Gravitational Waves from Chiral Plasma Instability in the Standard Cosmology

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Relic gravitational waves from the chiral plasma instability in the standard cosmological model

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In the primordial plasma, at temperatures above the scale of electroweak symmetry breaking, the presence of chiral asymmetries is expected to induce the development of helical hypermagnetic fields through the phenomenon of chiral plasma instability. It results in magnetohydrodynamic turbulence due to the high conductivity and low viscosity and sources gravitational waves that survive in the universe today as a stochastic polarized gravitational wave background. In this article, we show that this scenario only relies on Standard Model physics, and therefore the observable signatures, namely the relic magnetic field and gravitational background, are linked to a single parameter controlling the initial chiral asymmetry. We estimate the magnetic field and gravitational wave spectra, and validate these estimates with 3D numerical simulations.

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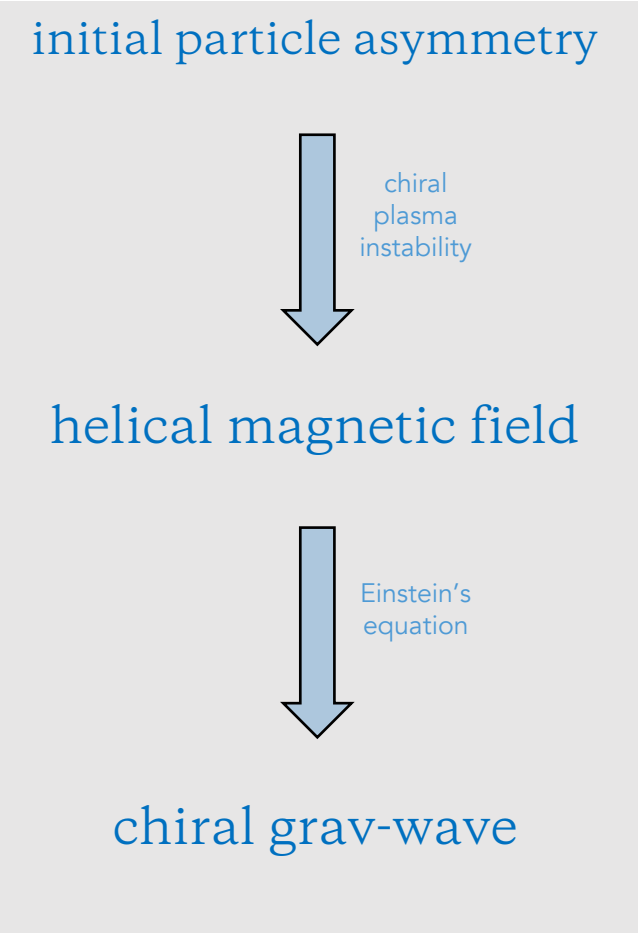
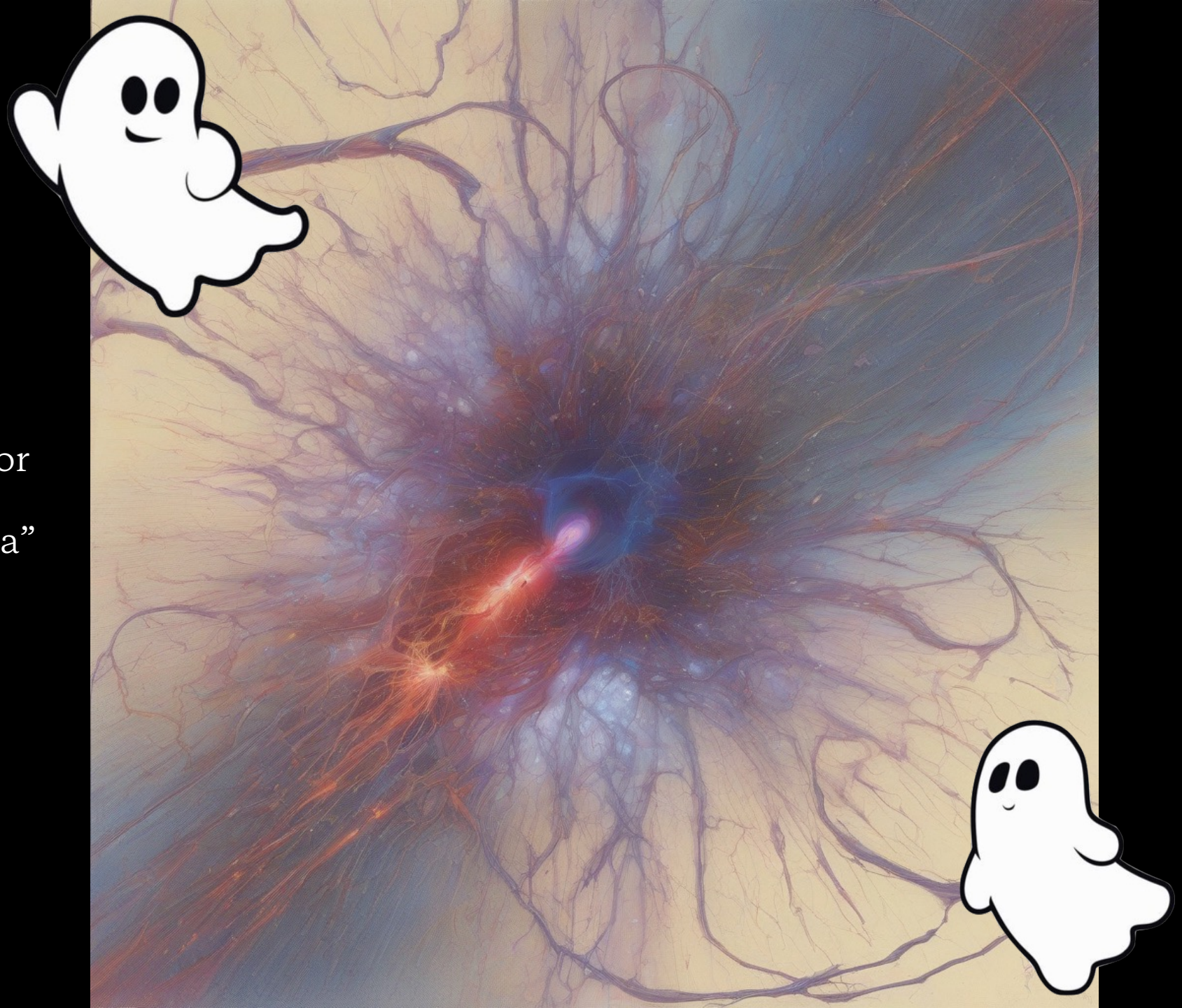


where I'm going with this:

chiral asymmetry is erased before magnetogenesis	viable window	baryon number is over-produced	
10^{-3}		10^{-2}	$(\lvert\tilde{\mu}_{Y,5}\rvert/T)_{\rm CPI}$
70 TeV		7000 TeV	$T_{\rm phys,CPI} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^2_{\rm CPI}$
7×10^{-16} G		1×10^{-15} G	$B_{\rm phys,0} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^{1/3}_{\rm CPI}$
9×10^{-4} pc		2×10^{-3} pc	$\xi_{M,\rm phys,0} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^{1/3}_{\rm CPI}$
7×10^{-39}		7×10^{-33}	$\Omega_{\rm GW}h_0^2 \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^6_{\rm CPI}$
0.1 MHz		1 MHz	$f_{\rm GW,0} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)_{\rm CPI}$

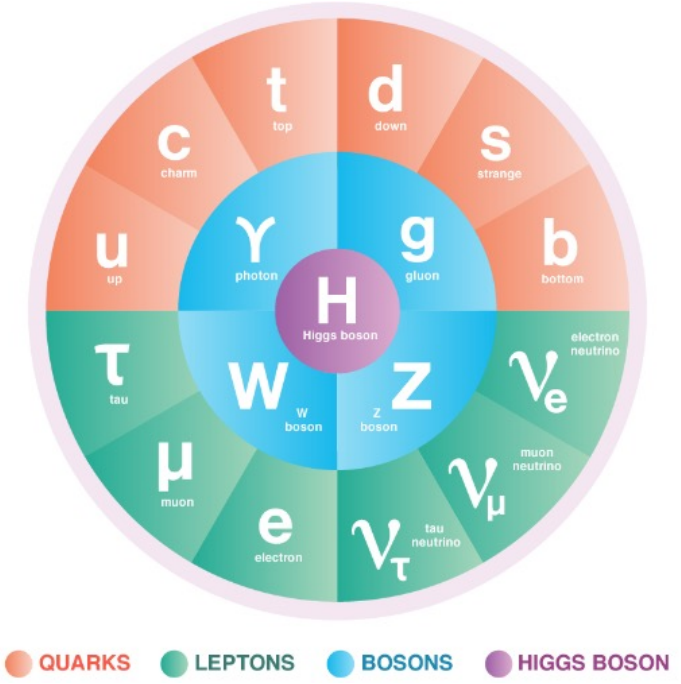
the electroweak plasma
& particle asymmetries

AI Image Generator
“primordial
electroweak plasma”



Particle content of the universe

The Standard Model of the Elementary Particles



electroweak unbroken phase

$$\langle \Phi \rangle = 0$$
$$\text{mass}(W^\pm, Z) = 0$$
$$\text{mass}(\text{quarks}, \text{leptons}) = 0$$

gauge theory

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

field content

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (\mathbf{3}\,,\,\mathbf{2}\,,\,1/3)$$
$$u_R \sim (\mathbf{3}\,,\,\mathbf{1}\,,\,4/3)$$
$$d_R \sim (\mathbf{3}\,,\,\mathbf{1}\,,\,-2/3)$$
$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\mathbf{1}\,,\,\mathbf{2}\,,\,-1)$$
$$e_R \sim (\mathbf{1}\,,\,\mathbf{1}\,,\,-2)$$
$$G \sim (\mathbf{8}\,,\,\mathbf{1}\,,\,0)$$
$$W \sim (\mathbf{1}\,,\,\mathbf{3}\,,\,0)$$
$$B \sim (\mathbf{1}\,,\,\mathbf{1}\,,\,0)$$
$$\Phi \sim (\mathbf{1}\,,\,\mathbf{2}\,,\,1)$$

Standard Model Boltzmann Equations w/ Anomalous Sources

$$\frac{d\eta_{u_L^i}}{dx} = -\mathcal{S}^i_{\rm UDW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Uu} + \mathcal{S}^{ij}_{\rm Uhd}\right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2}\mathcal{S}_{\rm w,sph} + \left(N_{\rm c} y_{Q_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2}\mathcal{S}^{\rm bkg}_{\rm w} + N_{\rm c}\frac{y_{Q_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{d_L^i}}{dx} = \mathcal{S}^i_{\rm UDW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Dd} + \mathcal{S}^{ij}_{\rm Dhu}\right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2}\mathcal{S}_{\rm w,sph} + \left(N_{\rm c} y_{Q_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2}\mathcal{S}^{\rm bkg}_{\rm w} - N_{\rm c}\frac{y_{Q_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{\nu_L^i}}{dx} = -\mathcal{S}^i_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}}\mathcal{S}^{ij}_{\nu\rm he} - \frac{1}{2}\mathcal{S}_{\rm w,sph} + \left(y_{L_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2}\mathcal{S}^{\rm bkg}_{\rm w} + \frac{y_{L_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{e_L^i}}{dx} = \mathcal{S}^i_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Ehe} + \mathcal{S}^{ij}_{\rm Ee}\right) - \frac{1}{2}\mathcal{S}_{\rm w,sph} + \left(y_{L_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2}\mathcal{S}^{\rm bkg}_{\rm w} - \frac{y_{L_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{u_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Uhu} + \mathcal{S}^{ji}_{\rm Uu} + \mathcal{S}^{ji}_{\rm Dhu}\right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y_{u_R}^2 \mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{d_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Dhd} + \mathcal{S}^{ji}_{\rm Dd} + \mathcal{S}^{ji}_{\rm Uhd}\right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y_{d_R}^2 \mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{e_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Ehe} + \mathcal{S}^{ji}_{\rm Ee} + \mathcal{S}^{ji}_{\nu\rm he}\right) - y_{e_R}^2 \mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{\phi^+}}{dx} = -\left(\mathcal{S}_{\rm hhW} + \mathcal{S}_{\rm hW}\right) + \sum_{i,j=1}^{N_{\rm g}}\left(-\mathcal{S}^{ij}_{\rm Dhu} + \mathcal{S}^{ij}_{\rm Uhd} + \mathcal{S}^{ij}_{\nu\rm he}\right)$$

$$\frac{d\eta_{\phi^0}}{dx} = \mathcal{S}_{\rm hhW} - \mathcal{S}_{\rm h} + \sum_{i,j=1}^{N_{\rm g}}\left(-\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Ehe}\right)$$

$$\frac{d\eta_{W^+}}{dx} = \left(\mathcal{S}_{\rm hhW} + \mathcal{S}_{\rm hW}\right) + \sum_{i=1}^{N_{\rm g}}\left(\mathcal{S}^i_{\rm UDW} + \mathcal{S}^i_{\nu\rm EW}\right) .$$

$$\mathcal{S}^{ij}_{\rm Dhu} \equiv \frac{\gamma^{ij}_{\rm Dhu}}{2}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) , \quad \mathcal{S}^{ij}_{\rm Uhu} \equiv \frac{\gamma^{ij}_{\rm Uhu}}{2}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Uhd} \equiv \frac{\gamma^{ij}_{\rm Uhd}}{2}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) , \quad \mathcal{S}^{ij}_{\rm Dhd} \equiv \frac{\gamma^{ij}_{\rm Dhd}}{2}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\nu\rm he} \equiv \frac{\gamma^{ij}_{\nu\rm he}}{2}\left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) , \quad \mathcal{S}^{ij}_{\rm Ehe} \equiv \frac{\gamma^{ij}_{\rm Ehe}}{2}\left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) ,$$

$$\mathcal{S}^i_{\rm UDW} \equiv \gamma^i_{\rm UDW}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}^i_{\nu\rm EW} \equiv \gamma^i_{\nu\rm EW}\left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm hhW} \equiv \gamma_{\rm hhW}\left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm s,sph} \equiv \gamma_{\rm s,sph}\sum_{i=1}^{N_{\rm g}}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{u_R^i}}{k_{u_R^i}} - \frac{\eta_{d_R^i}}{k_{d_R^i}}\right) ,$$

$$\mathcal{S}_{\rm w,sph} \equiv \gamma_{\rm w,sph}\sum_{i=1}^{N_{\rm g}}\left(\frac{N_{\rm c}}{2}\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{N_{\rm c}}{2}\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{1}{2}\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} + \frac{1}{2}\frac{\eta_{e_L^i}}{k_{e_L^i}}\right)$$

$$\mathcal{S}^{\rm bkg}_{\rm y} = \frac{1}{sT}\frac{\alpha_{\rm y}}{4\pi}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\langle Y_{\mu\nu}\rangle\langle Y_{\rho\sigma}\rangle}$$

$$\mathcal{S}^{\rm bkg}_{\rm w} = \frac{1}{sT}\frac{1}{2}\frac{\alpha_{\rm w}}{4\pi}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\langle W^a_{\mu\nu}\rangle\langle W^a_{\rho\sigma}\rangle}$$

$$\mathcal{S}^{\rm bkg}_{\rm yw} = \frac{1}{sT}\frac{gg'/4\pi}{4\pi}\epsilon^{\mu\nu\rho\sigma}\overline{\langle Y_{\mu\nu}\rangle\langle W^3_{\rho\sigma}\rangle} .$$

$$\mathcal{S}^{ij}_{\rm Uu} \equiv \gamma^{ij}_{\rm Uu}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Dd} \equiv \gamma^{ij}_{\rm Dd}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Ee} \equiv \gamma^{ij}_{\rm Ee}\left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) ,$$

$$\mathcal{S}_{\rm hW} \equiv \gamma_{\rm hW}\left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm h} \equiv \gamma_{\rm h}\frac{\eta_{\phi^0}}{k_{\phi^0}} .$$

$$\eta = n/s$$

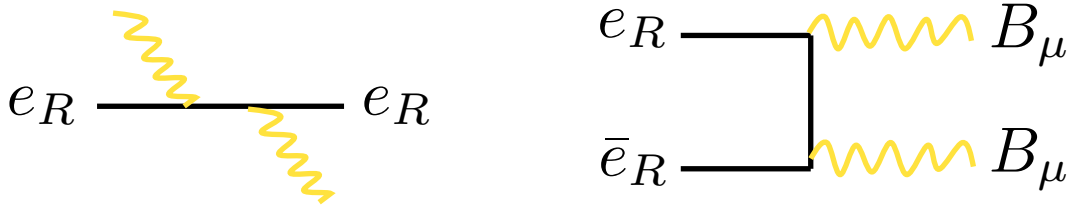
$$x = T/H \sim M_{\rm pl}/T$$

$$k = \#\ \text{degree of freedom}$$

Interactions & charge conservation

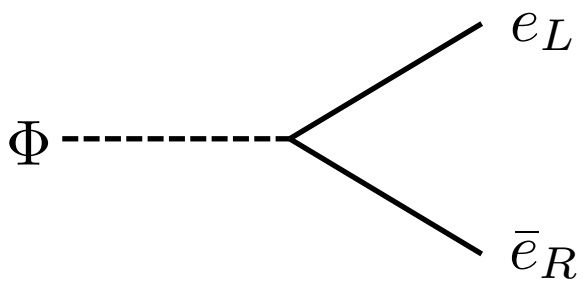
Some charges are exactly conserved. The corresponding particle asymmetries are static.

e.g., hypercharge … no interaction can change this



But some charges are only approximately conserved (on short time scales).

e.g., the Yukawa interactions change chiral charge



	hypercharge	Y-chiral
for Φ:	$Y = +1$	$\varepsilon Y^2 = 0$
for e_L:	$Y = -1$	$\varepsilon Y^2 = -1$
for $\bar{e}_R$:	$Y = +2$	$\varepsilon Y^2 = -4$
	(conserved)	(violated)

Chiral charge erasure

rate comes from: Bodeker & Schroder (2019)
see also: Cambbell, Davidson, Ellis, & Olive (1993)
Joyce & Shaposhnikov (1997)

The approximately conserved charges tend to be erased over time.

The limiting factor is the right-chiral electron, since it has $y_e \sim 3 * 10^{-6}$.

chiral charge evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_{Y,5}(t) = -\Gamma_{\mathrm{f}}(t)\,\mu_{Y,5}(t)$$

interaction rate

$$\Gamma_{\mathrm{f}}(t) \sim \left(1.3 \times 10^{-2}\right) y_e^2 T(t)$$

an exponential decay:

$$\mu_{Y,5}(t) = \mu_{Y,5,0}\,\exp\left[-\int_{t_0}^{t} \mathrm{d}t'\,\Gamma_f(t')\right]$$

chiral-erasure epoch

$$T_{\mathrm{phys,f}} \simeq (80\ \mathrm{TeV})\left(\frac{g_*}{106.75}\right)^{-1/2}$$

$$t_{\mathrm{phys,f}} \simeq (3.7 \times 10^{-17}\ \mathrm{sec})\left(\frac{g_*}{106.75}\right)^{1/2}$$

$$a_{\mathrm{f}} \simeq (9.7 \times 10^{-19}\ a_0)\left(\frac{g_*}{106.75}\right)^{1/6}$$

$$d_{H,\mathrm{phys,f}} \simeq (2.2 \times 10^{-6}\ \mathrm{cm})\left(\frac{g_*}{106.75}\right)^{1/2}$$

magnetic field evolution
& chiral magnetic effect

Evolution of hypermagnetic fields

conductivity: Arnold, Moore, & Yaffe (2000)

How do hypermagnetic fields evolve in the primordial plasma?

hyper-Maxwell equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}_Y = \rho_Y$$

$$\boldsymbol{\nabla} \times \boldsymbol{E}_Y = -\frac{\partial}{\partial t} \boldsymbol{B}_Y$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}_Y = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{B}_Y = \boldsymbol{J}_Y + \frac{\partial}{\partial t} \boldsymbol{E}_Y$$

continuity equation

$$\frac{\partial}{\partial \eta} \rho_Y + \boldsymbol{\nabla} \cdot \boldsymbol{J}_Y = 0$$

constitutive relation

$$\boldsymbol{J}_Y = \underbrace{\sigma_Y \left(\boldsymbol{E}_Y + \boldsymbol{v} \times \boldsymbol{B}_Y \right)}_{\text{Ohm's Law}} + \underbrace{\frac{2}{\pi} \alpha_Y \mu_{Y,5} \boldsymbol{B}_Y}_{\text{chiral magnetic effect (CME)}}$$

fine structure constant

$$\alpha_Y = g'^2 / 4\pi \approx 1/100$$

conductivity

$$\sigma_Y \sim T/\alpha_Y \approx 100T$$

magnetic diffusivity

$$\eta_Y = 1/\sigma_Y \approx 0.01 T^{-1}$$

Chiral magnetic effect

Vilenkin (1980), Fukushima, Kharzeev, & Warringa (2008),
Alekseev, Cheianov, & Frohlich (1998)
Boyarsky, Frohlich, & Ruchaiskiy (2011)

First studied for a relativistic electron-positron plasma in QED.

CME = in the presence of a chiral asymmetry, a magnetic field induces an electric current.

From the symmetry perspective, the CME could be correct.

$$\boldsymbol{J} \propto \mu_5 \boldsymbol{B}$$

mass dimensions: ✓

parity properties: ✓

$\boldsymbol{J}$ = vector = P-odd

$\boldsymbol{B}$ = axial-vector = P-even

m_e = scalar = P-even

μ_5 = pseudo-scalar = P-odd

$\mu_5 \propto [\#\text{ R-chiral}] - [\#\text{ L-chiral}]$

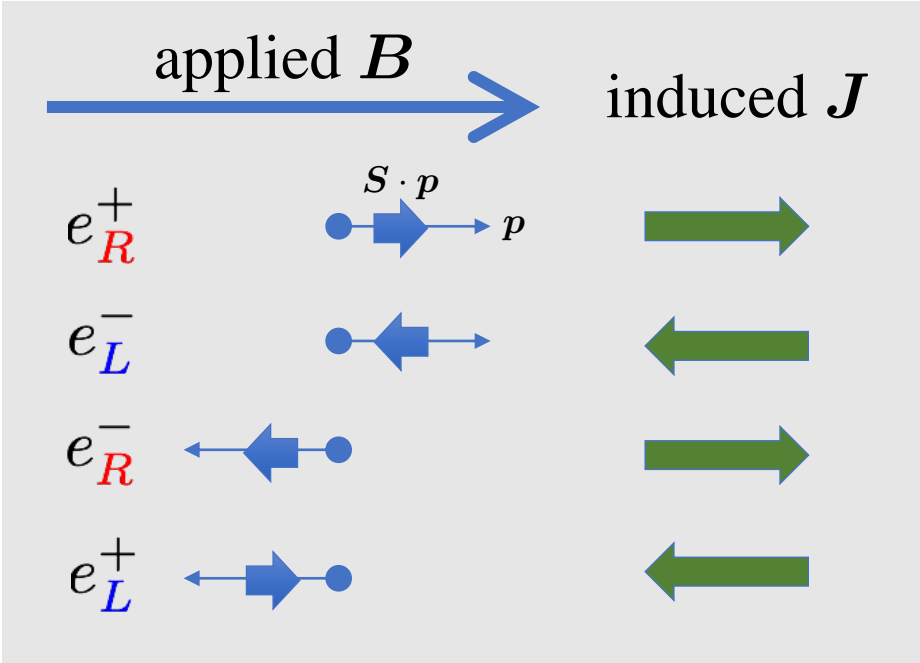
Semiclassical understanding

Tashiro, Vachaspati, & Vilenkin (2012)

Consider, applying a B-field to a collection of (relativistic) electrons & positrons.

The magnetic dipole moment μ ~ qS wants to align with B … the Hamiltonian is: $H = -\boldsymbol{\mu} \cdot \boldsymbol{B}$

without chiral asymmetry



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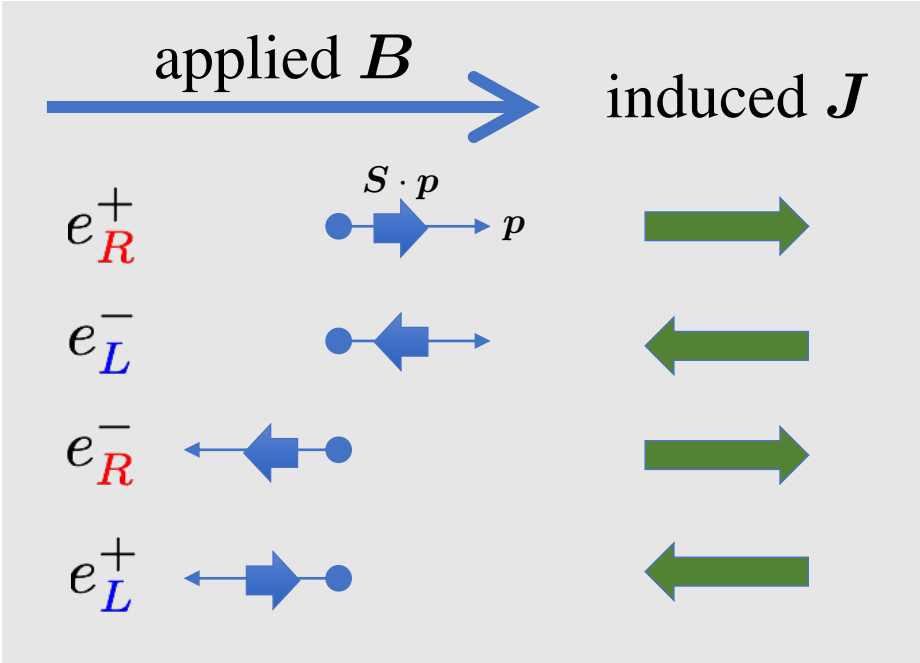
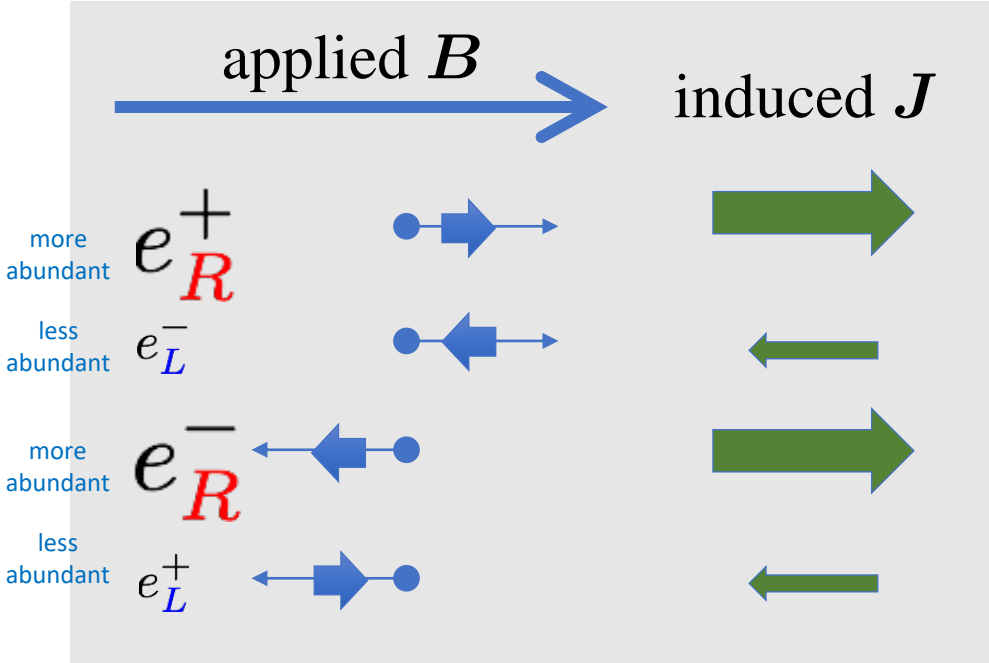


with chiral asymmetry



implies: $\boldsymbol{J} \propto q^2 \mu_5 \boldsymbol{B}$

CME recap

CME = in the presence of a chiral asymmetry, a magnetic field induces an electric current.

for electromagnetism: $\boldsymbol{J} = \frac{2}{\pi}\alpha\mu_5\boldsymbol{B}$

for hypercharge:

$$\boldsymbol{J}_Y = \frac{2}{\pi}\alpha_Y\mu_{Y,5}\boldsymbol{B}_Y$$

$$\mu_{Y,5}(t) = \sum_{\text{particles}} g_i\varepsilon_i Y_i^2\mu_i(t)$$

$\varepsilon_i = \pm 1$ for R- and L-chiral particles

$Y_i =$ hypercharge of particle species i

magnetogenesis
via chiral plasma instability

Linearized field equations

similar equations of motion to axion inflation

Combining the hyper-Maxwell equations (and neglecting velocity v)

$$0 = \dot{\boldsymbol{B}}_Y - \eta_Y \nabla^2 \boldsymbol{B}_Y - \frac{2}{\pi}\alpha_Y \eta_Y \mu_{Y,5} \boldsymbol{\nabla} \times \boldsymbol{B}_Y$$

Move to Fourier space (and assume homogenous μ_{Y5}).

Circular polarization modes decouple

$$0 = \dot{B}^{(R)}_{Y,k} + \eta_Y \left(k^2 - \mathrm{sgn}(\mu_{Y,5})\, k_{\mathrm{CPI}} k\right) B^{(R)}_{Y,k}$$

$$0 = \dot{B}^{(L)}_{Y,k} + \eta_Y \left(k^2 + \mathrm{sgn}(\mu_{Y,5})\, k_{\mathrm{CPI}} k\right) B^{(L)}_{Y,k}$$

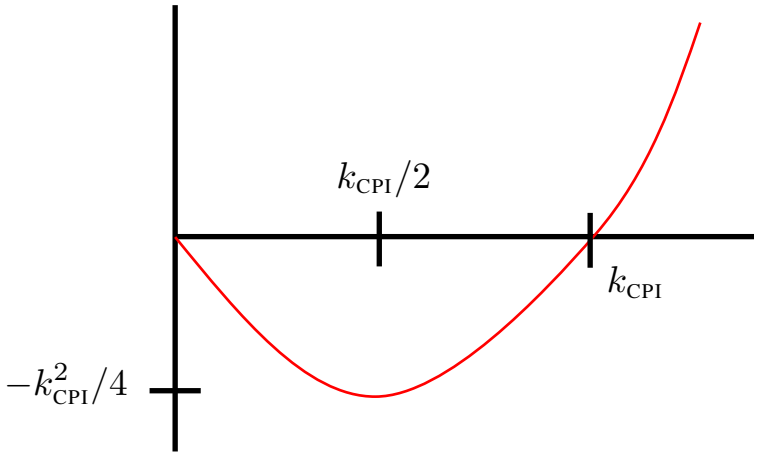
where $k_{\mathrm{CPI}} = \frac{2}{\pi}\alpha_Y |\mu_{Y,5}|$

Depending on the sign of μ_{Y5}, one mode or the other can be unstable.

Chiral plasma instability

Explore the dependence on wavenumber

$$0 = \dot{B}^{(R)}_{Y,k} + \eta_Y \left(k^2 - k_{\rm CPI} k \right) B^{(R)}_{Y,k}$$



- modes with $k > k_{\rm CPI}$ are decaying (Ohmic dissipation)
- modes with $k < k_{\rm CPI}$ have a tachyonic instability
- modes with $k = k_{\rm CPI}/2$ grow fastest

$$B^{(R)}_{Y,k}(t) \propto \exp\left[\tfrac{1}{4} \eta_Y k^2_{\rm CPI} t \right]$$

- typically $k_{\rm CPI}$ is deep inside the horizon

CPI = tachyonic growth of a helical magnetic field due to the CME

Helicity saturation

Since either the R- or L-polarized modes grow, the resultant magnetic field is helical.

As the helicity grows, the chiral asymmetry is depleted:

$$\partial_\mu j^\mu_{Y,5} \sim -\frac{g'^2}{16\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} \quad \Rightarrow \quad \frac{\partial}{\partial t} n_{Y,5} \sim -\frac{1}{\pi}\alpha_Y \frac{\partial}{\partial t}\mathcal{H}_M$$

The exponential growth must saturate when the initial asymmetry is `used up'

helicity density: $\mathcal{H}_{M,\mathrm{CPI}} \sim \frac{\pi}{6}\frac{1}{\alpha_Y}|\mu_{Y,5}|T^2_{\mathrm{CPI}}$

coherence length: $\xi_{M,\mathrm{CPI}} \approx 2\pi/(k_{\mathrm{CPI}}/2)$

field strength: $B_{\mathrm{CPI}} \approx \sqrt{\mathcal{H}_{M,\mathrm{CPI}}/\xi_{M,\mathrm{CPI}}}$

(We will validate these estimates using MHD simulations.)

using

$n_{Y,5} = \frac{1}{6}\mu_{Y,5}T^2$

$Y\tilde{Y} = -4\boldsymbol{E}_Y \cdot \boldsymbol{B}_Y = 4\frac{\partial}{\partial t}(\boldsymbol{A}_Y \cdot \boldsymbol{B}_Y) + \cdots$

Magnetogenesis

Typical scales

$$a_{\rm CPI} \simeq \left(1.1\times 10^{-18} a_0\right)\left(\frac{g_*}{106.75}\right)^{1/6}\left(\frac{\eta_Y}{0.01T^{-1}}\right)^{-1}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-2}$$

$$T_{\rm phys,CPI} \simeq \left(7.2\times 10^{4}\ {\rm GeV}\right)\left(\frac{g_*}{106.75}\right)^{-1/2}\left(\frac{\eta_Y}{0.01T^{-1}}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{2}$$

$$d_{H,\rm CPI} \simeq \left(2.5\times 10^{12}\ {\rm cm}\, a_0^{-1}\right)\left(\frac{g_*}{106.75}\right)^{1/3}\left(\frac{\eta_Y}{0.01T^{-1}}\right)^{-1}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-2}$$

$$\xi_{M,\rm CPI} \simeq \left(5.0\times 10^{5}\ {\rm cm}\, a_0^{-1}\right)\left(\frac{g_*}{106.75}\right)^{1/3}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{-1}$$

$$B_{\rm CPI} \simeq \left(5.0\times 10^{-11}\ {\rm G}\, a_0^{2}\right)\left(\frac{g_*}{106.75}\right)^{-2/3}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$

$$\Omega_{B,\rm CPI} \simeq \left(3.8\times 10^{-10}\right)\left(\frac{g_*}{106.75}\right)^{-1}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{2}$$

If $\mu_{Y5}/T >\sim 10^{-3}$ then the CPI develops before chiral charge erasure.

Inverse cascade

A maximally-helical field co-evolves with the plasma subject to the turbulent inverse-cascade:

$$B_0 = B_{\rm CPI}\left(\frac{t_{\rm rec}}{t_{\rm CPI}}\right)^{-1/3} \quad \text{and} \quad \xi_{B,0} = \xi_{B,\rm CPI}\left(\frac{t_{\rm rec}}{t_{\rm CPI}}\right)^{2/3} \quad \text{so} \quad \mathcal{H}_{B,0} = \mathcal{H}_{B,\rm CPI}$$

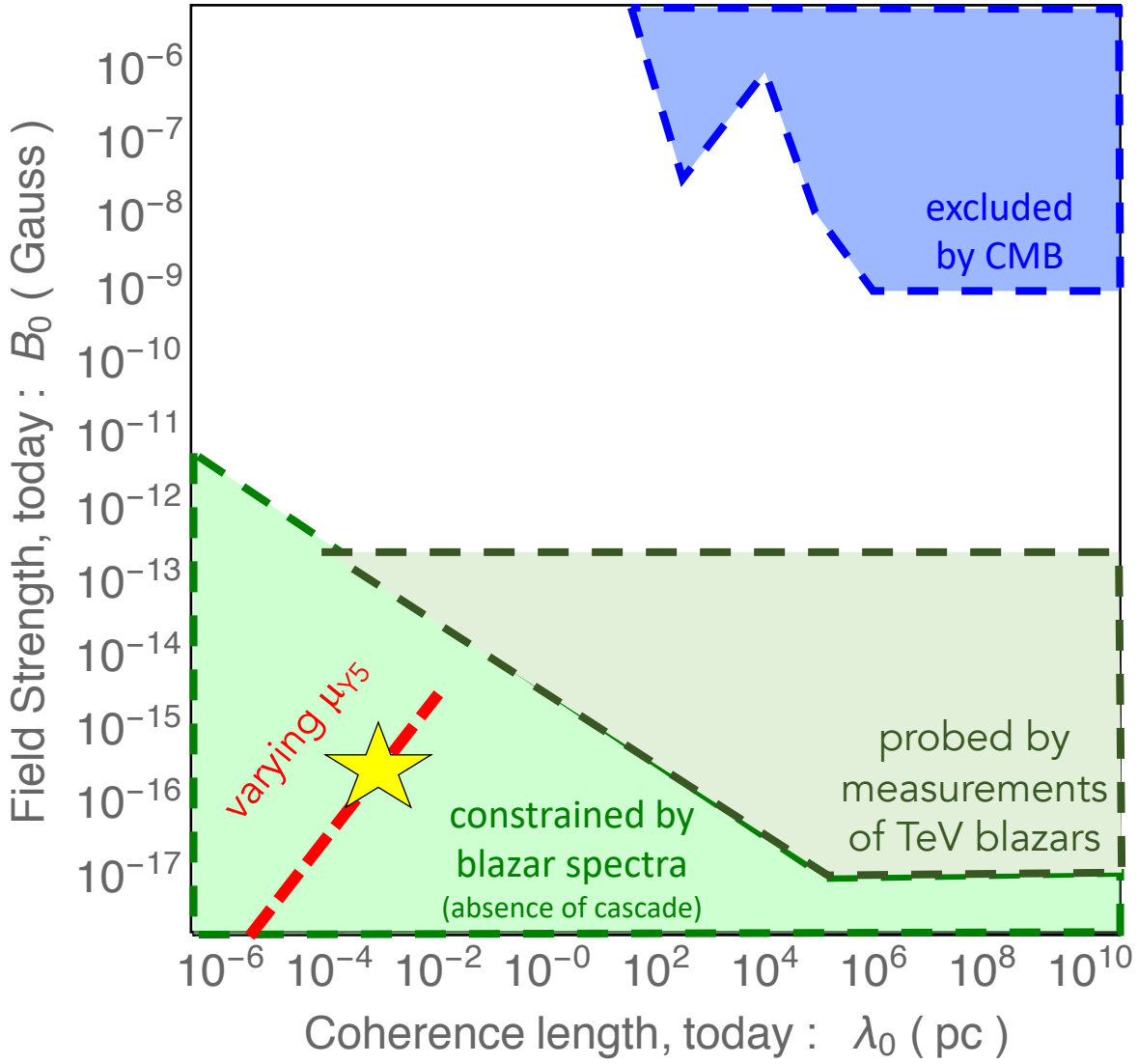
The field today is expected to be

$$\xi_{M,\rm phys,0} \simeq \left(9.5\times 10^{-4}\ \text{pc}\right)\left(\frac{g_*}{106.75}\right)^{1/9}\left(\frac{\eta_Y}{0.01T^{-1}}\right)^{2/3}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

$$B_{\rm phys,0} \simeq \left(6.6\times 10^{-16}\ \text{G}\right)\left(\frac{g_*}{106.75}\right)^{-5/9}\left(\frac{\eta_Y}{0.01T^{-1}}\right)^{-1/3}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

IGMF parameter space

adapted from: Durrer & Neronov (2013)



The relic of the
primordial magnetic field
is an intergalactic
magnetic field today

$$\xi_{M,\mathrm{phys},0} \simeq \left(9.5 \times 10^{-4}\ \mathrm{pc}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

$$B_{\mathrm{phys},0} \simeq \left(6.6 \times 10^{-16}\ \mathrm{G}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

- viable seeds for galactic dynamo
- too weak to explain blazars

grav-wave generation
& stochastic GW background

Einstein's equation

for details: Roper Pol, Brandenburg, Kahniashvili, Kosowsky, & Mandal (2018)

The primordial magnetic field sources gravitational wave radiation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\partial_t^2 h_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}$$

$$T_{ij} \sim B_i B_j \quad \text{(transverse / traceless part)}$$

$$\mathcal{E}_{\rm gw} = \langle \partial_t h_{ij} \partial_t h_{ij} \rangle / (32\pi G)$$

parametrically:

$$h \sim (GB^2)/\xi^2$$

$$\mathcal{E}_{\rm gw} \sim \xi^2 h^2 / G \sim G \xi^2 B^4$$

GW estimates

At the time of production

$$f_{\mathrm{gw,CPI}} \approx 2/\xi_{M,\mathrm{CPI}}$$
$$\mathcal{E}_{\mathrm{gw,CPI}} \sim (G/2\pi) a_{\mathrm{CPI}}^{-2} \xi_{M,\mathrm{CPI}}^{2} B_{\mathrm{CPI}}^{4}$$

there's a 2 because the source is quadratic in the field

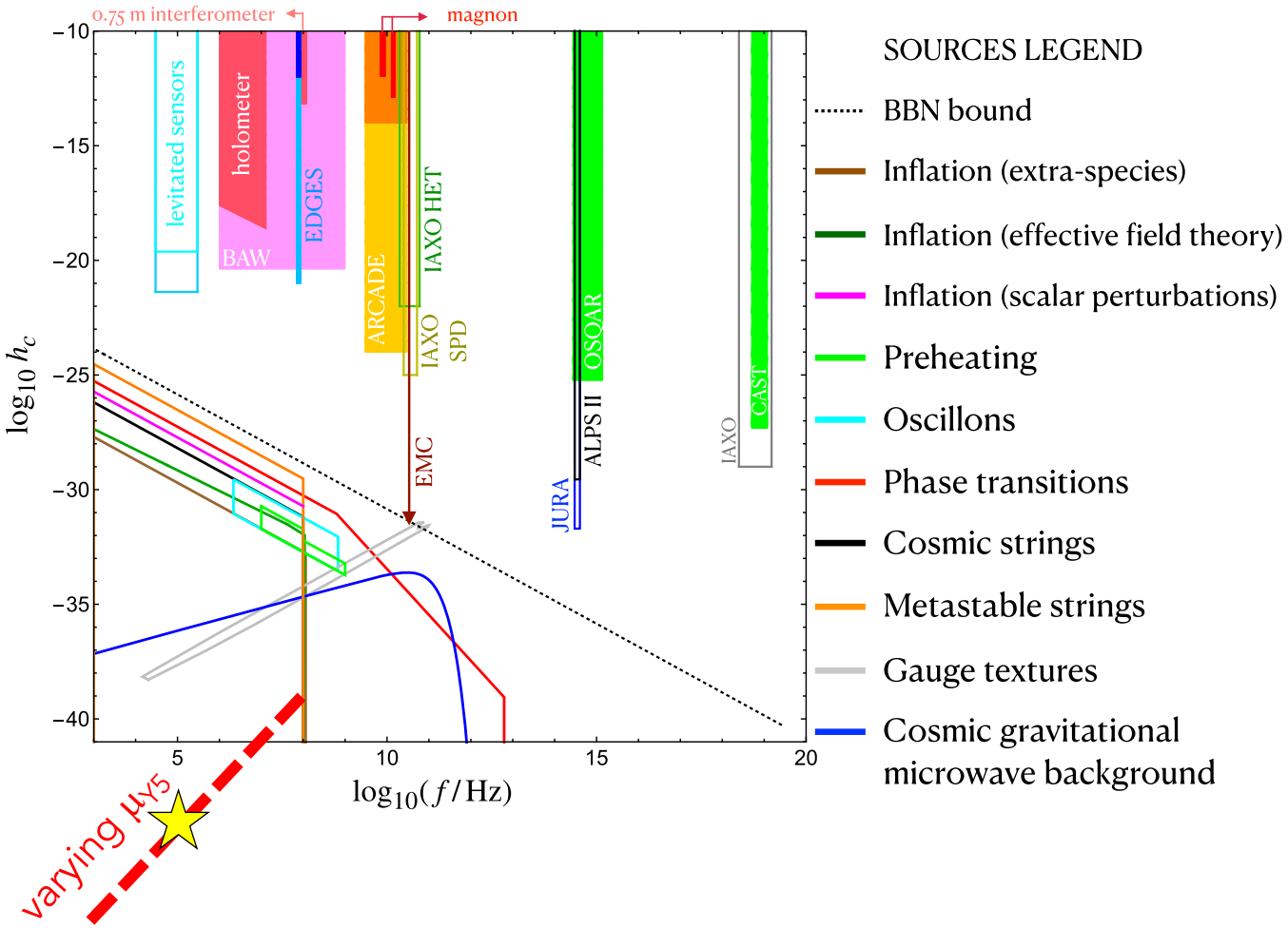
Frequency and cosmological energy fraction of stochastic GW today:

$$f_{\mathrm{gw,phys},0} = \left(1 \times 10^{5}\ \mathrm{Hz}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$
$$(\Omega_{\mathrm{gw}} h^{2})_{0} = \left(7 \times 10^{-39}\right)\left(\frac{\eta_{Y}}{0.01 T^{-1}}\right)^{2}\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{6}$$

The GW signal is high-frequency and very low amplitude (even for $\mu_{Y5}/T = 1$)

Searches for high-freq grav waves

review on high-freq grav wave: Aggarwal et al (2021)



$$f_{\mathrm{gw},0} = \left(1 \times 10^5\ \mathrm{Hz}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)$$

$$(\Omega_{\mathrm{gw}} h^2)_0 = \left(7 \times 10^{-39}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^6$$

$$h_c = \left(1 \times 10^{-45}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^2$$

using

$$\Omega_{\mathrm{gw}} = \frac{\pi f^2 h_c^2}{4G\mathcal{E}_{\mathrm{cr}}}$$

numerical validation
with PENCIL CODE sims

MHD equations + CME

magnetic diffusivity	η
kinematic viscosity	ν
chiral diffusion coeff.	D_5
rescaled chem. pot.	μ_5
chiral depletion param.	λ

Fully nonlinear system of equations

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}), \tag{5}$$

$$\frac{\partial \mu_5}{\partial t} = -\boldsymbol{\nabla} \cdot (\mu_5 \boldsymbol{u}) - \lambda\eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}) \cdot \boldsymbol{B} + D_5 \nabla^2 \mu_5, \tag{6}$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = \frac{2}{\rho}\boldsymbol{\nabla} \cdot (\rho\nu\mathsf{S}) - \frac{1}{4}\boldsymbol{\nabla}\ln\rho + \frac{\boldsymbol{u}}{3}(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho) - \frac{\boldsymbol{u}}{\rho}\left[\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \eta\boldsymbol{J}^2\right] + \frac{3}{4\rho}\boldsymbol{J}\times\boldsymbol{B}, \tag{7}$$

$$\frac{\partial \ln\rho}{\partial t} = -\frac{4}{3}(\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho) + \frac{1}{\rho}\left[\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) + \eta\boldsymbol{J}^2\right], \tag{8}$$

$$\mathsf{S}_{ij} = (\partial_j u_i + \partial_i u_j)/2 - \delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{u}/3$$

$$\frac{\partial^2}{\partial t^2}\tilde{h}_{+/\times}(\boldsymbol{k},t) + k^2\tilde{h}_{+/\times}(\boldsymbol{k},t) = \frac{6\,H_*}{t\,\mathcal{E}_{\mathrm{cr}}}\tilde{T}_{+/\times}(\boldsymbol{k},t),$$

PENCIL CODE
parallelized
6th order finite differences
3rd order time-stepping
1024^3 mesh points
initially: weak seed field, vanishing plasma velocity, homogenous chem. pot.

Parameters

cf, Brandenburg, He, Kahniashivili, Rheinhardt, & Schober (2021)

Dynamic range issues prevent studying the "expected" parameters.

Run	ηH_*	$(\mathcal{E}_{\rm cr}\lambda)^{1/2}/H_*$	μ_{50}/H_*	v_μ	v_λ	$\eta\mu_{50}^2/H_*$	k_1/H_*	$\mathcal{E}_{\rm M}^{\rm max}/\mathcal{E}_{\rm cr}$	$\mathcal{E}_{\rm GW}^{\rm sat}/\mathcal{E}_{\rm cr}$	q
B1	1×10^{-6}	2×10^{4}	10^{4}	1×10^{-2}	5×10^{-1}	1×10^{2}	1×10^{2}	1.6×10^{-2}	4.7×10^{-12}	0.027
B10	1×10^{-3}	2×10^{4}	10^{4}	1×10^{1}	5×10^{-1}	1×10^{5}	1×10^{2}	6.0×10^{-2}	6.0×10^{-9}	12
A1	1×10^{-6}	5×10^{4}	10^{4}	1×10^{-2}	2×10^{-1}	1×10^{2}	1×10^{2}	4.6×10^{-3}	8.9×10^{-14}	0.032
A12	5×10^{-3}	5×10^{4}	10^{4}	5×10^{1}	2×10^{-1}	5×10^{5}	5×10^{1}	9.2×10^{-3}	3.0×10^{-10}	18
X1	5×10^{-8}	10^{10}	10^{6}	5×10^{-2}	1×10^{-4}	5×10^{4}	5×10^{3}	2.4×10^{-9}	8.8×10^{-31}	0.39
X2	5×10^{-9}	10^{10}	10^{6}	5×10^{-3}	1×10^{-4}	5×10^{3}	5×10^{3}	2.4×10^{-9}	1.6×10^{-30}	0.53
X3	5×10^{-10}	10^{10}	10^{6}	5×10^{-4}	1×10^{-4}	5×10^{2}	5×10^{3}	2.4×10^{-9}	1.1×10^{-30}	0.44
X4	5×10^{-11}	10^{10}	10^{6}	5×10^{-5}	1×10^{-4}	5×10^{1}	5×10^{3}	2.3×10^{-9}	3.1×10^{-31}	0.12
Y1	5×10^{-8}	7×10^{11}	10^{6}	5×10^{-2}	1×10^{-6}	5×10^{4}	5×10^{3}	4.9×10^{-13}	3.6×10^{-38}	0.39
Y2	5×10^{-8}	7×10^{11}	10^{6}	5×10^{-2}	1×10^{-6}	5×10^{4}	2×10^{3}	4.4×10^{-13}	3.2×10^{-37}	1.3
Y3	5×10^{-8}	7×10^{11}	10^{6}	5×10^{-2}	1×10^{-6}	5×10^{4}	1×10^{3}	3.3×10^{-13}	6.9×10^{-37}	2.5
expected	$10^{-15}\eta_2$	6×10^{12}	$5\times10^{7}\mu_3$	$6\times10^{-8}\eta_2\mu_3$	$8\times10^{-6}\mu_3$	$3\eta_2\mu_3^2$	—	$6\times10^{-15}\mu_3^2$	$7\times10^{-39}\eta_2^2\mu_3^6$	—

$$v_\mu = \mu_{50}\eta$$

$$v_\lambda = \mu_{50}/(\mathcal{E}_{\rm cr}\lambda)^{1/2}$$

$$\mu_{50} = 2\alpha_Y\tilde{\mu}_{Y,5}/\pi$$

$$\lambda = 192\alpha_Y^2/T^2$$

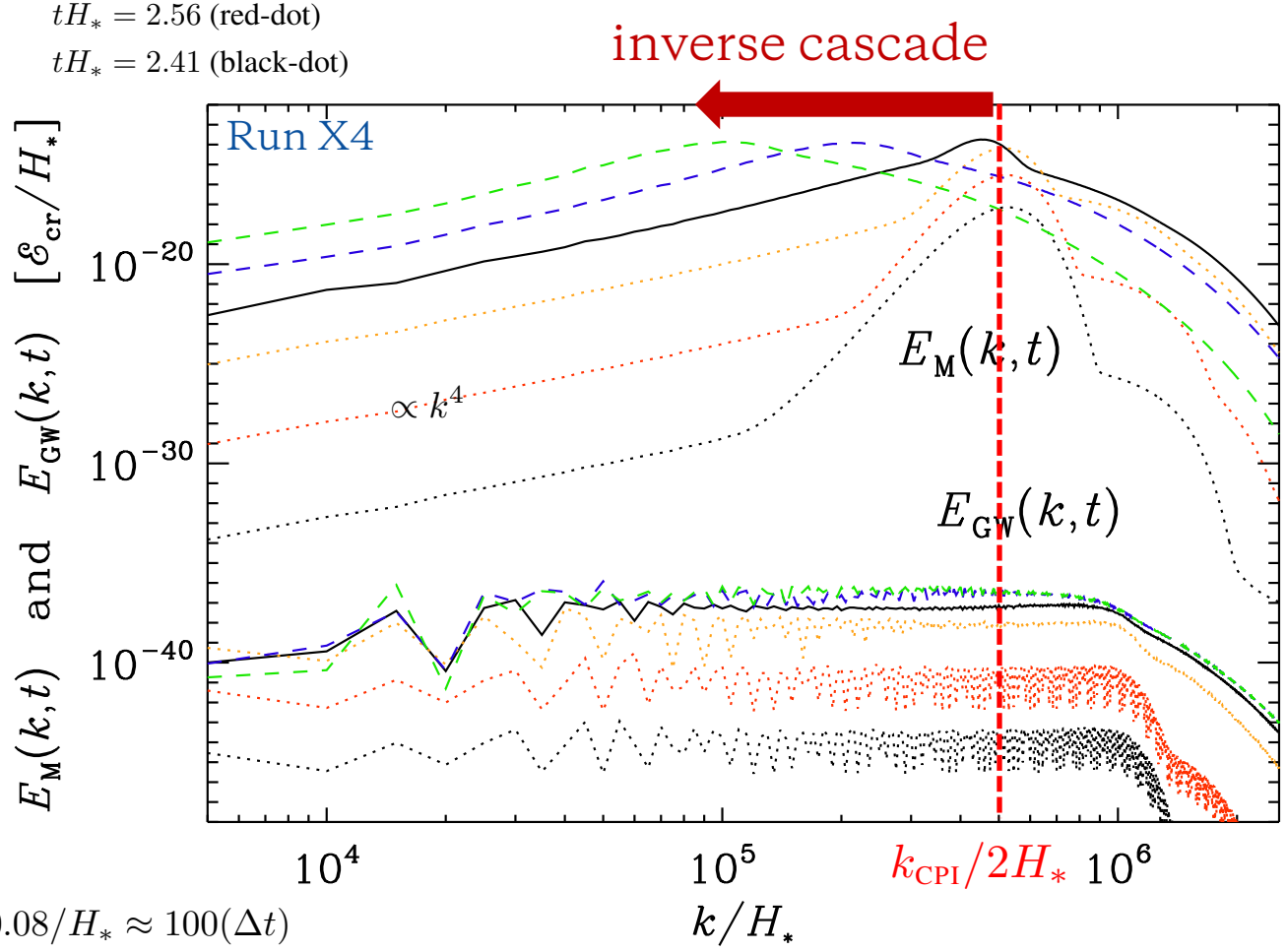
$$D_5 = \nu = \eta = 1/\sigma$$

$$\eta_2 = \eta_Y/(0.01T^{-1})$$

$$\mu_3 = \tilde{\mu}_{Y,5}/(10^{-3}T)$$

Spectra for PMF and GWs

$tH_* = 5.37$ (green-dashed)
$tH_* = 3.66$ (blue-dashed)
$tH_* = 2.98$ (solid)
$tH_* = 2.71$ (orange-dot)
$tH_* = 2.56$ (red-dot)
$tH_* = 2.41$ (black-dot)



$t_{CPI} = 0.08/H_* \approx 100(\Delta t)$

NB : $\mathcal{E}_M(\boldsymbol{x},t) = \frac{1}{2}\langle|\boldsymbol{B}(\boldsymbol{x},t)|^2\rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} P_B(\boldsymbol{k}) = \int \frac{dk}{k} \underbrace{\frac{k^3}{2\pi^2} P_B(k)}_{= d\mathcal{E}_M/d\ln k} = \int dk\, E_M(k)$

- mag energy grows at $k_{CPI}/2$
- GW energy grows while mag energy grows
- mag energy reaches maximum and GW energy spectrum saturates (solid curves)
- subsequently mag energy evolves with the inverse cascade scaling (expected for helical):

$$B \propto t^{-1/3} \quad \text{so} \quad \mathcal{E}_M \propto t^{-2/3}$$

$$\xi_M \propto t^{2/3} \quad \text{so} \quad k_M \propto t^{-2/3}$$

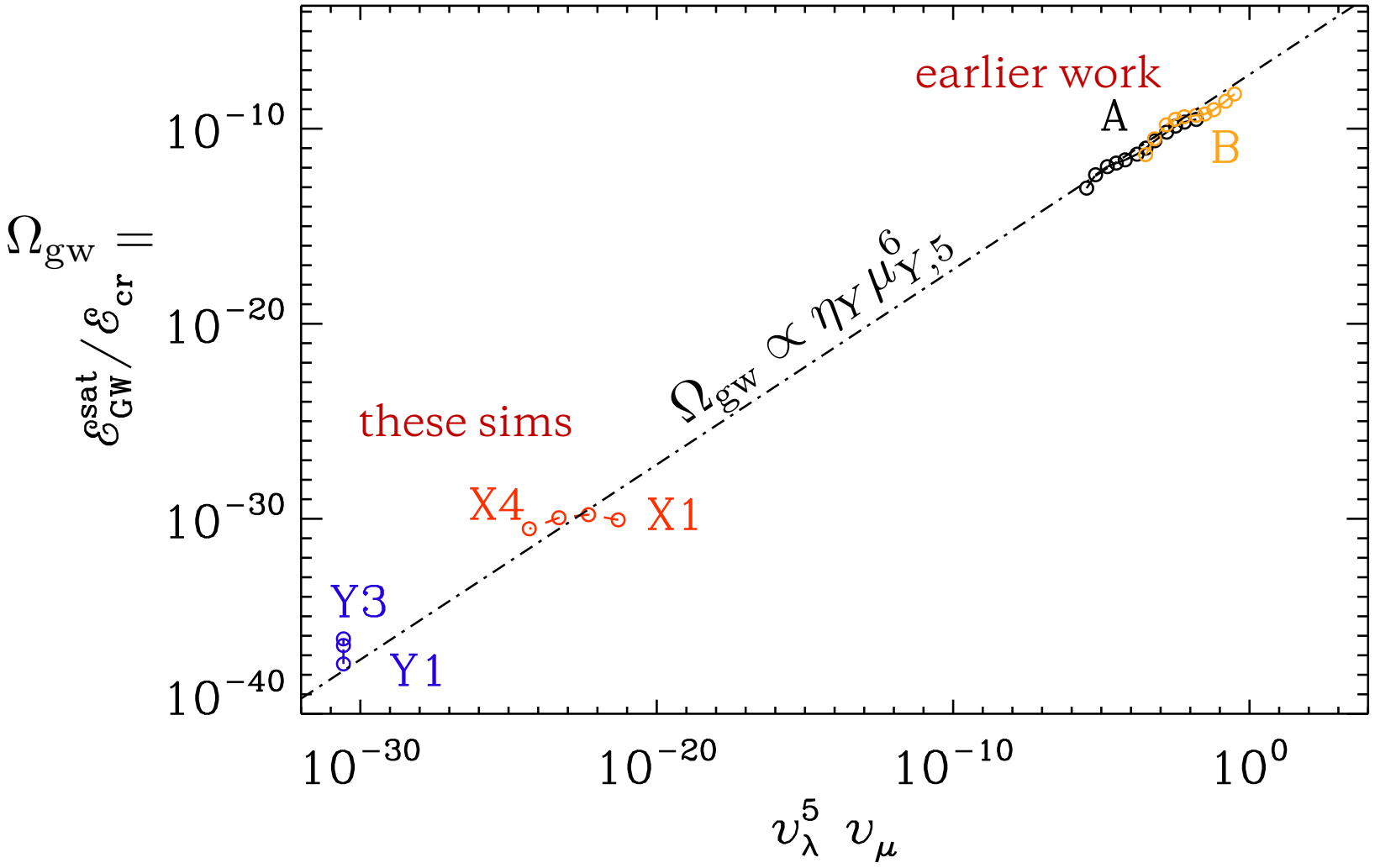
$$E_M \sim \mathcal{E}_M/k_M \propto t^0$$

- resultant GW energy spectrum is blue-tilted

$$\frac{d\mathcal{E}_{gw}}{d\ln k} = kE_{GW} \propto k^1 \quad (k \lesssim k_{CPI})$$

  - (scaling should extend down to k_{IR}~H)
  - (most energy at k_{UV}~k_{CPI})

Comparision w/ previous studies



Earlier work studied GW from CME at the EW epoch, assuming no chiral asymmetry washout (see Run Series A & B)

We explore parameters that are closer to the expectation for early universe cosmology at T > 80 TeV (see Run Series X and Y)

Main difference is the choice of magnetic diffusivity: η_Y. (see the table on prev. slide)

For those parameters, the GW signal is found to be weaker.

Scaling with $v_\lambda{}^5 \, v_\mu$ from earlier work, confirmed over wider range.

B-number overproduction
& an upper bound on μ_{Y5}

Sourcing baryon number

Fujita & Kamada (2016), Kamada & AL (2016a,b),
Jimenez, Kamada, Schmitz,Xu (2017)
see also: Giovannini & Shaposhnikov
(also gravitational anomaly: cf, Evangelos's talk)

Varying hypermagnetic helicity sources B and L-number

$$\partial_\mu j^\mu_{\mathsf{B}} = \partial_\mu j^\mu_{\mathsf{L}} = 3\frac{g^2}{16\pi^2} W^a \tilde{W}^a - 3\frac{g'^2}{16\pi^2} Y\tilde{Y}$$

$$\dot{n}_{\mathsf{B}} = \dot{n}_{\mathsf{L}} = -3\frac{g'^2}{4\pi^2}\dot{\mathcal{H}}_Y + \cdots$$

At the EW crossover, there is a change in helicity when the B_Y field converts to B_{em}

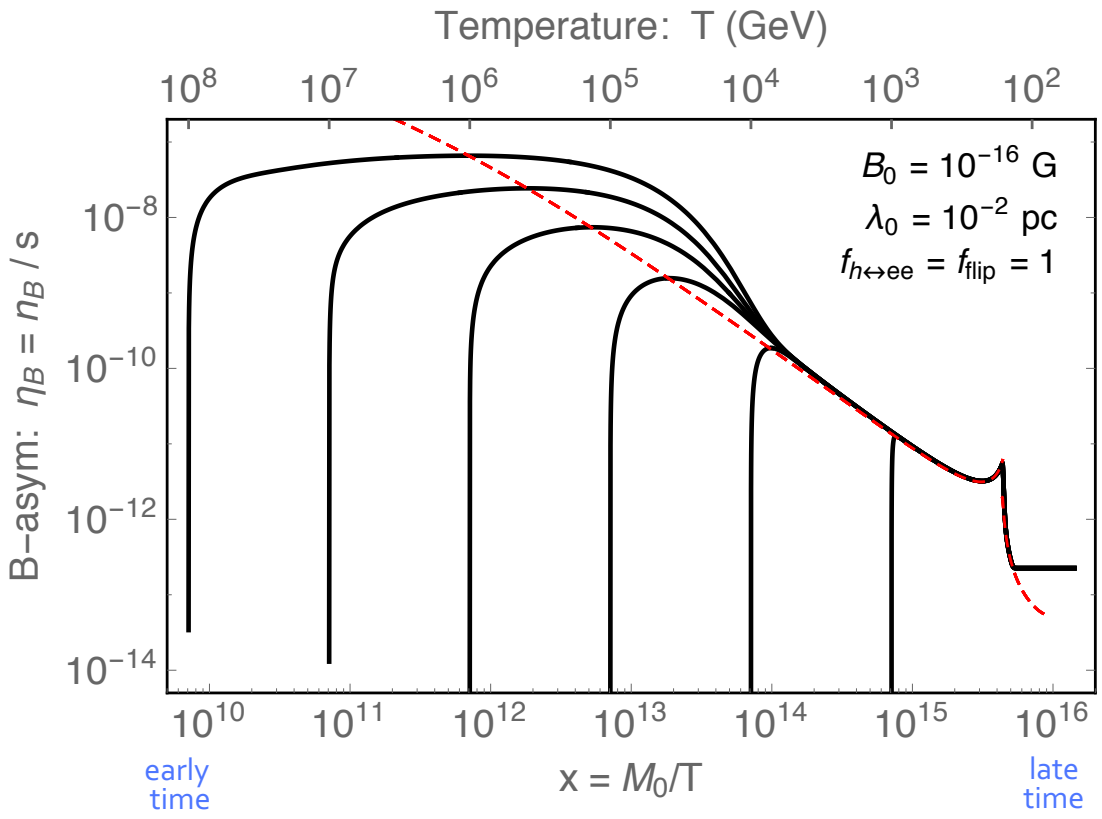
- ➔ this could explain the matter/antimatter asymmetry!
- ➔ but it would be a problem if too many baryons are created

(B+L) washout is avoided

Kamada & AL (2016)

basic idea: $\dot{n}_{\rm B} = -\Gamma_{\rm sphaleron}\, n_{\rm B} + S_{\rm helicity}$ ⇨ $n_{\rm B} \sim S_{\rm helicity} \,/\, \Gamma_{\rm sphaleron}$

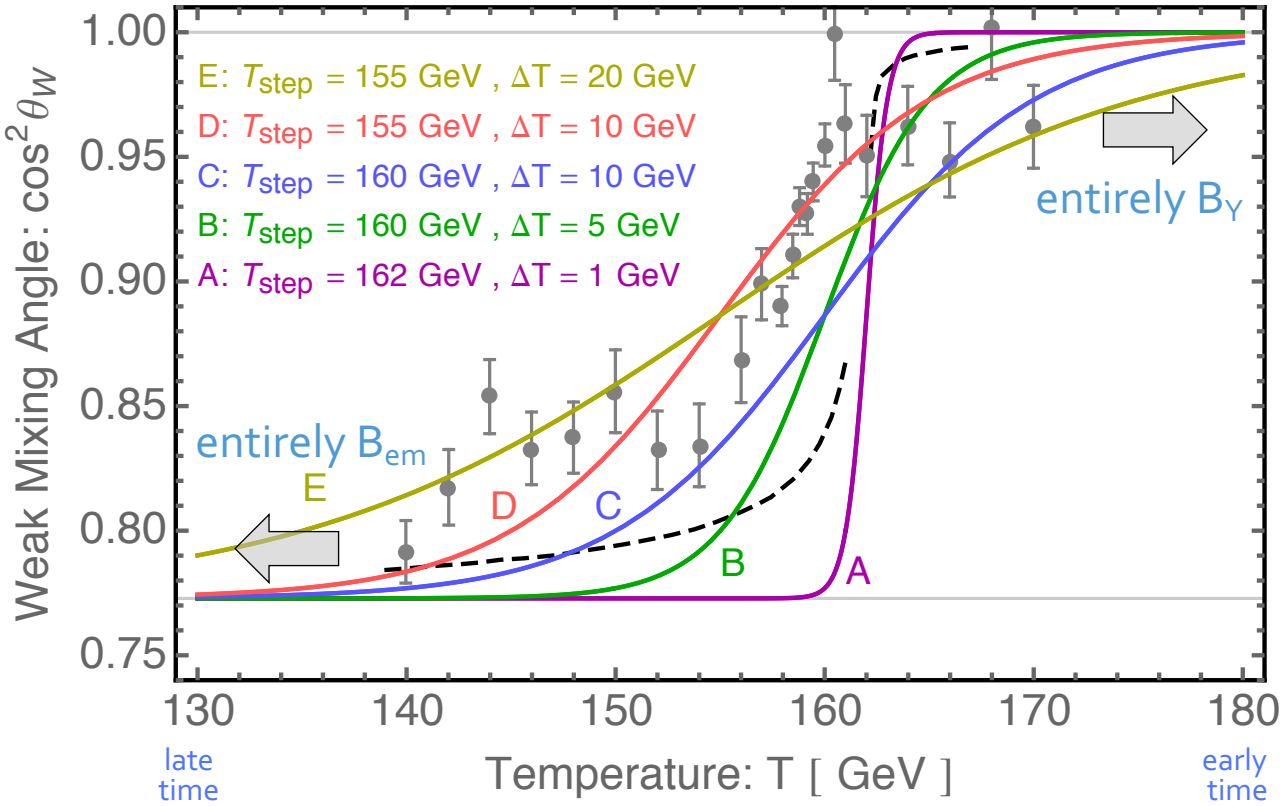
time evolution



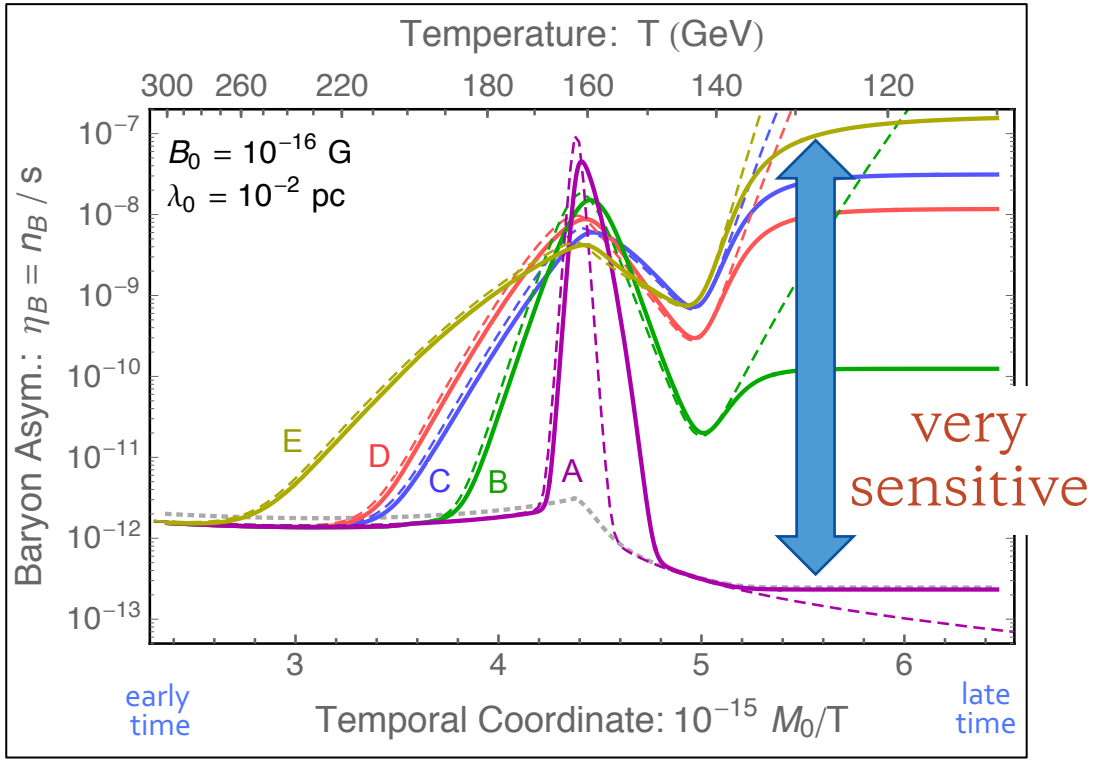
Sensitivity to the EW crossover

Kamada & AL (2016)

Z-γ mixing is proxy for B_Y to B_{em}



time evolution



We should work to develop a better understanding of how hypermagnetic fields are converted into electromagnetic fields at the EW epoch (even if it is a smooth crossover)

Avoiding baryon overproduction

[Domcke, Kamada, Mukaida, Schmitz, Yamada (2022)]

If μ_{Y5} is too large, the helical B_Y-field becomes too strong, and B-number is over-produced.

The upper limit is subject to uncertainties associated with modeling B_Y -> B_{EM} at the EW crossover.

We take $\mu_{Y,5}/T \lesssim 10^{-2}$ as a guide.

recall

$$B_{\rm phys,0} \simeq \left(6.6 \times 10^{-16}\ {\rm G}\right)\left(\frac{|\mu_{Y,5}|/T}{10^{-3}}\right)^{1/3}$$

summary graphic
& remarks

Summary

assume: chiral asymmetry (Baryo-genesis)

↓

Chiral Plasma Instability

↓

Magneto-genesis

↓

Gravito-genesis

chiral asymmetry is erased before magnetogenesis	viable window	baryon number is over-produced	
10^{-3}		10^{-2}	$(\lvert\tilde{\mu}_{Y,5}\rvert/T)_{\rm CPI}$
70 TeV		7000 TeV	$T_{\rm phys,CPI} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^2_{\rm CPI}$
7×10^{-16} G		1×10^{-15} G	$B_{\rm phys,0} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^{1/3}_{\rm CPI}$
9×10^{-4} pc		2×10^{-3} pc	$\xi_{M,\rm phys,0} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^{1/3}_{\rm CPI}$
7×10^{-39}		7×10^{-33}	$\Omega_{\rm GW}h_0^2 \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)^6_{\rm CPI}$
0.1 MHz		1 MHz	$f_{\rm GW,0} \propto (\lvert\tilde{\mu}_{Y,5}\rvert/T)_{\rm CPI}$

- chiral plasma instability develops at T_{CPI} > 80 TeV before washout
- development of CPI leads to helical PMF and chiral GWs
- PMF could provide seeds for dynamo – explain galactic magnetic field
- GWs are high frequency and too weak to detect

backup slides

Stochastic B-field power spectra & energy

$$\langle B_i(\boldsymbol{k},t)B_j^*(\boldsymbol{k}',t)\rangle = (2\pi)^3\,\delta(\boldsymbol{k}-\boldsymbol{k}')\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k) - \mathrm{i}\epsilon_{ijm}\hat{k}_m P_{aB}(k)\right]$$

$$\langle B_i(\boldsymbol{x},t)B_j(\boldsymbol{y},t)\rangle = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3}\langle B_i(\boldsymbol{k},t)B_j^*(\boldsymbol{k}',t)\rangle\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{y}}$$

$$= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)P_B(k) - \mathrm{i}\epsilon_{ijm}\hat{k}_m P_{aB}(k)\right]\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}$$

$$\mathcal{E}_M(\boldsymbol{x},t) = \frac{1}{2}\sum_i\langle B_i(\boldsymbol{x},t)B_i(\boldsymbol{x},t)\rangle$$

$$= \frac{1}{2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\left(3-1\right)P_B(k)\right]$$

$$= \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}P_B(k)$$

$$= \int_0^\infty\frac{\mathrm{d}k}{k}\underbrace{\frac{k^3}{2\pi^2}P_B(k)}_{=\,kE_M(k)}$$