

INCLUDING A SOURCE TERM IN HYDRODYNAMIC

SIMULATIONS

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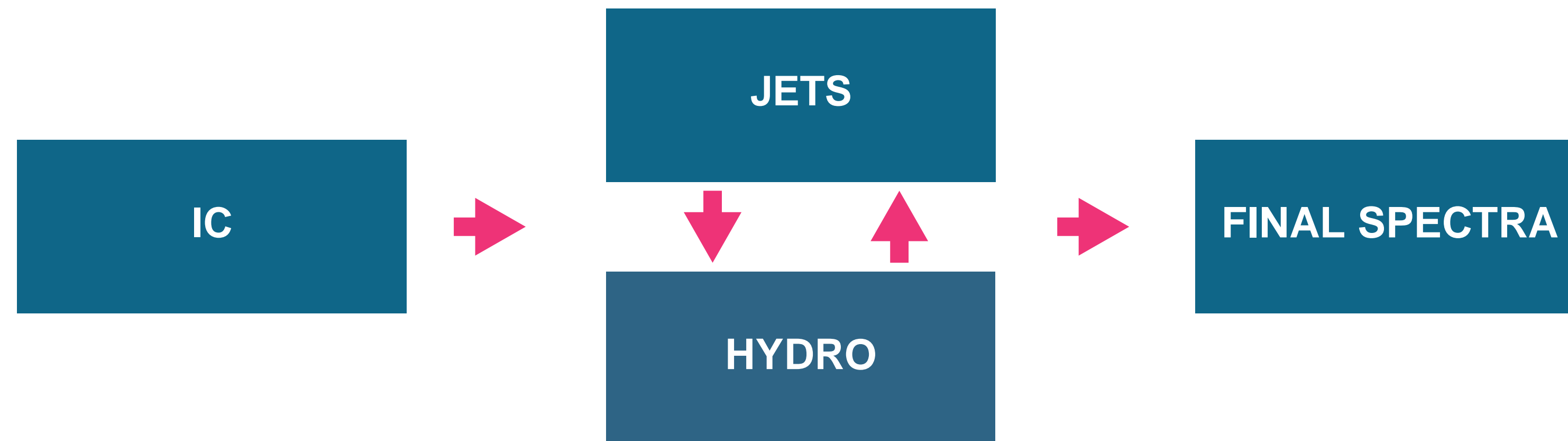
Project 2020/15893-4

Project 2024/08903-4



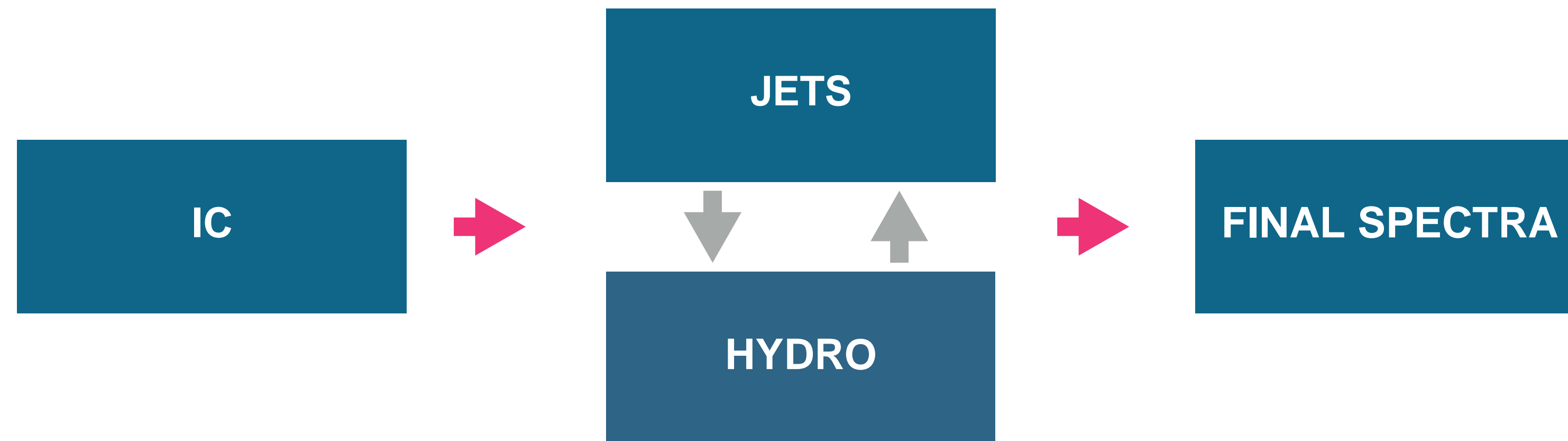
HYDRODYNAMICS AND JETS

- ▶ Hydrodynamics describe the evolution of the thermalized QGP
- ▶ Numerical simulations
- ▶ Objectives



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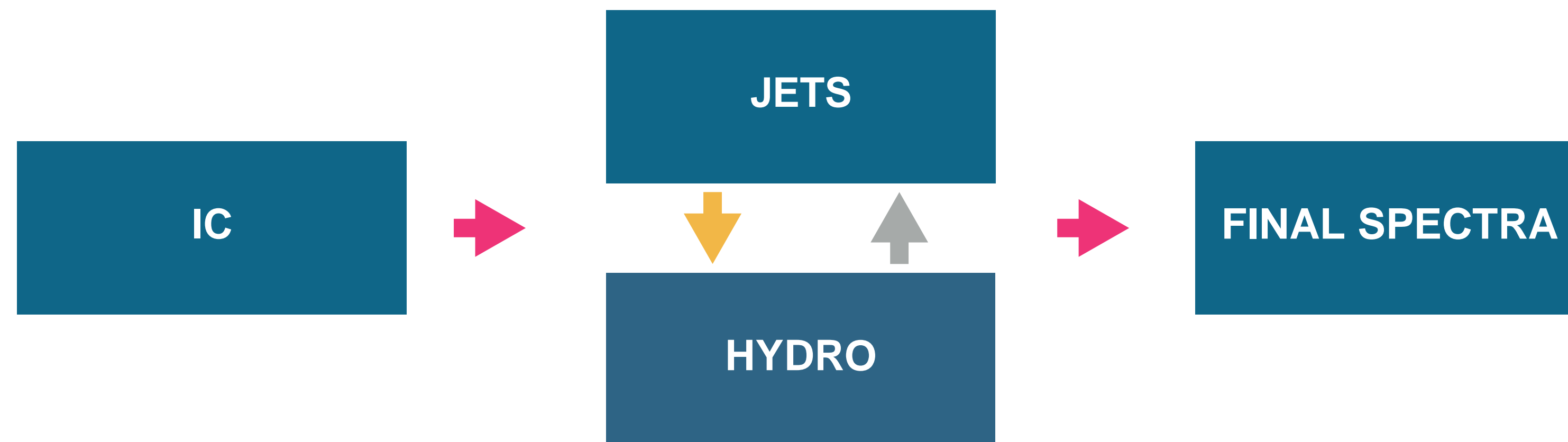


Jet affects the medium

Medium affects jets

HYDRODYNAMICS AND JETS

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PRC 95 044909
PRC 95 054914
PRC 97 064918
PLB 777 (2018) 86-90
PRC 90 024914

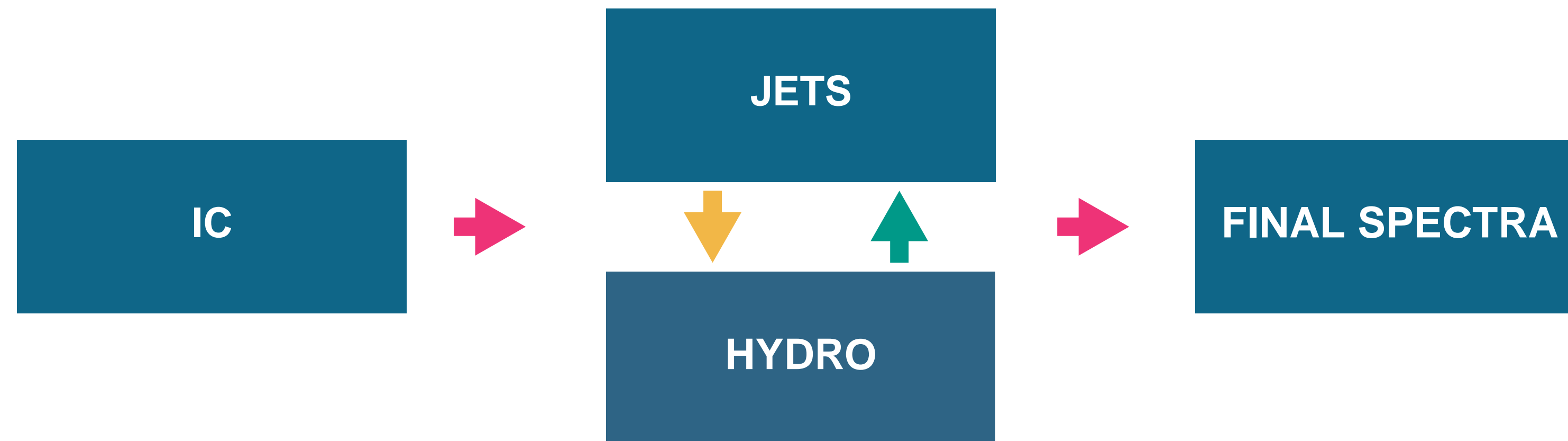
Jet affects the medium

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How far-from-equilibrium fluids behaves close to the jet?

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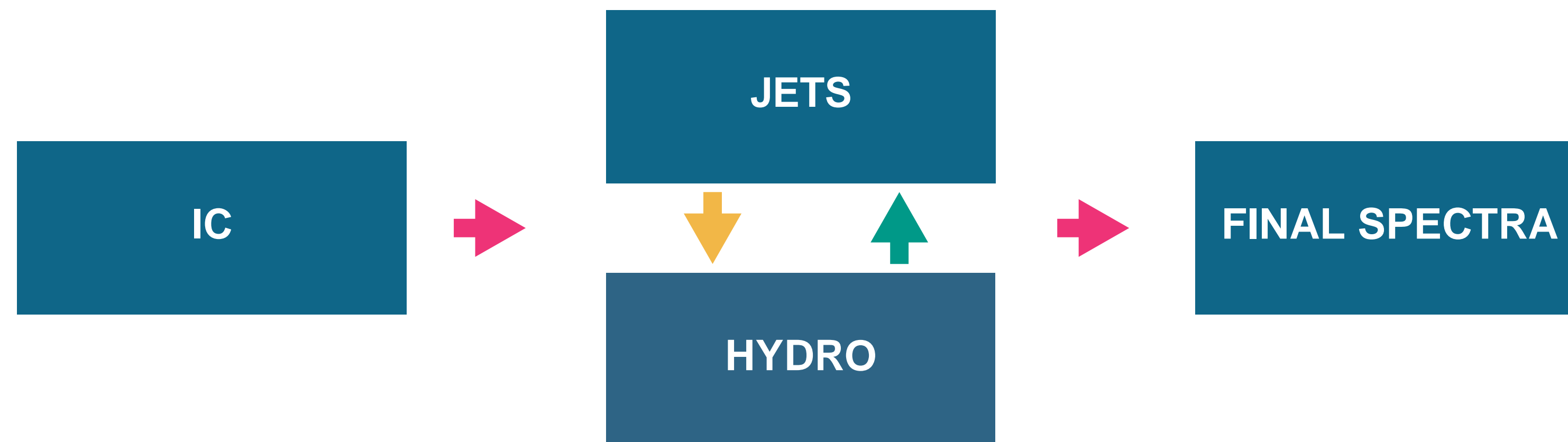
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- ▶ Hydrodynamics describe the evolution of the thermalized QGP
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Will be coupled soon

Jet affects the medium

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CCAKE

- ▶ Relativistic Viscous Hydrodynamic
- ▶ Lagrangian - SPH
- ▶ Significantly improved version of v-USPhydro

CCAKE

- ▶ Relativistic Viscous Hydrodynamic
- ▶ Lagrangian - SPH
- ▶ Significantly improved version of v-USPhydro →
 - BSQ charges evolution
 - (3+1)D simulations
 - Performance portability
 - Improved equations of motion
 - Hyperbolic and cartesian coordinates
 - Source terms

SPH METHOD



SPH METHOD



SPH METHOD



SPH METHOD



CLASSICAL SPH

$$A(\mathbf{r}) = \int_V A(\mathbf{r}') \delta(\mathbf{r}' - \mathbf{r}) dV'$$

- ▶ Define the kernel: $W(\mathbf{r}, h): \int_V W(\mathbf{r}, h) dV = 1$, $\lim_{h \rightarrow 0} W(\mathbf{r}, h) = \delta(\mathbf{r})$

$$A(\mathbf{r}) \approx \sum_{i=1}^N A(\mathbf{r}_i) W(\mathbf{r}_i - \mathbf{r}, h) \Delta V_i$$

$$\nabla A(\mathbf{r}) \approx \sum_{i=1}^N A(\mathbf{r}_i) \nabla W(\mathbf{r}_i - \mathbf{r}, h) \Delta V_i$$

RELATIVISTIC SPH

- ▶ q any quantity density at rest frame
- ▶ $q^* = qu^0 \sqrt{-g}$
- ▶ M some conserved quantity

$$M_{tot} = \sum_{b=1}^N m_b$$

$$\sigma_a^* = \sum_{b=1}^N m_b W(\mathbf{r}_a - \mathbf{r}_b, h)$$

$$q_a^* = \sum_{b=1}^N m_b \frac{q_b^*}{\sigma_b^*} W(\mathbf{r}_a - \mathbf{r}_b, h)$$

CCAKE CENTRAL LIBRARY

- ▶ Exascale Computing Project (ECP) - Co-design Center for Particle Applications (COPA)
- ▶ ECP-COPA/Cabana used in CCAKE
- ▶ Performance portability
 - ▶ - Personal Computers
 - ▶ - Workstations
 - ▶ - Clusters w/wo GPUs

The screenshot shows the website for the Exascale Computing Project (ECP) Co-design Center for Particle Applications (COPA). The page title is "PARTICLE-BASED APPLICATIONS" and it features a navigation menu with links for Home, About, Research, News, Training, and Library. A blue button labeled "< Co-design" is visible. The main content area is divided into two columns: "Project Details" and "Principal Investigator(s)".

Project Details

The Co-design Center for Particle Applications (CoPA) provides contributions to enable application readiness as we move toward exascale architectures for the "motif" of particle-based applications. CoPA focuses on co-design of several "sub-motifs," including short-range particle-particle interactions (e.g., those that often dominate molecular dynamics (MD) and smoothed particle hydrodynamics methods), long-range particle-particle interactions (e.g., electrostatic MD and gravitational N-body), particle-in-cell (PIC) methods, and O(N) complexity electronic structure and quantum MD (QMD) algorithms. Relevant particle applications are represented within CoPA and help drive the co-design process. Exascale Computing Project (ECP) application projects—such as EXAALT (LAMMPS-SNAP), WDMApp (XGC), ExaSky (HACC/SWFFT), and ExaAM (MPM)—serve as application partners as well as non-ECP applications.

Principal Investigator(s):

Susan Mniszewski, PI, Los Alamos National Laboratory; Jim Belak, Co-PI, Lawrence Livermore National Laboratory

Collaborators:

<https://www.exascaleproject.org/research-project/particle-based-applications/>

CCAKE: SPH

- ▶ Particle number as conserved quantity
- ▶ Ideal and dissipative

$$m_b = 1$$

$$\frac{1}{V_a^*} = \sigma_a^* = \sum_{b=1}^N m_b W(\mathbf{r}_a - \mathbf{r}_b, h)$$

- ▶ Specific density $\sigma = \frac{1}{V}$

KERNEL

► Cubic spline

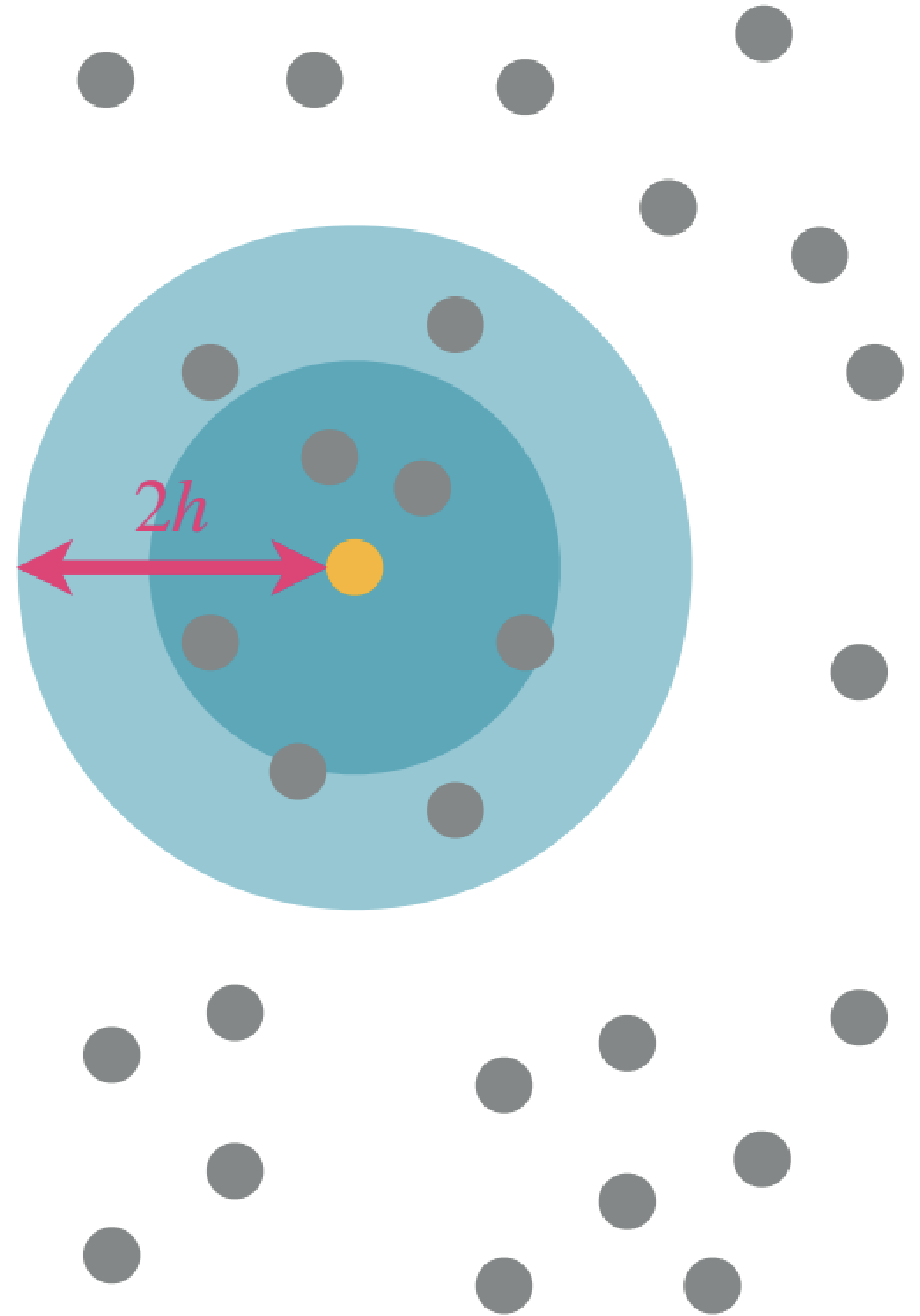
- $C = \frac{1}{6h}, D = 1$

- $C = \frac{15}{14h^2}, D = 2$

- $C = \frac{1}{4h^3}, D = 3$

$$W(\mathbf{r}, h) = C \begin{cases} (2-q)^3 - 4(1-q)^3 & , 0 \leq q < 1 \\ (2-q)^3 & , 1 \leq q < 2 \\ 0 & , q \geq 2 \end{cases}$$

$q = |\mathbf{r}|/h$



CCAKE: SPH

- ▶ $\sigma = \frac{1}{V}$

Dynamical variables

- ▶ $S = \frac{s}{\sigma}, N_X = \frac{n_X}{\sigma}, \tilde{\Pi} = \frac{\Pi}{\sigma}, \tilde{\pi} = \frac{\pi}{\sigma}$

- ▶ x^μ, u^μ

$$s^* = \sum_{b=1}^N m_b S_b W(\mathbf{r}_a - \mathbf{r}_b, h)$$

$$n_X^* = \sum_{b=1}^N m_b N_{X,b} W(\mathbf{r}_a - \mathbf{r}_b, h)$$

CCAKE: EQUATIONS OF MOTION

■ Source terms

■ Turned off

- ▶ Hyperbolic and cartesian coordinates

$$d_{\mu} N_X^{\mu} = \rho_X$$

$$d_{\mu} T^{\mu\nu} = j^{\nu}$$

$$\tau_{\Pi} D\Pi + \Pi = -\zeta\Theta + \mathcal{J} + \mathcal{R} - \frac{\tau_{\Pi}\beta_{\Pi}}{2\beta_{\Pi}} \dot{\Pi}$$

arXiv:2209.11210

$$\mathcal{J} = -\delta_{\Pi\Pi}\Pi\Theta - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\mathcal{R} = \varphi_1\Pi^2 + \varphi_3\pi_{\mu\nu}\pi^{\mu\nu}$$

$$\tau_{\pi}\Delta_{\alpha\beta}^{\mu\nu} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{J}^{\mu\nu} + \mathcal{R} - \frac{\tau_{\pi}\beta_{\pi}}{2\beta_{\pi}} \dot{\pi}^{\mu\nu}$$

$$\mathcal{J}^{\mu\nu} = -\delta_{\pi\pi}\pi^{\mu\nu}\Theta - 2\tau_{\pi}\Delta_{\alpha\beta}^{\mu\nu}\pi_{\lambda}^{\alpha}\omega^{\beta\lambda} - \tau_{\pi\pi}\Delta_{\alpha\beta}^{\mu\nu}\pi^{\lambda\alpha}\sigma_{\lambda}^{\beta} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

$$\mathcal{R} = \varphi_6\Pi\pi^{\mu\nu} + \varphi_7\Delta_{\alpha\beta}^{\mu\nu}\pi^{\lambda\alpha}\pi_{\lambda}^{\beta}$$

CCAKE: EQUATIONS OF MOTION

■ Source terms

■ Turned off

- ▶ Hyperbolic and cartesian coordinates

$$d_{\mu} N_X^{\mu} = \rho_X$$

$$d_{\mu} T^{\mu\nu} = j^{\nu}$$

Invert equations and isolate variables

$$\tau_{\Pi} D\Pi + \Pi = -\zeta\Theta + \mathcal{J} + \mathcal{R} - \frac{\tau_{\Pi}\beta_{\Pi}}{2\beta_{\Pi}} \dot{\Pi}$$

arXiv:2209.11210

$$\mathcal{J} = -\delta_{\Pi\Pi}\Pi\Theta - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\mathcal{R} = \varphi_1\Pi^2 + \varphi_3\pi_{\mu\nu}\pi^{\mu\nu}$$

$$\tau_{\pi}\Delta_{\alpha\beta}^{\mu\nu}D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{J}^{\mu\nu} + \mathcal{R} - \frac{\tau_{\pi}\beta_{\pi}}{2\beta_{\pi}} \dot{\pi}^{\mu\nu}$$

$$\mathcal{J}^{\mu\nu} = -\delta_{\pi\pi}\pi^{\mu\nu}\Theta - 2\tau_{\pi}\Delta_{\alpha\beta}^{\mu\nu}\pi_{\lambda}^{\alpha}\omega^{\beta\lambda} - \tau_{\pi\pi}\Delta_{\alpha\beta}^{\mu\nu}\pi^{\lambda\alpha}\sigma_{\lambda}^{\beta} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

$$\mathcal{R} = \varphi_6\Pi\pi^{\mu\nu} + \varphi_7\Delta_{\alpha\beta}^{\mu\nu}\pi^{\lambda\alpha}\pi_{\lambda}^{\beta}$$

CCAKE: EQUATION OF STATE

- ▶ QCD EoS based on a Taylor series expansion up to $O(\mu_X^4)$
- ▶ Coupled to a Hadron Resonance Gas using the PDG2016+ list
- ▶ Lattice EOS doesn't cover all necessary regions

CCAKE: EQUATION OF STATE

- ▶ QCD EoS based on a Taylor series expansion up to $O(\mu_X^4)$
- ▶ Coupled to a Hadron Resonance Gas using the PDG2016+ list
- ▶ Fallback equations

$$p(T, \vec{\mu}) = \frac{1}{2} A_0 T_0^4 \left[\left(\frac{T}{T_0} \right)^2 + \sum_X \left(\frac{\mu_X}{\mu_{X,0}} \right)^2 \right]^2$$

LATTICE-BASED



TANH-
CONFORMAL



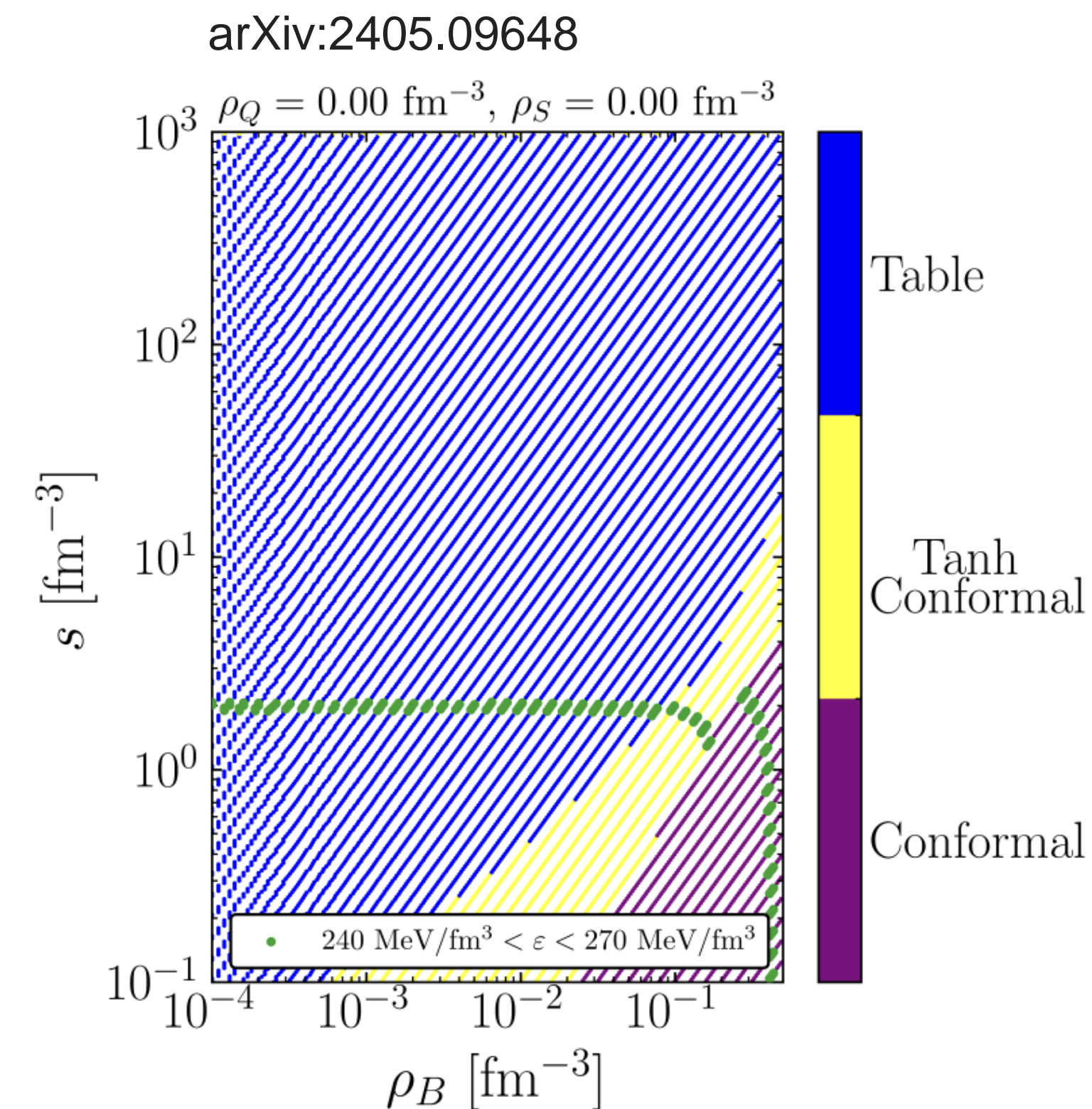
CONFORMAL



CONFORMAL
DIAGONAL

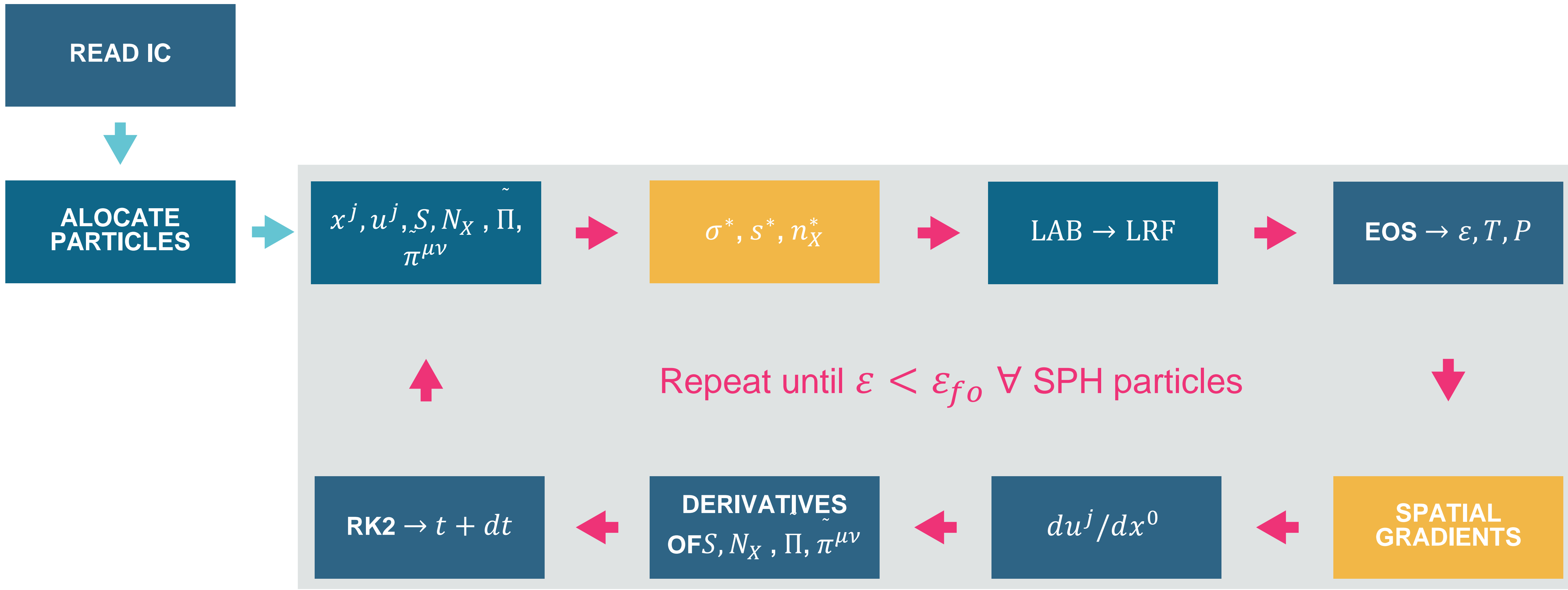
$$p(T, \vec{\mu}) = \frac{1}{2} A_0 T_0^4 \left(1 + \tanh \left(\frac{T - T_c}{T_s} \right) \right) \left[\left(\frac{T}{T_0} \right)^2 + \sum_X \left(\frac{\mu_X}{\mu_{X,0}} \right)^2 \right]^2$$

$$p(T, \vec{\mu}) = \frac{1}{2} A_0 T_0^4 \left[\left(\frac{T}{T_0} \right)^2 + \sum_X \left(\frac{\mu_X}{\mu_{X,0}} \right)^4 \right]$$



CCAKE: CODE STRUCTURE

■ SPH



ANALYTICAL CHECKS

- ▶ $(1 + 1)D$ longitudinal analytical
- ▶ $(2 + 1)D$ transversal analytical

(1+1)D ANALYTICAL

$$a = 1$$

$$\eta_0 = 0$$

PRC 105, L021902

▶ (1 + 1)D longitudinal expansion check

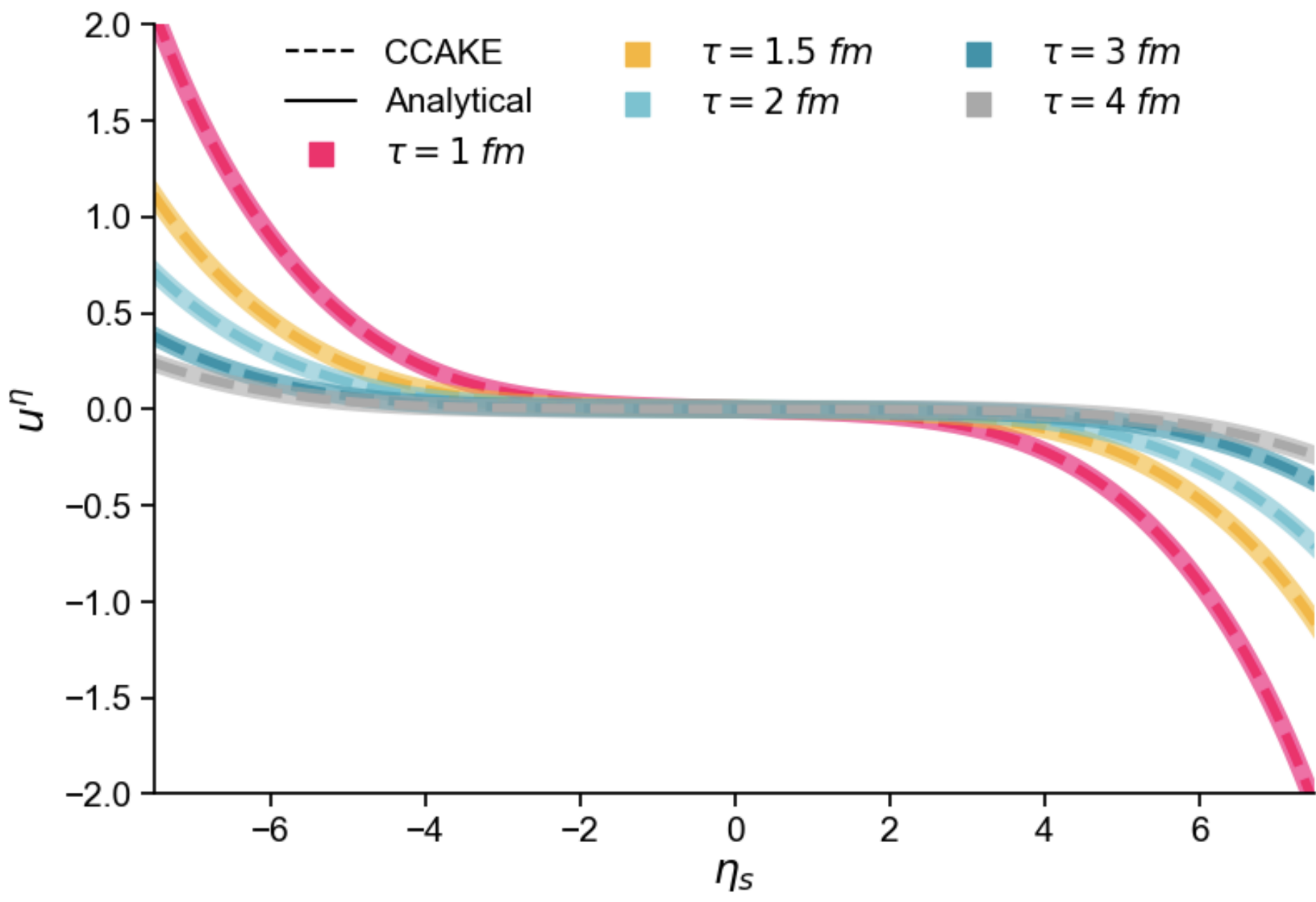
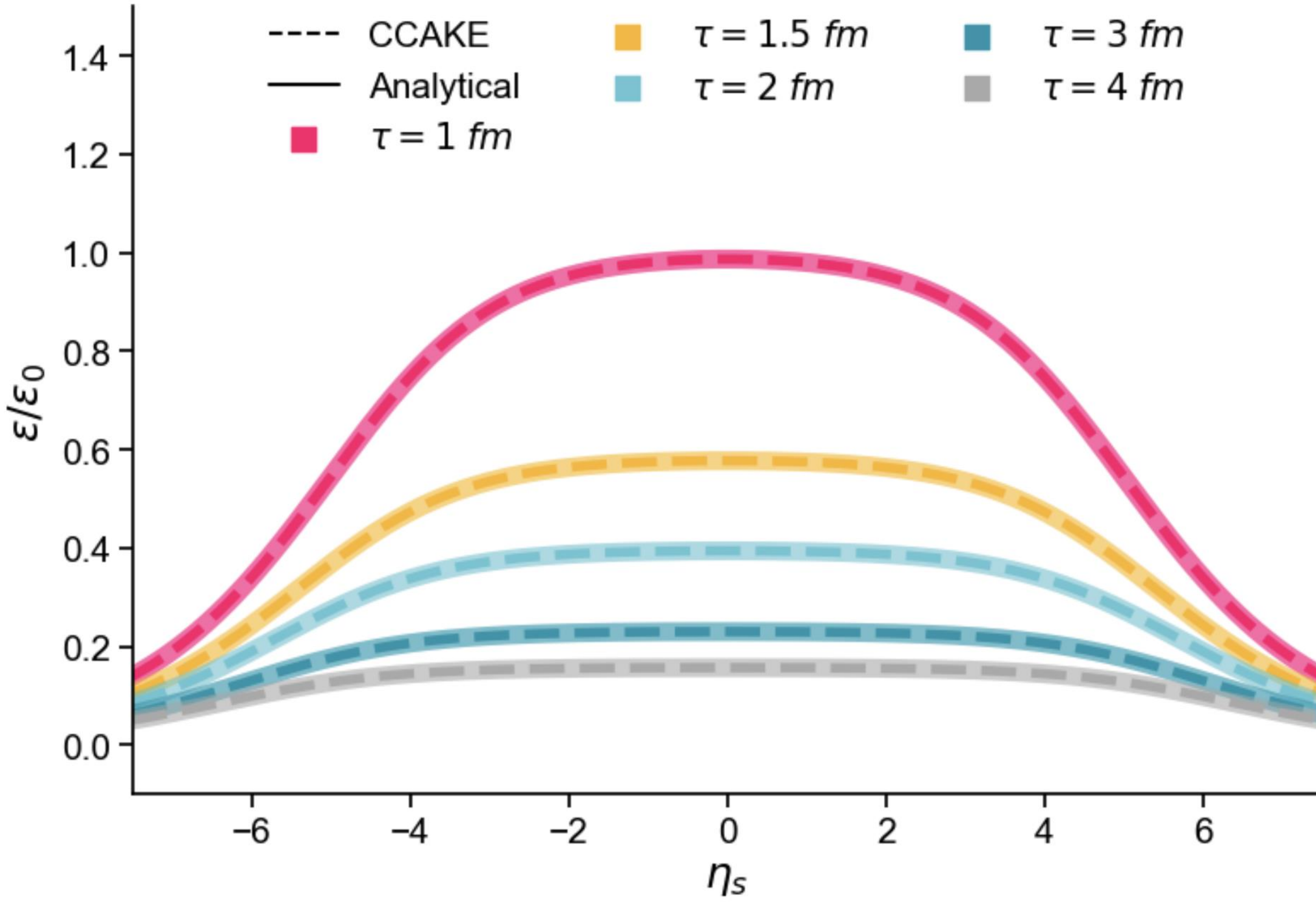
▶ Conformal EOS ($\mu_X = 0$)

▶ Ideal

$$u^\eta(\eta, \tau) = \frac{1}{2\tau} \left(\sqrt{\frac{t_0 e^{\eta_0 - \eta} + \tau a}{t_0 e^{\eta - \eta_0} + \frac{\tau}{a}}} - \sqrt{\frac{t_0 e^{\eta - \eta_0} + \frac{\tau}{a}}{t_0 e^{\eta_0 - \eta} + \tau a}} \right)$$

$$\epsilon(\eta, \tau) = \epsilon_0 \left(\frac{t_0}{\tau_0} + \frac{a\tau}{\tau_0} e^{\eta - \eta_0} \right)^{\frac{(1-c_s^4)}{4c_s^2} \frac{1}{a^2} - \frac{(1+c_s^2)^2}{4c_s^2}} \left(\frac{t_0}{\tau_0} + \frac{\tau}{a\tau_0} e^{\eta_0 - \eta} \right)^{\frac{(1-c_s^4)}{4c_s^2} a^2 - \frac{(1+c_s^2)^2}{4c_s^2}}$$

(1+1)D ANALYTICAL



(2+1)D ANALYTICAL

- ▶ (2 + 1)D transversal - Gubser
- ▶ Conformal EOS
- ▶ Shear viscosity

$$\frac{1}{\hat{T}} \frac{d\hat{T}}{d\rho} = \left[\frac{1}{3} (\hat{\pi} - 2) + \hat{\pi} \sum_Y \left(\frac{\mu_Y}{T} \right)^2 \right] \tanh(\rho)$$

$$\frac{1}{\hat{\mu}_Y} \frac{d\hat{\mu}_Y}{d\rho} = -\frac{2}{3} (1 + \hat{\pi}) \tanh(\rho)$$

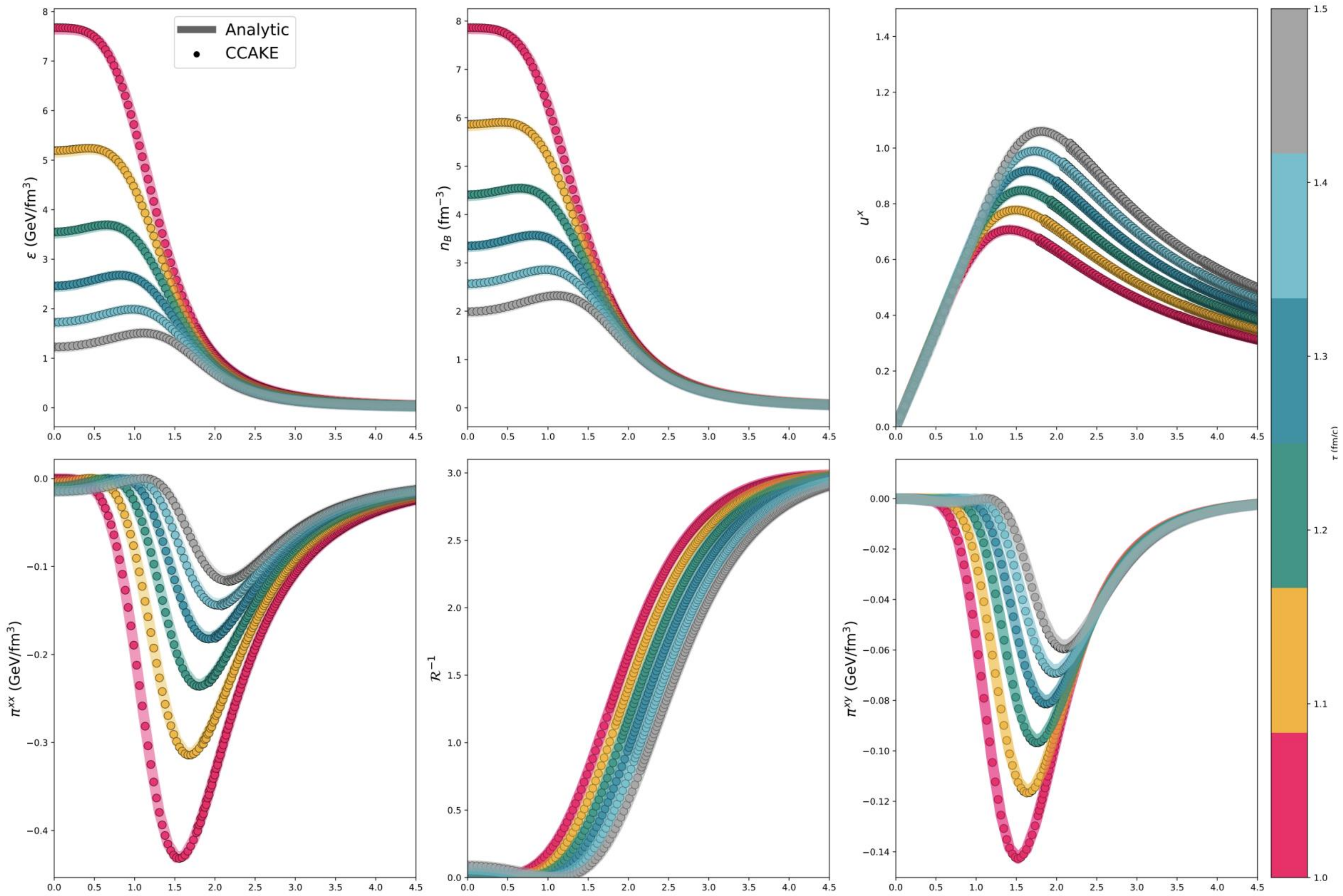
$$\frac{d\hat{\pi}}{d\rho} = \frac{4}{12} \tanh(\rho) - \frac{\hat{\pi}}{\tau_R} - \frac{4}{3} \hat{\pi}^2 \tanh(\rho)$$

$$q = 1$$

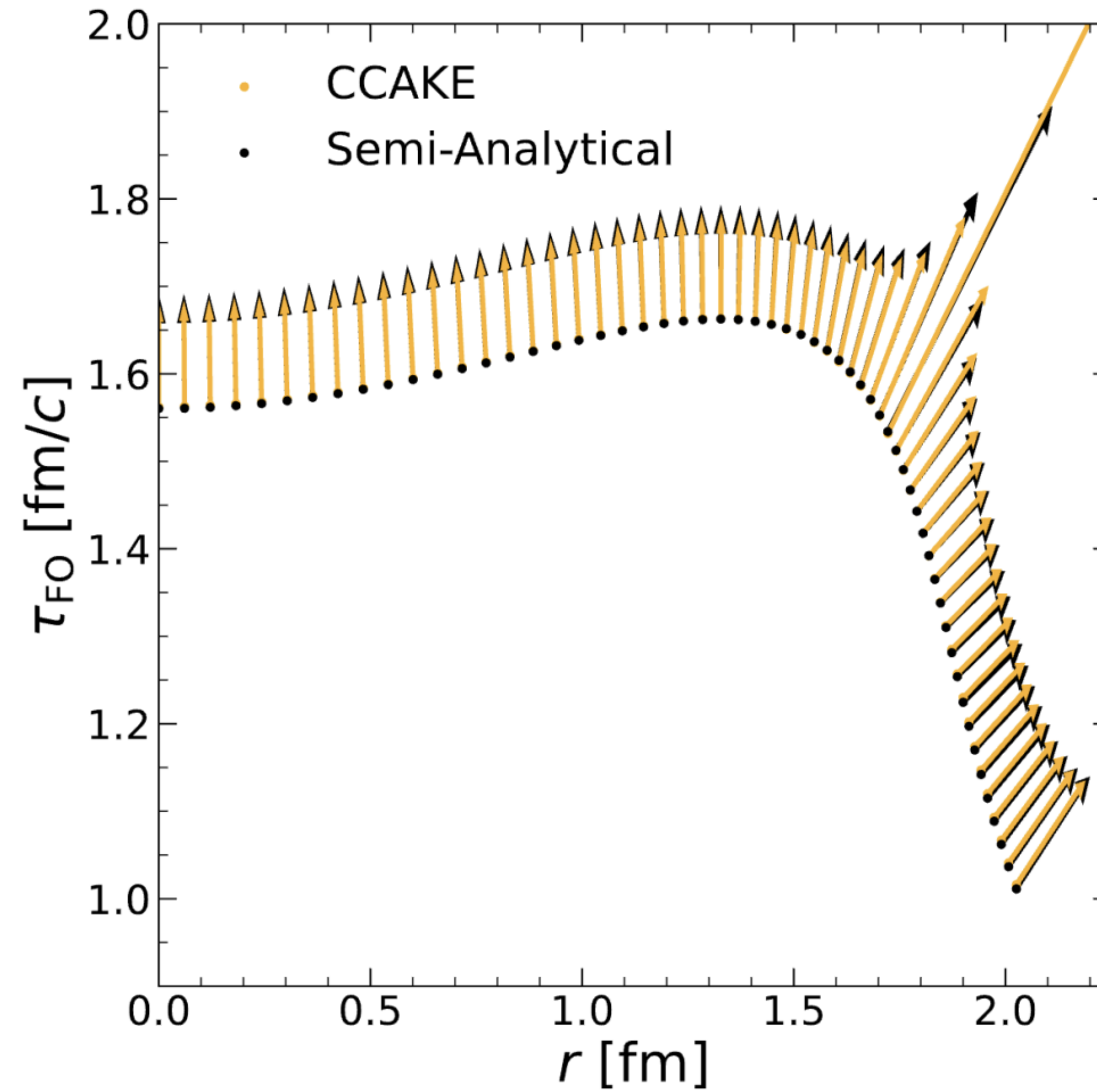
$$r = \sqrt{x^2 + y^2}$$

$$\sinh(\rho) = -\frac{1 - q^2\tau^2 + q^2r^2}{2q\tau}$$

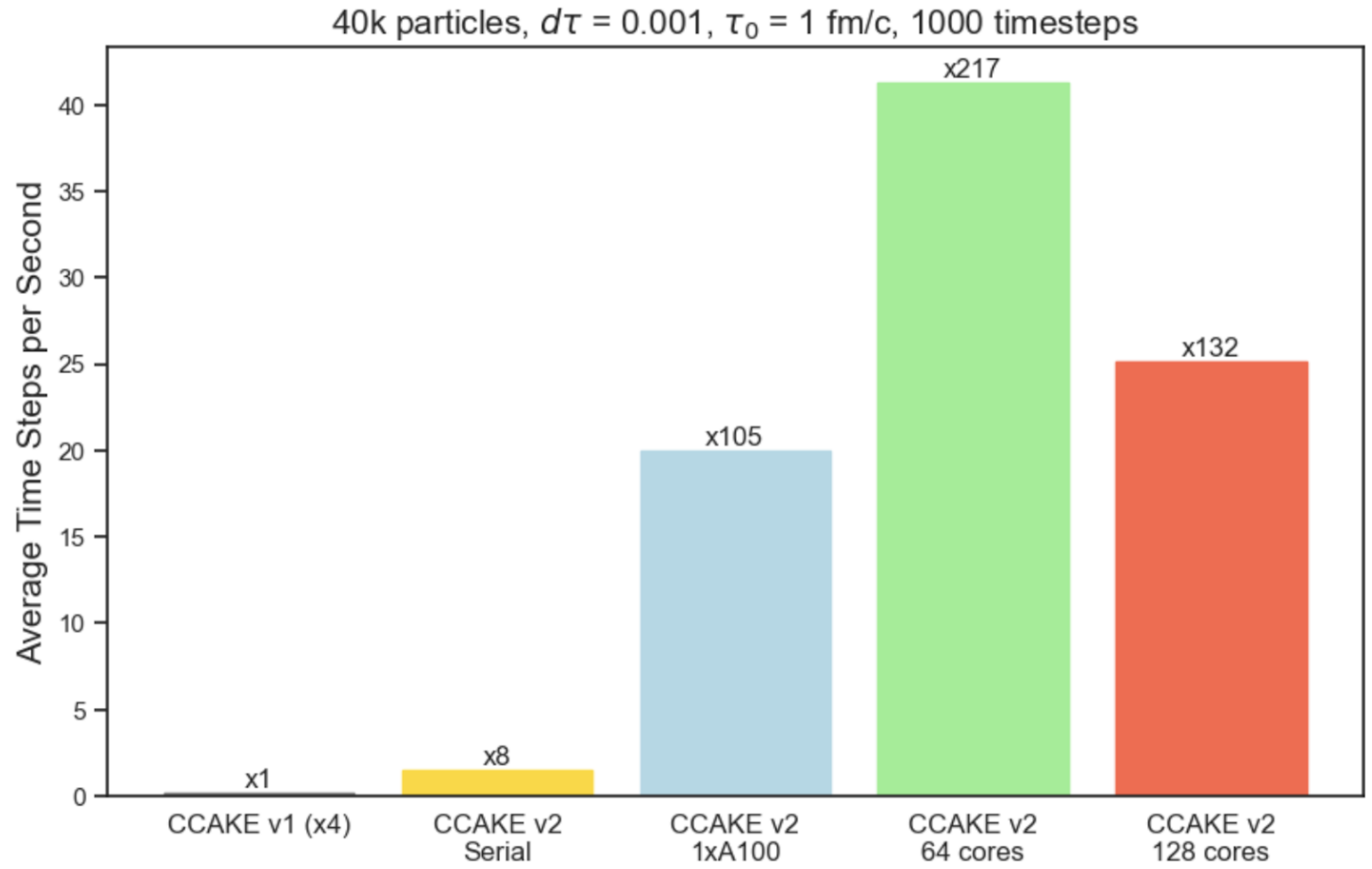
$$\tan(\theta) = \frac{2qr}{1 + q^2\tau^2 - q^2r^2}$$



(2+1)D ANALYTICAL



PERFORMANCE

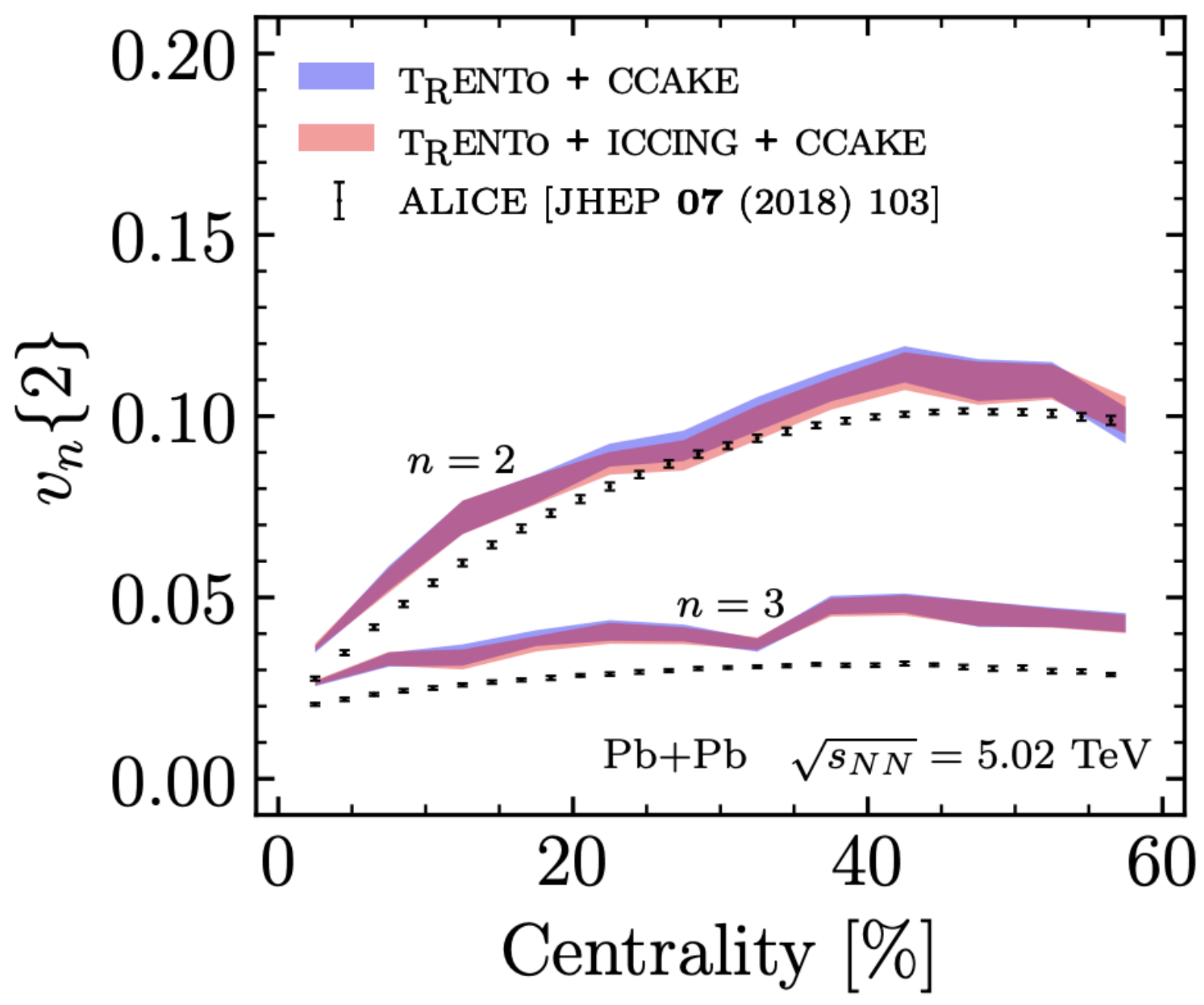
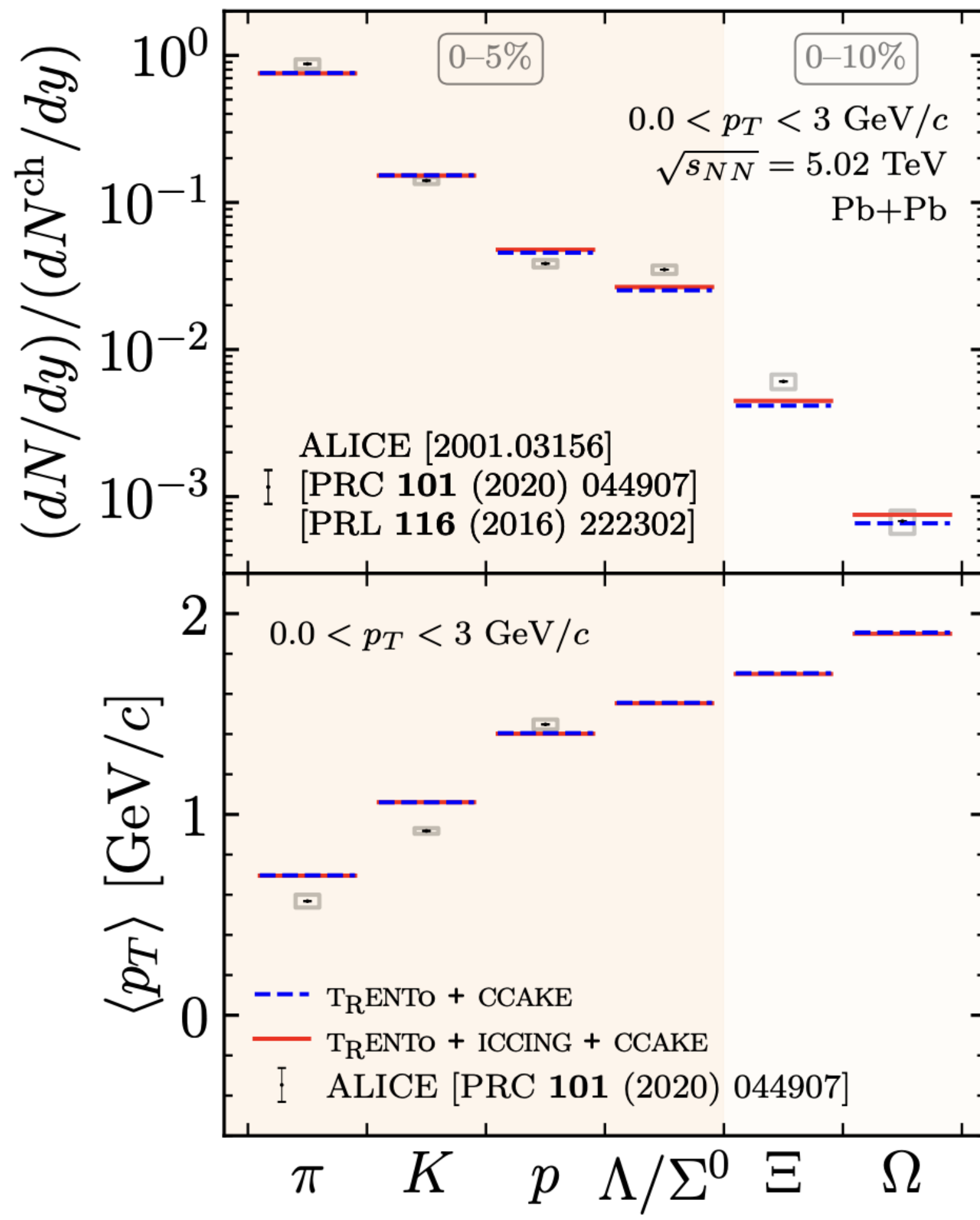
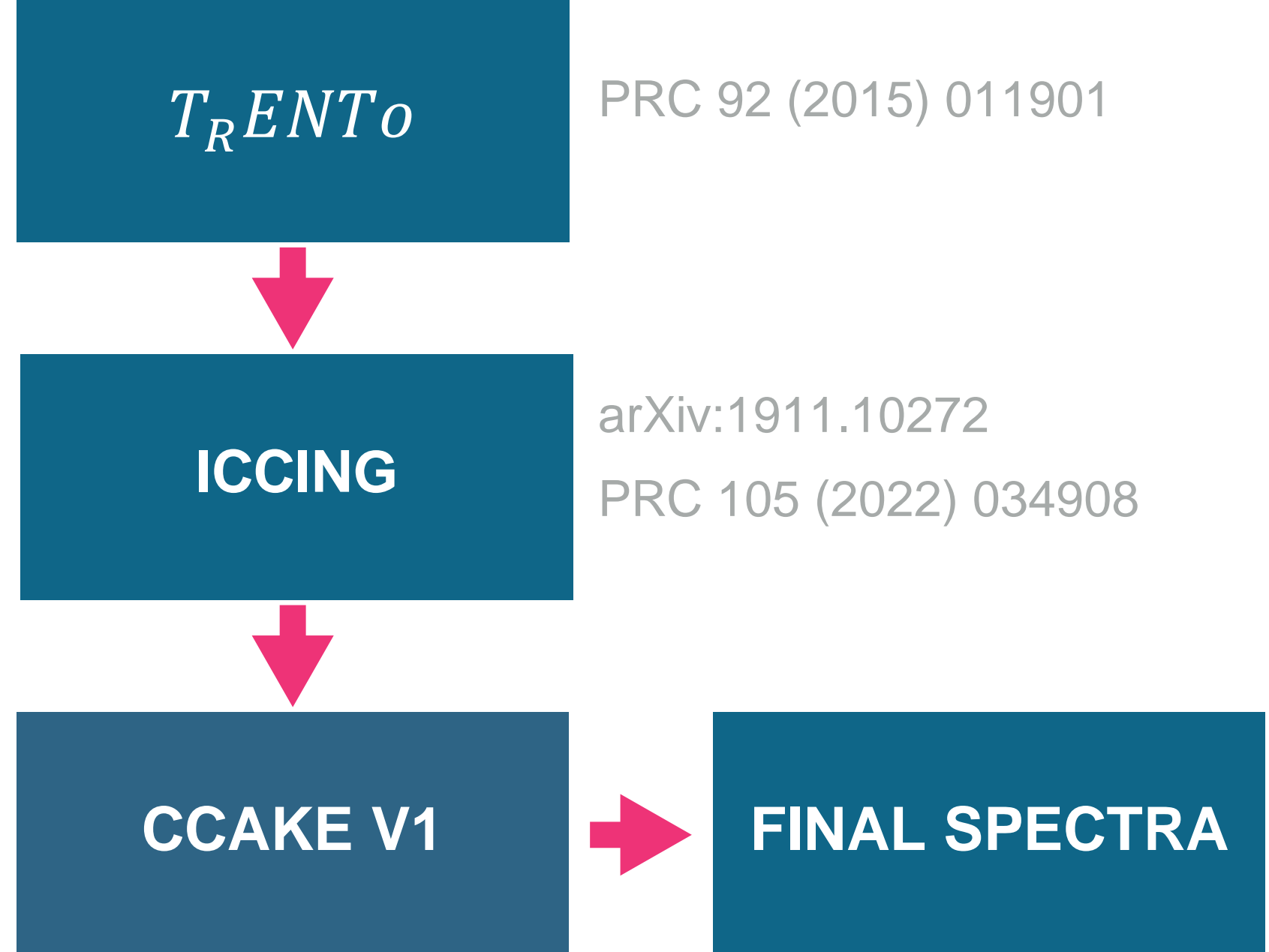


(2+1)D EBE SIMULATIONS

arXiv:2405.09648

▶ (2 + 1)D with shear viscosity

▶ Full FOS



SOURCE TERMS

PRC 90 (2014) 2, 024914
PRL 97 (2006) 062301

- ▶ Di-jet in the transverse plane
- ▶ Moving at the speed of light

$$\vec{r}_{jet} = \vec{r}_{jet,0} + (\tau - \tau_0)\vec{v}$$

$$j^\nu(\tau, \vec{r}) = \frac{dE}{dl} \delta^3(\vec{r} - \vec{r}_{jet}) \frac{u^\nu}{\gamma}$$

SOURCE TERMS

PRC 90 (2014) 2, 024914
PRL 97 (2006) 062301

- ▶ Di-jet in the transverse plane
- ▶ Moving at the speed of light

$$\vec{r}_{jet} = \vec{r}_{jet,0} + (\tau - \tau_0)\vec{v}$$

Normalization $\int \frac{W}{\tau} \tau dx dy d\eta = \Delta\eta$

$$W(\vec{r} - \vec{r}_{jet}, h) \frac{1}{\tau}$$

$$j^\nu(\tau, \vec{r}) = \frac{dE}{dl} \delta^3(\vec{r} - \vec{r}_{jet}) \frac{u^\nu}{\gamma}$$

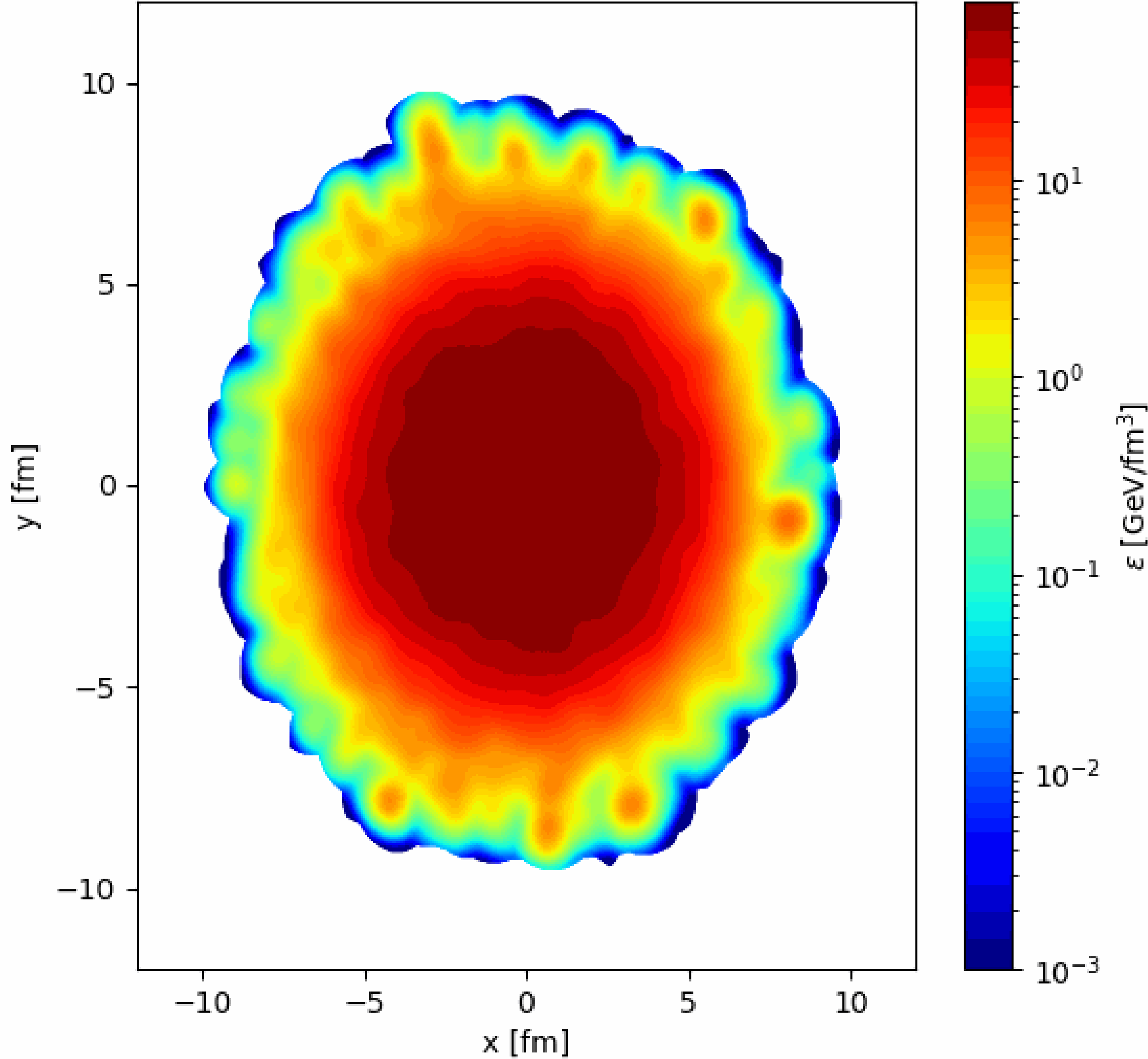
$$\frac{dE}{dl} = \frac{s(\vec{r}_{jet})}{s_0} \frac{dE}{dl} \Big|_0$$

SOURCE TERMS

- ▶ 300 ICCING events averaged
- ▶ PbPb $\sqrt{s}_{NN} = 5.02 TeV$, 0-5%
- ▶ Toy model to explore effects in hydro
- ▶ Di-jet forming at (0,0) at τ_0
- ▶ Moving in $\pm x$ direction

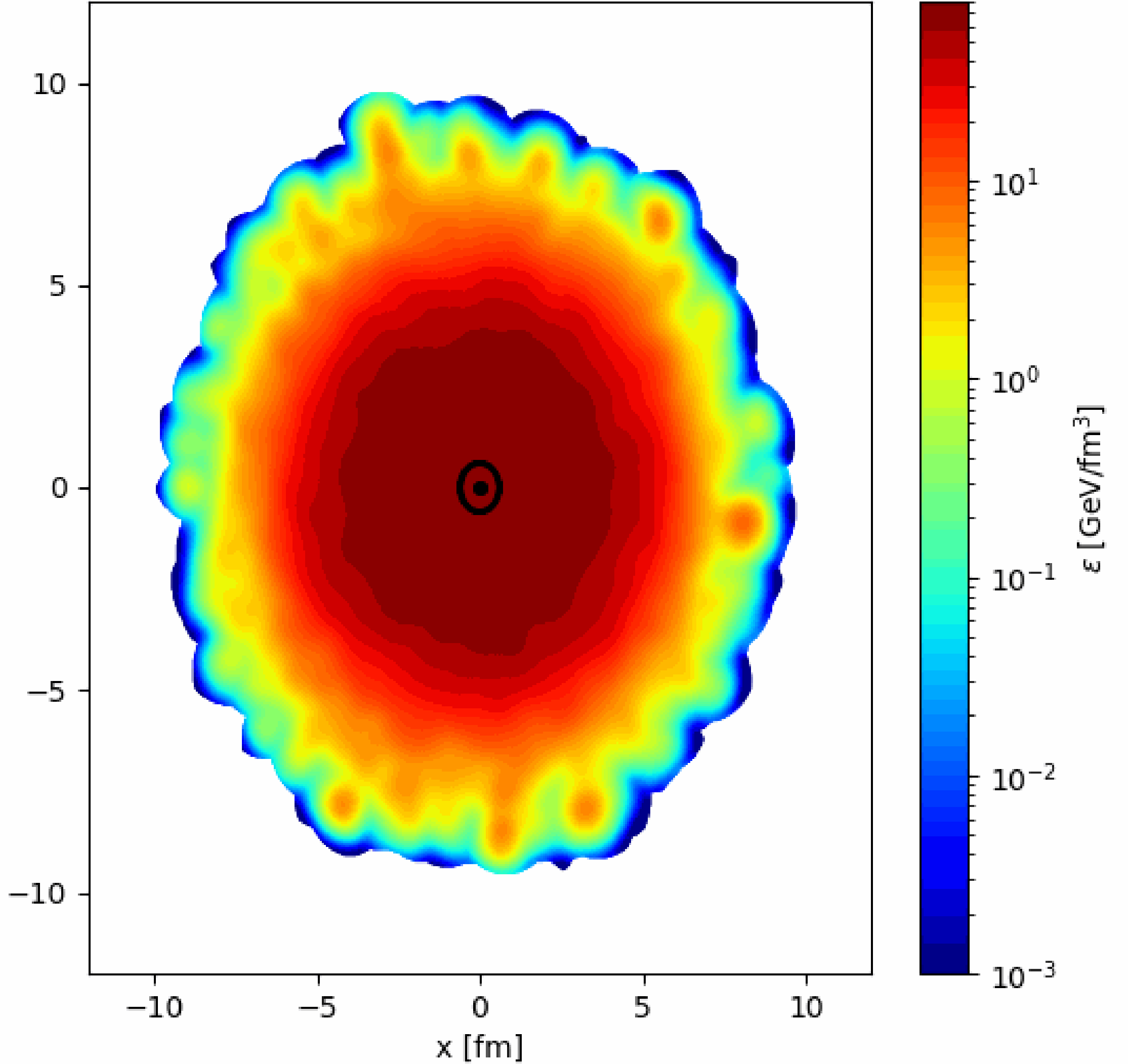
SOURCE OFF

0.60 fm/c



SOURCE ON

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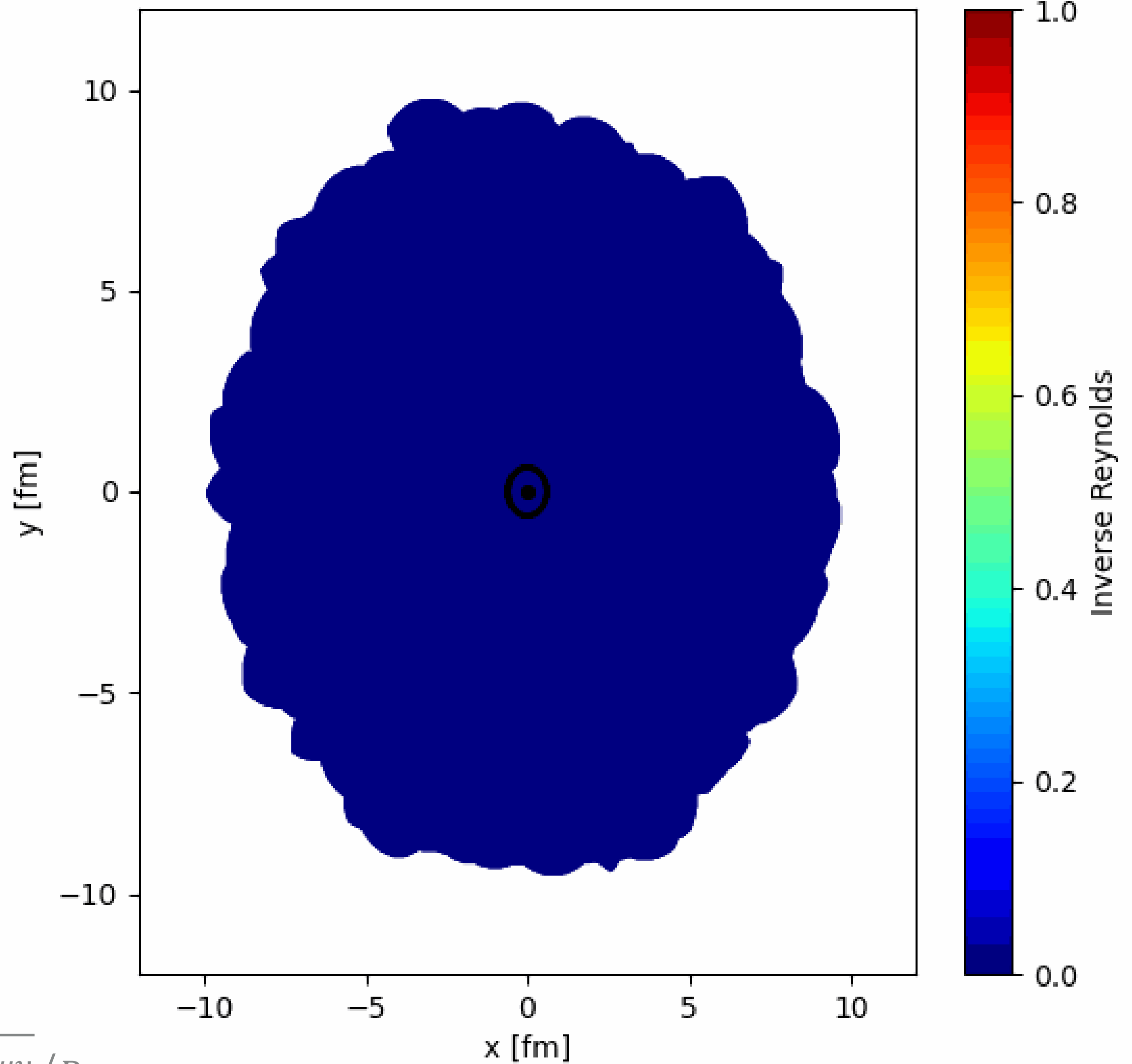
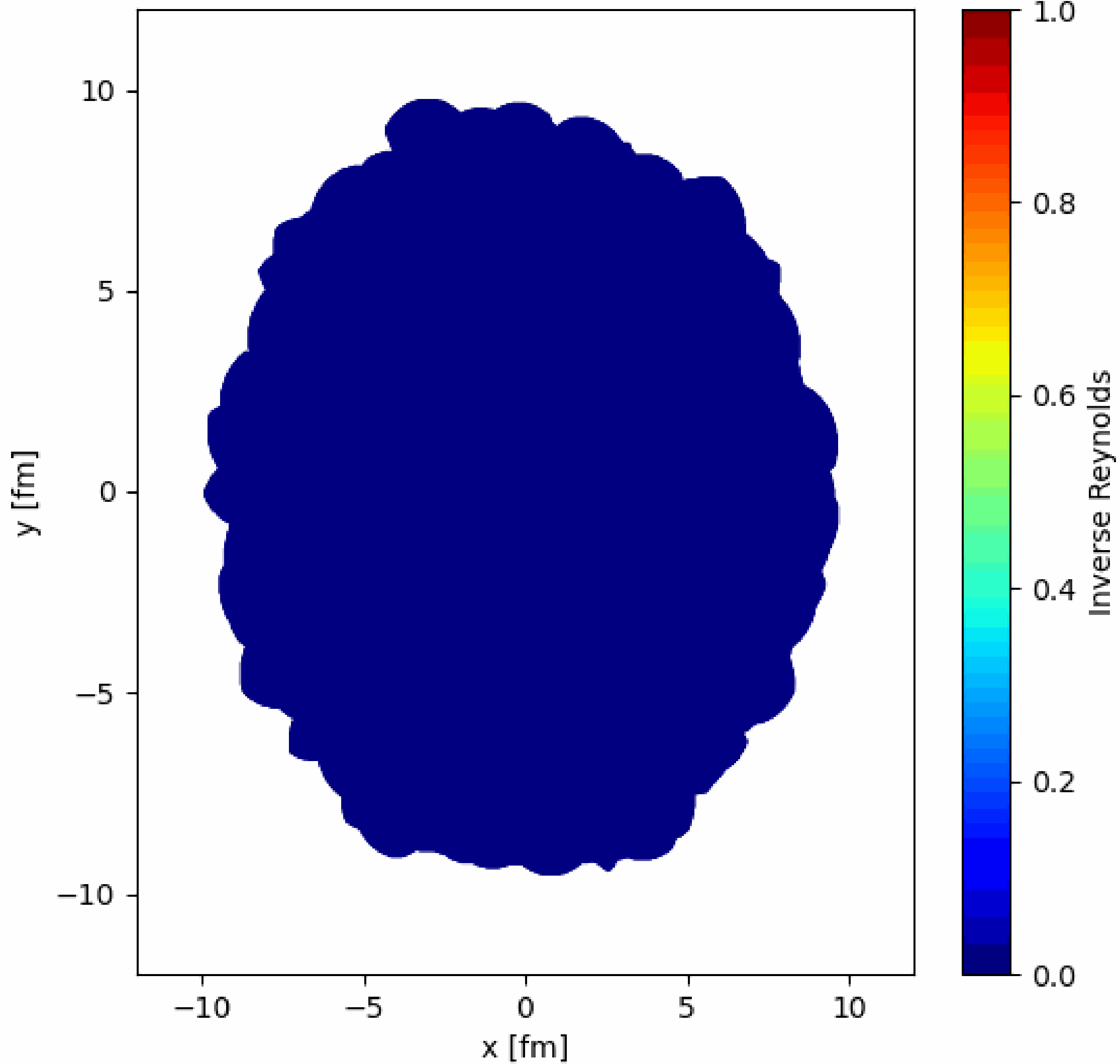


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0.60 fm/c

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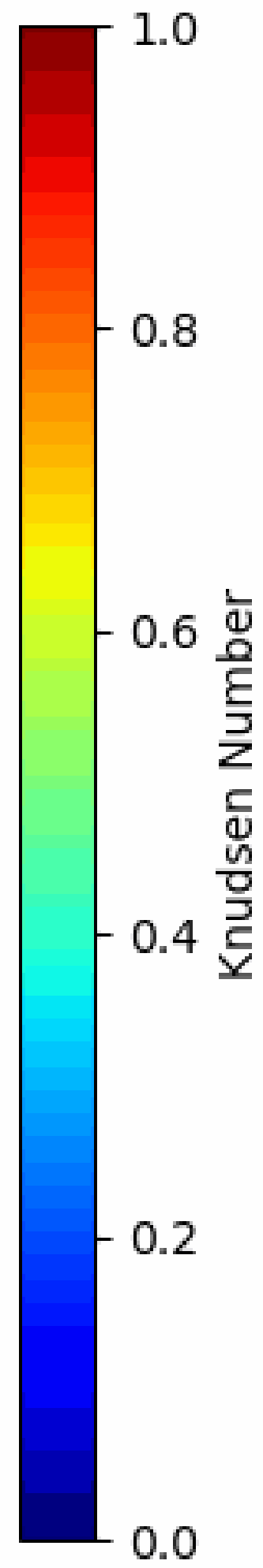
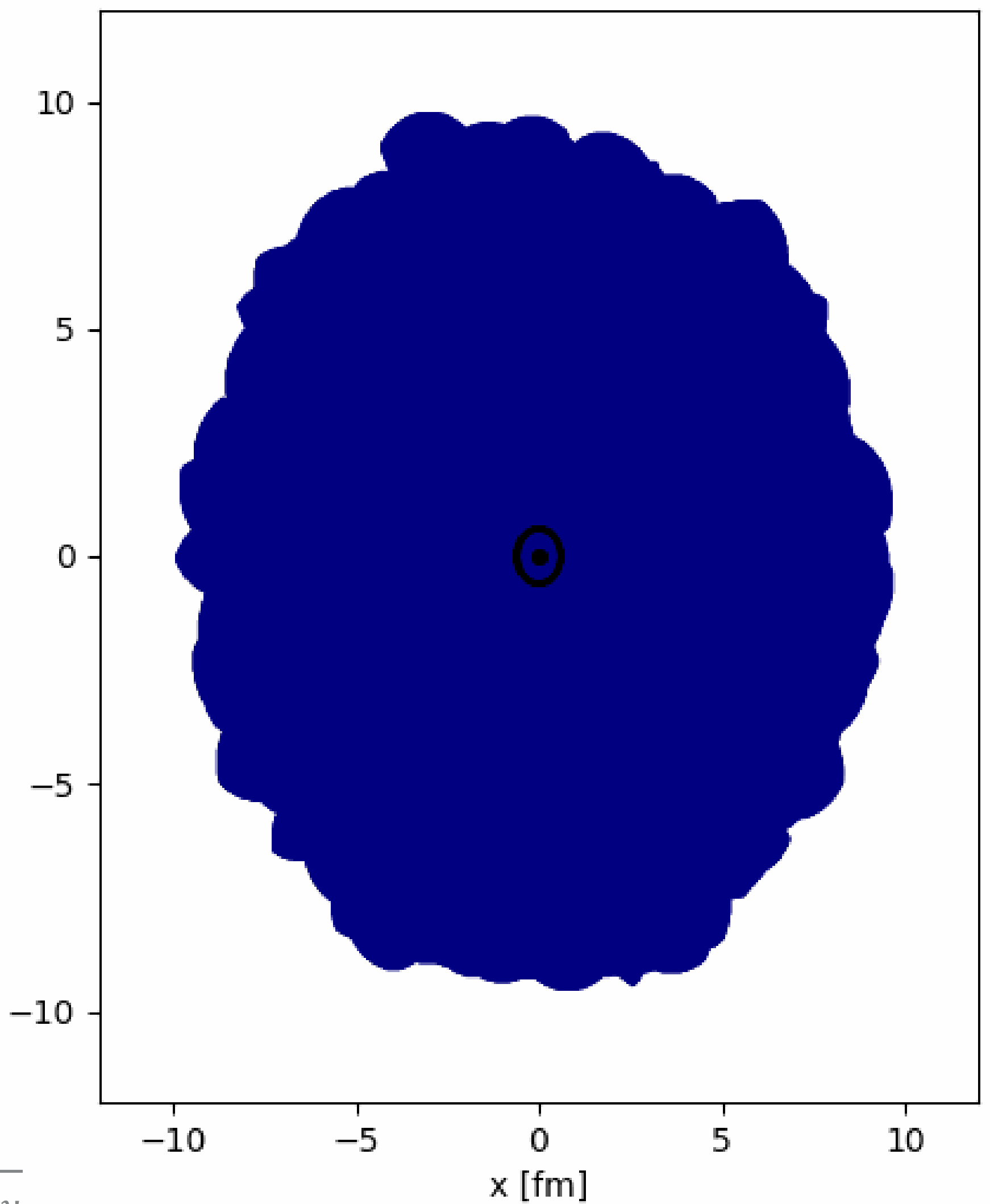
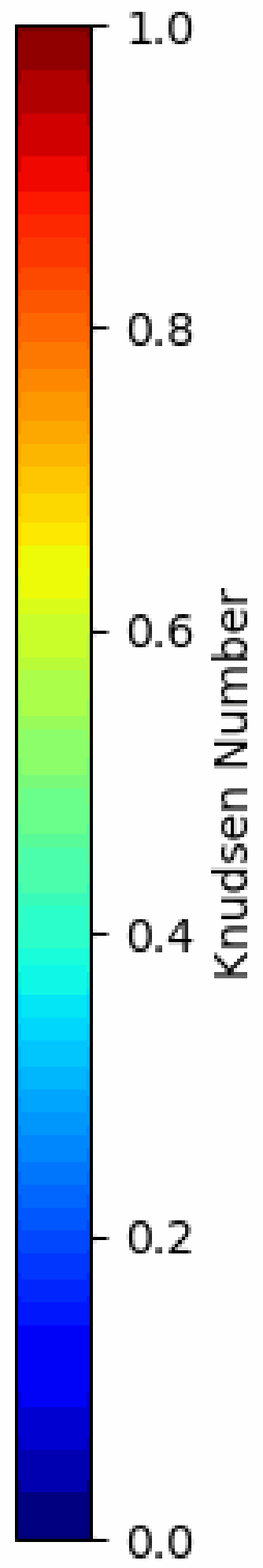
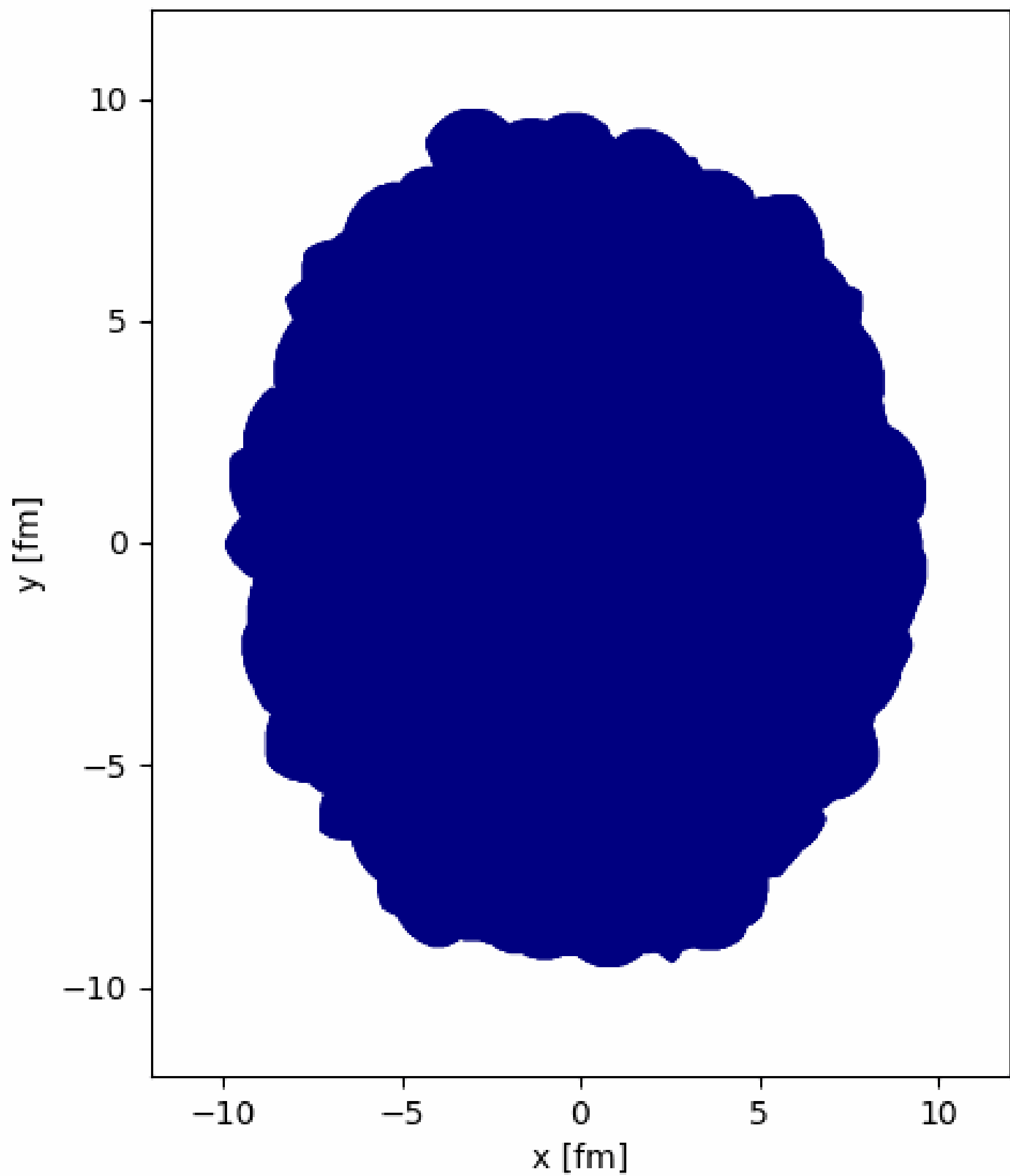
$$\sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/P$$

SOURCE OFF

SOURCE ON

0.60 fm/c

0.60 fm/c



$$\tau_\pi \sqrt{\sigma_{\mu\nu} \sigma^{\mu\nu}}$$

CONCLUSIONS AND OUTLOOK

- ▶ Performance portability (GPU/CPU)
- ▶ Excellent results for the $(2+1)D$ and $(1+1)D$ checks
- ▶ Reproduce data
- ▶ Hydrodynamics with sources terms
- ▶ Check causality near the jet
- ▶ Simulate with more physical parameters
- ▶ Couple with jet simulator

BACKUP

COORDINATE SYSTEM

- ▶ $c = \hbar = k_b = 1$
- ▶ $x^\mu = \{\tau, x, y, \eta_s\}$
- ▶ $\tau = \sqrt{t^2 - z^2}$
- ▶ $\eta_s = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right)$

DERIVATIVES

▶ Partial derivatives

$$\partial_k q^* = \sum_{b=1}^N m_b \left[q_b^* \frac{(\sigma_a^*)^{-1}}{(\sigma_b^*)^n} - q_a^* \frac{(\sigma_a^*)^{-1}}{(\sigma_b^*)^n} \right] \partial_k W_{ab}$$

- ▶ $n = 0$ usual
- ▶ $n = 1$ antisymmetric
- ▶ $n = -1$ symmetric

DERIVATIVES

- ▶ Partial time derivatives depends on total derivatives

$$d_0 A^\mu = \frac{dA^\mu}{dx_0} - v^i \partial_i A^\mu + \Gamma_{0\sigma}^\mu A^\sigma,$$

$T^{\mu\nu}$ AND N^μ

► Full

$$N^\mu = nu^\mu + q^\mu$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - g^{\mu\nu}(P + \Pi) + \pi^{\mu\nu}$$

$$\begin{aligned} u^\mu d_\mu &= D, \\ \Delta^{\mu\nu} &= g^{\mu\nu} - u^\mu u^\nu \\ \nabla_\mu &= \Delta_\mu^\alpha d_\alpha \end{aligned}$$

CCAKE: FREEZE-OUT SURFACE

- ▶ Surface $\varepsilon_{fo} = \varepsilon(x = (x^0, x^1, x^2, x^3))$

$$n_\mu = \frac{N_\mu}{\sqrt{|N_\mu N^\mu|}} = -\frac{\partial_\mu \varepsilon_{fo}(x)}{\sqrt{|\partial_\mu \varepsilon_{fo}(x) \partial^\mu \varepsilon_{fo}(x)|}}$$

EXAFLOP

- ▶ 10^{18} operations / second

JET SIMULATION PARAMETERS

$$s_0 = 200 \text{fm}^{-3}$$

$$\frac{dE}{dl} \Big|_0 = 30 \text{GeV}$$

$$h_{jet} = 0.3 \text{fm}$$

$$\Delta\eta = 0.1$$

$$\Delta E \sim 500 \text{GeV}$$

$$\tau_0 = 0.6 \text{fm}/c$$

$$\frac{\eta}{s} = 0.2$$

$$\tau_\pi = \frac{\eta}{5w}$$