrum
$$E = \frac{n(d-2)}{2r}$$

Scaling Dimensions
$$\Delta = rE \quad \Rightarrow \quad \Delta_n = \frac{d-2}{2} n$$

Free Theory CFT/Canovaccio

Summary

Heavy composite operators admit semiclassical descriptions

Periodic homogeneous solutions on cylinder

Bohr-Sommerfeld quantization condition introduces n

Energy spectrum reproduces scaling

$$\Delta = rE \qquad \Rightarrow \qquad \Delta_n = \frac{d-2}{2}\, n$$

Bridge between semiclassical dynamics and operator dimensions

Interacting dynamics

When the fun starts!

Ising CFT

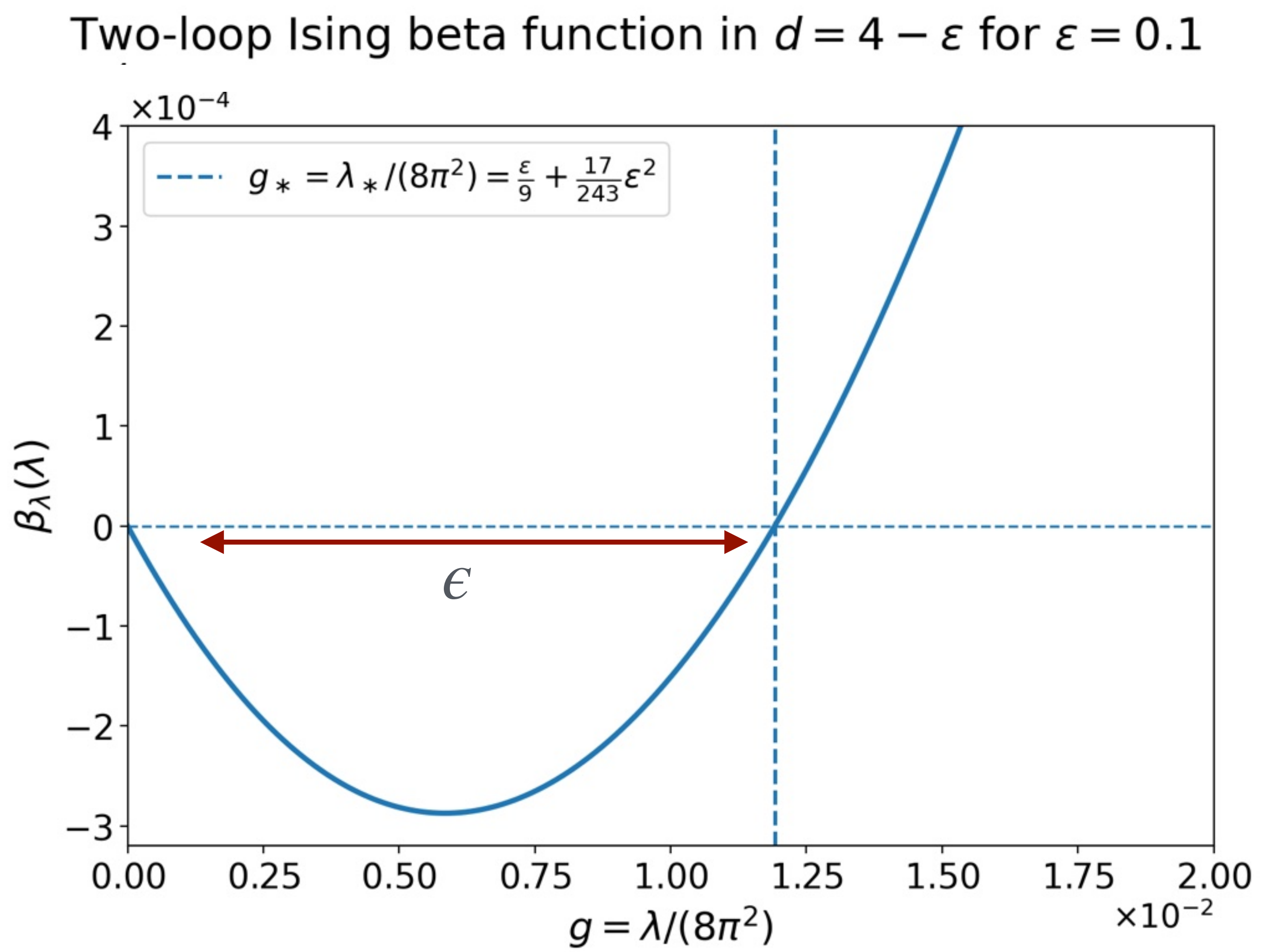
Setting up the stage

CFT in $d-\epsilon$ dimensions

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4$$

Fixed point coupling

$$\lambda_* = \frac{8\pi^2\epsilon}{9} + \frac{136\pi^2\epsilon^2}{243} + \mathcal{O}(\epsilon^3)$$



$$\beta_\lambda(\lambda) \equiv \mu\frac{d\lambda}{d\mu} = -\epsilon\,\lambda + \frac{9}{8\pi^2}\lambda^2 - \frac{51}{64\pi^4}\lambda^3 + \mathcal{O}(\lambda^4)$$

Operator Spectrum for Ising CFT

Dynamics

Conformal primary:

Scaling dimension eigenvector/eigenvalue

Operator mix: same spin s and classical dim.

$$\partial^s \, \Box^p \, \phi^n$$

Classical dimension: $s + 2p + n$

$$\Delta_{n,q_\ell} = \left(\frac{d-2}{2}\right) n + s + 2p + \gamma_{n,q_\ell} = r\, E_{n,q_\ell}$$

$\{q_\ell\}$ integers specifying eigenoperators $\partial^s \, \Box^p \, \phi^n$

Semiclassical 1/n expansion

Operator ordering

$$\partial^s \Box^p \phi^n$$

$$\Delta_{n,q_\ell} = rE_{n,q_\ell} = n \sum_{i=0} \frac{C_i(\lambda n, q_\ell)}{n^i}$$

$$\Delta_{n,q_\ell} = n\, C_0(\epsilon n) + C_1(\epsilon n, q_\ell) + \dots$$

$$\lambda_* = \frac{8\pi^2\epsilon}{9} + \mathcal{O}(\epsilon^2)$$

On the cylinder

Dynamics

$$\mathscr{L}^{(\mathrm{cyl})} = \frac{1}{2}(\partial\phi)^2 - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 \qquad \text{with} \qquad \mu = \left(\frac{d-2}{2r}\right)$$

$$\phi_{class} = v(t)$$

$$v(t) = \mu\sqrt{\frac{2m}{\lambda(1-2m)}}\,\mathrm{cn}\left(\frac{\mu\ t}{\sqrt{1-2m}}\,\middle|\, m\right)$$

Period $\mathscr{T} = \frac{4}{\mu}\sqrt{1-2m}\,\mathbb{K}$

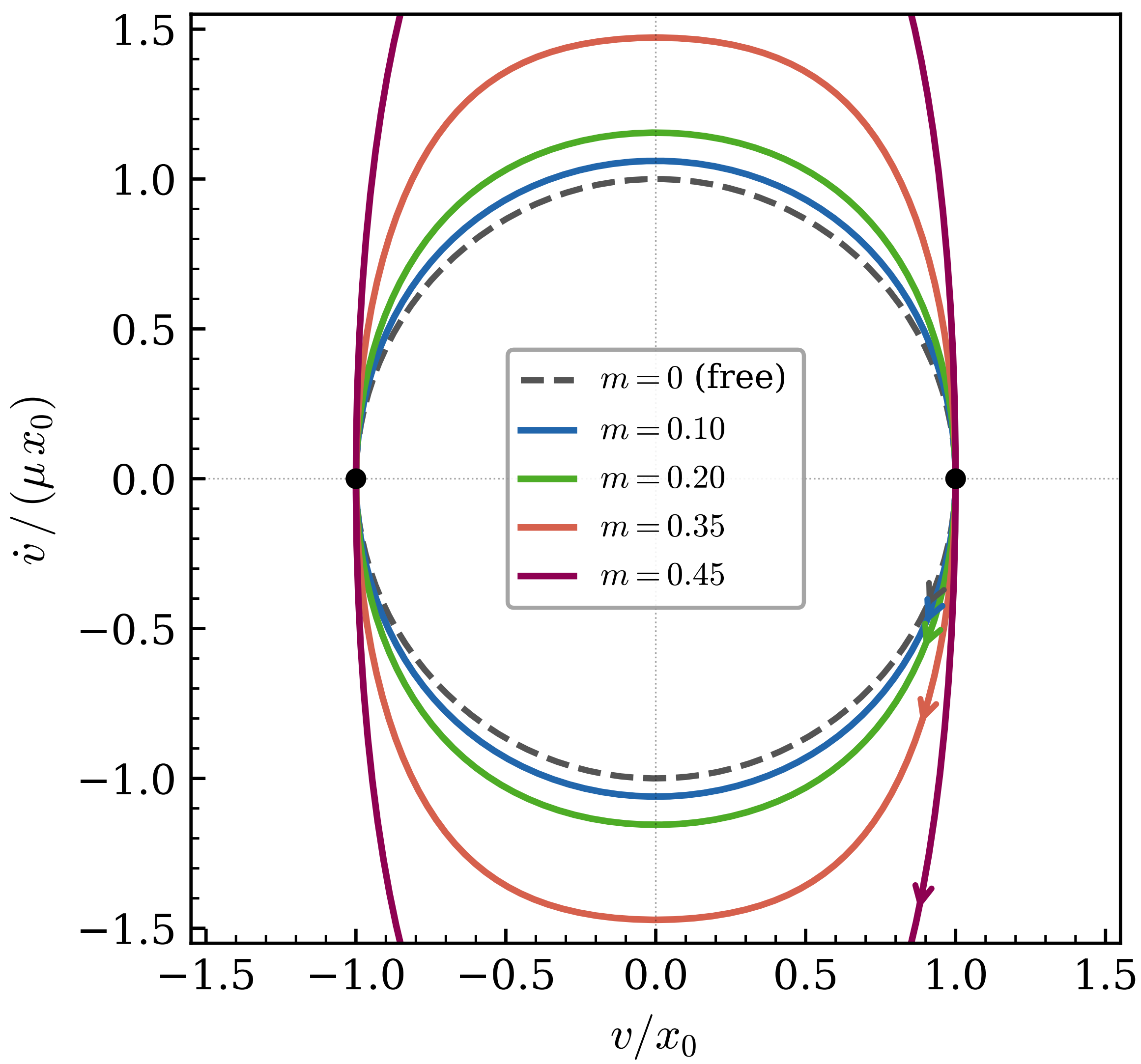
$\mathrm{cn}(\cdot\,|\,\mathrm{m})$ Jacobi elliptic cosine function

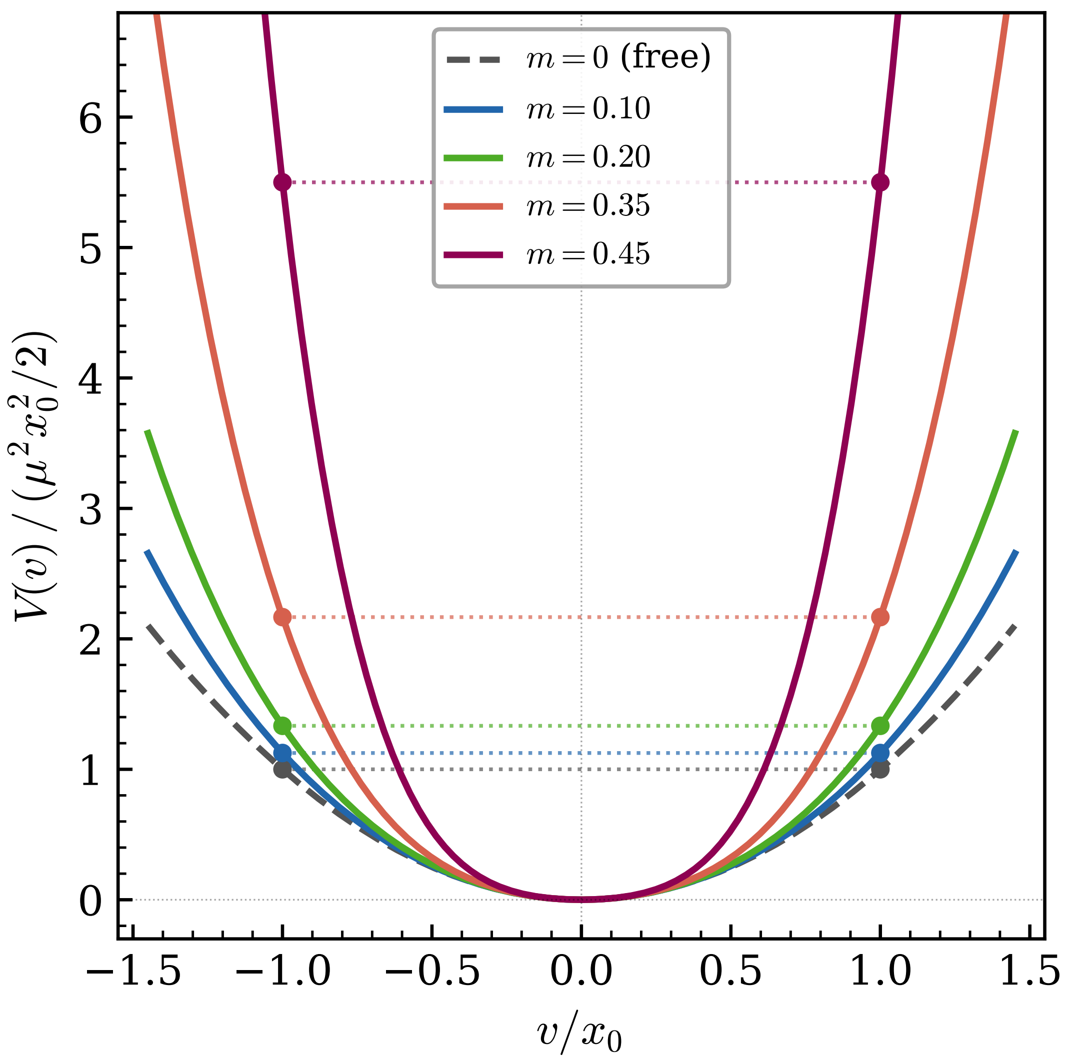
$\mathbb{K}(m)$ complete elliptic integral of the 1st kind

$m \to m(\lambda n)$ via Bohr-Sommerfeld Quantization

Action variable - interacting theory

Bohr-Sommerfeld quantization



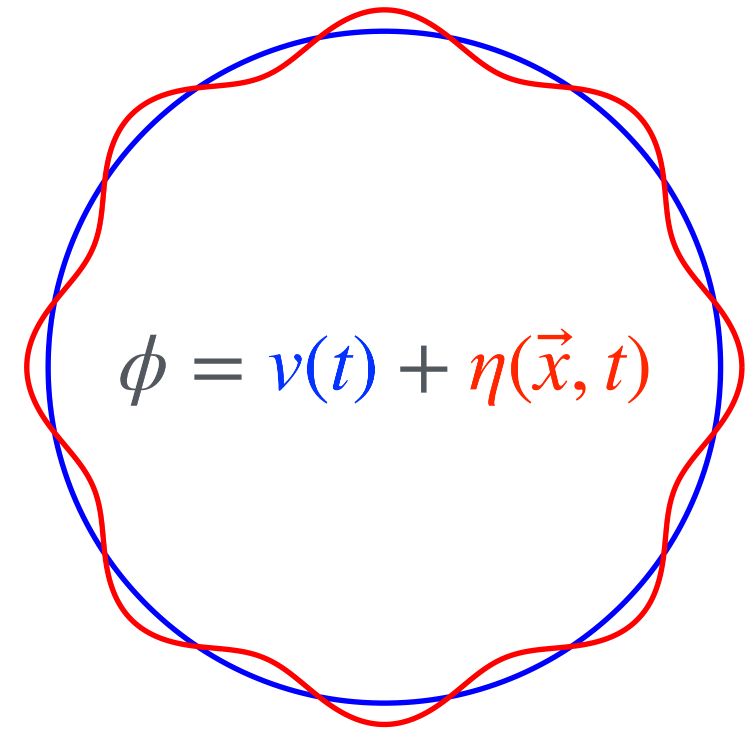


$$I = \oint \Pi \, dv$$

Naive understanding LO/NLO

Going beyond the leading order dynamics

$$E = E_{\rm cl} + E_{\rm fluct}$$



Generalized Bohr-Sommerfeld Quantization

$$I(E) + \sum_{\nu_\ell > 0} \left(q_\ell + \frac{1}{2} \right) \left(\mathcal{T} \frac{d\nu_\ell}{d\mathcal{T}} - \nu_\ell \right) = 2\pi\, n$$

ν_ℓ Stability angles

q_ℓ integers specifying eigenoperators $\partial^s \Box^p \phi^n$

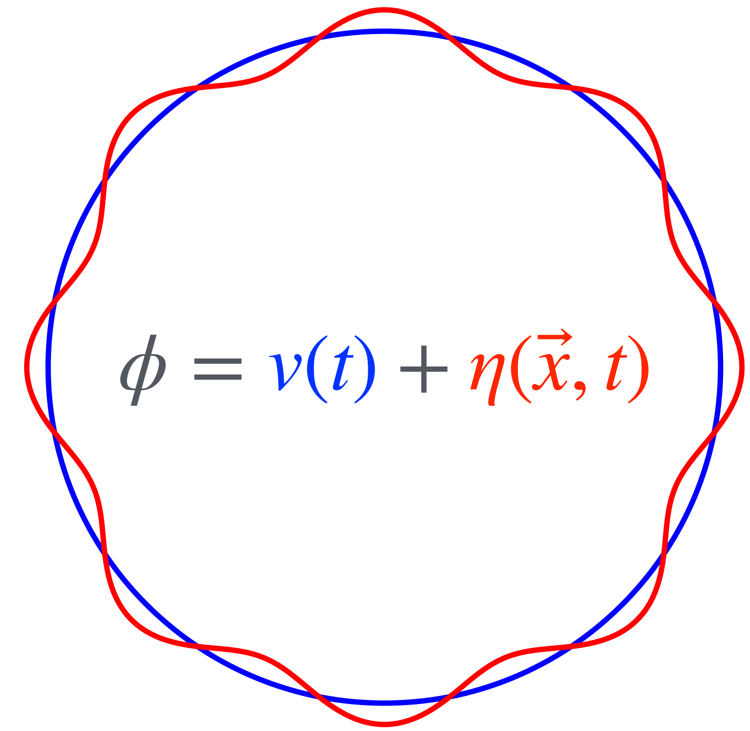
Classical limit $I(E_{\rm cl}) = 2\pi\, n$

See Oleg's talk for extra details!

Precise relations LO/NLO

Going beyond the leading order dynamics

$$E = E_{\mathrm{cl}} + E_{\mathrm{fluct}}$$



$$E_{n,q_\ell} = E_{cl} - \frac{\lambda_*}{2}\beta_0 E_{cl} + \frac{1}{\mathcal{T}} \sum_{\nu_\ell > 0} \left(q_\ell + \frac{1}{2} \right) \nu_\ell$$

$$\beta_0 = \frac{9}{8\pi^2}$$ 1-loop coefficient

$$\partial^s \,\Box^p\, \phi^n$$

$\{q_\ell\}$ integers specifying eigenoperators $\partial^s \,\Box^p\, \phi^n$

See Oleg's talk for extra details!

LO and NLO scaling

Where the physics happens

Set $r = 1$

$$\Delta_{n,q_\ell} = \left(\frac{d-2}{2}\right) n + s + 2p + \gamma_{n,q_\ell} = r\, E_{n,q_\ell}$$

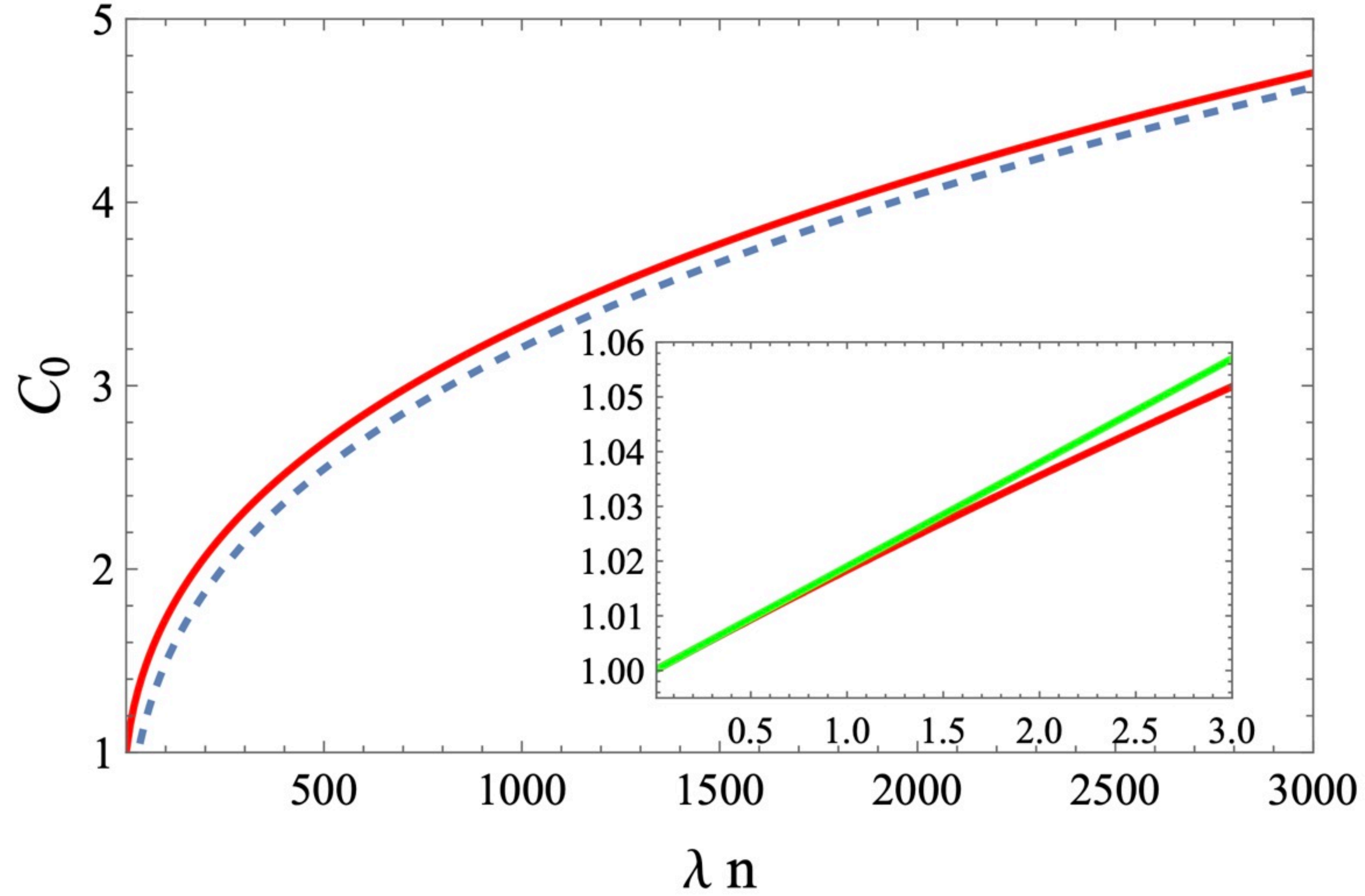
$$= n\, C_0(\epsilon n) + C_1(\epsilon n, q_\ell) + \dots$$

$$n\, C_0 = E_{cl}$$

$$C_1 = -\frac{\lambda_*}{2}\beta_0 E_{cl} + \frac{1}{\mathcal{T}} \sum_{\nu_\ell > 0} \left(q_\ell + \frac{1}{2}\right) \nu_\ell$$

LO

Full dynamics known



Strong coupling

$$\lambda n \to \infty$$

$$\lim_{n\to\infty} \Delta_n \sim n^{\frac{d}{d-1}}$$

Infinite order expansion

Perturbation theory

$$\lambda_* = \frac{8\pi^2}{9}\epsilon$$

$$\Delta_{n,q_\ell} = n + \sum_{\ell=1}^{\infty} q_\ell \ell$$

$$+\frac{1}{6}\left[n^2 - 2\left(2 + \sum_{\ell=1}^{\infty}\frac{(\ell-1)q_\ell}{\ell+1}\right)n + \mathcal{O}\left(n^0\right)\right]\epsilon$$

$$-\frac{1}{324}\left[17n^3 - \left(67 + 3\sum_{\ell=1}^{\infty}\frac{(\ell-1)\left(17\ell^4+78\ell^3+135\ell^2+98\ell+12\right)q_\ell}{\ell(\ell+1)^3(\ell+2)}\right)n^2 + \mathcal{O}\left(n, n^0\right)\right]\epsilon^2 + \mathcal{O}\left(\epsilon^3\right),$$

$$\Delta_{n,q_\ell} = n\, C_0(\epsilon n) + C_1(\epsilon n, q_\ell) + \ldots$$

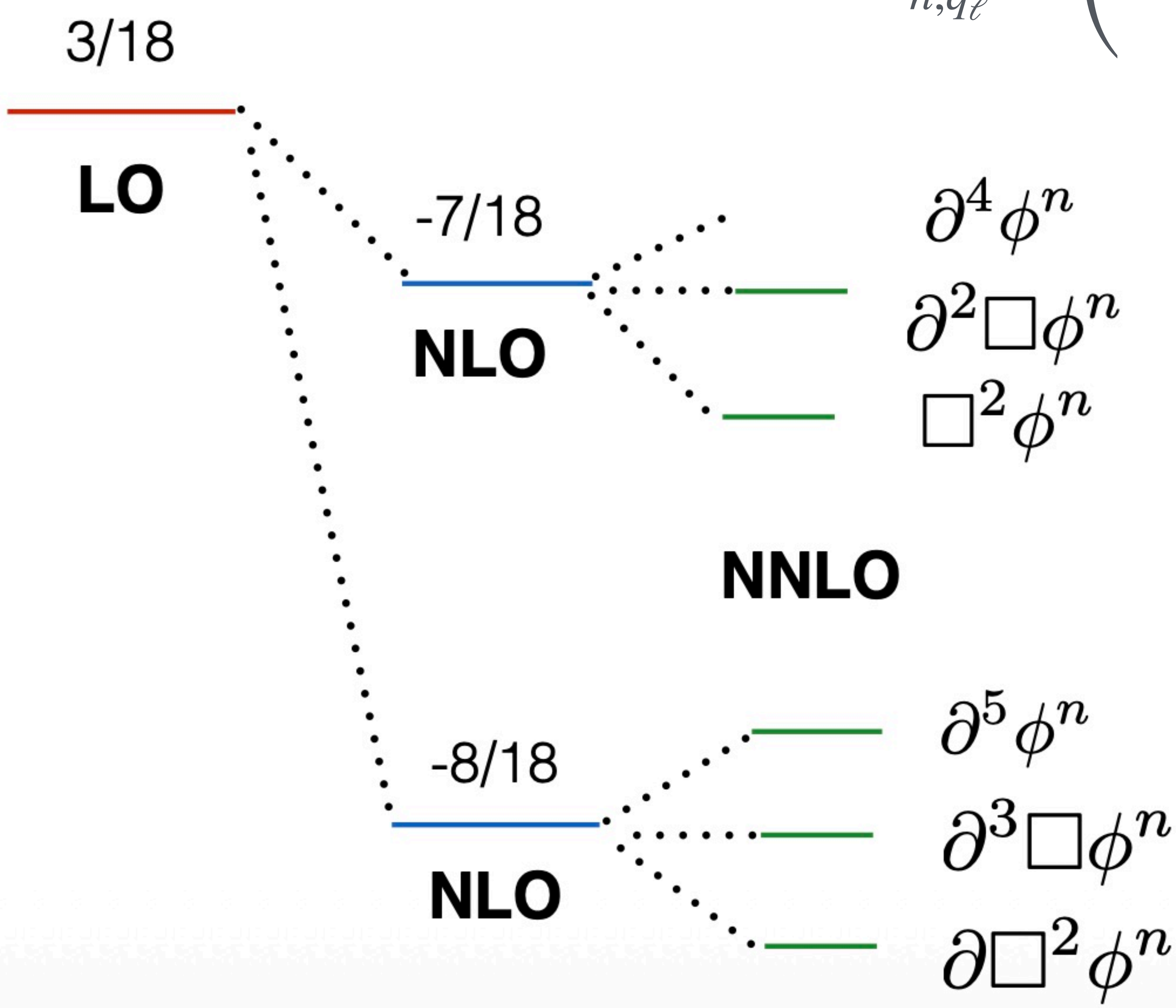
We can generate analytically “any” number of loops

Family of primaries for $q_\ell = 0$ akin to ϕ^n

Splitting of operators at NLO

Spectral analogy

$$\Delta_{n,q_\ell} = \left(\frac{d-2}{2}\right) n + s + 2p + \gamma_{n,q_\ell}$$



Models investigated:

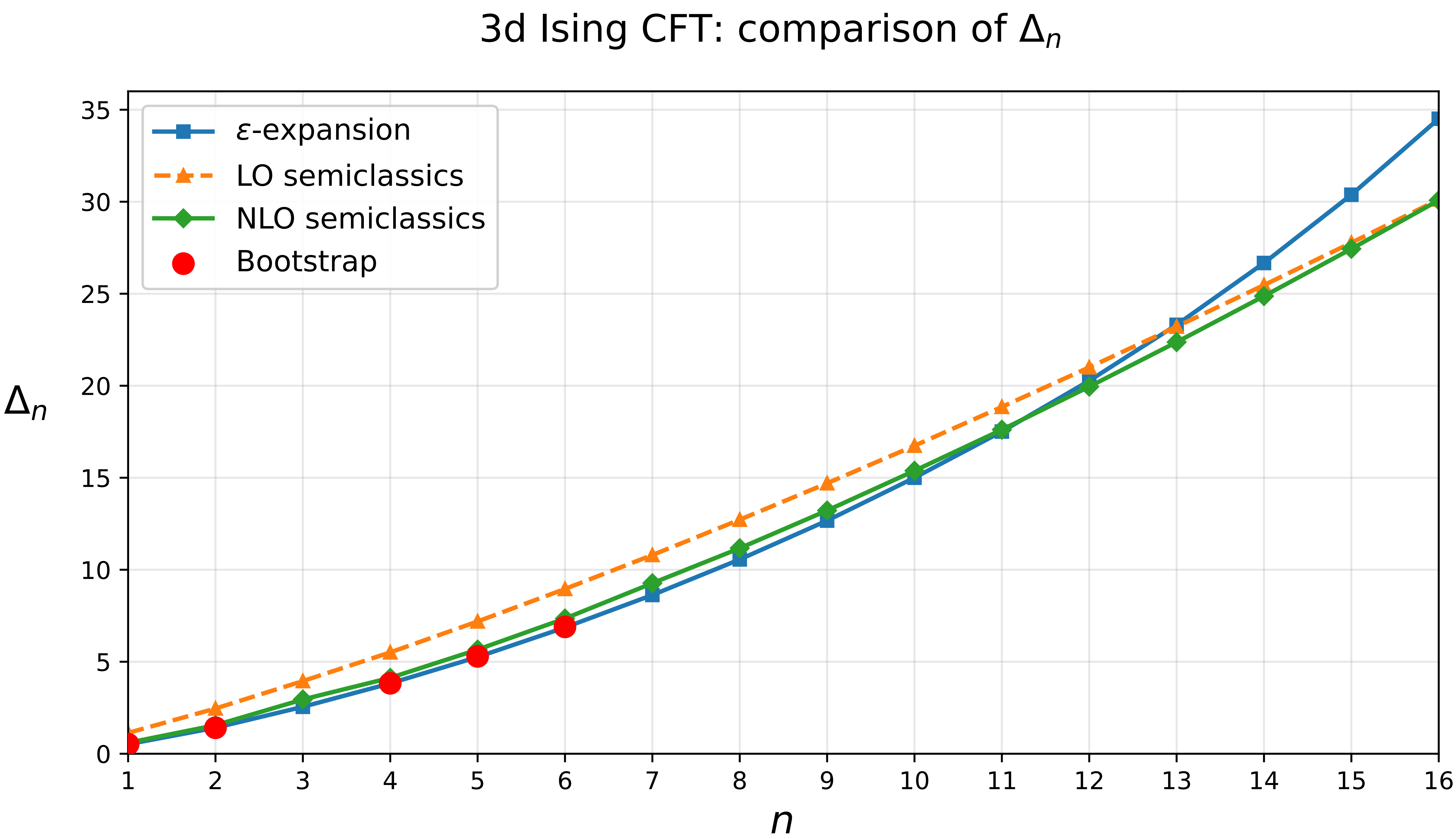
$O(N) - \phi^4$ in $4 - \epsilon$

$O(N) - \phi^6$ in $3 - \epsilon$

The operators split at NLO in 1/n!

The large n Frontier

NLO is the new state-of-the-art for physical Ising 3D



Semiclassical Canovaccio

Tackling QFT/CFT dynamics

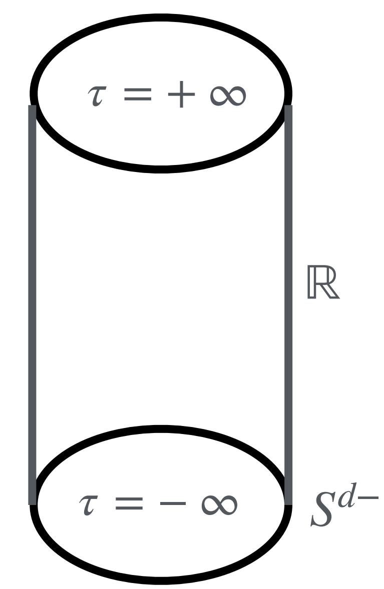
Choose your favorite: QFT/CFT

$$\partial^s \, \Box^p \, \phi^n$$

Act I

Sectors admitting $\hbar$-like expansion

Dynamics: Scaling dimensions



Act II

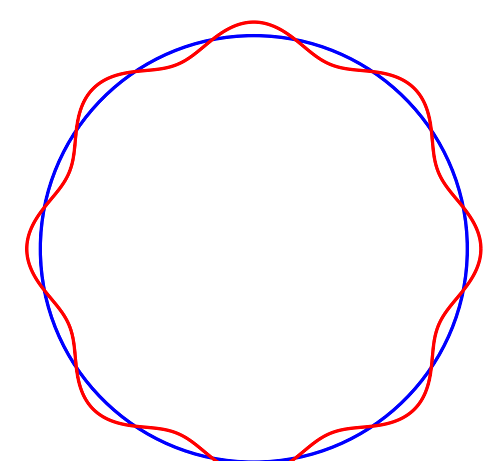
Conformal mapping onto the Cylinder

Spectrum on cylinder $\Rightarrow$ Scaling dimensions

Act III

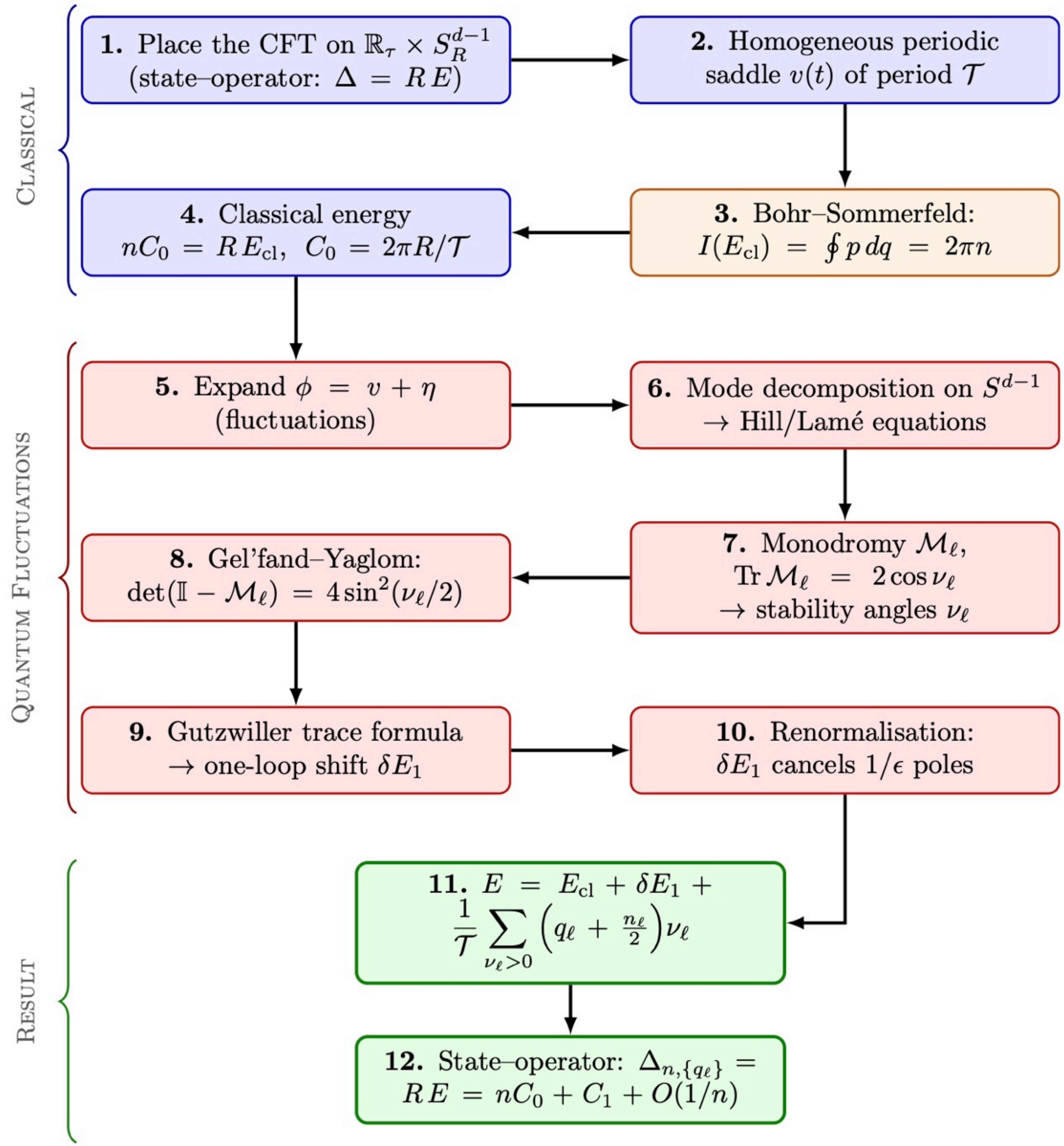
Expand around the classical solution

Quantize Action variable

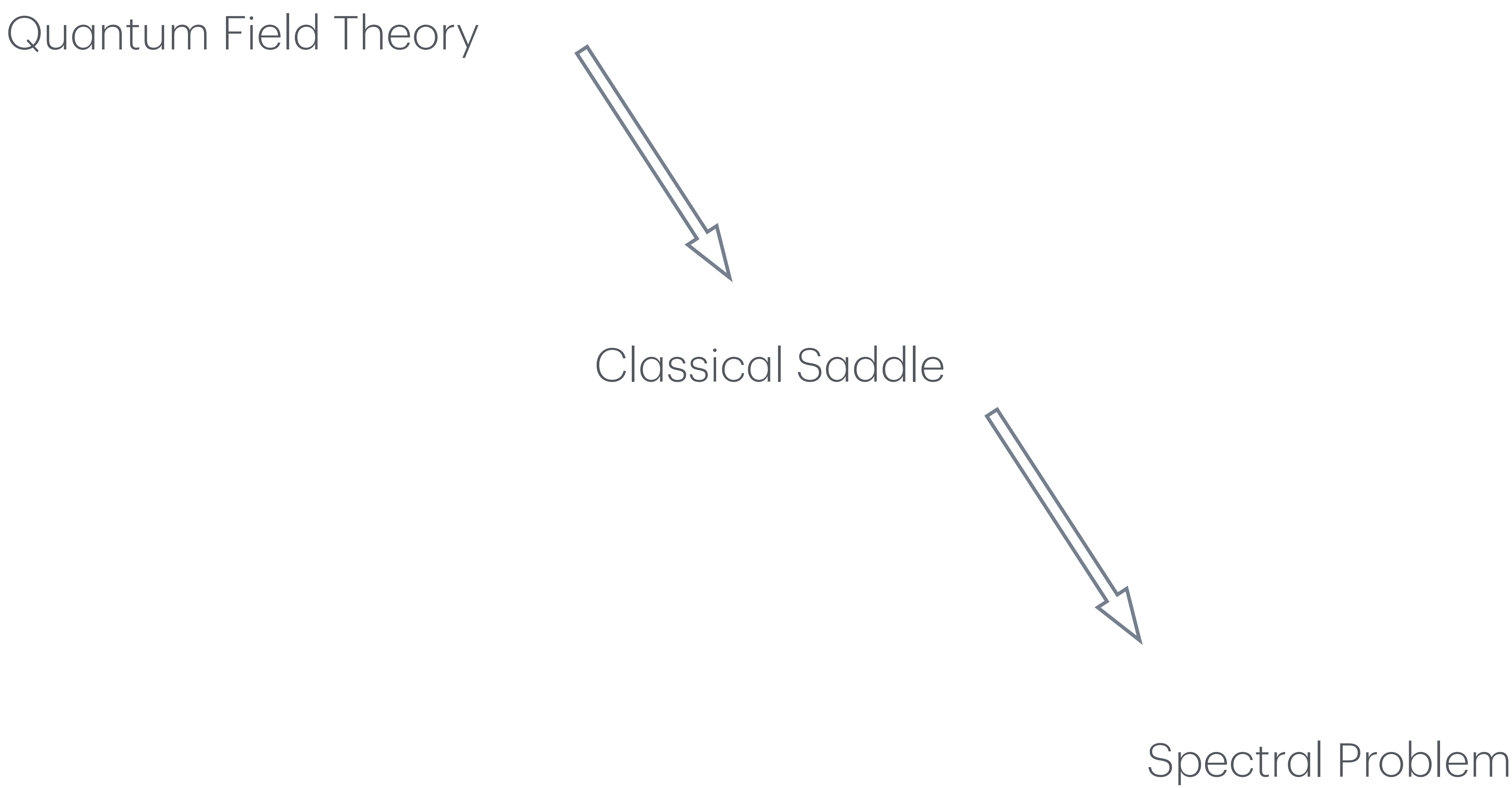


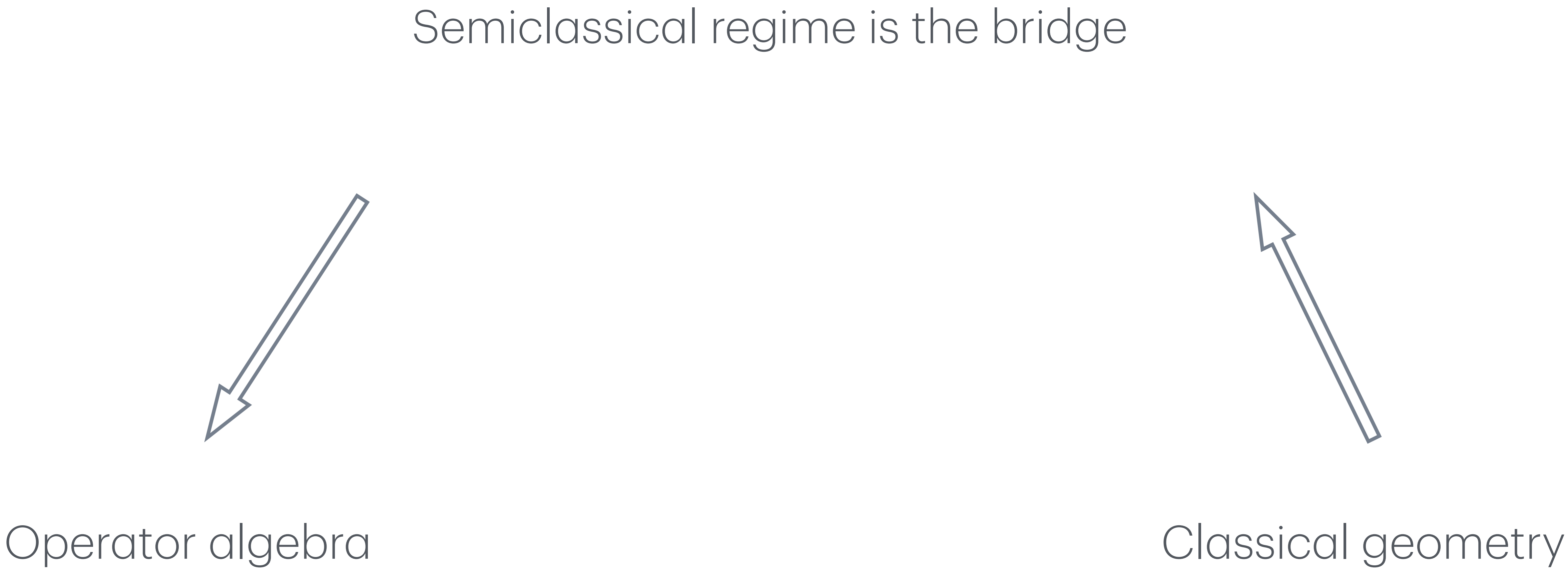
Finale

$$\Delta_{n,q_\ell} = \left(\frac{d-2}{2}\right) n + s + 2p + \gamma_{n,q_\ell} = r\, E_{n,q_\ell}$$



Classical
1. Place the CFT on $\mathbb{R}_\tau \times S_R^{d-1}$ (state–operator: $\Delta = R\,E$)
2. Homogeneous periodic saddle $v(t)$ of period $\mathcal{T}$
3. Bohr–Sommerfeld: $I(E_{\rm cl}) = \oint p\,dq = 2\pi n$
4. Classical energy $nC_0 = R\,E_{\rm cl},\ C_0 = 2\pi R/\mathcal{T}$
Quantum Fluctuations
5. Expand $\phi = v + \eta$ (fluctuations)
6. Mode decomposition on S^{d-1} $\to$ Hill/Lamé equations
7. Monodromy $\mathcal{M}_\ell$, $\mathrm{Tr}\,\mathcal{M}_\ell = 2\cos\nu_\ell$ $\to$ stability angles ν_ℓ
8. Gel'fand–Yaglom: $\det(\mathbb{I} - \mathcal{M}_\ell) = 4\sin^2(\nu_\ell/2)$
9. Gutzwiller trace formula $\to$ one-loop shift δE_1
10. Renormalisation: δE_1 cancels $1/\epsilon$ poles
Result
11. $E = E_{\rm cl} + \delta E_1 + \dfrac{1}{\mathcal{T}} \sum_{\nu_\ell > 0} \left(q_\ell + \frac{n_\ell}{2}\right)\nu_\ell$
12. State–operator: $\Delta_{n,\{q_\ell\}} = R\,E = nC_0 + C_1 + O(1/n)$


Quantum Field Theory
Classical Saddle
Spectral Problem


Semiclassical regime is the bridge
Operator algebra
Classical geometry

Semiclassics as Spectral Problem

Emerging mathematical structures

Classical hamiltonian dynamics $\partial^s \Box^p \phi^n$

Floquet theory and stability angles

Trace and spectral asymptotics

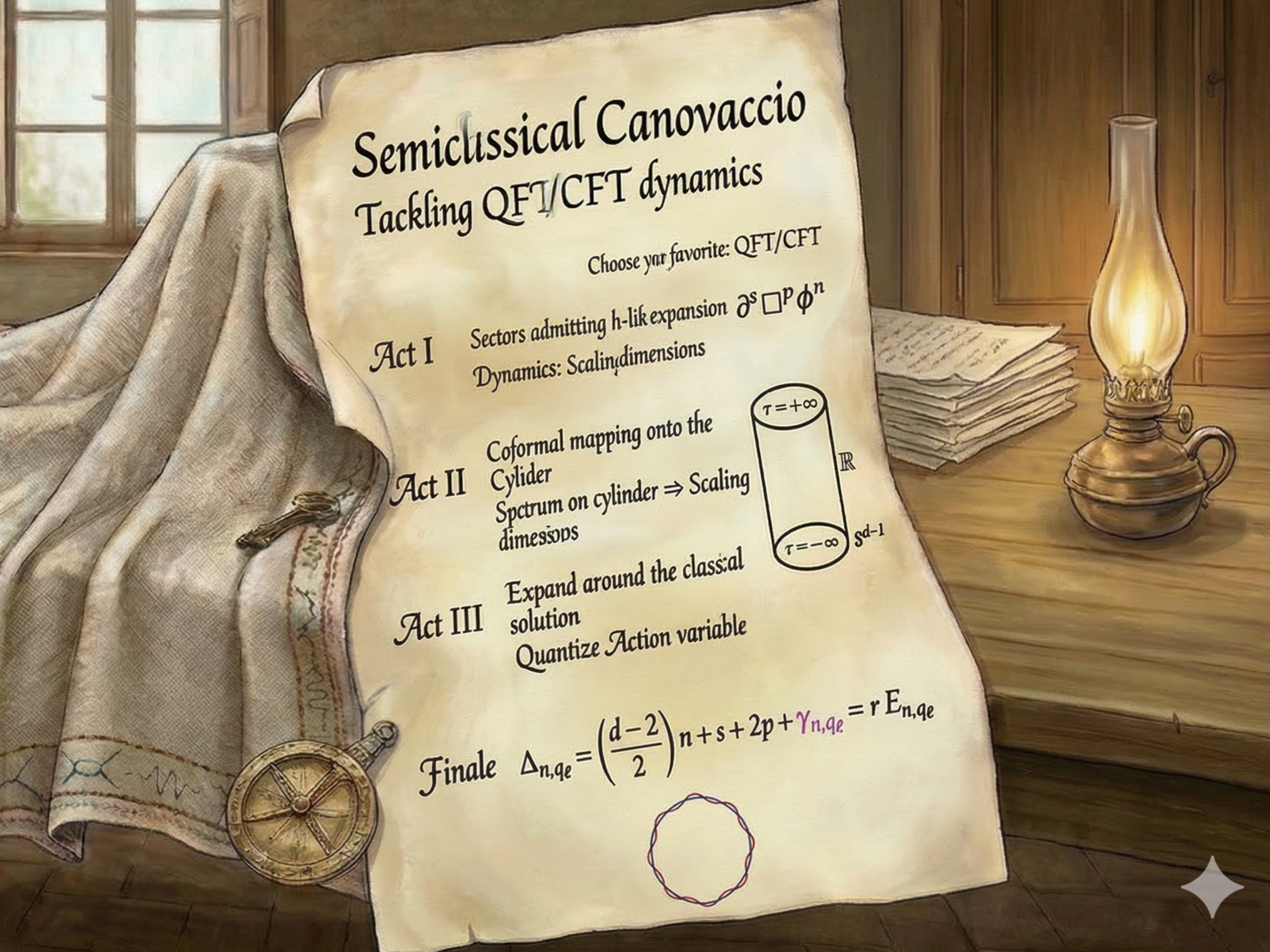
(Non) perturbative information for Q/CFTs hidden in spectral data

$$L = -\partial_t^2 + \mathcal{O}_{S^{d-1}} + V_{\mathrm{cl}}(t)$$

Can heavy-operator sectors emerge via operator-albraic structures?

Universal classical regimes where classical dynamics emerges from asymptotic spectral properties of quantum states

Semiclassical Theory of Quantum Fields


Semiclussical Canovaccio
Tackling QFT/CFT dynamics
Choose yar favorite: QFT/CFT
Act I
Sectors admitting h-lik expansion $\partial^s \Box^p \phi^n$
Dynamics: Scalin dimensions
Act II
Coformal mapping onto the Cylider
Spctrum on cylinder ⇒ Scaling dimessons
$\tau=+\infty$
$\mathbb{R}$
$\tau=-\infty$
S^{d-1}
Act III
Expand around the classical solution
Quantize Action variable
Finale $\Delta_{n,qe} = \left(\frac{d-2}{2}\right) n + s + 2p + \gamma_{n,qe} = r\, E_{n,qe}$

What next?

Wish list

Gross-Neveu model

Near conformal dynamics/conformal window

Standard Model Higgs composite operators

Large N, O(N) in 3 dimensions

Large order behavior of the series (resurgence)

Composite operators made by gauge fields, (gravity included?)!

Relevance:

Every realm of physics!

Condensed matter: Magnetic insulators, binary alloys, lattice model for critical behavior in fluids

Phase Transitions: including QCD phase transitions, Bose Einstein, etc.

Standard Model: Effective theory operators, multi-particle processes

Cosmology: Inflaton's composite operator dynamics

Potential new handle on Gravity

A powerful canovaccio for novel QFT dynamics

Relevance:

Every realm of science!

Biological & Social Systems: Neural networks in neuroscience, opinion formation in social dynamics

Machine Learning & Computing: Test computing techniques, Montecarlo & ML (percolation transitions)

Structural engineering: Applied to model structural behavior of sea ice

Quantum computing: Error correcting (analytic control over an infinite number of operators) ?

A powerful canovaccio for novel QFT dynamics

Semiclassical Canovaccio
Tackling QFT/CFT dynamics
Choose your favorite: QFT/CFT
Act 1
Sectors admitting h-like expansion
$\partial^s \Box^h \phi^n$
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$\tau = +\infty$
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Expand around the classical solution
Quantize Action variable
Finale
$\Delta_{n,q_e} = \left(\frac{d-2}{2}\right)n + s + 2p + \gamma_{n,q_e} = r E_{n,q_e}$
Thank you!