Cinvestav

IB corrections to CVC predictions of τ decay and e^+e^- annihilation into 2 pions

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Virtual Mini Workshop on τ decays: theoretical aspects

December 9th, 2024

Content

- Isospin symmetry/breaking and CVC
- Isospin Breaking corrections ($\Delta a_\mu^{\mathrm{LO},\pi\pi}$, $\Delta B_{\pi\pi}$)
- Long-distance (LD) Electromagnetic corrections $G_{\mathrm{EM}}(s)$
- Corrections in pion FF: $\rho - \omega$ mixing, mass and width differences of $\rho^\pm - \rho^0$ mesons
- Impact on $a_\mu^{\mathrm{LO},\pi\pi}[\tau]$, $\mathscr{B}_{\pi\pi^0}(e^+e^-)$

In collaboration with Davier, Flores-Baez, Flores-Tlalpa, Malaescu, Miranda, Roig, Toledo, Zhang,

PRD 74, 071301 (2006); PRD 76, 096010 (2007)
EPJC 66, 127 (2010)
Eprint 2411.07696
Cirigliano et al, JHEP 02, (2002); PLB (2001)
Miranda+Roig, PRD 102, 114017 (2020)

Motivation ($a_\mu^{\mathrm{LO},\pi\pi}$ in data-driven approach)

- Relation CVC $\tau^\pm \to \pi^\pm\pi^0\nu_\tau / e^+e^- \to \pi^+\pi^-$ proposed 1971. Precise data require Isospin Breaking (IB). Recent exp/theory advances $\Rightarrow a_\mu^{\mathrm{LO},\,\pi\pi}$

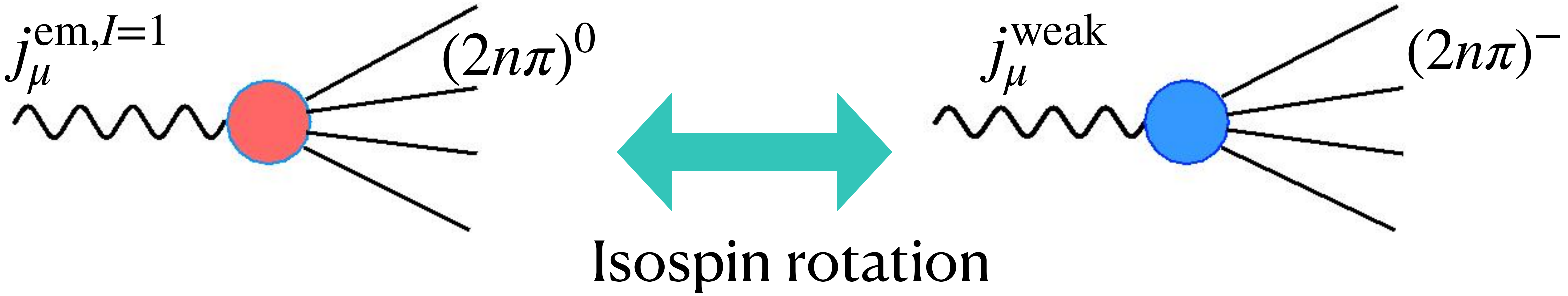
- $e^+e^- \to \pi^+\pi^-(\gamma)$ data should be used in $a_\mu^{\mathrm{HVP},2\pi}$; experimental results precise but not very well consistent (KLOE vs CMD3)

- $\tau^\pm \to \pi^\pm\pi^0\nu_\tau(\gamma)$ data consistent between 4 different experiments; precise Good understanding of IB and uncertainties required. Many detailed studies in last 25 years.

- If consistency is achieved and IB is understood, gain precision in $a_\mu^{\mathrm{LO},\,\pi\pi}$

Isospin Symmetry and CVC hypothesis

- Isospin properties non-strange currents: $\left[I_{\pm}, j_{\mu}^{\mathrm{em},I=1}(0)\right] = \mp V_{\mu}^{\mathrm{weak},\,\pm}(0)$



- Isospin symmetry: $\partial^{\mu}(j_{\mu}^{\mathrm{em}}, V_{\mu}^{\mathrm{weak}}) = 0$, CVC ; exact relation between had. matrix elements/observables.

- Isospin is broken (IB): $m_u \neq m_d$ $[\partial^{\mu} V_{\mu}^{\mathrm{weak}} \neq 0]$ and EM interactions. But still good symmetry: IB effects of order $(m_u - m_d)/\Lambda$, $\alpha = e^2/4\pi \sim O(\mathrm{few}\ \%)$.

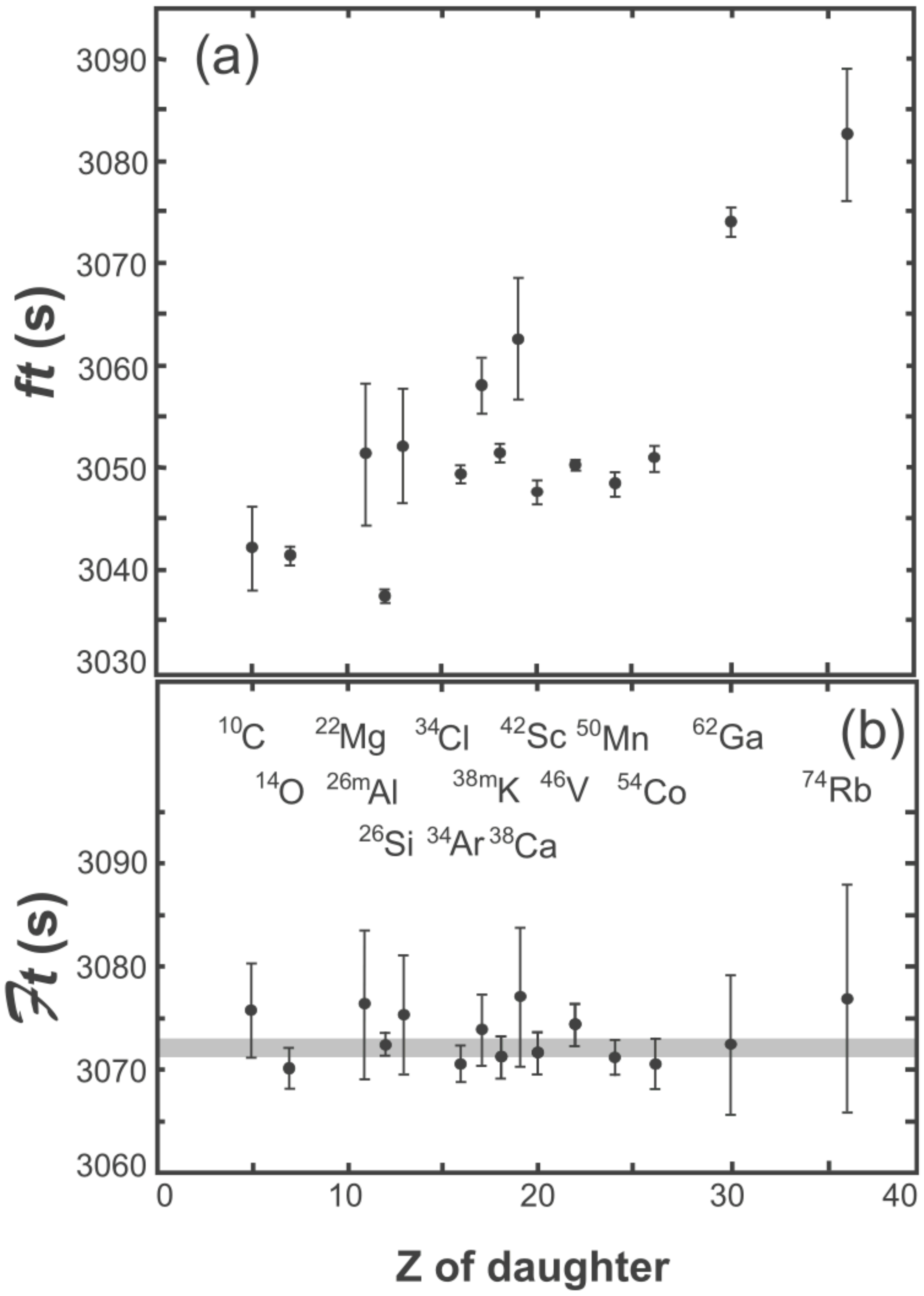
$|V_{ud}|$: The most precise test of CVC

15 $0^+(Z, A) \to 0^+(Z-1, A)e^+\nu_e$ transitions

$= 2$

Isospin symmetry

$$ft\,|F^{0^+\to 0^+}(0)|^2 = \frac{2\pi^3 \ln 2}{G_F^2 m_e^5 |V_{ud}|^2}$$



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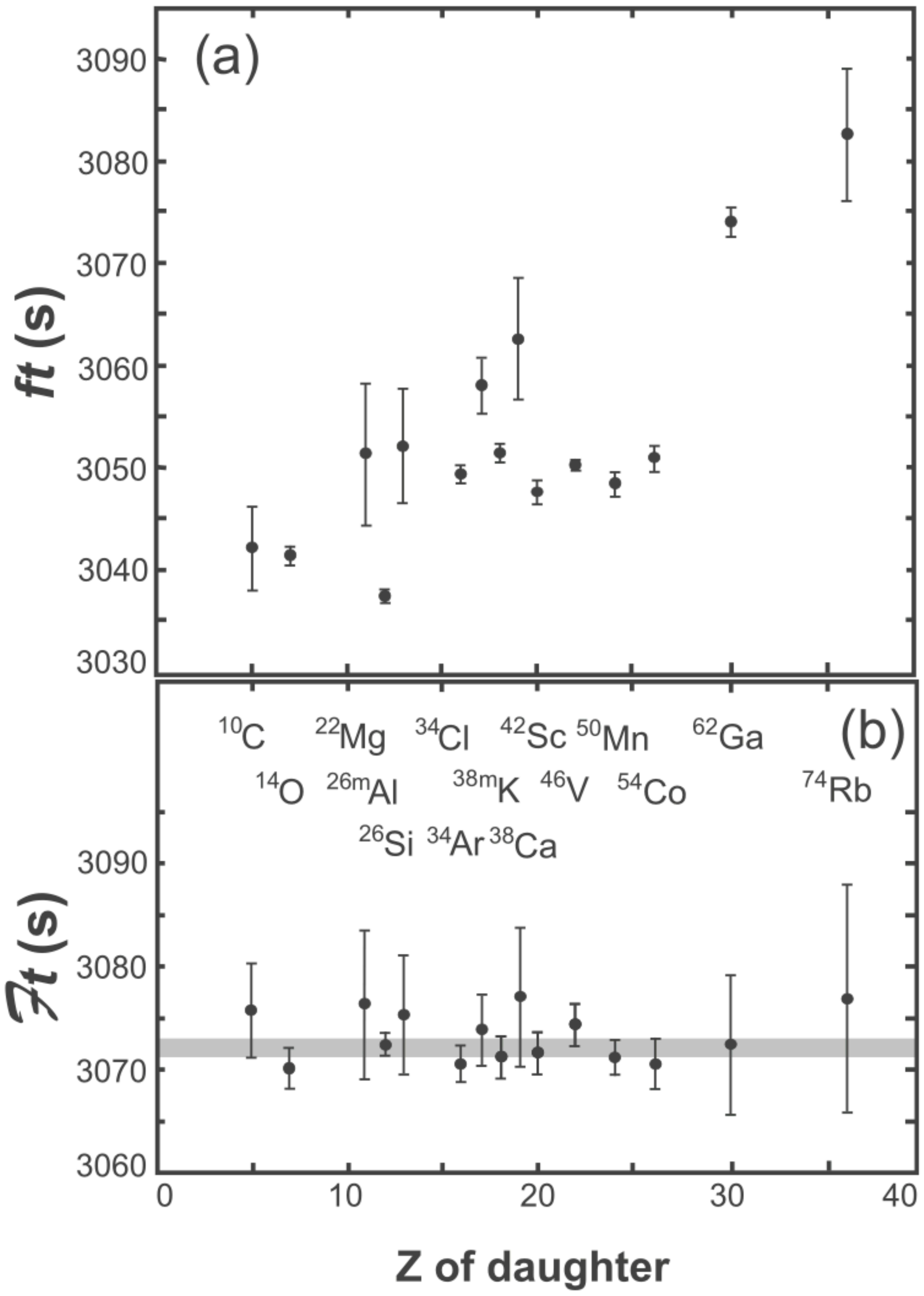
$$\mathcal{F}t \equiv ft(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{2\pi^3 \ln 2}{2G_F^2 m_e^5 |V_{ud}|^2(1+\Delta_R^V)}$$

Test of CVC $|F^{0^+\to 0^+}(0)| = \sqrt{2}$

$\Longrightarrow \quad |V_{ud}| = 0.97367(32)$



IB in pion form factors

- CVC hypothesis predicts $F_\pi^{\mathrm{em}}(s) = F_\pi^{\mathrm{weak}}(s)$ for all $s = (p_{\pi_1} + p_{\pi_2})^2$.
- IB effects modify form factors. In the time-like $\sqrt{s} \leq 1$ GeV region:

$$F_\pi^{\mathrm{em}}(s) \to F_0(s) = f_{\rho^0}(s)\left[1 + \delta_{\rho\omega}\frac{s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}\right]$$

$$F_\pi^{\mathrm{weak}}(s) \to F_-(s) = f_{\rho^-}(s)$$

* More isovector resonances $(\rho', \rho'', \cdots)$ required at higher s.

* IB effects:

1) rho-omega mixing $(\delta_{\rho\omega} = |\delta_{\rho\omega}| e^{i\phi_{\rho\omega}})$; data indicates $\phi_{\rho\omega}$ small.

2) $f_{\rho^0}(s) \neq f_{\rho^-}(s)$ lineshapes: $\delta m_\rho = m_{\rho^-} - m_{\rho^0}$ and $\delta\Gamma_\rho = \Gamma_{\rho^0} - \Gamma_{\rho^-}$

Experiments provide final-state photon inclusive observables

- $\tau^{\pm} \to \pi^{\pm}\pi^{0}\nu_{\tau}(\gamma)$ decays, $(m_{\pi^+} + m_{\pi^0})^2 \leq s \leq m_{\tau}^2$

$$\frac{d\Gamma_{2\pi(\gamma)}}{ds} = \frac{\Gamma_e |V_{ud}|^2}{2m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2\left(1 + \frac{2s}{m_\tau^2}\right)\beta_-^3(s)\,\left|F_-'(s)\right|^2 S_{EW} G_{EM}(s)$$

Universal SD- EW corr. =1.0233(3).
Cirigliano et al, PRD, 2023

LD EM corr. (2π specific)

- $e^+e^- \to \pi^+\pi^-(\gamma_{\mathrm{FSR}})$, with $s \geq 4m_{\pi^+}^2$

Final State Radiation, S-QED;
Schwinger Book+ Dres-Hikasa

$$\sigma_{2\pi(\gamma)}(s) = \frac{\pi\alpha^2}{3s}\beta_0^3(s)\left|F_0'(s)\right|^2\ \mathrm{FSR(s)}$$

$\beta_{-,0}(s)$, $\pi-$velocity

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$\beta_{-,0}(s)$, $\pi-$velocity

- IB corrected CVC relation

$$\frac{d\Gamma_{2\pi(\gamma)}}{ds} = \frac{3}{2}\frac{\Gamma_e |V_{ud}|^2}{\pi\alpha^2 m_\tau^2} s\sigma_{2\pi(\gamma)}(s)\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\cdot\frac{S_{EW}}{R_{IB}(s)}$$

s-dependent IB correction

$$R_{\mathrm{IB}}(s) = \frac{\mathrm{FSR}(s)}{G_{\mathrm{EM}}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

IB effects. Different sensitivities to the pion FF.

$$\Delta^{\mathrm{IB}} a_\mu^{\mathrm{LO},\pi\pi}[\tau] = \frac{\alpha^2 m_\tau^2}{6|V_{ud}|^2\pi^2} \frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e} \int_{4m_\pi^2}^{m_\tau^2} ds \frac{K(s)}{s} \frac{dN_{\pi\pi^0}}{N_{\pi\pi^0} ds} \left(1 - \frac{s}{m_\tau^2}\right)^{-2} \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \left[\frac{R_{\mathrm{IB}}(s)}{S_{EW}} - 1 \right]$$

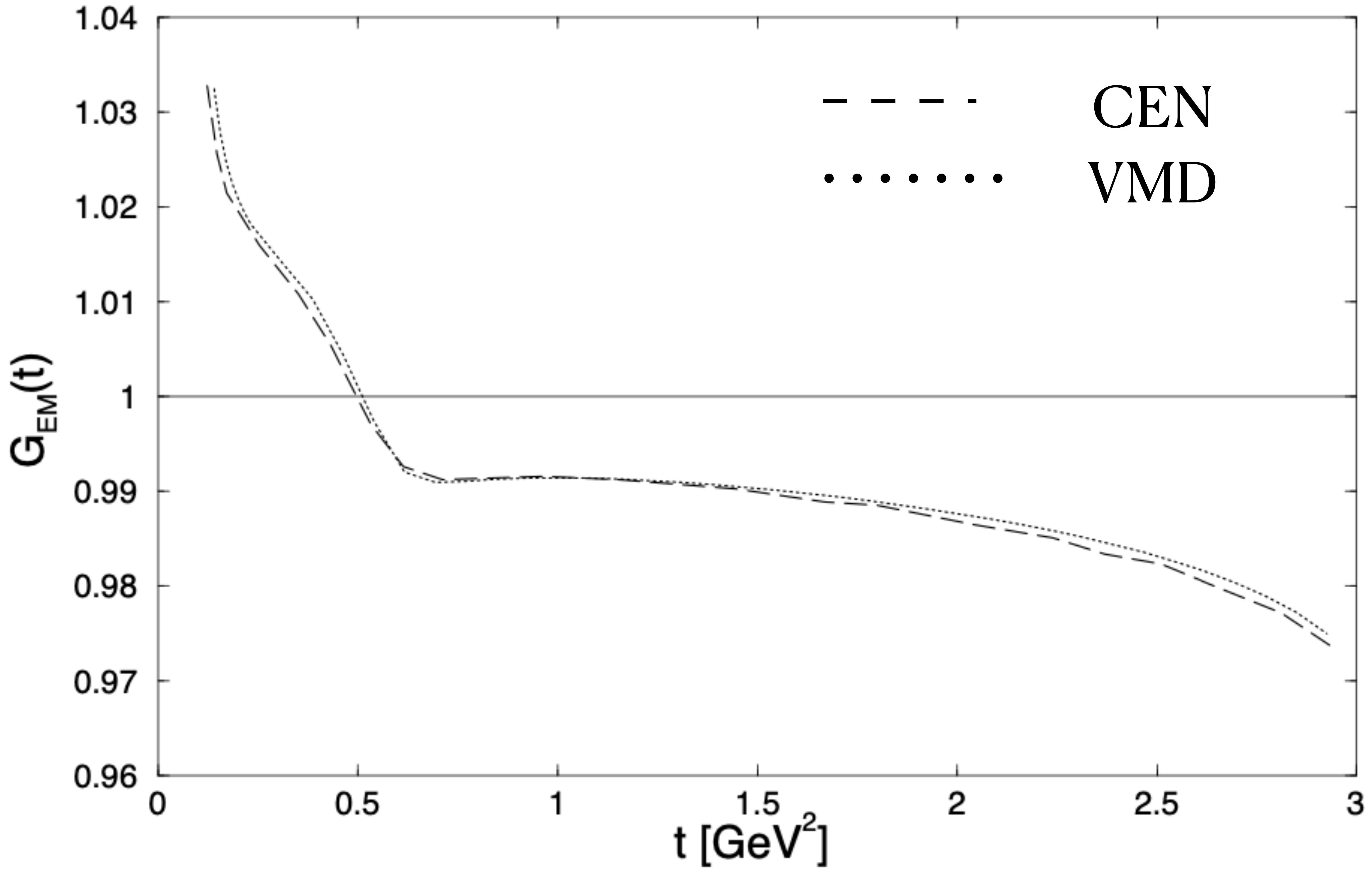
$$\Delta^{\mathrm{IB}} \mathcal{B}_{\pi\pi^0}[e^+e^-] = \frac{3}{2} \frac{\mathcal{B}_e |V_{ud}|^2}{\pi\alpha^2 m_\tau^2} \int_{4m_\pi^2}^{m_\tau^2} s ds \sigma_{2\pi(\gamma)}(s) \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \left[\frac{S_{EW}}{R_{\mathrm{IB}}(s)} - 1 \right]$$

$$\mathcal{B}_{\pi\pi^0,e} = \mathrm{BR}[\tau^- \to (\pi^-\pi^0, e^-\bar{\nu}_e)\nu_\tau]$$

LD electromagnetic corrections of $O(\alpha)$ in $\tau^- \to \pi^-\pi^0\nu_\tau$: $G_{\rm EM}(s)$

$$\frac{d\Gamma_{2\pi(\gamma)}}{ds} = \frac{d\Gamma^0_{2\pi}}{ds} \times G_{EM}(s)$$

- CEN: virtual+real-photon corrections in RChT to $O(p^4)$. FF Guerrero Pich for $\sqrt{s} \leq$ GeV [1]
- VMD: radcorr in VMD . FF BW with $\rho(770)$ [2]
- MR: uncertainty of CEN estimated (SD constraints)+excited ρ in pion FF [3]



ALEPH, Belle, exclude $\omega \to \pi^0\gamma$

[1] Cirigliano et al, JHEP (2002);
[2] Flores-Tlalpa et al, PRD (2006);
[3] Miranda+Roig, PRD (2020)

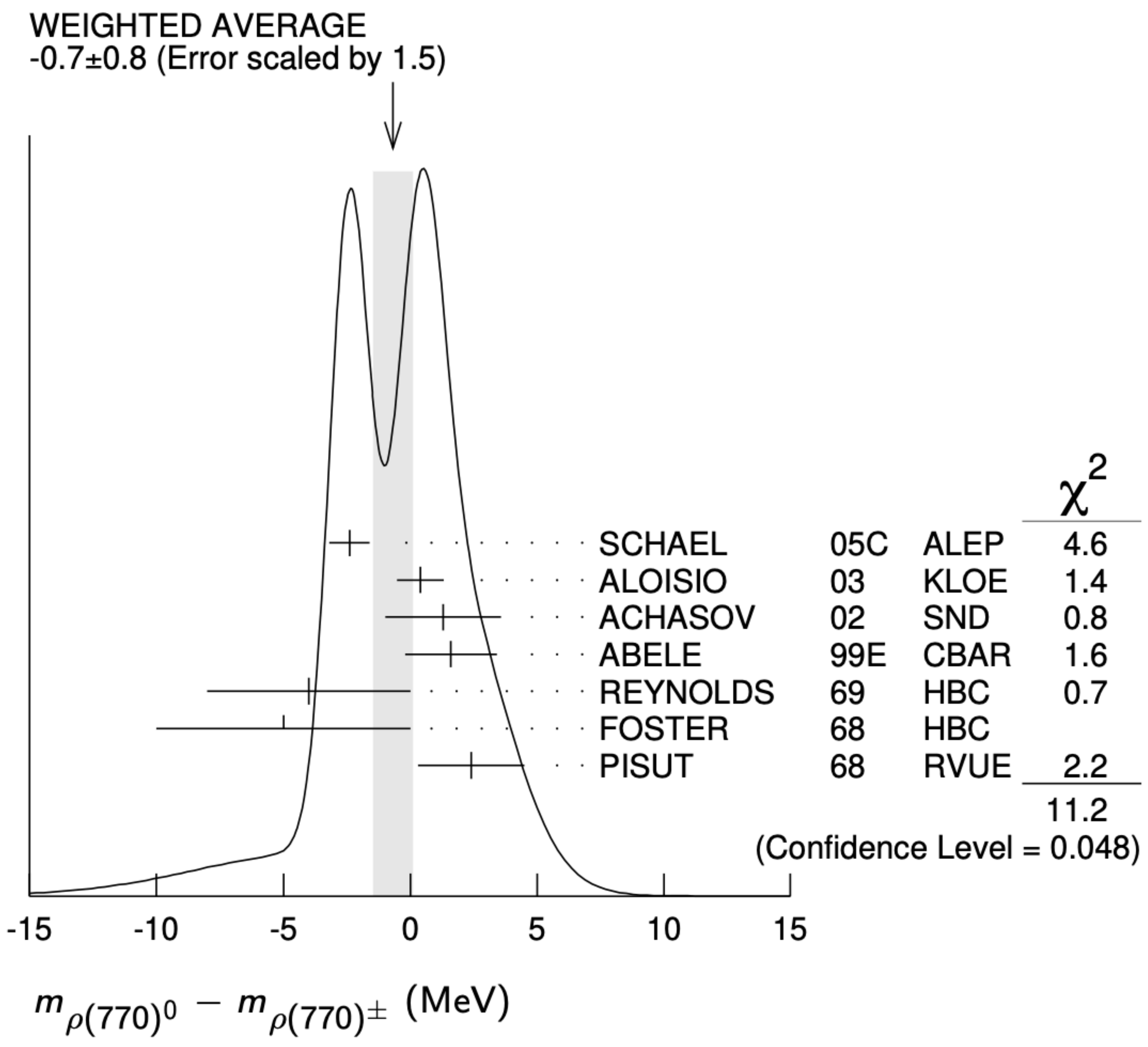
IB in rho resonance lineshape: rho mass /width difference

1. Rho mass difference in $|f_0(s)/f_-(s)|^2$

$$\delta m_\rho \equiv m_{\rho^\pm} - m_{\rho^0}$$

δm_ρ (MeV)	Method/Reference
0.02 ± 0.02	VMD (Feuillat et al, 2001)
$(-0.7, +0.4)$	$1/N_C$ (Bijnens+ Gosdzinsky 1996)
$+0.014$	from $\delta m_\rho^2 = \delta m_\pi^2$
-0.7 ± 0.8	PDG average ($S = 1.5$)

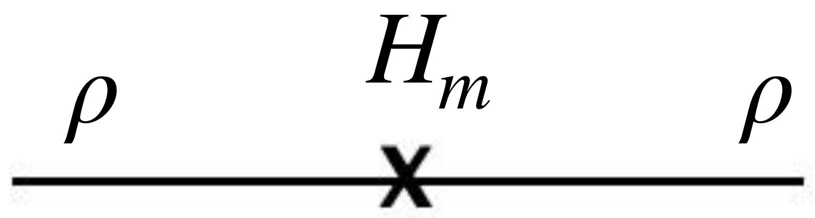
- Consistent with very small/ vanishing(?) IB.
- Model- & process-dependent



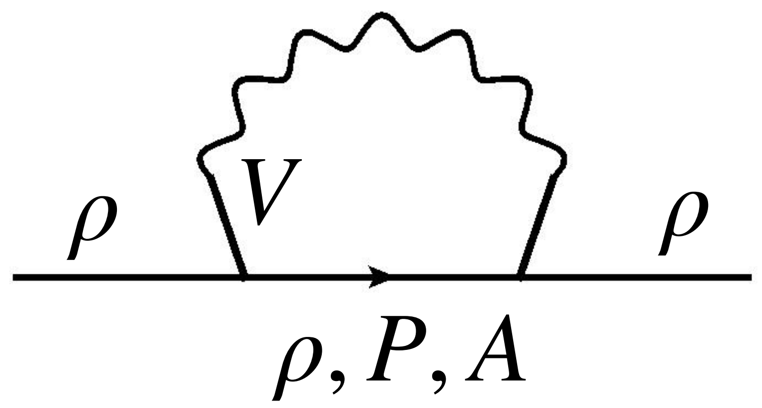
Mass diff. in a VMD model: $m_\rho = m_0 + \delta m_\rho^{m_u \neq m_d} + \delta m_\rho^\gamma$

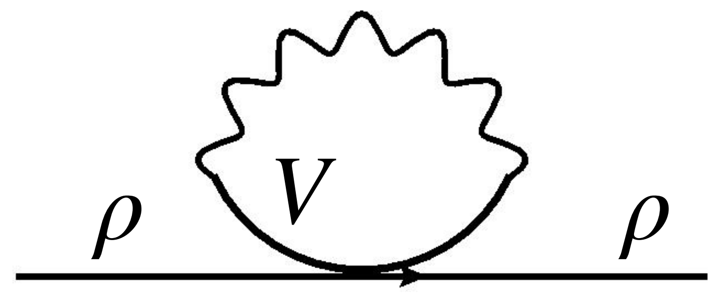
Feuillat et al, PLB, 2001

- LO in u-d mass diff. $\rightarrow \delta m_\rho^{m_d \neq m_u} = 0$

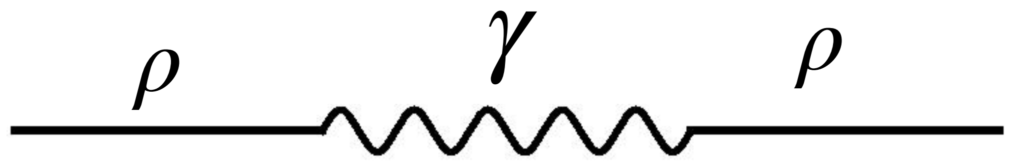


- $\delta m^\gamma_{\rho^\pm} = \frac{\alpha m_\rho}{8\pi}\left[2 + \pi\sqrt{3} - \frac{2}{3}\right] = 1.49 \text{ MeV}$





- $\delta m^\gamma_{\rho^0} = \frac{3\Gamma(\rho^0 \to e^+e^-)}{2\alpha} = 1.43 \text{ MeV}$



$$\delta m_\rho = \delta m^\gamma_{\rho^\pm} - \delta m^\gamma_{\rho^0} \approx 0.06 \text{ MeV}$$

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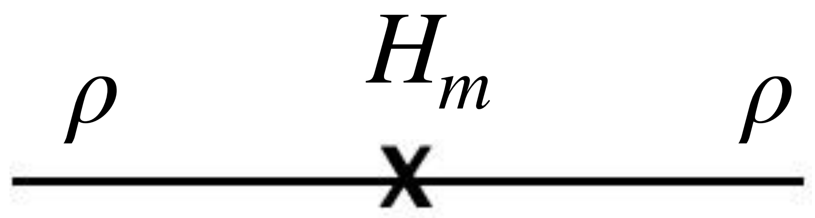
Feuillat et al, PLB, 2001

- LO in u-d mass diff. $\rightarrow \delta m_\rho^{m_d \neq m_u} = 0$



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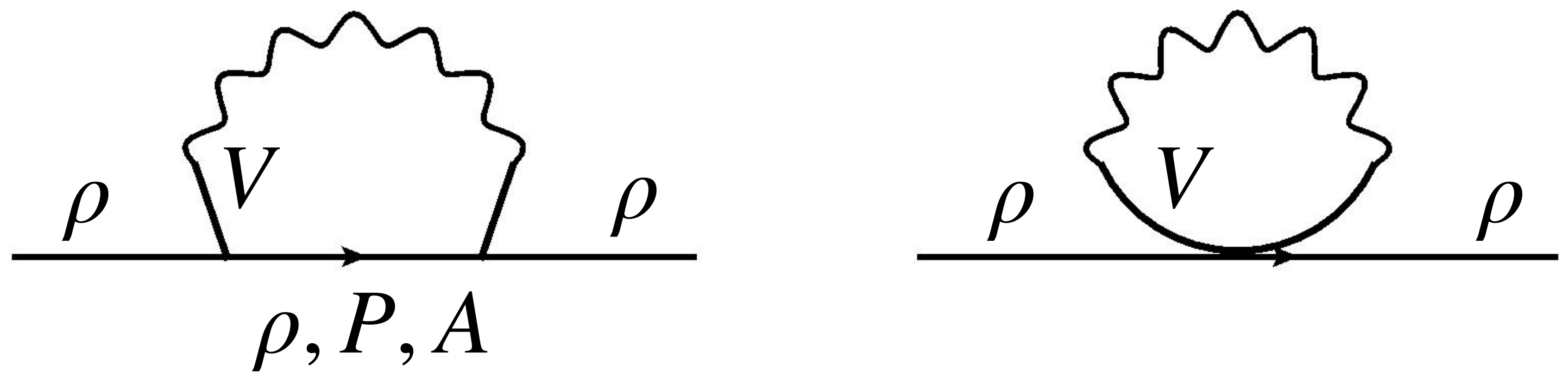

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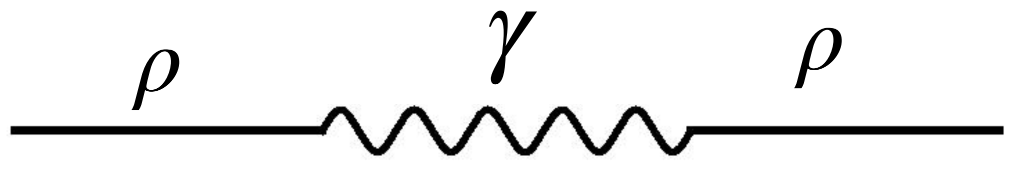


$$\delta m_\rho = \delta m^\gamma_{\rho^\pm} - \delta m^\gamma_{\rho^0} \approx 0.06 \text{ MeV}$$

Davier et al, EPJC 2010

$$\begin{aligned} m_{\rho_+} - m_{\rho^0_{\text{bare}}} &= m_{\rho^+} - m_{\rho^0} + \delta m^\gamma_{\rho^0} \\ &= (-0.4 \pm 0.9) + 1.43 \\ &= (1.0 \pm 0.9) \text{ MeV} \end{aligned}$$

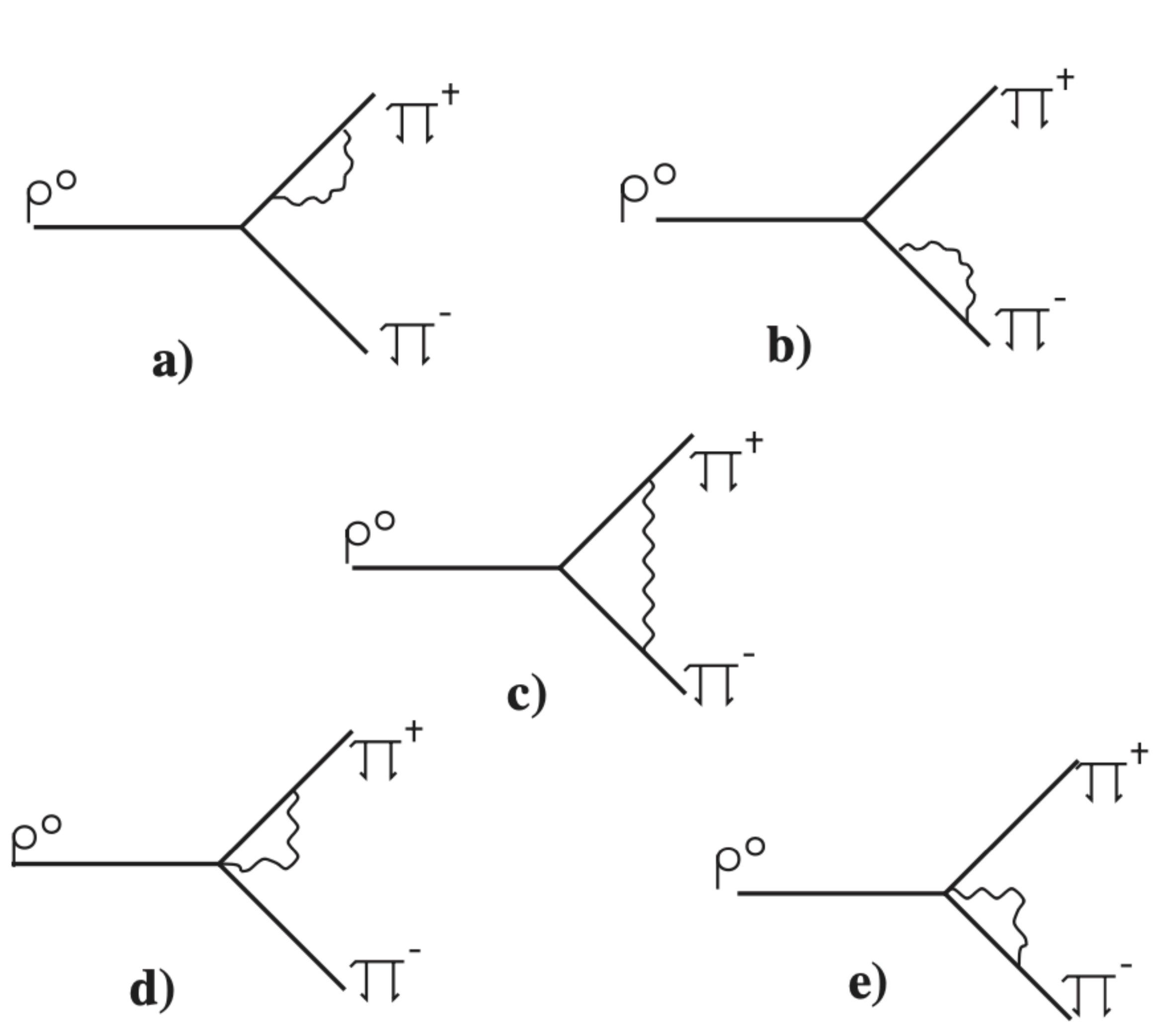
2. Rho width difference: $\delta\Gamma_\rho \equiv \Gamma_{\rho^0} - \Gamma_{\rho^\pm}$

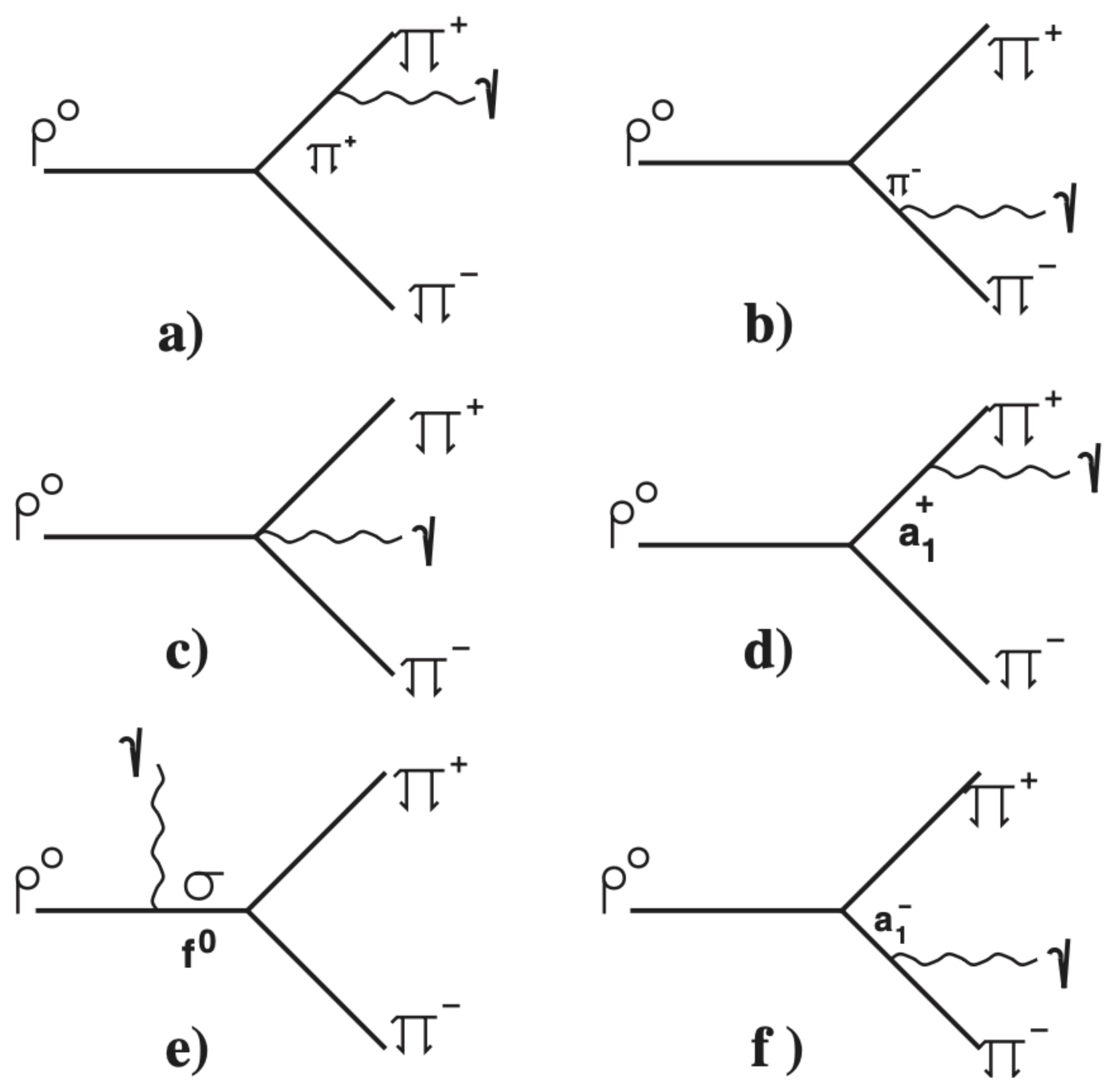
Scalar QED+ canonical $\rho\rho\gamma$ vertex

$$\Gamma_{\rho^\pm} = \Gamma[\pi^\pm\pi^0(\gamma_<)] + \Gamma[\pi^\pm\pi^0\gamma_>] + \Gamma^\pm[\text{rest}]$$
$$\Gamma_{\rho^0} = \Gamma[\pi^+\pi^-(\gamma_<)] + \Gamma[\pi^+\pi^-\gamma_>] + \Gamma^0[\text{rest}]$$

Virtual + real foton $E_\gamma \leq \omega_0$

Real photon $E_\gamma > \omega_0$





$$(\Gamma^\pm - \Gamma^0)\Big|_{\text{rest}} = (0.083 \pm 0.008)\ \text{MeV}$$

$$\text{rest} = \pi\gamma, \ell^+\ell^-, \eta\gamma, 3\pi, 4\pi \ \text{PDG}$$

Only the sum $\pi\pi + \pi\pi\gamma$ is independent of ω_0. Therefore we compute the inclusive photon $\rho \to \pi\pi$ rates

Flores-Baez et al, PRD76 (2007)

To a very good approximation, $\delta\Gamma_\rho$ from photon inclusive rates : $\Gamma_\rho = \Gamma[\rho \to \pi\pi(\gamma)]$

$$\delta\Gamma_\rho(s) = \frac{g^2_{\rho\pi\pi}\sqrt{s}}{48\pi}\left[\beta_0^3(s)(1+\delta_0) - \beta_-^3(s)(1+\delta_-)\right]$$

$$= \begin{cases} 0.76 \text{ MeV} \\ (-0.61 \pm 0.45) \text{ MeV } [*], \quad \delta\Gamma_\rho(\pi\pi) + \delta\Gamma(\pi\pi\gamma + \text{ rest}) \\ (-0.42 \pm 0.58) \text{ MeV } [**], \quad \text{same} \end{cases}$$

[*] Cirigliano et al, JHEP (2002);
[**] Alemany et al, EPJC (1998)

Validation:

☑ $\Gamma(\rho^0 \to \pi^+\pi^-\gamma) - \Gamma(\rho^+ \to \pi^+\pi^0\gamma)\Big|_{\omega_0 > 50 \text{ MeV}} = 1.1 \text{ MeV} \quad (0.45 \pm 0.45)[*]$

☑ $\text{BR}(\rho^0 \to \pi^+\pi^-\gamma)\Big|_{\omega_0 > 50 \text{ MeV}} = 11.5 \times 10^{-3} \quad (9.9 \pm 1.6) \times 10^{-3}$ [PDG]

Flores-Baez et al, PRD76 (2007)

- $F_i(s) = f_{\rho^i}(s)[1 + \delta_{i0}\delta_{\rho\omega}BW_\omega(s)[+\rho' + \ldots]$, IB correction factor

$$\left|\frac{f_{\rho^0}(s)}{f_{\rho_-}(s)}\right|^2 = 1 + A(s)\delta m_\pi + B(s)\delta m_\rho + C(s)\delta\Gamma_\rho + \cdots$$

- $A(s), B(s), C(s)$ expected to be “mildly” model-dependent.
- At least four (model) parameterizations used for the isovector pion FF:
  - Gounaris-Sakurai (**GS**), **PRL21 (1968)**
  - Kuhn-Santamaria (**KS**), **ZPC48 (1990)**
  - Guerrero-Pich (**GP**), **PLB 412 (1996)**
  - Dispersive representation (**Disp**)

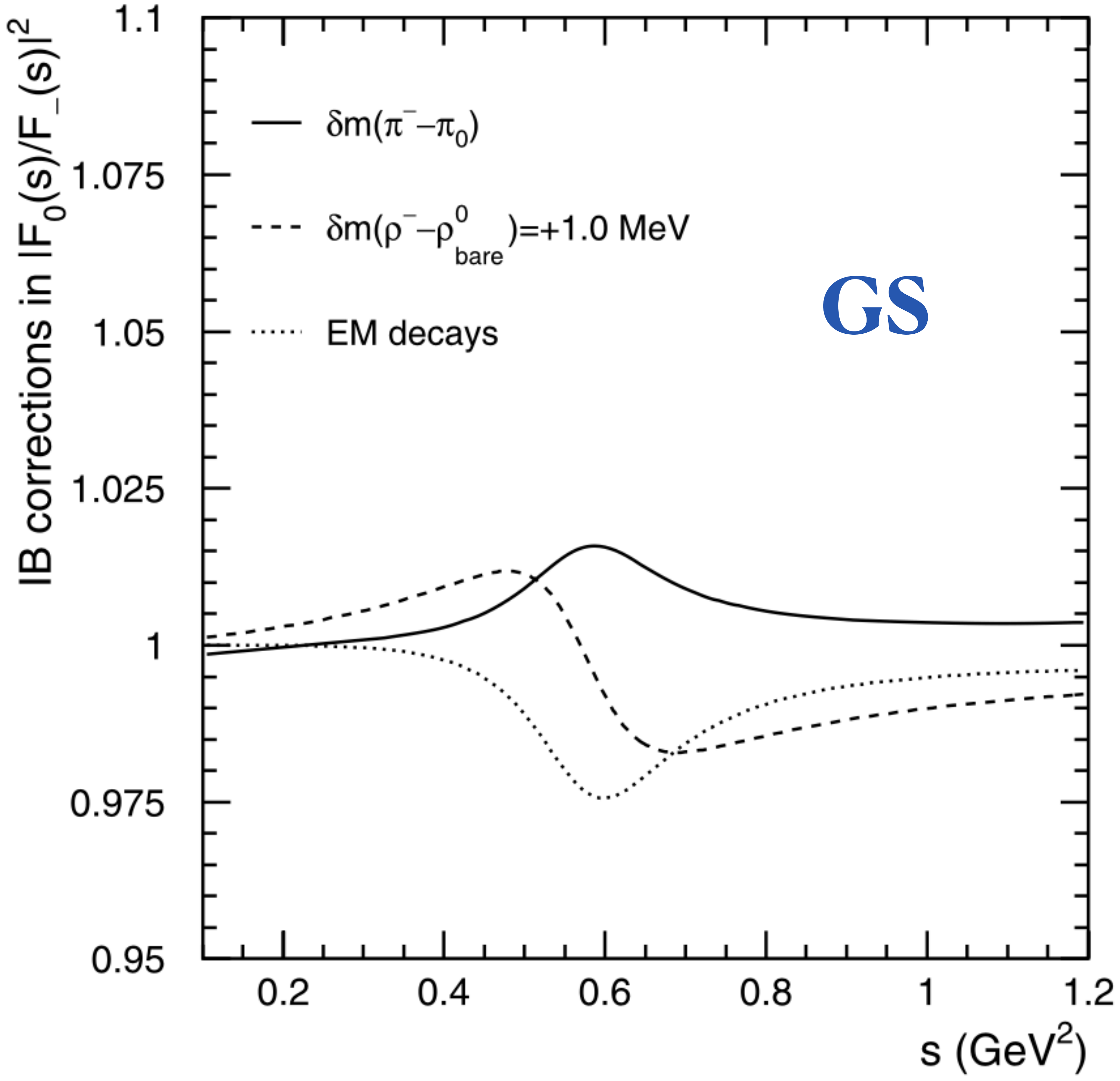
GS, KS → Davier et al, (2010, 2023)

GP → Cirigliano et al (2002)

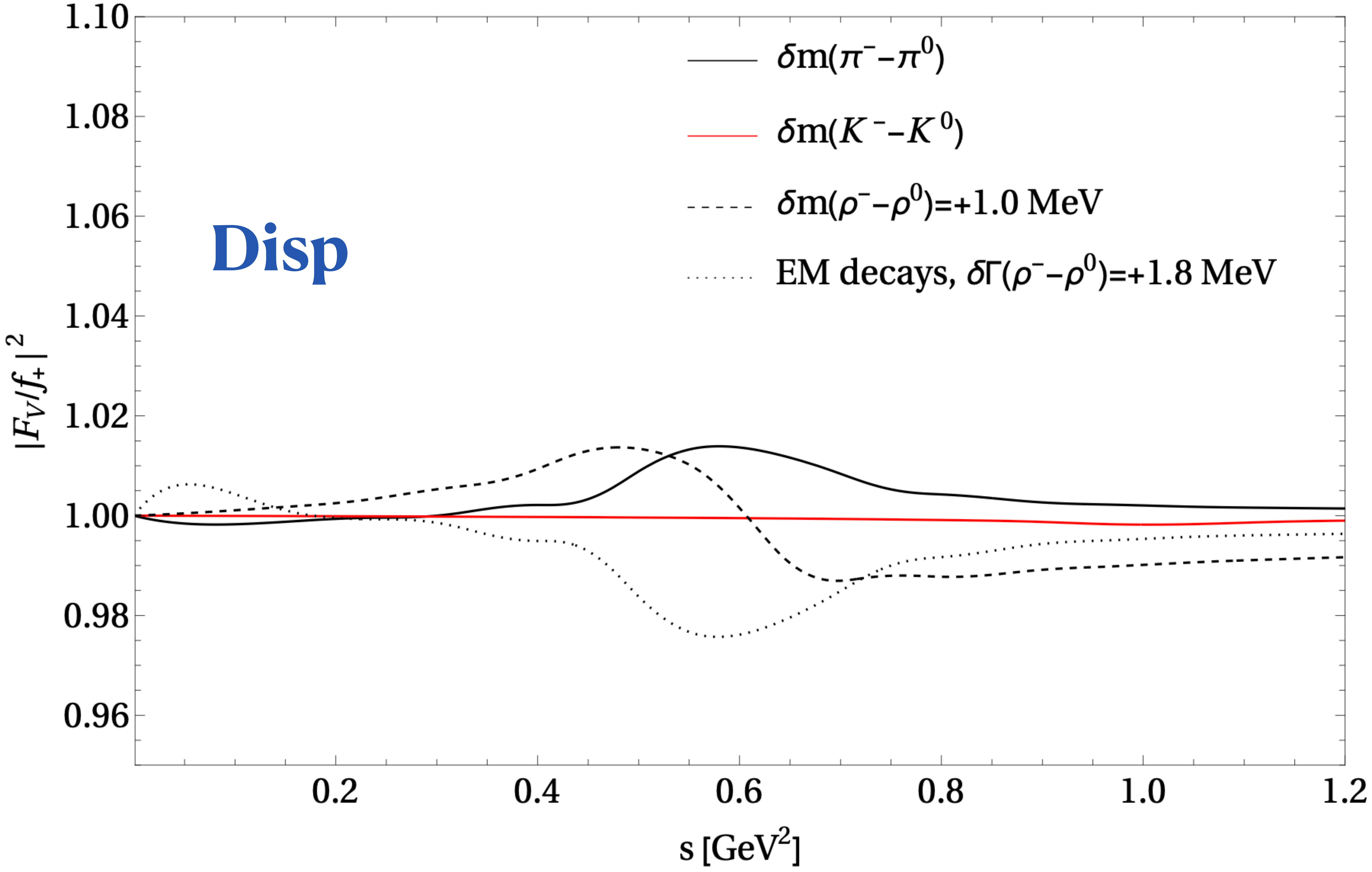
Disp → Miranda et al (2024)

Comparison of IB corrections in $|f_{\rho^0}(s)/f_{\rho^-}(s)|^2$

Davier et al (2010)



Miranda et al (2024)



Numerical IB corrections: independent of pion FF

$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Source	$\Delta a_\mu^{{\rm LO},\pi\pi}[\tau](10^{-10})$ Davier et al (2010)	$\Delta a_\mu^{{\rm LO},\pi\pi}[\tau](10^{-10})$ Miranda et al (2024)	$\Delta \mathcal{B}_{\pi\pi^0}^{e^+e^-}$ (%) Davier et al (2010)	$\Delta \mathcal{B}_{\pi\pi^0}^{e^+e^-}$ (%) Miranda et al (2024)
$S_{\rm EW}$	$-12.21(15)$	$-11.96(15)$	$+0.57(1)$	$+0.57(1)$
$G_{\rm EM}(s)$	$-1.92(90)$	$-1.71(^{61}_{48})$	$-0.07(17)$	$-0.09(^{3}_{1})$
${\rm FSR}(s)$	$+4.67$	$+4.56$	$-0.19(2)$	$-0.19(2)$
Δm_π in $\sigma(ee)$	-7.88	-7.47	$+0.19$	$+0.20$
FF	$+2.38(0.79)$	$+1.53(^{1.95}_{2.0})$	$+0.19(9)$	$+0.14(^{9}_{6})$
TOTAL	$-14.9(1.9)$	$-15.0(^{2.0}_{2.2})$	$+0.69(22)$	$+0.63(^{10}_{7})$

2.9 % correction

2.5 % correction

Isospin breaking in FF lineshapes $|F_0(s)/F_-(s)|^2$

Davier et al EJP C84, 721(2024)

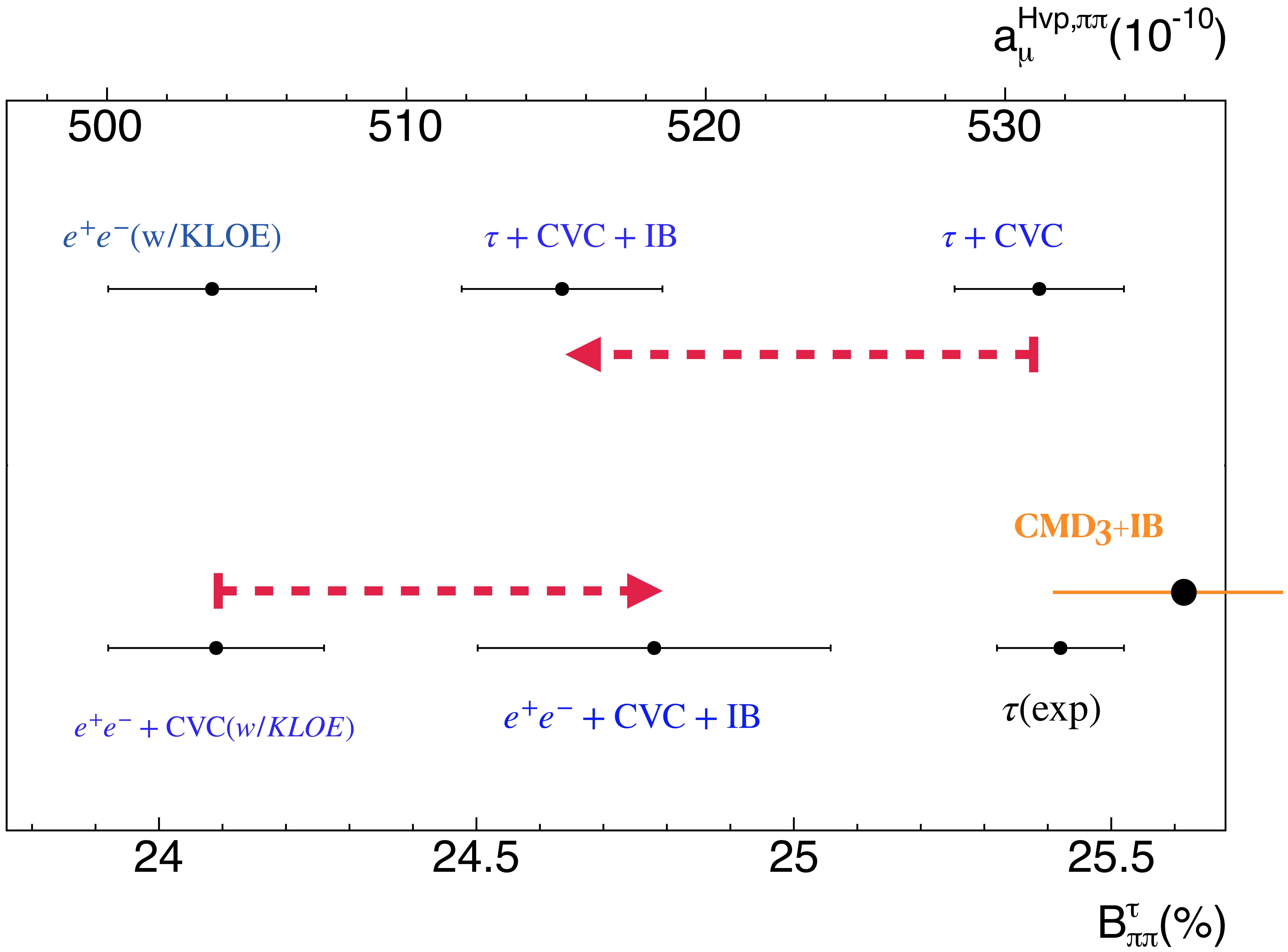
Source	$\Delta a_\mu^{LO,\pi\pi}[\tau](10^{-10})$ Davier et al 2010 GS	Davier et al 2010 KS	Miranda et al 2024 GS	Miranda et al 2024 KS	Miranda et al 2024 Disp p_{4-1}
$\rho-\omega$	$+4.0(4)$	$+2.80(15)$	$+3.84(8)\binom{0.23}{2.17}$	$+4.00(8)\binom{0.25}{1.96}$	$+2.72(8)\binom{0.65}{1.15}$
Δm_π	$+4.09$	$+4.02$	$+3.74$	$+4.12$	$+3.58$
Δm_ρ	$+0.20\binom{27}{19}$	$+0.11\binom{19}{11}$	$+0.10\binom{18}{9}$	$-0.04\binom{6}{0}$	$+1.85\binom{1.69}{1.66}$
$\pi\pi\gamma$	$-5.91(59)$	$-6.39(64)$	$-6.09(67)$	$-6.68(74)$	$-6.62(73)$
Total	$+2.38$	$+0.54$	$+1.59$	$+1.40$	$+1.53$

Source	$\Delta\mathcal{B}_{\pi\pi^0}(\%)$ Davier et al 2010 GS	Davier et al 2010 KS	Miranda et al 2024 GS	Miranda et al 2024 KS	Miranda et al 2024 Disp p_{4-1}
$\rho-\omega$	$-0.01(1)$	$-0.02(1)$	$-0.08(8)$	$-0.09(0)\binom{11}{0}$	$-0.01(0)\binom{6}{4}$
Δm_π	-0.22	-0.22	-0.21	-0.22	-0.20
Δm_ρ	$+0.08(8)$	$+0.09(8)$	$+0.08(8)$	$+0.09(8)$	$-0.02\binom{2}{1}$
$\pi\pi\gamma$	$+0.34(3)$	$+0.37(4)$	$+0.34(3)$	$+0.37(4)$	$+0.37(4)$
Total	$+0.19(9)$	$+0.22(9)$	$+0.13(12)$	$+0.15\binom{11}{9}$	$+0.14\binom{7}{6}$

IB corrections in good direction to restore agreement of τ vs e^+e^- with CVC

Davier et al, EPJC (2010)

Miranda et al, 2411.07696 (2024)



Final comments

- Current precision measurements of $\tau^{\pm} \to \pi^{\pm}\pi^{0}\nu_{\tau}$ and $e^{+}e^{-} \to \pi^{+}\pi^{-}$ require leading order (LO) Isospin breaking (IB) corrections for testing CVC predictions;
- LO IB effects help to achieve better agreement between the predictions of $a_{\mu}^{\mathrm{LO},\pi\pi}$ from τ- and $e^{+}e^{-}$ data in dispersive approach (KLOE vs CMD3?);
- LO IB effects added to the CVC prediction of the $\mathrm{B}(\tau^{\pm} \to \pi^{\pm}\pi^{0}\nu_{\tau}(\gamma))$ is in better agreement with experiment;
- IB effects are model-dependent ($|F_0(s)/F_-(s)|$). But different sources ($\rho\omega$, δm_{ρ}, $\delta\Gamma_{\rho}$) tend to cancel and render small this contribution

Backup

Isospin symmetry $f_0(s) = f_+(s)$. Isospin Breaking ($f_+(s) \neq f_0(s)$):

$$\langle \pi^+(p_+)\pi^-(p_-)|j_\mu^{\rm em}(0)|0\rangle = F_V(s)\,(p_+ - p_-)_\mu$$

$$\langle \pi^+(p_+)\pi^0(p_0)|j_\mu^{\rm weak}(0)|0\rangle = \sqrt{2}f_+(s)\left[(p_+ - p_0)_\mu - \frac{\Delta^{+0}}{s}(p_+ + p_0)_\mu\right] + \sqrt{2}f_0(s)\frac{\Delta^{+0}}{s}(p_+ + p_0)_\mu$$

Below $\sqrt{s} \leq 1$ GeV

$$F_V(s) = f_{\rho^0}(s)\left[1 + \delta_{\rho\omega}\frac{m_\omega^2}{m_\omega^2 - s - i m_\omega \Gamma_\omega}\right]$$

$$f_+(s) = f_{\rho^+}(s)$$

Isospin breaking in $\rho\omega$ mixing and in rho mass and width differences that determines the rho resonance lineshape

Mass and width difference: pole position vs BW forms

Mass and width in BW forms (**Conven.**)

$$F_\pi(s) = \frac{A(s)}{D(s)},$$
$$D(s) = s - \widetilde{M}_\rho^2(s) + i\widetilde{M}_\rho(s)\widetilde{\Gamma}_\rho(s)$$
$$\mathrm{Re}(D(m_\rho^2)) = 0, \quad \text{and} \quad \mathrm{Im}(D(m_\rho^2)) = m_\rho \Gamma_\rho$$

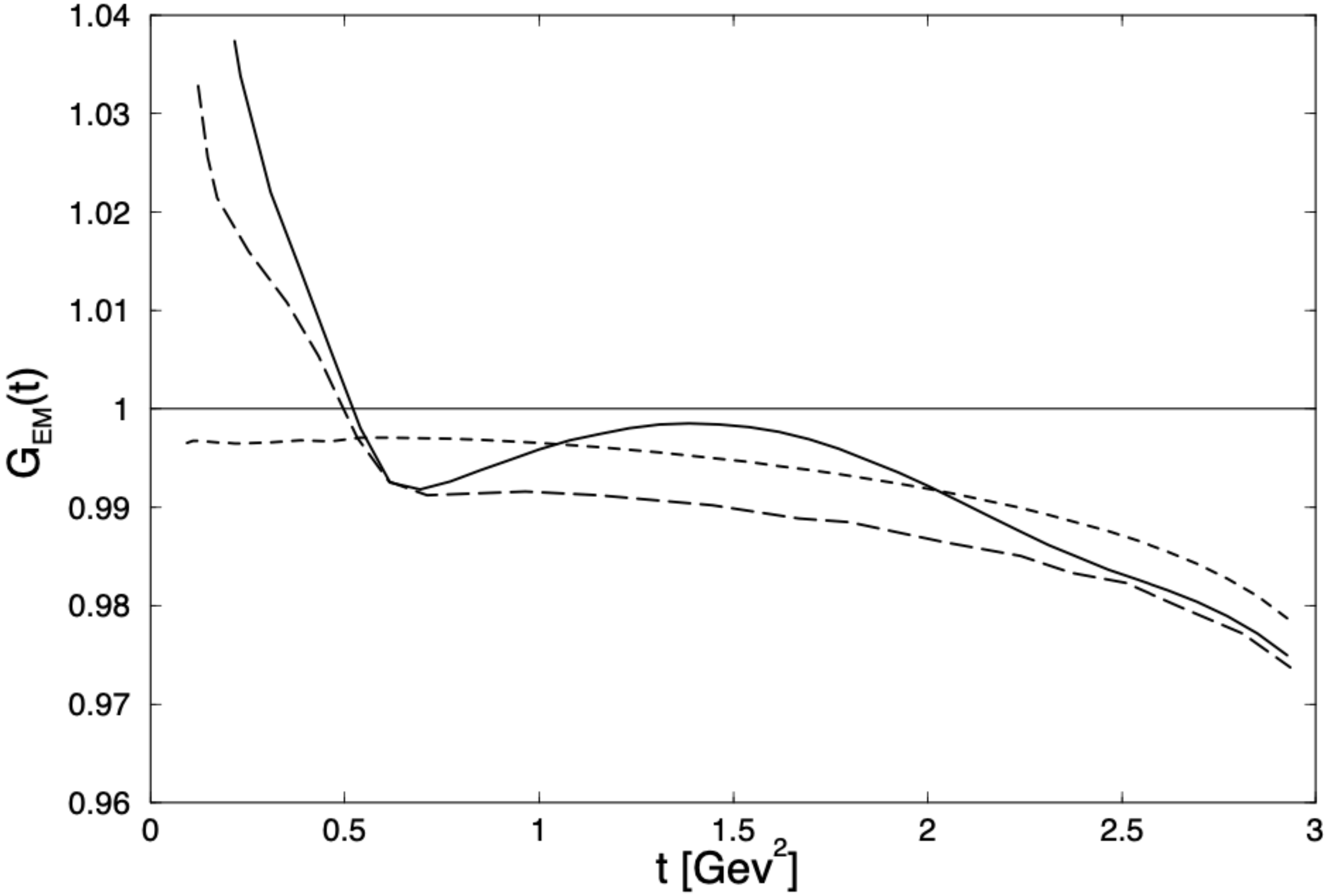
Mass and width from **pole** position

$$F_\pi(s) = \frac{R_p}{s - s_p} + B(s)$$
$$s_p = M^2 - iM\Gamma$$

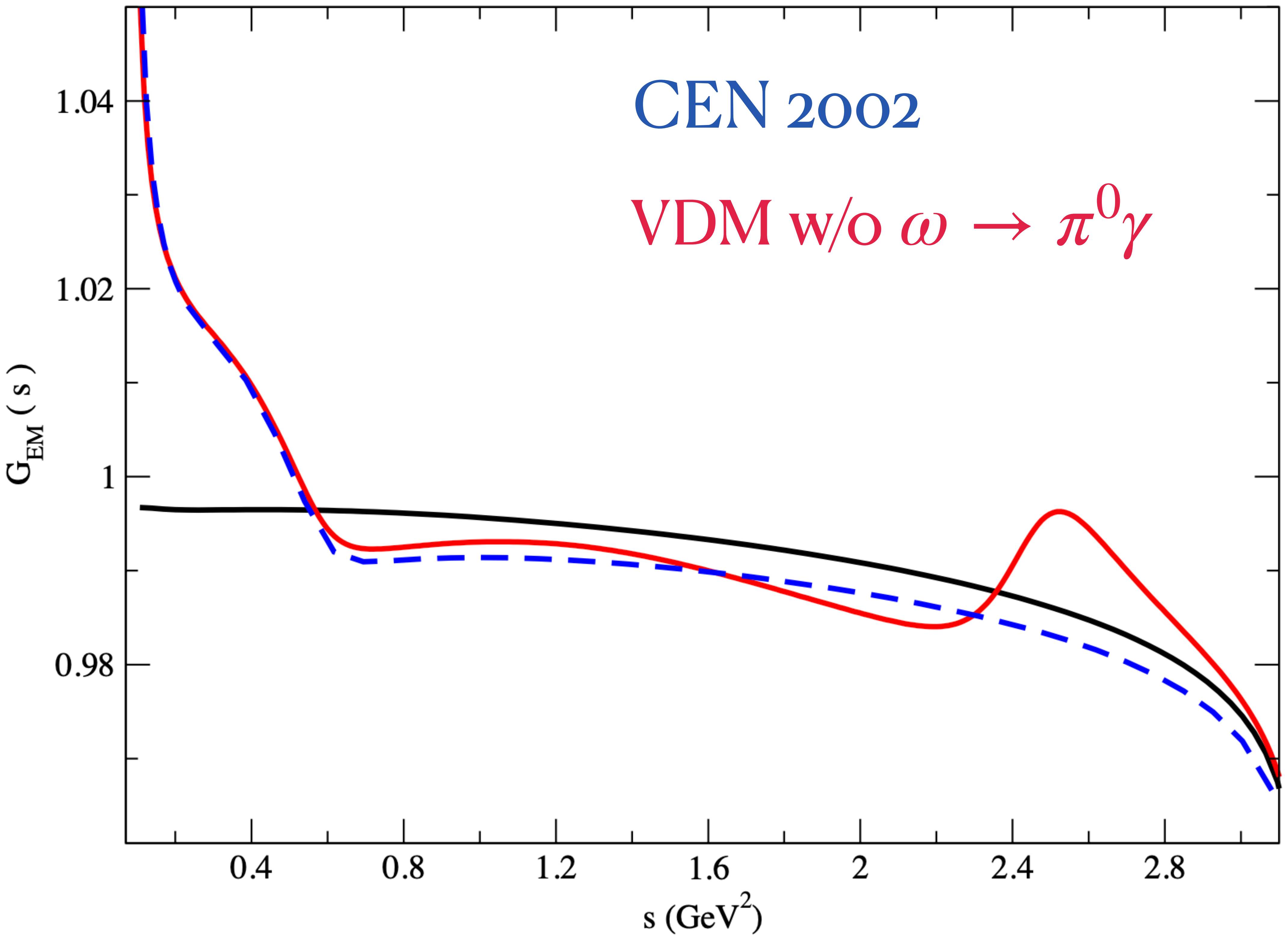
Conven. from **ALEPH, ZP C76, 15 (1997),** combined fit to e^+e^- & $\tau \to$ 2pi data, including IB corr.

Fit	$m_{\rho^\pm}$ (MeV)		m_{ρ^0} (MeV)		$m_{\rho^\pm} - m_{\rho^0}$ (MeV)	
	Conven.	Pole	Conven.	Pole	Conven.	Pole
KS($\lambda = 1$)	773.4 ± 0.9	757.0 ± 1.3	773.4 ± 0.7	756.9 ± 1.0	0.0 ± 1.0	0.1 ± 1.6
GS($\lambda = 1$)	775.7 ± 0.9	758.1 ± 1.3	775.7 ± 0.7	758.0 ± 1.0	0.0 ± 1.0	0.1 ± 1.6
GS($\lambda = 0.45 \pm 0.11$)	783.8 ± 3.0	758.3 ± 5.4	783.8 ± 3.0	758.1 ± 5.4	0.0 ± 1.2	0.2 ± 7.6

Fit	$\Gamma_{\rho^\pm}$ (MeV)		Γ_{ρ^0} (MeV)		$\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$ (MeV)	
	Conven.	Pole	Conven.	Pole	Conven.	Pole
KS($\lambda = 1$)	147.7 ± 1.6	143.2 ± 1.5	147.3 ± 1.3	142.7 ± 1.2	0.4 ± 1.0	0.5 ± 1.9
GS($\lambda = 1$)	150.8 ± 1.7	145.3 ± 1.5	150.8 ± 1.3	145.2 ± 1.2	0.0 ± 2.0	0.1 ± 1.9
GS($\lambda = 0.45 \pm 0.11$)	162.0 ± 5.3	145.1 ± 6.3	162.4 ± 5.0	145.2 ± 6.5	-0.4 ± 2.5	-0.1 ± 8.7



1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0
0.5
1
1.5
2
2.5
3
$G_{EM}(t)$
t [Gev^2]



CEN 2002
VDM w/o $\omega \to \pi^0\gamma$
1.04
1.02
1
0.98
0.4
0.8
1.2
1.6
2
2.4
2.8
G_{EM} (s)
s (GeV^2)

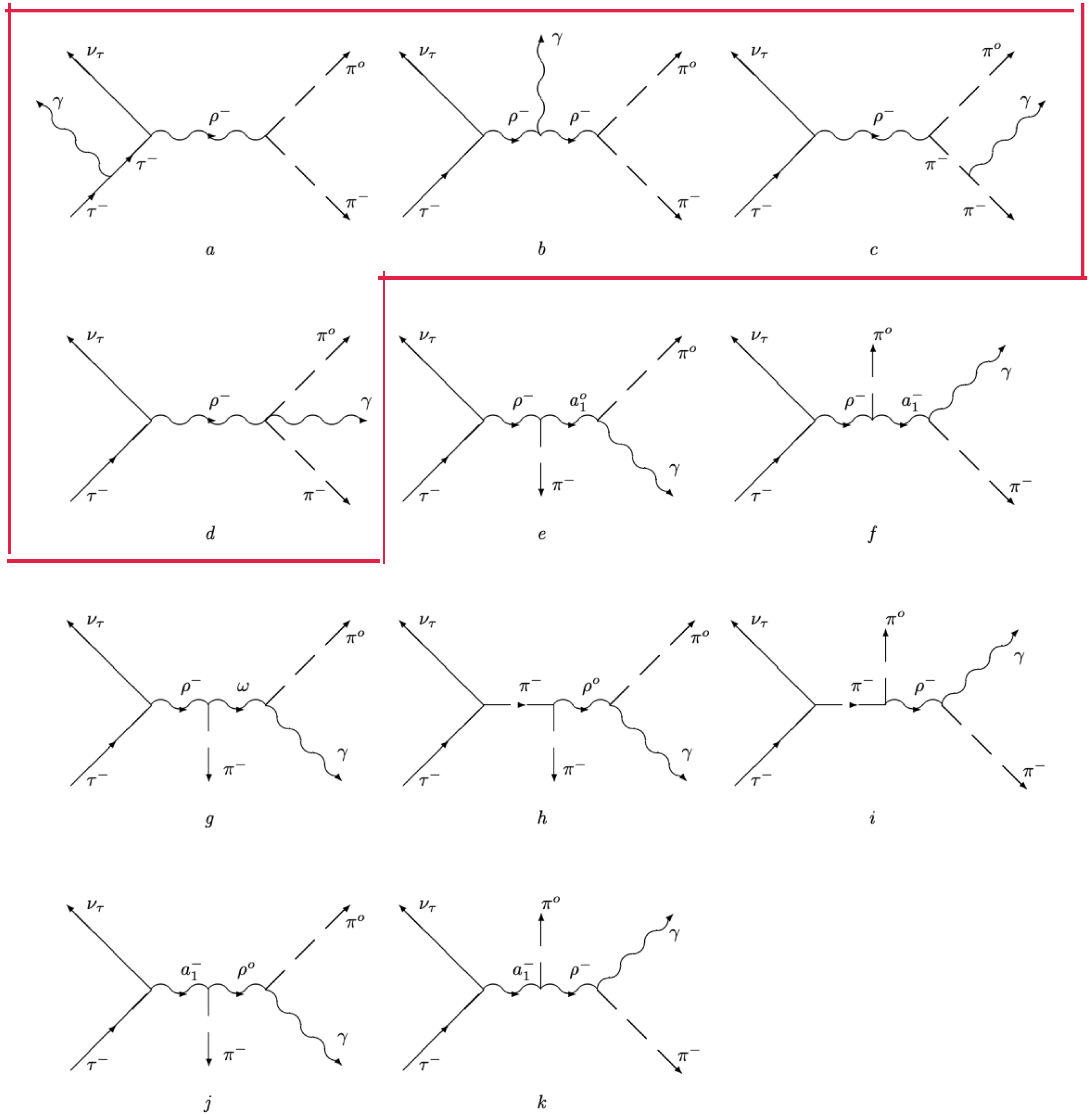
$G_{\rm EM}(s)$ in VMD

$$\mathscr{M}(a \to b + \gamma(k)) = \frac{A}{k} + Bk^0 + Ck$$

A, B model-independent ($a \to b$);
C is model-dependent

$$G_{EM}(t) = 1 + \frac{\frac{d\Gamma_v^1}{dt} + \frac{d\Gamma_r^1}{dt}}{\frac{d\Gamma^0}{dt}}$$

$$\frac{d\Gamma_r^1}{dt} = \frac{d\Gamma_r^1(m.i.)}{dt} + \frac{d\Gamma_r^1(m.d.)}{dt}$$



Pion form factor

$$F_V(s) = \Big[BW_\rho(s, m_\rho, \Gamma_\rho) \Big(1 + \delta_{\rho\omega} \frac{s}{m_\omega^2} BW_\omega^{NR}(s, m_\omega, \Gamma_\omega) + \delta_{\rho\phi} \frac{s}{m_\phi^2} BW_\phi^{NR}(s, m_\phi, \Gamma_\phi) \Big)$$

$$+ c_{\rho'} BW_{\rho'}(s, m_{\rho'}, \Gamma_{\rho'}) + c_{\rho''} BW_{\rho''}(s, m_{\rho''}, \Gamma_{\rho''}) \Big] / (1 + c_{\rho'} + c_{\rho''}),$$

Gounaris-Sakurai

$$BW^{GS}(s, m, \Gamma) = \frac{m^2 \left[1 + d(m)\Gamma/m\right]}{m^2 - s + f(s, m, \Gamma) - im\Gamma(s, m, \Gamma)}$$

Kuhn-Santamaría

$$BW^{KS}(s, m, \Gamma) = \frac{m^2}{m^2 - s - im\Gamma(s, m, \Gamma)}$$

$$m\Gamma(s, m, \Gamma) = \Gamma \frac{s}{m} \left(\frac{\beta_\pi(s)}{\beta_\pi(m^2)} \right)^3 \theta(s - 4m_\pi^2)$$

Dispersive

$$F_\pi^V(s) = \Omega_1^1(s) G_\omega(s) G_{\rm in}^N(s)$$

$$\Omega_1^1(s) = \exp\left\{ \frac{s}{\pi} \int_{4m_\pi^2}^\infty ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}$$

$$G_\omega(s) = 1 + \frac{s}{\pi} \int_{9m_\pi^2}^\infty ds' \frac{\operatorname{Im} g_\omega(s')}{s'(s'-s)} \left(\frac{1 - \frac{9m_\pi^2}{s'}}{1 - \frac{9m_\pi^2}{m_\omega^2}} \right)^4$$

$$g_\omega(s) = 1 + \epsilon_\omega \frac{s}{(m_\omega - \frac{i}{2}\Gamma_\omega)^2 - s}$$

$$G_{\rm in}^N(s) = 1 + \sum_{k=1}^N c_k \left(z^k(s) - z^k(0) \right)$$

$$c_1 = -\sum_{k=2}^N k\, c_k .$$

J. Miranda et al, 2411.07696

	BABAR12+Belle	KLOE12+Belle	KLOEc+Belle	CMD3+Belle	Global fit 1	Global fit 2
data points	322 + 62	60 + 62	85 + 62	209 + 62	322+60+209+62	322+85+209+62
χ^2	493.4	356.4	442.9	584.5	2192.2	2683.0
χ^2/d.o.f	1.3	2.9	3.0	2.2	3.4	4.0
m_ρ	774.9 ± 0.1 MeV	774.9 ± 0.1 MeV	774.7 ± 0.1 MeV	774.9 ± 0.2 MeV	774.3 ± 0.1 MeV	774.1 ± 0.1 MeV
$\lvert\delta_{\rho\omega}\rvert$	$(2.0 \pm 0.0) \cdot 10^{-3}$	$(2.0 \pm 0.0) \cdot 10^{-3}$	$(2.0 \pm 0.0) \cdot 10^{-3}$	$(2.1 \pm 0.0) \cdot 10^{-3}$	$(2.1 \pm 0.0) \cdot 10^{-3}$	$(2.1 \pm 0.0) \cdot 10^{-3}$
$\arg[\delta_{\rho\omega}]$	$(17.2 \pm 0.9)^\circ$	$(16.1 \pm 1.3)^\circ$	$(25.0 \pm 2.3)^\circ$	$(14.3 \pm 1.2)^\circ$	$(13.3 \pm 0.4)^\circ$	$(10.2 \pm 0.4)^\circ$
$\lvert\delta_{\rho\phi}\rvert$	$0^\dagger$	$0^\dagger$	$0^\dagger$	$(2.0 \pm 0.1) \cdot 10^{-4}$	$(1.3 \pm 0.1) \cdot 10^{-4}$	$(1.7 \pm 0.2) \cdot 10^{-4}$
$\arg[\delta_{\rho\phi}]$	–	–	–	$(56.5 \pm 2.9)^\circ$	$(22.9 \pm 5.3)^\circ$	$(40.4 \pm 6.3)^\circ$
C_2	-0.63 ± 0.00	-0.26 ± 0.00	-0.42 ± 0.01	-0.43 ± 0.00	$(-0.27 \pm 0.00) \cdot 10^{-1}$	-0.39 ± 0.02
C_3	0.37 ± 0.00	0.17 ± 0.00	0.29 ± 0.01	0.29 ± 0.01	$(0.28 \pm 0.00) \cdot 10^{-1}$	0.26 ± 0.01
C_4	-0.10 ± 0.00	-0.05 ± 0.00	-0.09 ± 0.00	-0.09 ± 0.01	$(-0.15 \pm 0.00) \cdot 10^{-1}$	$(-0.76 \pm 0.03) \cdot 10^{-1}$
$m_{\rho'}$	1603.5 ± 3.1 MeV	1564.8 ± 1.5 MeV	1636.1 ± 7.0 MeV	1600.5 ± 1.6 MeV	1482.3 ± 5.1 MeV	1552.5 ± 6.7 MeV
$\Gamma_{\rho'}$	426 ± 5 MeV	426 ± 7 MeV	611 ± 9 MeV	405 ± 8 MeV	313 ± 9 MeV	356 ± 10 MeV
$\mathrm{Re}[c_{\rho'}]$	0.30 ± 0.00	0.65 ± 0.01	1.85 ± 0.02	0.36 ± 0.01	0.49 ± 0.01	0.26 ± 0.01
$\mathrm{Im}[c_{\rho'}]$	0.31 ± 0.01	0.31 ± 0.01	0.25 ± 0.01	0.40 ± 0.01	0.27 ± 0.01	0.34 ± 0.02
$m_{\rho''}$	1899.4 ± 6.2 MeV	$1730^\dagger$ MeV	$1730^\dagger$ MeV	$1730^\dagger$ MeV	1822.2 ± 4.0 MeV	1896.2 ± 3.9 MeV
$\Gamma_{\rho''}$	406 ± 9 MeV	$260^\dagger$ MeV	$260^\dagger$ MeV	$260^\dagger$ MeV	125 ± 10 MeV	239 ± 11 MeV
$\mathrm{Re}[c_{\rho''}]$	-0.11 ± 0.00	-0.28 ± 0.00	-1.00 ± 0.04	-0.18 ± 0.01	$(-0.13 \pm 0.11) \cdot 10^{-1}$	$(0.78 \pm 0.14) \cdot 10^{-1}$
$\mathrm{Im}[c_{\rho''}]$	-0.35 ± 0.01	-0.32 ± 0.01	-0.29 ± 0.01	-0.31 ± 0.01	-0.25 ± 0.01	-0.31 ± 0.02