

Lower limit on dark matter mass from phase space density in dwarf galaxies

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**High Energy Astrophysics and Cosmology
in the era of all-sky surveys**

Ani Plaza Hotel, Yerevan, Armenia

Dark Matter properties from astrophysics

- 1 **stable** on cosmological time-scale
 - 2 (almost) **collisionless** to form ellipsoidal halos
 - 3 (almost) electrically **neutral** to be Dark
 - 4 **stability of globular stellar clusters** $M_X \lesssim 10^3 M_\odot \approx 10^{61} \text{ GeV}$ otherwise too strong tidal forces
 - 5 **confinement in a galaxy:** quantum physics!
- de Broglie wavelength: $\lambda = 2\pi / (M_X v_X) < l_{\text{galaxy}}$, for bosons
 in a galaxy $v_X \sim 0.5 \cdot 10^{-3}$ \longrightarrow $M_X \gtrsim 3 \cdot 10^{-22} \text{ eV}$ for fermions
- Pauli blocking: Tremaine–Gunn limit $M_X \gtrsim 750 \text{ eV}$

$$f(\mathbf{p}, \mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \Bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Microscopic processes in the expanding Universe

A **competition** between **scattering, decays, etc** and **expansion**

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \sum(\text{production} - \text{destruction})$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}(na^3) = a^3 \int \dots$

production:

$$\sigma(A + B \rightarrow X + C)n_A n_B, \quad \Gamma(D \rightarrow E + X)n_D \cdot M_D/E_D, \quad \text{etc}$$

destruction:

$$\sigma(A + X \rightarrow C + B)n_A n_X, \quad \Gamma(X \rightarrow F + G)n_X \cdot M_X/E_X, \quad \text{etc}$$

Fast direct and inverse processes, $\Gamma \gtrsim H$, are in equilibrium,
 $\Sigma(\) = 0$ and thermalize particles

Gas of free particles in the expanding Universe

homogeneous gas
in comoving coordinates:

$$dN = f(\mathbf{p}, t) d^3\mathbf{X} d^3\mathbf{p}$$

$$d^3\mathbf{x} = \text{const}, \quad d^3\mathbf{k} = \text{const}, \quad f(k) = \text{const}$$

$$f(k) d^3\mathbf{x} d^3\mathbf{k} = \text{const}$$

comoving volume equals physical volume

$$d^3\mathbf{x} d^3\mathbf{k} = d^3(a\mathbf{x}) d^3\left(\frac{\mathbf{k}}{a}\right) = d^3\mathbf{X} d^3\mathbf{p}$$

$$f(\mathbf{p}, t) = f(\mathbf{k}) = f[a(t) \cdot \mathbf{p}].$$

$$t = t_i : f_i(\mathbf{p}) \longrightarrow f(\mathbf{p}, t) = f_i\left(\frac{a(t)}{a(t_i)} \mathbf{p}\right)$$

$$f_i(\mathbf{p}) = f_{\text{Pl}}\left(\frac{|\mathbf{p}|}{T_i}\right) = \frac{1}{(2\pi)^3} \frac{1}{e^{|\mathbf{p}|/T_i} - 1}$$

$$f(\mathbf{p}, t) = f_{\text{Pl}}\left(\frac{a(t)|\mathbf{p}|}{a_i T_i}\right) = f_{\text{Pl}}\left(\frac{|\mathbf{p}|}{T_{\text{eff}}(t)}\right)$$

$$T_{\text{eff}}(t) = \frac{a_i}{a(t)} T_i$$

decoupling at $T \gg m$:

neutrinos, hot(warm) dark matter

decoupling at $T \ll m$: $f(\mathbf{p}) = \frac{1}{(2\pi)^3} \exp\left(-\frac{m-\mu_i}{T_i}\right) \exp\left(-\frac{a^2(t)\mathbf{p}^2}{2ma_i^2 T_i}\right)$

$$f(\mathbf{p}, t) = \frac{1}{(2\pi)^3} \exp\left(-\frac{m-\mu_{\text{eff}}}{T_{\text{eff}}}\right) \exp\left(-\frac{\mathbf{p}^2}{2mT_{\text{eff}}}\right)$$

$$T_{\text{eff}}(t) = \left(\frac{a_i}{a(t)}\right)^2 T_i, \quad \frac{m-\mu_{\text{eff}}(t)}{T_{\text{eff}}} = \frac{m-\mu_i}{T_i}$$

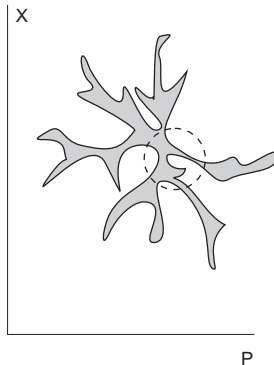
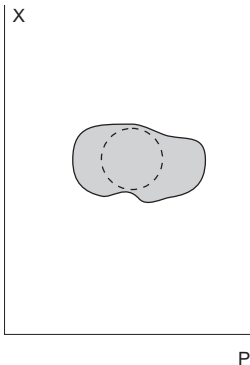
Liouville theorem for Dark Matter PSD

For a **decoupled component**

$$\frac{d}{dt} f(\mathbf{x}, \mathbf{p}, t) = \frac{\partial}{\partial t} f + \dot{\mathbf{p}} \frac{\partial}{\partial \mathbf{p}} f + \dot{\mathbf{x}} \frac{\partial}{\partial \mathbf{x}} f = 0$$

e.g. in the **expanding homogeneous Universe**, $p \propto 1/a(t)$ and $H(t) = \dot{a}/a$

$$\frac{\partial}{\partial t} f - H \mathbf{p} \frac{\partial}{\partial \mathbf{p}} f = 0$$

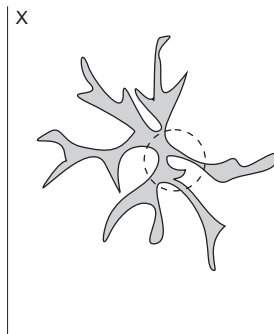
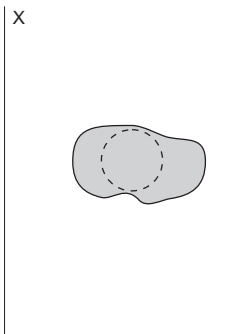


Observations: Coarse Grained PSD

- can ultimately determine the DM model !!
- in reality only averaged quantities are available:
coarse grained phase space density

$$Q \equiv \frac{\rho}{\sigma^3} \rightarrow f_{obs}, \quad Q_{theory} \rightarrow \frac{m_{DM}^4 \times n_{DM}}{p_{DM}^3}$$

- Take maximum of PSD, naturally $f_{obs}^{max} \leq f_{theory}^{max} \rightarrow m_{DM} > \dots$



Galactic dark halos:

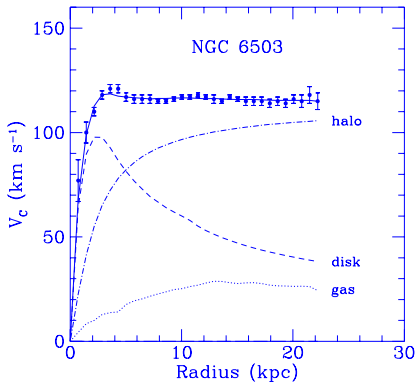
flat rotation curves

$$v(R) = \sqrt{G \frac{M(R)}{R}}$$

$$M(R) = 4\pi \int_0^R \rho(r) r^2 dr$$

we observe v_{los} by
Doppler and fit with
e.g.

$$\rho = \frac{\rho_0}{r(r^2 + r_0^2)}$$



fit to data:

$$r_0, \sigma = \sqrt{3} v_{los}, \rho_0, \rightarrow Q$$

DM model:

production mechanism gives $f(k, t_d) \rightarrow Q_{theory}$

Refined constraint for DM: phase space density

after decoupling $f_i = f_i(\kappa) = \text{const}$ and defines **psd**, which remains intact due to the Liouville theorem even in galaxies with inhomogeneous distribution in space

coarse grained phase space density:

$$f(\kappa, \mathbf{x}, t) \leq \max_{\kappa} f_i(\kappa)$$

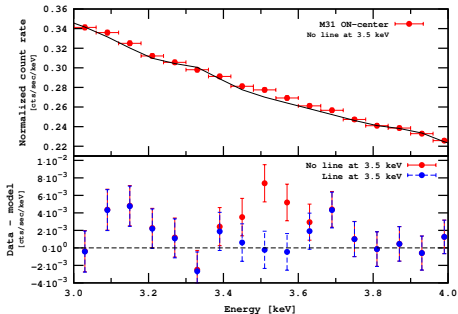
observation:

$$Q = \frac{\rho}{\langle v_{\parallel}^2 \rangle^{3/2}} \equiv \mathcal{Q} \cdot 1 \frac{M_{\odot}/\text{pc}^3}{(\text{km/s})^3} = \left(5 \cdot 10^{-3} - 2 \cdot 10^{-2} \right) \frac{M_{\odot}/\text{pc}^3}{(\text{km/s})^3}.$$

$$Q \simeq 3^{3/2} \frac{\rho_{DM}}{\langle v_{DM}^2 \rangle^{3/2}} = 3^{3/2} m^4 \frac{n}{\langle P^2 \rangle^{3/2}} = 3^{3/2} m^4 f(\mathbf{P}, \mathbf{x}).$$

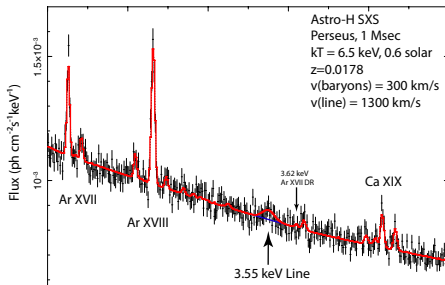
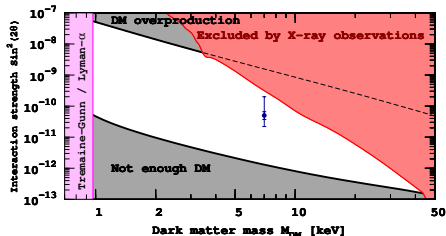
$$m^4 \gtrsim \frac{Q}{3^{3/2} \max f_i}$$

... 10 years ago: **Dark Matter** decay observed in X-ray?



Stacking signals from many galaxies, especially Perseus cluster, then Andromeda

1402.2301, 1402.4119



Refined constraint for DM: phase space density

for non-resonance production

$$f_\nu = f_{FD}(p/T_{prod}^{resc})$$

D.Gorbunov, A.Khmel'nitsky, V.Rubakov (2008)

F.Bezrukov, D.Gorbunov

$$m \gtrsim 6 \text{ keV} \cdot \left(\frac{0.2}{\Omega_{DM}} \right)^{1/3} \left(\frac{2}{5 \cdot 10^{-3}} \right)^{1/3} \left(\frac{g_*(T_d)}{43/4} \right)^{1/3},$$

for a set of darkest dwarf galaxies...

Can we improve today?

see e.g. 2010.03572

- much more data dozens of dwarf galaxies
- numerical tools to estimate galaxy DM profiles and velocities GravSphere
- alternative observables Excess Mass Function
- more realistic DM models many even for sterile neutrino DM

F.Bezrukov, D.Gorbunov, E.Koreshkova (2024...)

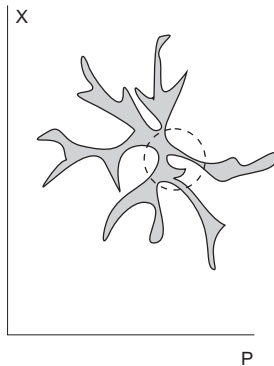
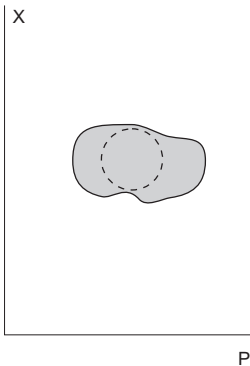
Observations: Excess Mass Function

astro-ph/0507300

- can ultimately determine the DM model !!
- in reality only averaged quantities are available:
coarse grained phase space density

$$D(F) \equiv \int d^3\mathbf{x} d^3\mathbf{p} (f(\mathbf{x}, \mathbf{p}, t) - F) \Theta(f - F)$$

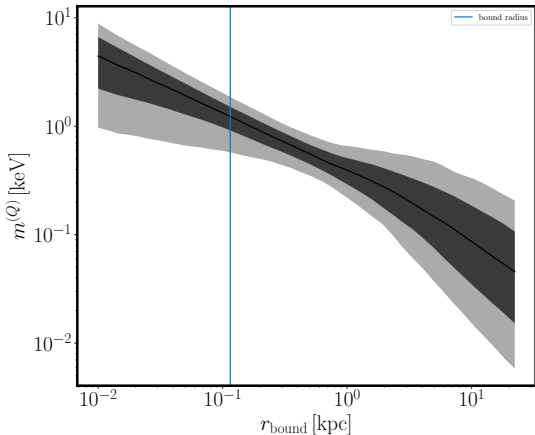
- Take maximum of EMF, naturally for any F we get $D_{obs} \leq D_{theory} \rightarrow m_{DM} > \dots$



Object	M	$+e_M$	$-e_M$	m	$+e_m$	$-e_m$
Sculptor	4.75	0.23	0.26	3.76	0.19	0.19
Fornax	2.11	0.05	0.06	1.67	0.04	0.04
Carina	4.98	0.29	0.32	3.95	0.24	0.24
NGC 2419	34.81	1.29	1.40	27.59	1.06	1.06
UMa II	8.90	0.62	0.72	7.05	0.52	0.52
Leo T	7.61	0.70	0.85	6.03	0.60	0.60
Segue 1	32.34	3.05	3.76	25.63	2.64	2.64
Leo I	4.87	0.24	0.26	3.86	0.20	0.20
Sextans	3.86	0.20	0.23	3.06	0.17	0.17
UMa I	5.70	0.28	0.32	4.52	0.24	0.24
Willman 1	28.61	4.60	3.82	22.68	4.25	2.63
Leo II	7.37	0.26	0.28	5.84	0.21	0.21
Leo V	19.82	4.75	7.25	15.71	4.67	3.90
Leo IV	12.17	1.94	2.89	9.64	1.79	1.79
ComBer	14.78	0.94	1.08	11.72	0.79	0.79
CVn II	14.98	1.72	2.24	11.87	1.52	1.52
CVn I	3.68	0.09	0.10	2.92	0.08	0.08
Bootes II	17.00	3.52	6.33	13.47	3.38	3.38
Bootes I	9.59	1.09	0.74	7.60	0.96	0.54
Munoz 1	196.14	36.24	58.82	155.46	34.00	34.00
UMi	3.66	0.15	0.16	2.90	0.12	0.12
Hercules	7.65	0.68	0.83	6.06	0.59	0.59
Draco	5.70	0.24	0.27	4.52	0.20	0.20
NGC 7492	87.58	19.81	39.48	69.42	19.30	19.30

1806.06891

Improved GravSphere and DM profiles



The code solves the Radial Jeans Equation for stars

$$\frac{1}{\rho_*} \frac{\partial}{\partial r} (\rho_* \sigma_{r*}^2) + 2 \frac{\beta_* \sigma_{r*}^2}{r} = - \frac{GM_{tot}(r)}{r^2}$$

we incorporated into the code the DM part

$$\frac{1}{\rho} \frac{\partial}{\partial r} (\rho \sigma_r^2) + 2 \frac{\beta \sigma_r^2}{r} = - \frac{GM_{tot}(r)}{r^2}$$

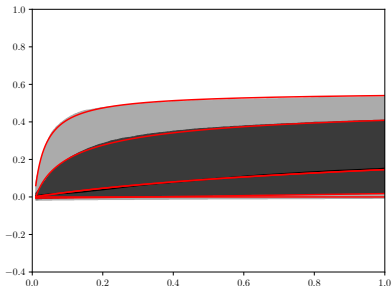
with possible **asymmetry** in velocities

$$\beta = 1 - \frac{\sigma_\tau^2}{\sigma_r^2}$$

Sculptor

for dwarf galaxies we always find $M_* \ll M_{DM} \approx M_{tot}$

Different β s: observations vs simulations



2010.03572

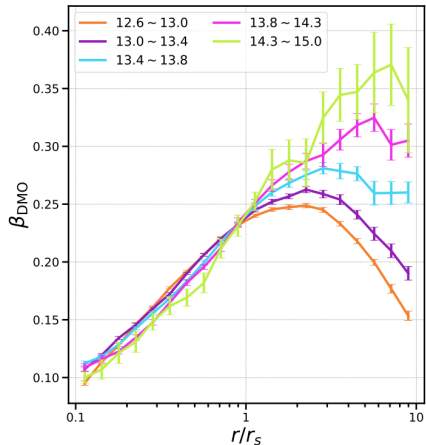
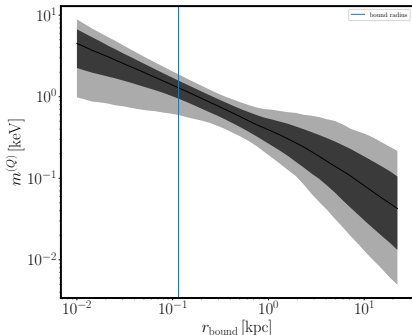
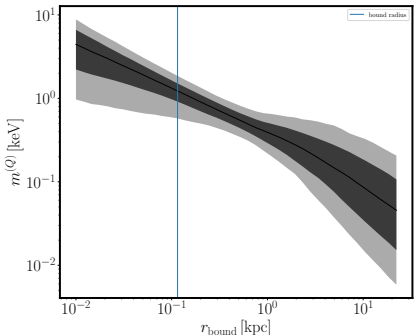


Figure 5. Similar to Figure 2, but for dark matter particles from TNG300-Dark.

Different priors for anisotropy β : Sculptor



changes in the estimates are very mild

Method	$m(\text{old})/\text{keV}$	\rightarrow	$m(\text{new})/\text{keV}$
Q	1.23	\rightarrow	1.28
EMF	1.77	\rightarrow	1.96

Thermal-like spectrum

Name	m_Q [keV]	1sl	1su	2sl	2su	m_EMF [keV]	1sl	1su	2sl	2su
Andromeda V	1.225203	0.915404	1.508837	0.574870	1.861966	3.284792	2.644467	3.855428	1.778734	4.748085
Aquarius Dwarf	1.044913	0.625182	1.545089	0.361816	2.047351	2.695354	1.726186	3.754703	1.048958	4.832258
Bootes Dwarf Spheroidal Galaxy	1.251969	0.957058	1.569541	0.598466	1.933975	3.122932	2.594244	3.686279	1.679004	4.477893
Carina dSph	1.225575	0.689700	1.822694	0.381416	2.185220	3.063947	1.791050	4.360144	1.015114	5.213309
Cetus Dwarf Galaxy	1.294163	0.970726	1.635405	0.613755	2.037867	3.453562	2.826858	4.160696	2.007532	5.154994
Coma Dwarf Galaxy	0.599414	0.489215	0.752504	0.359349	0.996540	1.348781	1.175112	1.663179	0.885092	2.275956
CVn I dSph	0.968654	0.559121	1.668725	0.307919	2.287392	2.521491	1.521478	4.243809	0.878673	5.813495
Dra dSph	0.975814	0.599496	1.361359	0.350361	1.723500	2.352959	1.530362	3.078809	0.925938	3.881244
Fornax Dwarf Spheroidal	0.801772	0.564298	1.081818	0.346644	1.301402	2.087832	1.516204	2.763630	0.943984	3.401473
Hercules Dwarf Galaxy	3.071394	1.712936	4.951560	0.766637	7.366827	8.444337	5.101316	12.857755	2.651494	18.541500
Leo A	1.259895	0.929238	1.645328	0.617181	2.113699	3.197134	2.505659	4.002589	1.782953	5.184168
NGC 6822	0.190343	0.112368	0.391769	0.064555	0.614813	0.439151	0.256330	0.963746	0.145410	1.618612
PegDIG	0.398304	0.271357	0.518844	0.163097	0.668478	1.042921	0.734041	1.337502	0.452132	1.749286
Sculptor Dwarf Galaxy	1.767212	1.262249	2.169552	0.819836	2.463576	4.562855	3.470384	5.333576	2.376466	5.864807
Sextans dSph	0.901622	0.409543	1.312771	0.215919	1.600582	2.358999	1.071764	3.313751	0.569674	4.049504
Sgr dIG	1.123976	0.561329	1.595040	0.246589	2.228213	2.872287	1.514923	3.733869	0.681142	5.134155
UMI Galaxy	1.886494	1.134251	2.610104	0.558499	3.220944	4.759525	3.072335	5.992665	1.574418	6.923074
WLM Galaxy	0.413615	0.260354	0.600323	0.169230	0.786573	1.046321	0.691461	1.467004	0.471625	1.943515
Z 64-73	1.205084	0.917226	1.458488	0.580151	1.702723	3.141063	2.546489	3.614556	1.700633	4.196149
Z 126-111	1.675490	1.094263	2.162382	0.632936	2.619640	4.316711	3.000453	5.166667	1.875650	5.940151

a model from 1909.13328

Name	m_K [keV]	1sl	1su	2sl	2su	m_EMF(K) [keV]	1sl	1su	2sl	2su
Andromeda V	2.166578	1.796353	2.476913	1.332008	2.835453	3.497362	3.013851	3.891382	2.285047	4.460284
Aquarius Dwarf	1.955848	1.405823	2.515008	0.989099	3.013848	3.086111	2.305718	3.834070	1.661166	4.525860
Bootes Dwarf Spheroidal Galaxy	2.196888	1.848482	2.540523	1.366902	2.905469	3.398433	3.000867	3.795834	2.260441	4.316036
Carina dSph	2.167001	1.497445	2.796861	1.023219	3.142795	3.356434	2.371411	4.223046	1.643635	4.749298
Cetus Dwarf Galaxy	2.244203	1.865410	2.608554	1.389250	3.004866	3.613793	3.142022	4.089167	2.408145	4.708254
Coma Dwarf Galaxy	1.368294	1.200776	1.583733	0.984759	1.897149	1.995507	1.817161	2.291240	1.511719	2.810364
CVn I dSph	1.862849	1.308433	2.642596	0.891676	3.236487	2.954950	2.128702	4.137574	1.486951	5.077721
Dra dSph	1.871689	1.368414	2.318433	0.968853	2.698037	2.837803	2.145387	3.387067	1.549997	3.942571
Fornax Dwarf Spheroidal	1.649634	1.316208	1.999979	0.962233	2.252265	2.617940	2.126266	3.141072	1.566014	3.595396
Hercules Dwarf Galaxy	3.911617	2.687394	5.317275	1.602791	6.864464	6.401905	4.561080	8.426099	2.795671	10.698317
Leo A	2.205819	1.813758	2.618718	1.394230	3.076278	3.448015	2.936049	3.996924	2.338088	4.736780
NGC 6822	0.654507	0.466410	1.040988	0.326606	1.390789	0.967468	0.684420	1.600621	0.475612	2.232013
PegDIG	1.052118	0.822086	1.247035	0.592631	1.467659	1.677072	1.333966	1.973172	0.974077	2.349510
Sculptor Dwarf Galaxy	2.741830	2.208468	3.128291	1.673432	3.394612	4.328428	3.613612	4.800666	2.810353	5.122451
Sextans dSph	1.778920	1.071108	2.264894	0.709763	2.572709	2.830309	1.703953	3.530583	1.134394	4.025837
Sgr dIG	2.049740	1.311752	2.566979	0.773028	3.177446	3.216783	2.124347	3.828938	1.267948	4.718027
UMi Galaxy	2.859409	2.061766	3.523065	1.307497	4.033015	4.452598	3.345519	5.191568	2.167458	5.729640
WLM Galaxy	1.077942	0.800498	1.369627	0.606863	1.629462	1.682441	1.284760	2.097395	1.000164	2.520167
Z 64-73	2.143640	1.798651	2.423457	1.339861	2.677083	3.403471	2.958881	3.738168	2.260460	4.127944
Z 126-111	2.649478	2.014739	3.121641	1.417007	3.531334	4.177470	3.291756	4.711143	2.400282	5.183088

a model from 1909.13328

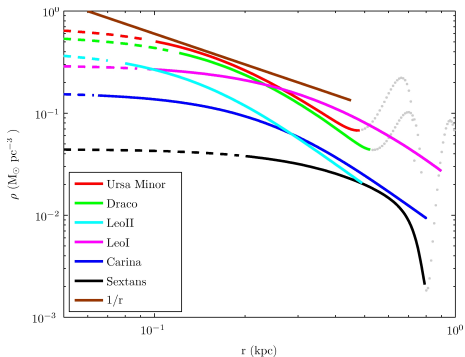
Name	m_LRT [keV]	1sl	1su	2sl	2su	m_EMF(LRT) [keV]	1sl	1su	2sl	2su
Andromeda V	4.507249	3.622139	5.269110	2.555246	6.169278	6.752091	5.649359	7.693878	4.095486	9.065615
Aquarius Dwarf	4.000052	2.721200	5.363776	1.805599	6.244449	5.852572	4.141401	7.593104	2.819200	9.281037
Bootes Dwarf Spheroidal Galaxy	4.580899	3.745064	5.427315	2.633511	6.347368	6.569339	5.639058	7.531188	4.042654	8.806768
Carina dSph	4.508276	2.929211	6.071428	1.878472	6.956278	6.472618	4.299422	8.507876	2.797349	9.806104
Cetus Dwarf Galaxy	4.696208	3.785106	5.597248	2.683810	6.601421	7.020006	5.930011	8.160952	4.370107	9.679779
Coma Dwarf Galaxy	2.636639	2.264021	3.127067	1.796358	3.860349	3.580224	3.178412	4.235578	2.555389	5.402206
CVn I dSph	3.779045	2.502562	5.682561	1.599863	7.198815	5.558407	3.776442	8.259350	2.478283	10.536007
Dra dSph	3.799976	2.636910	4.877924	1.762554	5.821890	5.340369	3.831135	6.617380	2.613791	7.946463
Fornax Dwarf Spheroidal	3.279395	2.519921	4.105549	1.748511	4.715895	4.827822	3.775394	5.990737	2.639348	7.034315
Hercules Dwarf Galaxy	8.979604	5.795106	12.847306	3.171012	17.306765	13.637979	9.160178	18.891757	5.224936	25.115715
Leo A	4.602633	3.663117	5.622700	2.695038	6.784813	6.673873	5.499923	7.978406	4.200892	9.794507
NGC 6822	1.115342	0.751169	1.916585	0.495682	2.687279	1.530600	1.022093	2.744211	0.669068	4.038444
PegDIG	1.940513	1.455162	2.366100	0.993322	2.861349	2.875632	2.191797	3.493320	1.515154	4.299150
Sculptor Dwarf Galaxy	5.932285	4.609081	6.918837	3.334654	7.610795	8.682951	6.997018	9.854807	5.206341	10.709432
Sextans dSph	3.581162	1.981437	4.746760	1.225953	5.507620	5.283636	2.921857	6.868583	1.816239	8.041357
Sgr dIG	4.224967	2.509971	5.493312	1.354365	7.045839	6.148904	3.769990	7.614397	2.061130	9.792785
UMI Galaxy	6.230130	4.253901	7.947836	2.500474	9.305569	8.991945	6.401576	10.868336	3.848440	12.334142
WLM Galaxy	1.996194	1.410681	2.639637	1.021206	3.232658	2.892036	2.099899	3.763137	1.562020	4.687933
Z 64-73	4.451625	3.627545	5.136682	2.572831	5.769173	6.555397	5.538444	7.359589	4.038036	8.315071
Z 126-111	5.699830	4.140920	6.901681	2.746473	7.969604	8.332748	6.276608	9.670832	4.331168	10.933744

- It seems indeed the present data can be used to place a lower limit on DM mass
- For sterile neutrino it is in 3-6 keV range depending on the production mechanism
- EMF always gives noticeably stronger bounds (upto 50%) than widely used Q_{max}

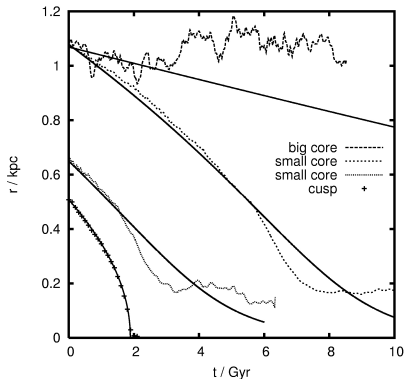
CDM Problems at small-scales ... ?

- NFW profile fits nicely DM in galaxy clusters $\rho \propto r^{-1}(r+r_c)^{-2}$
- Dwarf galaxy density profiles: $\rho_M(r) \propto r^{-(0.5-1.5)}$ cusp
most DM-dominated objects

Cores observed (?)

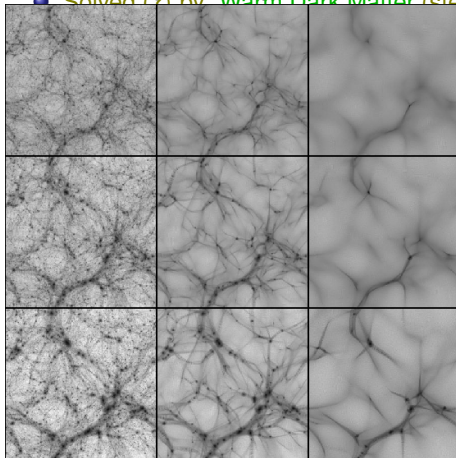


5 Clusters in the Fornax dSph



CDM Problems ... ?

- Missing satellites: $\frac{dN_{obj}}{d \ln M} \propto \frac{1}{M}$ no-scale 100 instead of 1000
- “Too big to fail” problem
- Solved (?) by Warm Dark Matter (sterile neutrino, gravitino) free-streaming



$$\left(\frac{dN_{obj}}{d \ln M} \right)_{WDM} / \left(\frac{dN_{obj}}{d \ln M} \right)_{CDM}$$

Three Generations of Matter (Fermions) spin $\frac{1}{2}$

	I	II	III
mass →	2.4 MeV	1.27 GeV	171.2 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
name →	Left u Right up	Left c Right charm	Left t Right top
Quarks	4.8 MeV	104 MeV	4.2 GeV
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
	Left d Right down	Left s Right strange	Left b Right bottom
Leptons	<0.0001 eV ~ 10 keV	~ 0.01 eV \sim GeV	~ 0.04 eV \sim GeV
	0	0	0
	Left ν_e Right N_1	Left ν_μ Right N_2	Left ν_τ Right N_3
	electron neutrino sterile neutrino	muon neutrino sterile neutrino	tau neutrino sterile neutrino
	0.511 MeV	105.7 MeV	1.777 GeV
	-1	-1	-1
Left e Right electron	Left μ Right muon	Left τ Right tau	

Bosons (Forces) spin 1	0	g	gluon	
	0	γ	photon	
	91.2 GeV	0	Z⁰	weak force
	80.4 GeV	± 1	W[±]	weak force
	>114 GeV	0	H	Higgs boson
				spin 0

Seesaw mechanism: $M_N \gg 1 \text{ eV}$

With $m_{\text{active}} \lesssim 1 \text{ eV}$ we work in the seesaw (type I) regime:

$$\mathcal{L}_N = \bar{N} i \not{\partial} N - f \bar{L}_e^c \tilde{H} N - \frac{M_N}{2} \bar{N}^c N + \text{h.c.}$$

Higgs gains $\langle H \rangle = v/\sqrt{2}$ and then

$$\mathcal{Y}_N = \frac{1}{2} (\bar{\nu}_e, \bar{N}^c) \begin{pmatrix} 0 & v \frac{f}{\sqrt{2}} \\ v \frac{f}{\sqrt{2}} & M_N \end{pmatrix} \begin{pmatrix} \nu_e \\ N \end{pmatrix} + \text{h.c.}$$

For a hierarchy $M_N \gg M^D = v \frac{f}{\sqrt{2}}$ we have

flavor state $\nu_e = U \nu_1 + \theta N$ with $U \approx 1$ and

active-sterile mixing: $\theta = \frac{M^D}{M_N} = \frac{v f}{2 M_N} \ll 1$

and mass eigenvalues

$$\approx M_N \quad \text{and} \quad -m_{\text{active}} = \theta^2 M_N \lll M_N$$

Sterile neutrino: a vast region of mass

Within the seesaw paradigm, as far as

$$m_a \sim \frac{f^2 v^2}{M_N^2} M_N \sim \theta^2 M_N$$

Any set

(mass scale M_N , Yukawa coupling f)

is viable

And with special tuning or symmetry larger (but not smaller) mixing

is viable

$$\hat{m}_a \sim \hat{f}^T \frac{1}{\hat{M}_N} \hat{f} v^2$$

Sterile neutrino: well-motivated keV-mass Dark Matter

- massive fermions giving mass to active neutrino through mixing (seesaw)

$$m_a \sim \frac{f^2 v^2}{M_N^2} M_N \sim \theta^2 M_N$$

- unstable, $N \rightarrow \nu\nu\nu$ is always open
but exceeding the age of the Universe if

(applicable for $M_N < M_W$)

$$\tau_{N \rightarrow 3\nu} \sim 1 / \left(G_F^2 M_N^5 \theta_{\alpha N}^2 \right) \implies \theta^2 < 1.5 \times 10^{-7} \left(\frac{50 \text{ keV}}{M_N} \right)^5$$

- with seesaw constraint $m_a \sim \theta^2 M_N$

$$\tau_{N \rightarrow 3\nu} \sim 1 / \left(G_F^2 M_N^4 m_\nu \right) \sim 10^{11} \text{ yr} (10 \text{ keV} / M_N)^4$$

Sterile neutrino: indirect searches

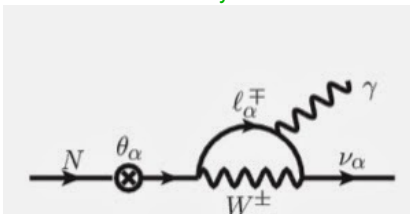
$$m_a \sim \frac{f^2 v^2}{M_N^2} M_N \sim \theta^2 M_N$$

- **unstable**, but exceeding the age of the Universe if

$$\frac{\theta^2}{3 \times 10^{-3}} < \left(\frac{10 \text{ keV}}{M_N} \right)^5$$

- **DM sterile neutrinos can be searched at X-ray telescopes because of two-body radiative decay** give limits in absence of the feature

a narrow line ($\delta E_\gamma / E_\gamma \sim \nu \sim 10^{-3}$)
 at photon frequency $E_\gamma = M_N/2$



$$\frac{\theta^2}{10^{-11}} \lesssim \left(\frac{10 \text{ keV}}{M_N} \right)^4$$

$$F_\gamma \propto \Gamma_N \rho_N / M_N \dots$$

Seesaw neutrino can not serve as DM !!

$$\frac{\theta^2}{10^{-11}} \lesssim \left(\frac{10 \text{ keV}}{M_N} \right)^4$$

one order down

$$\frac{m_a}{0.1 \text{ eV}} \sim \frac{\theta^2}{10^{-5}} \frac{M_N}{10 \text{ keV}}$$

$$\frac{\theta^2}{10^{-7}} \lesssim \left(\frac{1 \text{ keV}}{M_N} \right)^4$$

and one more...

$$\frac{m_a}{0.1 \text{ eV}} \sim \frac{\theta^2}{10^{-4}} \frac{M_N}{1 \text{ keV}}$$

$$\frac{\theta^2}{10^{-3}} \lesssim \left(\frac{0.1 \text{ keV}}{M_N} \right)^4$$

$$\frac{m_a}{0.1 \text{ eV}} \sim \frac{\theta^2}{10^{-3}} \frac{M_N}{0.1 \text{ keV}}$$

How light can be this dark matter ?

Sterile neutrino production in the early Universe

- before the EW transition, $T > T_{EW}$

$$H \rightarrow L + N, \quad \frac{\Gamma_{H \rightarrow \nu_a N}}{H} \simeq \frac{f_\nu^2}{16\pi} \frac{T}{H} \ll 1,$$

- after the EW transition, $T < T_{EW}$

- 1 r.h. neutrino production in scatterings

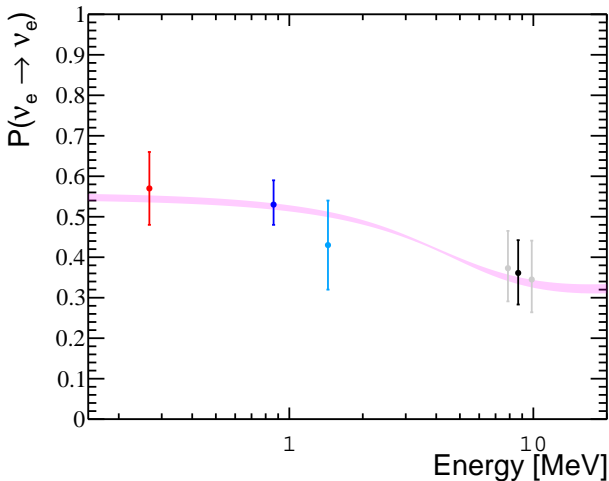
$$\nu_L + X \rightarrow N_R + Y, \quad \Gamma \propto \frac{M_D^2}{T^2}$$

- 2 sterile neutrino production in oscillations

Neutrino matter effect:

asymmetry

Mikheev–Smirnov–Wolfenstein effect



BOREXINO measurements of solar neutrino flux

Fermi charged currents

$$\mathcal{L} = -2\sqrt{2}G_F \bar{\nu}_e \gamma^\mu e \cdot \bar{e} \gamma_\mu \nu_e$$

only matter, no currents

$$\langle \langle \bar{e}_k \gamma_{kl}^0 e_l \rangle \rangle = \langle \langle e^\dagger e \rangle \rangle = n_e,$$

$$\langle \langle \bar{e}_k \gamma_{kl}^j e_l \rangle \rangle = 0.$$

$$\langle \langle e_k \bar{e}_l \rangle \rangle = -\frac{1}{4} \gamma_{kl}^0 \cdot n_e$$

Fermi interaction gives

$$\mathcal{L}_{eff} = -\sqrt{2}G_F n_e \bar{\nu}_e \gamma^0 \nu_e.$$

$$i\gamma^0 \partial_0 \rightarrow i\gamma^0 \partial_0 - \sqrt{2}G_F n_e \gamma^0,$$

effective potential

$$i\partial_0 - V, \text{ with } V = \sqrt{2}G_F n_e$$

competes with

$$H_{eff} = \Delta m^2 / 2E$$

Production in oscillations

$$\frac{\partial}{\partial t} f_s(t, \mathbf{p}) - H\mathbf{p} \frac{\partial}{\partial \mathbf{p}} f_s(t, \mathbf{p}) = \Gamma_\alpha P(v_\alpha \rightarrow v_s) f_\alpha(t, \mathbf{p}).$$

$\Gamma_\alpha \propto G_F^2 T^4 E$ is the **weak interaction** rate in plasma

$$P(v_\alpha \rightarrow v_s) = \sin^2 2\theta_\alpha^{\text{mat}} \cdot \sin^2 \left(\frac{t}{2t_\alpha^{\text{mat}}} \right),$$

$$t_\alpha^{\text{mat}} = \frac{t_\alpha^{\text{vac}}}{\sqrt{\sin^2 2\theta_\alpha + (\cos 2\theta_\alpha - V_{\alpha\alpha} \cdot t_\alpha^{\text{vac}})^2}},$$

$$\sin 2\theta_\alpha^{\text{mat}} = \frac{t_\alpha^{\text{mat}}}{t_\alpha^{\text{vac}}} \cdot \sin 2\theta_\alpha, \quad t_\alpha^{\text{vac}} = \frac{2E}{M_N^2}$$

sign of the **effective plasma potential** matters:

$$V_{\alpha\alpha} < 0 \implies \text{mixing gets suppressed}$$

$$V_{\alpha\alpha} > 0 \implies \text{amplification via resonance}$$

DM from oscillations:

(DW & ShF)

$$(\cos 2\theta_\alpha - V_{\alpha\alpha} \cdot t_\alpha^{\text{vac}})^2$$

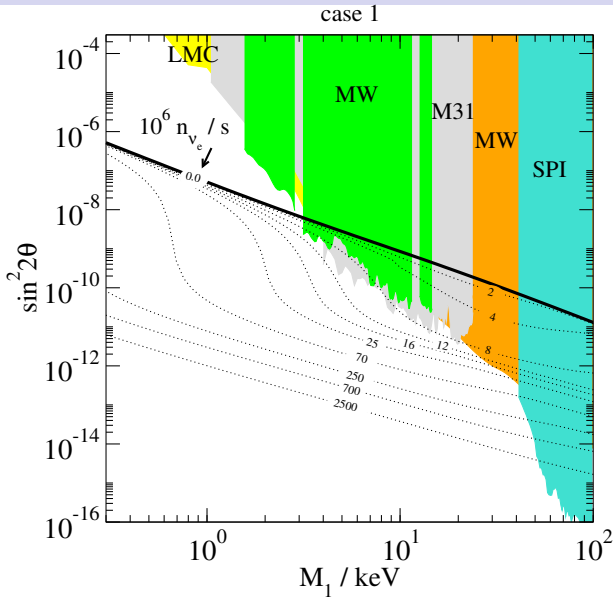
non-resonant:

$$V_{\alpha\alpha} \sim -\# G_F^2 T^4 E$$

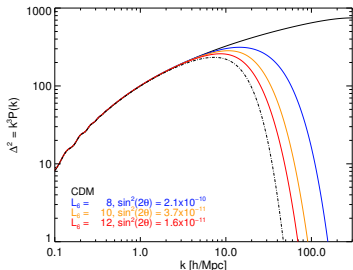
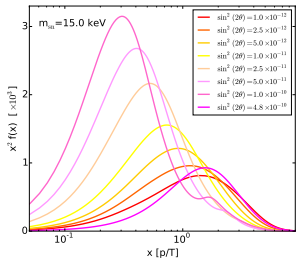
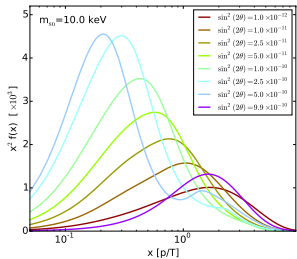
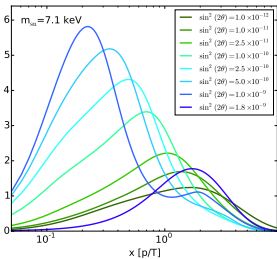
resonant production in the lepton asymmetric plasma

$$V_{\alpha\alpha} \sim +\# G_F T^2 \mu_{L_\alpha}$$

1601.07553



Sterile neutrino spectra from oscillations



1601.07553

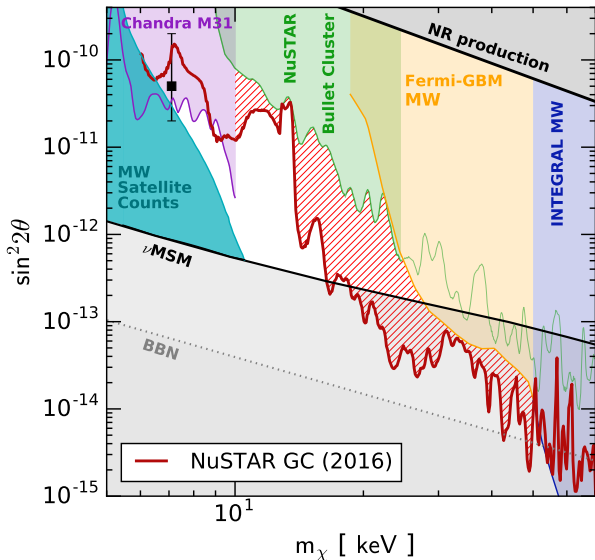
non-resonant production:
thermal-shape spectrum

1611.00005

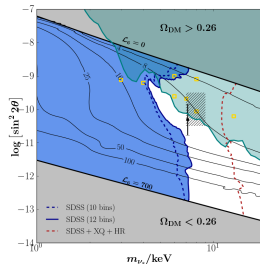
$$v = \frac{\langle p \rangle}{m} = 3.15 \frac{T}{m} \left(\frac{g_*.0}{g_*} \right)^{1/3}$$

... present searches

1609.00667, 1706.03118



- upper limits on mixing: from X-ray searches
- lower limits on mass: from structure formation with $p_N \sim T$, DM free streaming too fast at $T = 1$ eV



Closing sterile neutrino DM?

In a minimal variant, may be...

But situation changes with just 1 new d.o.f.

- reopening large mixings with $\Omega_N < \Omega_{DM}$

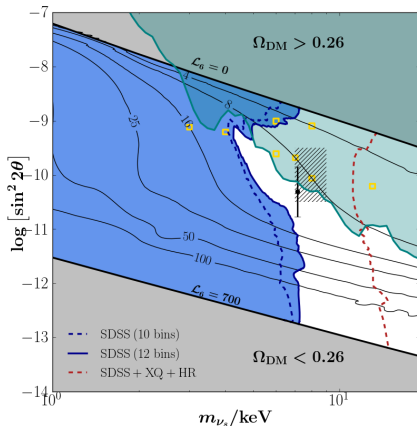
to avoid X-ray bounds:

$$\theta_{X\text{-ray}}^2 = \theta_{\alpha I}^2 \frac{\Omega_N}{\Omega_{DM}}$$

- reopening of small masses with $V_N \ll V_{WDM}$,

e.g. cold sterile neutrino

production not from the SM plasma particles



Larger mixing: Suppression of production

Form only a fraction of DM !!

$$P(\nu_\alpha \rightarrow \nu_s) = \sin^2 2\theta_\alpha^{\text{mat}} \cdot \sin^2 \left(\frac{t}{2t_\alpha^{\text{mat}}} \right), \quad \sin 2\theta_\alpha^{\text{mat}} = \frac{t_\alpha^{\text{mat}}}{t_\alpha^{\text{vac}}} \cdot \sin 2\theta_\alpha,$$

$$t_\alpha^{\text{mat}} = \frac{t_\alpha^{\text{vac}}}{\sqrt{\sin^2 2\theta_\alpha + (\cos 2\theta_\alpha - V_{\alpha\alpha} \cdot t_\alpha^{\text{vac}})^2}}, \quad t_\alpha^{\text{vac}} = \frac{2E}{M_N^2}$$

Most efficient production occurs at

(DW)

$$T_{\text{max}} \approx 133 \text{ MeV} \left(\frac{1 \text{ keV}}{M_N} \right)^{1/3}$$

It is suppressed if $T_{\text{reh}} \ll T_{\text{max}}$

G.Gelmini, S.Palomares-Ruiz, S.Pascoli (2004)

Suppression of cosmological production

Add more ingredients e.g.

$$\bar{L}\tilde{H}N + M_N\bar{N}^c N \rightarrow \bar{L}\tilde{H}N + \phi\bar{N}^c N$$

Scalar? Majoron?

(lepton symmetry)

$$P(\nu_\alpha \rightarrow \nu_s) = \sin^2 2\theta_\alpha^{\text{mat}} \cdot \sin^2 \left(\frac{t}{2t_\alpha^{\text{mat}}} \right), \quad \sin 2\theta_\alpha^{\text{mat}} = \frac{t_\alpha^{\text{mat}}}{t_\alpha^{\text{vac}}} \cdot \sin 2\theta_\alpha,$$

$$t_\alpha^{\text{mat}} = \frac{t_\alpha^{\text{vac}}}{\sqrt{\sin^2 2\theta_\alpha + (\cos 2\theta_\alpha - V_{\alpha\alpha} \cdot t_\alpha^{\text{vac}})^2}}, \quad t_\alpha^{\text{vac}} = \frac{2E}{M_N^2}$$

Coupling to scalar can change
the effective neutrino Hamiltonian in the primordial plasma

$$\begin{pmatrix} V_{\alpha\alpha} & M_D \\ M_D & V_{NN} + M_N \end{pmatrix}$$

Suppression of production with $\phi \bar{N}^c N$

- strong coupling to scalar or Majoron, which decreases the active-sterile mixing in primordial plasma

e.g. L.Bento, Z.Berezhiani (2001)

$$\phi NN \rightarrow G \bar{N} N \bar{N} N \rightarrow V_{NN}$$

- homogeneous $\phi = \phi(t)$ makes sterile neutrino mass changing in cosmology, which suppresses the early-time oscillations

F.Bezrukov, A.Chudaykin, D.G. (2017)

$$\phi(t) NN \rightarrow M_N = M_N(t) = M_N(T)$$

- ▶ sterile neutrinos are massless in the early Universe
- ▶ sterile neutrinos are superheavy in the early Universe

Massless in the early Universe

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \frac{f}{2} \phi \bar{N}^c N + \text{h.c.}$$

And may be more scalar fields in the hidden sector... to make the phase transition:

$$T > T_c \implies \langle \phi \rangle = 0, \quad M_N = 0$$

$$T < T_c \implies \langle \phi \rangle = v_\phi, \quad M_N = f v_\phi$$

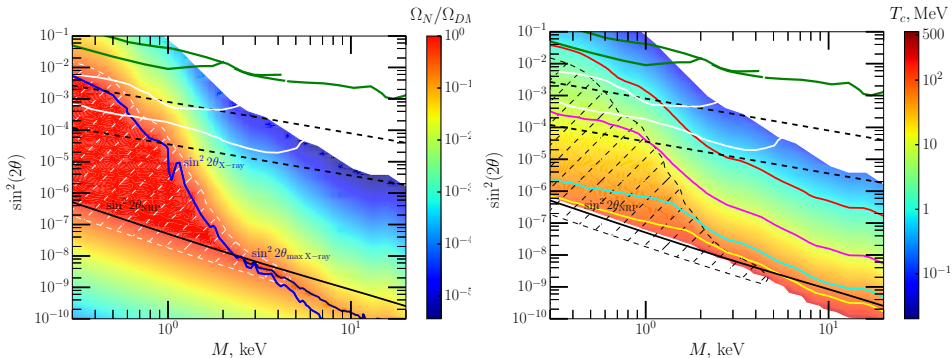
So the neutrino is pure Dirac fermion at the beginning...

The production in oscillations will be suppressed, if

$$T_c < T_{max} \approx 133 \text{ MeV} \left(\frac{1 \text{ keV}}{M_N} \right)^{1/3}$$

there is always a chirality flip contribution $\propto M_D^2/E^2$

Results: large mixing is allowed

for details see [1705.02184](#)

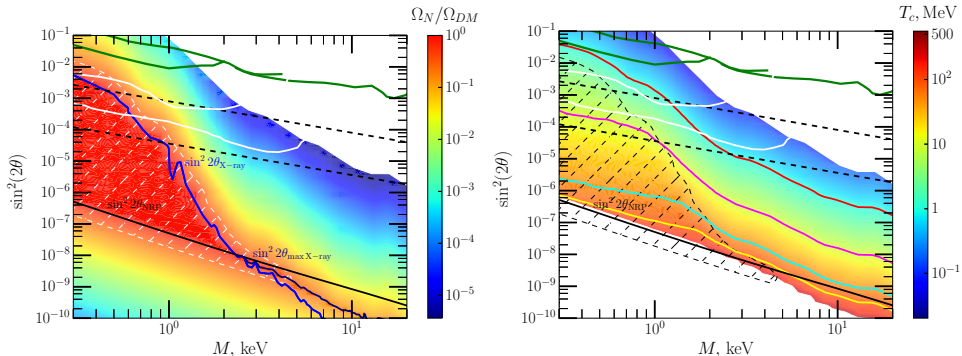
Important:

- 1 seesaw light sterile neutrino (dashed lines: $m_a \sim 0.008 - 0.2$ eV)
- 2 can be directly tested !! (between green and white lines)

$$m_a \sim \theta^2 M_N$$

Results

for details see [1705.02184](#)



Important:

- 1 seesaw light sterile neutrino (dashed lines for $m_a = 0.2$ eV and $m_a = 0.009$ eV)
- 2 can be directly tested !! (green and white lines)
- 3 produced sterile neutrinos are warm (not thermal-like spectrum !!), and hence most probably can form only a fraction of DM

Back to oscillations: superheavy at early times

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2 + \frac{f}{2} \phi \bar{N}^c N + \text{h.c.}$$

homogeneous scalar field in FLRW expanding Universe

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

two-stage evolution:

$$m_\phi < H(t) \implies \phi = \phi_i = \text{const}$$

$$m_\phi > H(t) \implies \rho = \langle E_k \rangle - \langle E_p \rangle = 0, \quad \rho \sim m_\phi^2 \phi^2 \propto 1/a^3$$

- At $m_\phi < H(t)$ sterile neutrino mass is $M = M_N + f\phi_i \gg M_N$
- At present sterile neutrino mass is $M_N \sim 1 \text{ keV}$
- If at $m_\phi > H(t)$ sterile neutrinos are nonrelativistic most time, $m_\phi = H_{osc} = \frac{T_{osc}^2}{M_{Pl}}$

$$M(t) = M_N + f\phi_i \frac{T^3}{T_{osc}^3} > T$$

Cool and Cold sterile neutrinos

1809.09123

sterile neutrino mass

$$M(t) = M_N + f\phi(t) = M_N + f\phi_i \frac{T^3}{T_{OSC}^3} \cos(m_\phi t)$$

1) sometimes crosses zero, which allows for sterile neutrino production by a 'slow' oscillator $m_\phi \ll M_N$

the produced sterile neutrinos are almost at rest

Cold Dark Matter

avoiding limits from structure formation

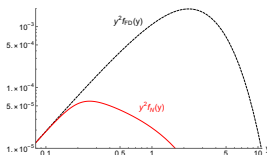
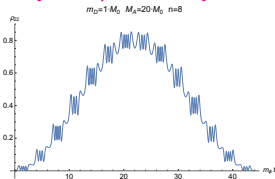
avoiding X-ray limits with tiny mixing angle

2) Both L_{OSC} and θ_{eff} change with $M(t)$, which oscillates !!

resonance

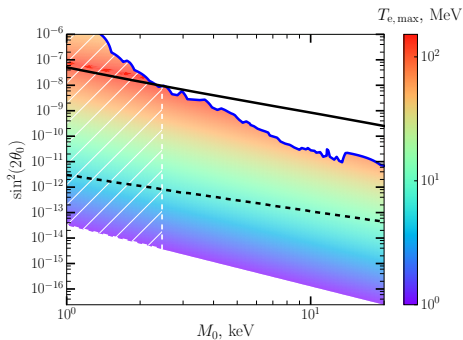
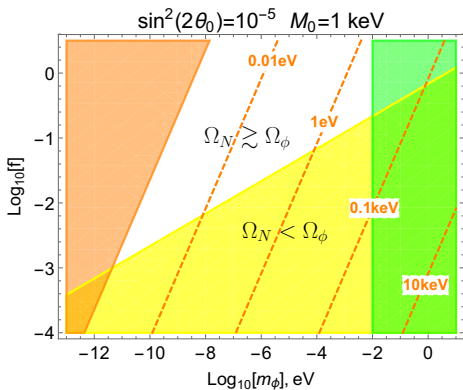
very complicated system: three oscillators with time-dependent couplings

cool



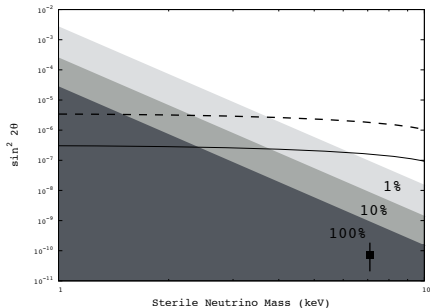
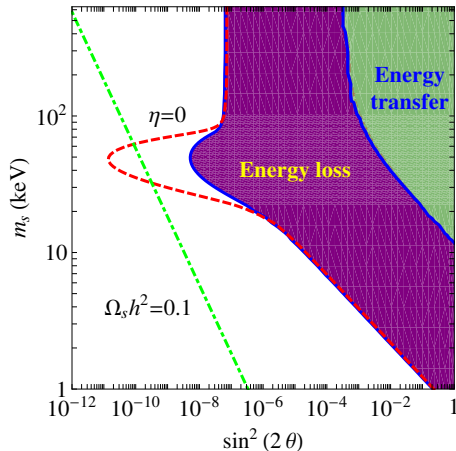
Allowed regions for each mechanism

1809.09123



Backup slides

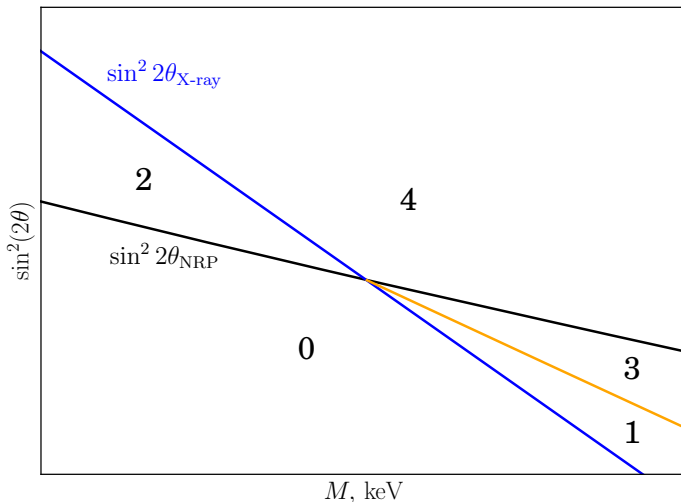
Limits from SN



1102.5124

1603.05503

A sketch of model parameter space

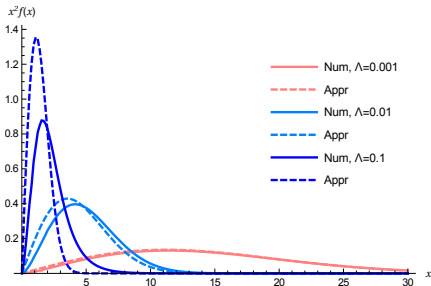


0,1: allowed even w/o scalar field

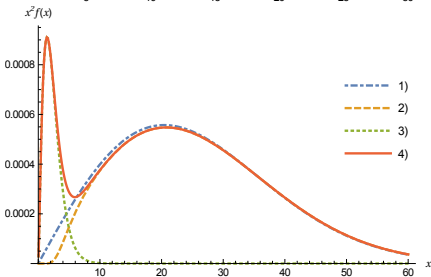
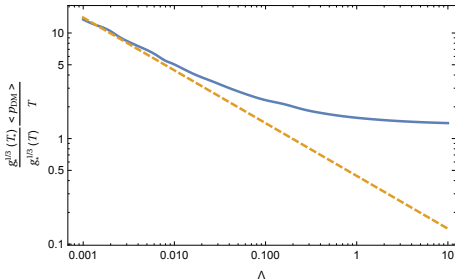
2: scalar helps to avoid X-ray bound and make $\Omega_N = \Omega_{DM}$, but free-streaming...

3,4: Ω_N is determined by X-ray bound

DM from Heavy scalar (Majoron?) decay



F.Bezrukov, D.G., 2014



$$\tau H(T = M/3) \equiv \frac{1}{18} \frac{1}{\Lambda}$$

$$x = \frac{p}{T} \left(\frac{g_*(T_*)}{g_*(T)} \right)^{1/3}$$

Decoupling of relativistic Dark Matter

Assumptions

- 1 DM particles are in equilibrium in plasma
- 2 DM decouple from plasma at temperature $T_d \gtrsim M_X$,
so they are **relativistic** (e.g. neutrino)

Later on

$$n_X(T_d) = g_X \cdot \left(\frac{1}{4}\right) \cdot \frac{\zeta(3)}{\pi^2} T_d^3$$

$$n_X a^3 = \text{const}, \quad s a^3 = \text{const} \quad \Rightarrow \quad \frac{n_X}{s} = \text{const} = \# \frac{g_X}{g_*(T_d)}$$

useful

DM particle mass M_X fixes Ω_X :

$$\Omega_X = \frac{M_X \cdot n_{X,0}}{\rho_c} = \frac{M_X \cdot s_0}{\rho_c} \frac{n}{s} \approx 0.2 \times \frac{M_X}{100 \text{ eV}} \left(\frac{g_X}{2}\right) \cdot \left(\frac{100}{g_*(T_d)}\right)$$

– NO heavy stable feebly coupled to SM particles !

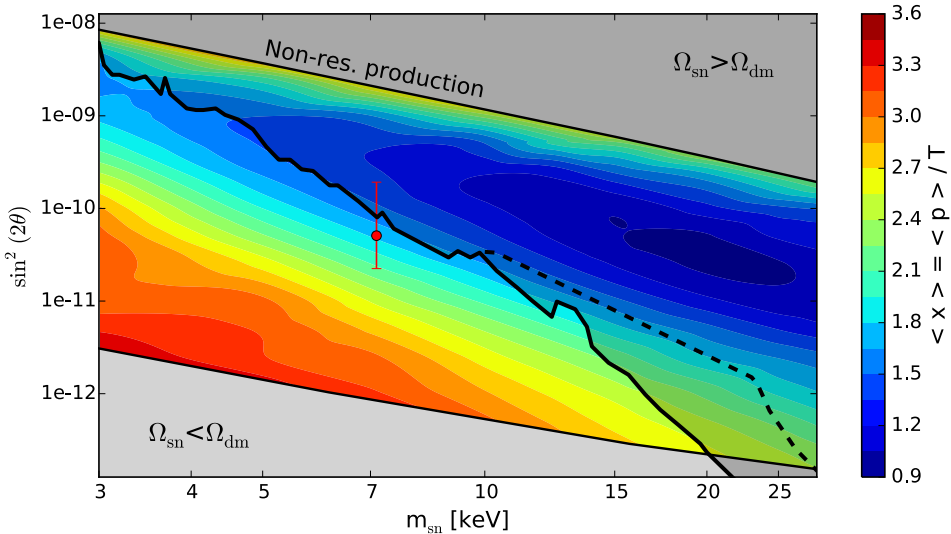
– NO realistic DM models:

Pauli blocking prevents fermionic DM

$$\frac{p_X}{M_X} \propto \frac{a_d}{a} \sim \frac{3T}{M_X} \left(\frac{g_*(T)}{g_*(T_d)}\right)^{1/3}$$

too energetic for the proper structure formation

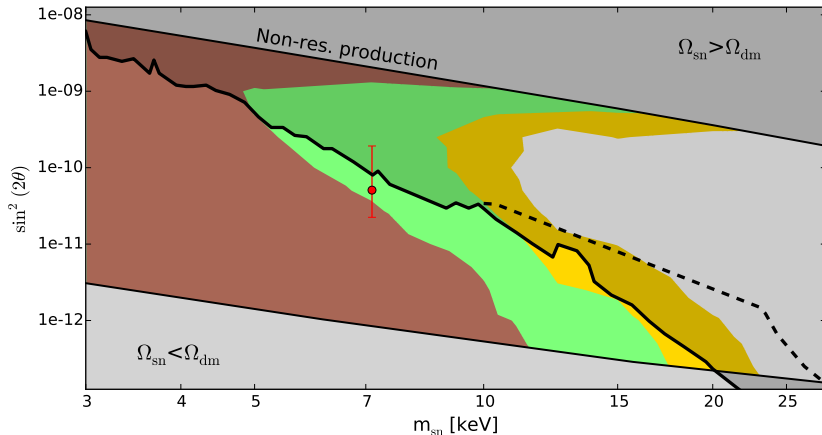
Sterile neutrino Dark Matter



A.Schneider (2016)

Sterile neutrino Dark Matter: ... gone?

A.Schneider (2016)



brown: MW satellite counts

green and yellow: Lyman- α

production by inflaton