# INCLUDING A SOURCE TERM IN HYDRODYNAMIC SIMULATIONS

[1] University of Illinois Urbana-Champaign
[2] University of São Paulo

Kevin Possendoro Pala [1,2]
Dekrayat Almaalol [1]
Jacquelyn Noronha-Hostler [1]
Jordi Salinas San Martín [1]
Kevin Ingles [1]
Surkhab Kaur [1]

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FAPESP
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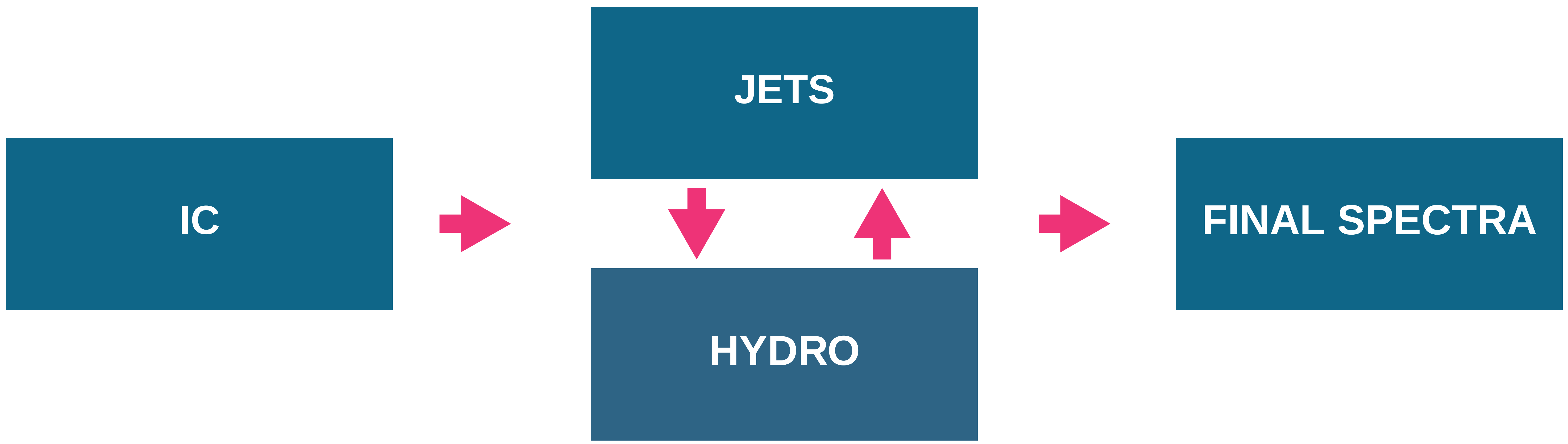
# HYDRODYNAMICS AND JETS

- Hydrodynamics describe the evolution of the thermalized QGP
- Numerical simulations
- Objectives



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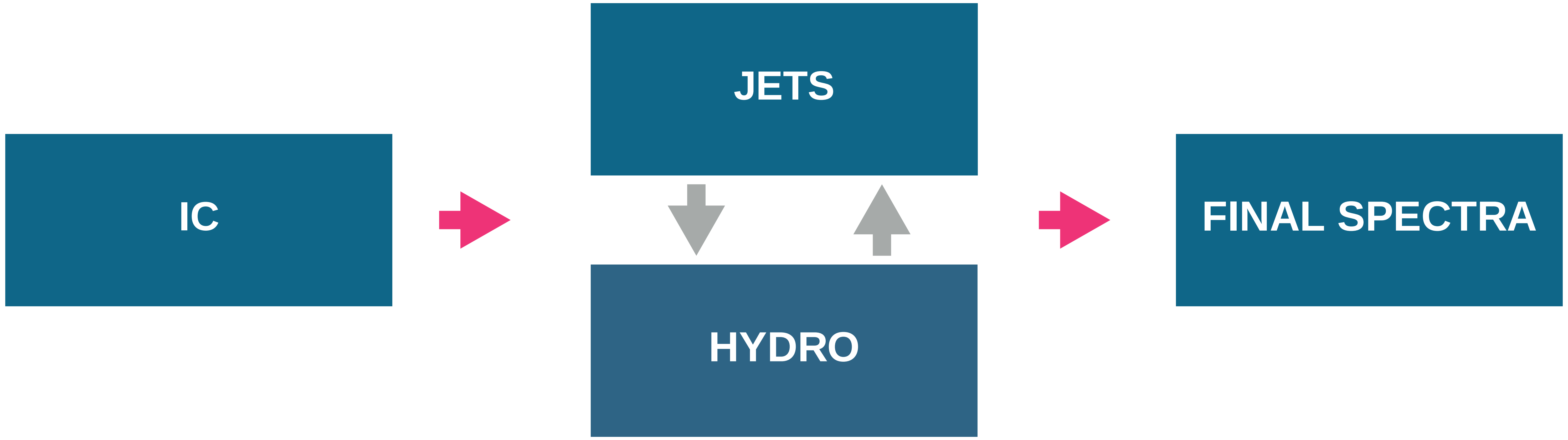


Jet affects the medium

Medium affects jets

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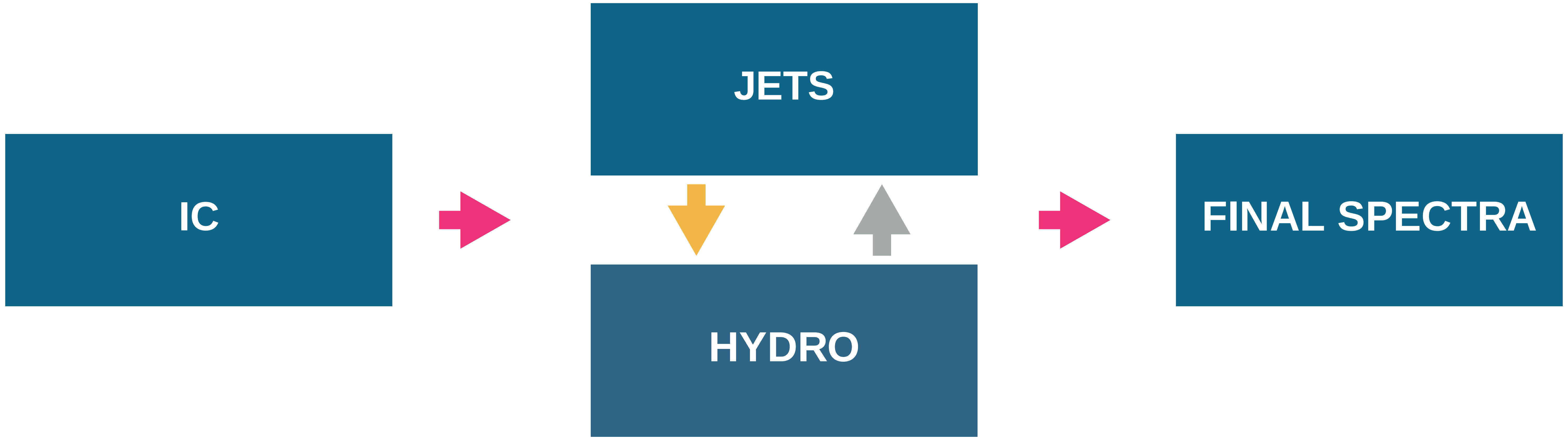


PRC 95 044909
PRC 95 054914
PRC 97 064918
*PLB* 777 (2018) 86-90
PRC 90 024914

Jet affects the medium

How far-from-equilibrium fluids behaves close to the jet?

Medium affects jets

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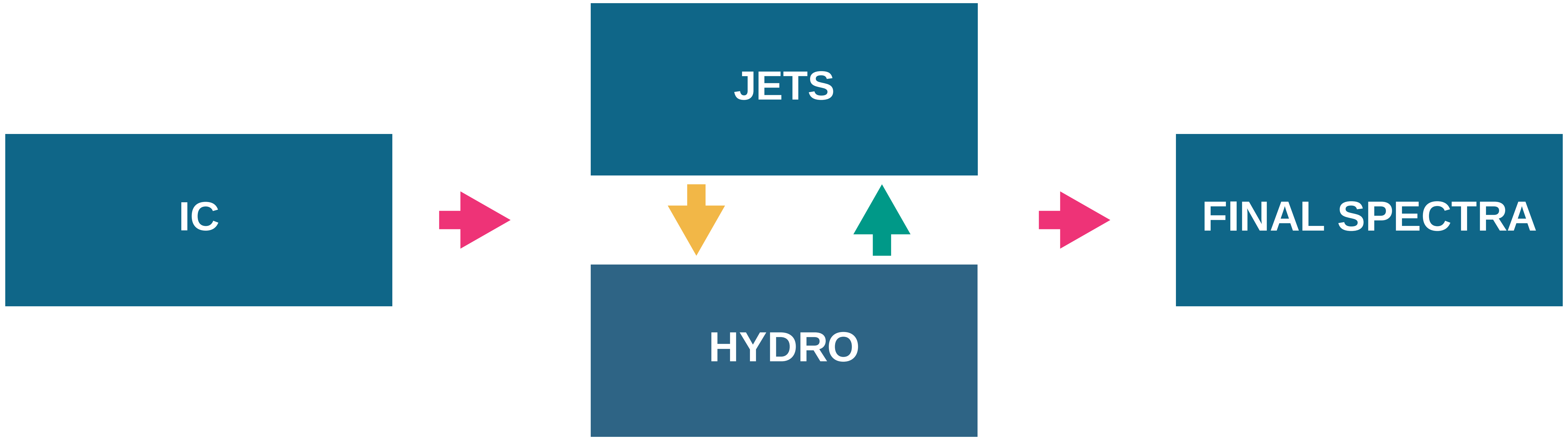


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Jet affects the medium — How far-from-equilibrium fluids behaves close to the jet?

Medium affects jets — Isaac Long talk

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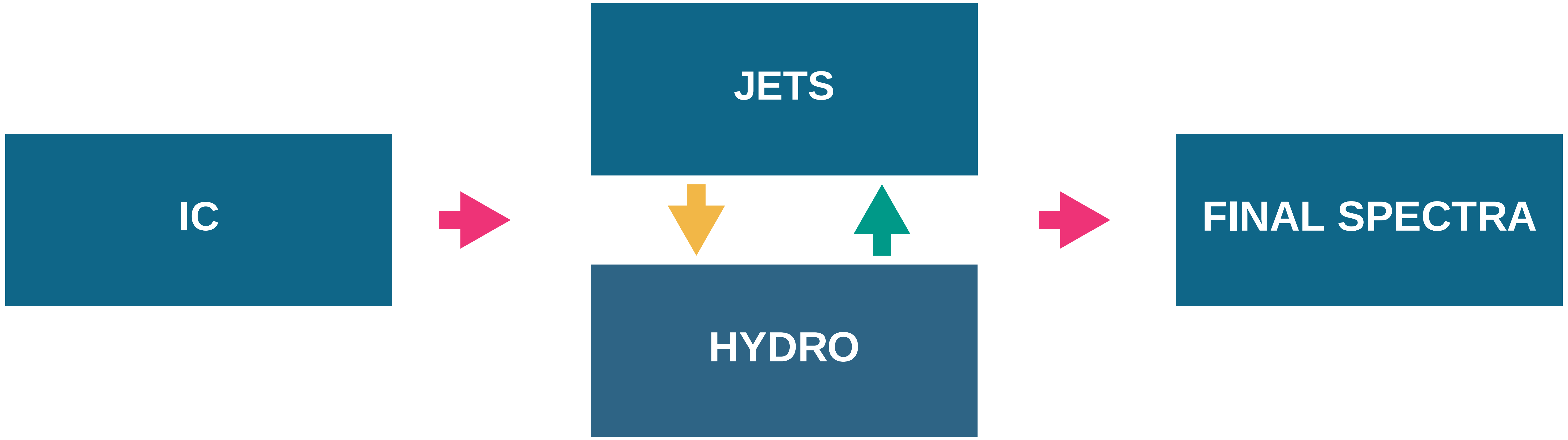


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Will be coupled soon

- Jet affects the medium — How far-from-equilibrium fluids behaves close to the jet?
- Medium affects jets — Isaac Long talk

# CCAKE

- Relativistic Viscous Hydrodynamic
- Lagrangian - SPH
- Significantly improved version of v-USPhydro

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BSQ charges evolution

(3+1)D simulations

Performance portability

Improved equations of motion

Hyperbolic and cartesian coordinates

Source terms

# SPH METHOD

# SPH METHOD

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# SPH METHOD

# CLASSICAL SPH

$$A(\mathbf{r}) = \int_V A(\mathbf{r}')\delta(\mathbf{r}' - \mathbf{r})dV'$$

- Define the kernel: $W(\mathbf{r}, h)$: $\int_V W(\mathbf{r}, h)dV = 1$ , $\lim_{h \to 0} W(\mathbf{r}, h) = \delta(\mathbf{r})$

$$A(\mathbf{r}) \approx \sum_{i=1}^{N} A(\mathbf{r_i})W(\mathbf{r_i} - \mathbf{r}, h)\Delta V_i$$

$$\nabla A(\mathbf{r}) \approx \sum_{i=1}^{N} A(\mathbf{r_i})\nabla W(\mathbf{r_i} - \mathbf{r}, h)\Delta V_i$$

# RELATIVISTIC SPH

- $q$ any quantity density at rest frame
- $q^* = qu^0\sqrt{-g}$
- M some conserved quantity

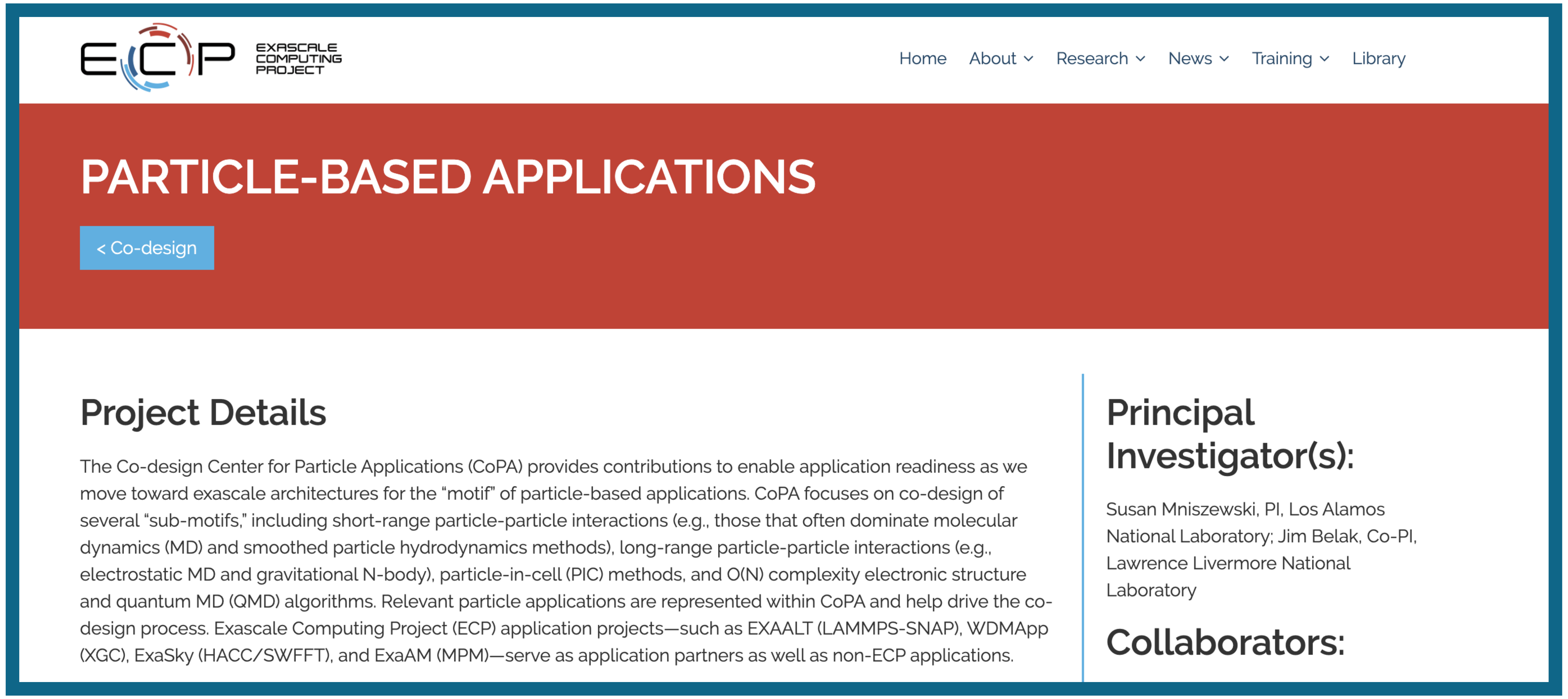
$$M_{tot} = \sum_{b=1}^{N} m_b$$

$$\sigma_a^* = \sum_{b=1}^{N} m_b W(\mathbf{r_a} - \mathbf{r_b}, h)$$

$$q_a^* = \sum_{b=1}^{N} m_b \frac{q_b^*}{\sigma_b^*} W(\mathbf{r_a} - \mathbf{r_b}, h)$$

# CCAKE CENTRAL LIBRARY

- Exascale Computing Project (ECP) - Co-design Center for Particle Applications (COPA)
- ECP-COPA/Cabana used in CCAKE
- Performance portability
  - - Personal Computers
  - - Workstations
  - - Clusters w/wo GPUs




https://www.exascaleproject.org/research-project/particle-based-applications/

# CCAKE: SPH

- Particle number as conserved quantity
- Ideal and dissipative

$$m_b = 1$$

$$\frac{1}{V_a^*} = \sigma_a^* = \sum_{b=1}^{N} m_b W(\mathbf{r_a} - \mathbf{r_b}, h)$$

- Specific density $\sigma = \frac{1}{V}$

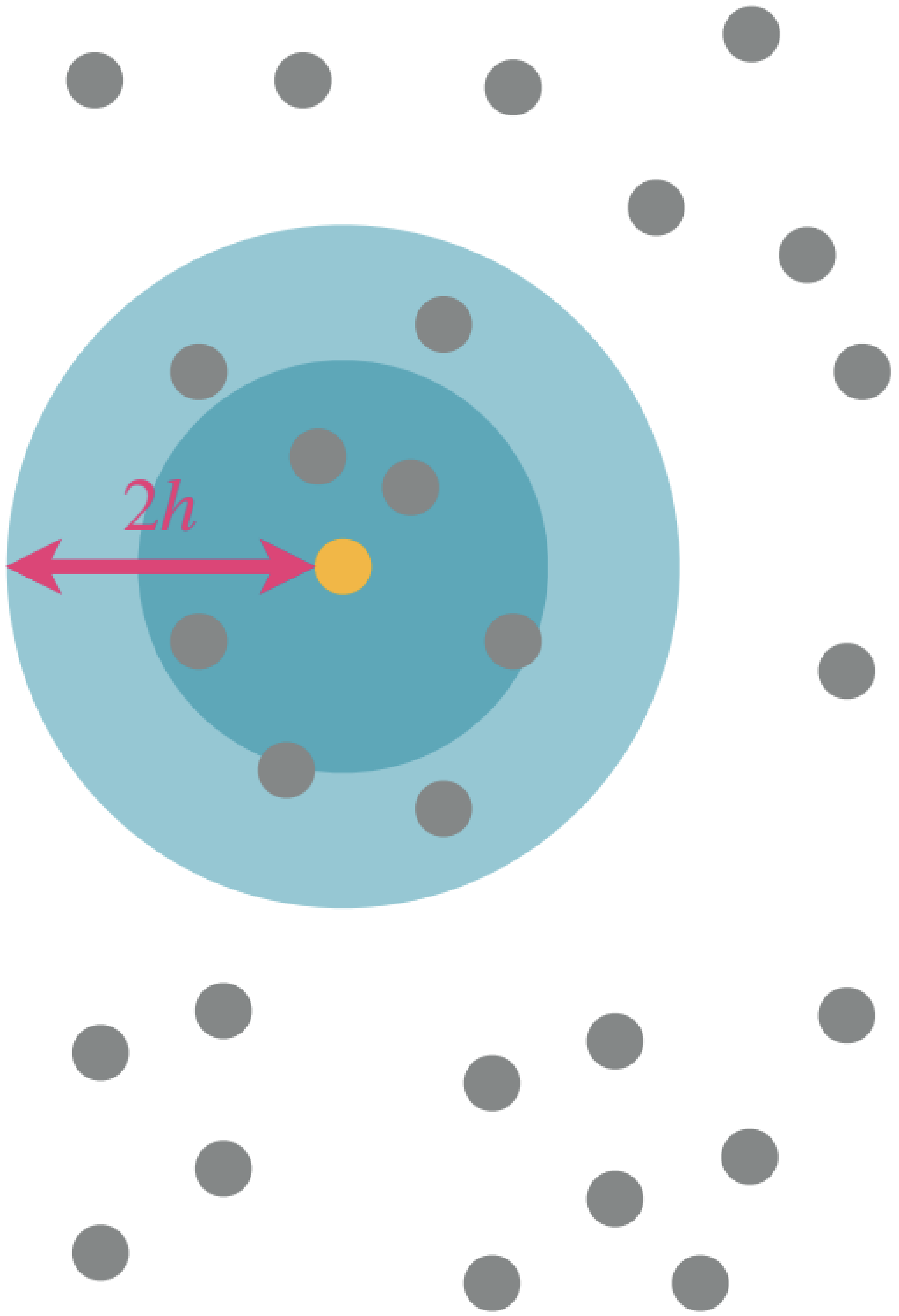
# KERNEL

- Cubic spline

- $C = \frac{1}{6h}, D = 1$
- $C = \frac{15}{14h^2}, D = 2$
- $C = \frac{1}{4h^3}, D = 3$

$$W(\mathbf{r}, h) = C \begin{cases} (2-q)^3 - 4(1-q)^3 & , 0 \le q < 1 \\ (2-q)^3 & , 1 \le q < 2 \\ 0 & , q \ge 2 \end{cases}$$

$q = |\mathbf{r}|/h$



# CCAKE: SPH

- $\sigma = \frac{1}{V}$

Dynamical variables

- $S = \frac{s}{\sigma}, N_X = \frac{n_X}{\sigma}, \tilde{\Pi} = \frac{\Pi}{\sigma}, \tilde{\pi} = \frac{\pi}{\sigma}$
- $x^\mu, u^\mu$

$$s^* = \sum_{b=1}^{N} m_b S_b W(\mathbf{r_a} - \mathbf{r_a}, h)$$

$$n_X^* = \sum_{b=1}^{N} m_b N_{X,b} W(\mathbf{r_a} - \mathbf{r_a}, h)$$

# CCAKE: EQUATIONS OF MOTION

Source terms

Turned off

- Hyperbolic and cartesian coordinates

$$d_\mu N_X^\mu = \rho_X$$

$$d_\mu T^{\mu\nu} = j^\nu$$

$$\tau_\Pi D\Pi + \Pi = -\zeta\Theta + \mathcal{J} + \mathcal{R} - \frac{\tau_\Pi \dot{\beta}_\Pi}{2\beta_\Pi}\Pi$$

arXiv:2209.11210

$$\mathcal{J} = -\delta_{\Pi\Pi}\Pi\Theta - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

$$\mathcal{R} = \varphi_1\Pi^2 + \varphi_3\pi_{\mu\nu}\pi^{\mu\nu}$$

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{J}^{\mu\nu} + \mathcal{R}^{\mu\nu} - \frac{\tau_\pi \dot{\beta}_\pi}{2\beta_\pi}\pi^{\mu\nu}$$

$$\mathcal{J}^{\mu\nu} = -\delta_{\pi\pi}\pi^{\mu\nu}\Theta - 2\tau_\pi\Delta^{\mu\nu}_{\alpha\beta}\pi^\alpha_\lambda\omega^{\beta\lambda} - \tau_{\pi\pi}\Delta^{\mu\nu}_{\alpha\beta}\pi^{\lambda\alpha}\sigma^\beta_\lambda + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}$$

$$\mathcal{R} = \varphi_6\Pi\pi^{\mu\nu} + \varphi_7\Delta^{\mu\nu}_{\alpha\beta}\pi^{\lambda\alpha}\pi^\beta_\lambda$$

# CCAKE: EQUATIONS OF MOTION

Source terms

Turned off

- Hyperbolic and cartesian coordinates

Invert equations and isolate variables

$$d_\mu N_X^\mu = \rho_X$$

$$d_\mu T^{\mu\nu} = j^\nu$$

$$\tau_\Pi D\Pi + \Pi = -\zeta\Theta + \mathcal{J} + \mathcal{R} - \frac{\tau_\Pi \dot{\beta}_\Pi}{2\beta_\Pi}\Pi$$

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$$\mathcal{J} = -\delta_{\Pi\Pi}\Pi\Theta - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

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$$\mathcal{R} = \varphi_6 \Pi\pi^{\mu\nu} + \varphi_7 \Delta^{\mu\nu}_{\alpha\beta}\pi^{\lambda\alpha}\pi^\beta_\lambda$$

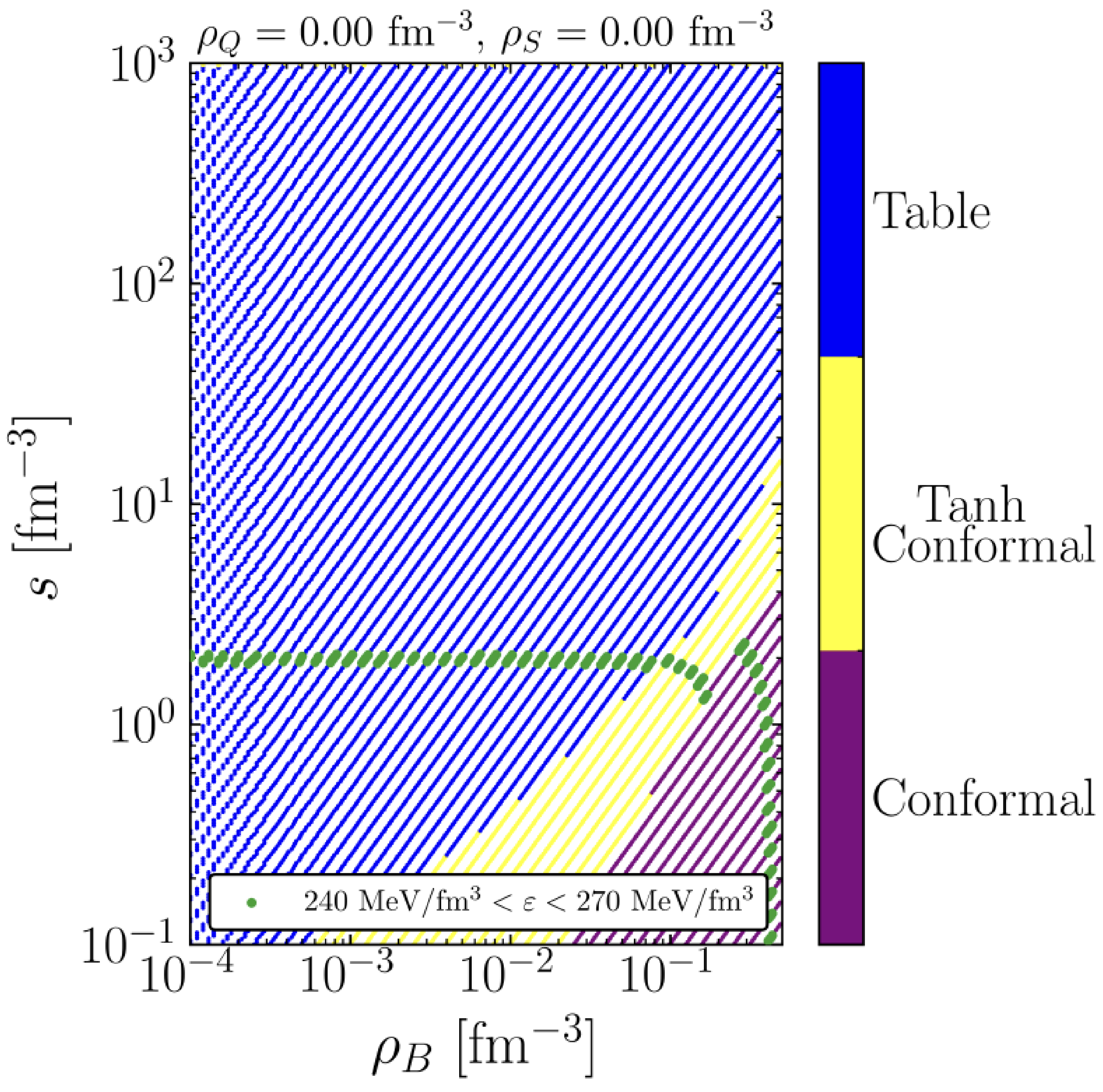
# CCAKE: EQUATION OF STATE

- QCD EoS based on a Taylor series expansion up to O($\mu_X^4$)
- Coupled to a Hadron Resonance Gas using the PDG2016+ list
- Lattice EOS doesn't cover all necessary regions

# CCAKE: EQUATION OF STATE

- QCD EoS based on a Taylor series expansion up to O($\mu_X^4$)
- Coupled to a Hadron Resonance Gas using the PDG2016+ list
- Fallback equations

arXiv:2405.09648




$$p(T,\vec{\mu}) = \frac{1}{2} A_0 T_0^4 \left[ \left(\frac{T}{T_0}\right)^2 + \sum_X \left(\frac{\mu_X}{\mu_{X,0}}\right)^2 \right]^2$$

LATTICE-BASED → TANH-CONFORMAL → CONFORMAL → CONFORMAL DIAGONAL

$$p(T,\vec{\mu}) = \frac{1}{2} A_0 T_0^4 \left(1 + \tanh\left(\frac{T - T_c}{T_s}\right)\right) \left[ \left(\frac{T}{T_0}\right)^2 + \sum_X \left(\frac{\mu_X}{\mu_{X,0}}\right)^2 \right]^2$$

$$p(T,\vec{\mu}) = \frac{1}{2} A_0 T_0^4 \left[ \left(\frac{T}{T_0}\right)^2 + \sum_X \left(\frac{\mu_X}{\mu_{X,0}}\right)^4 \right]$$

# CCAKE: CODE STRUCTURE

# ANALYTICAL CHECKS

- $(1+1)D$ longitudinal analytical
- $(2+1)D$ transversal analytical

# (1+1)D ANALYTICAL

$$a = 1$$
$$\eta_0 = 0$$

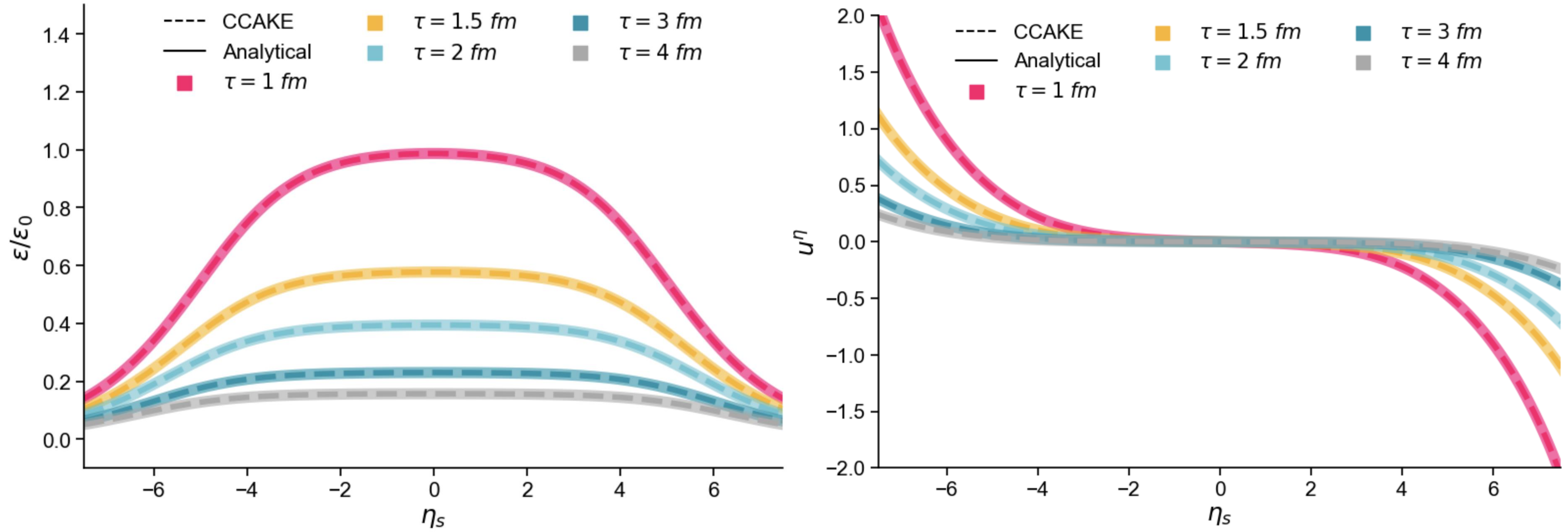
PRC **105**, L021902

- $(1+1)D$ longitudinal expansion check
- Conformal EOS ($\mu_X = 0$)
- Ideal

$$u^\eta(\eta,\tau) = \frac{1}{2\tau}\left(\sqrt{\frac{t_0 e^{\eta_0-\eta} + \tau a}{t_0 e^{\eta-\eta_0} + \frac{\tau}{a}}} - \sqrt{\frac{t_0 e^{\eta-\eta_0} + \frac{\tau}{a}}{t_0 e^{\eta_0-\eta} + \tau a}}\right)$$

$$\epsilon(\eta,\tau) = \epsilon_0 \left(\frac{t_0}{\tau_0} + \frac{a\tau}{\tau_0} e^{\eta-\eta_0}\right)^{\frac{(1-c_s^4)}{4c_s^2}\frac{1}{a^2} - \frac{(1+c_s^2)^2}{4c_s^2}} \left(\frac{t_0}{\tau_0} + \frac{\tau}{a\tau_0} e^{\eta_0-\eta}\right)^{\frac{(1-c_s^4)}{4c_s^2}a^2 - \frac{(1+c_s^2)^2}{4c_s^2}}$$

# (1+1)D ANALYTICAL



# (2+1)D ANALYTICAL

$$q = 1$$

$$r = \sqrt{x^2 + y^2}$$

$$\sinh(\rho) = -\frac{1 - q^2\tau^2 + q^2r^2}{2q\tau}$$

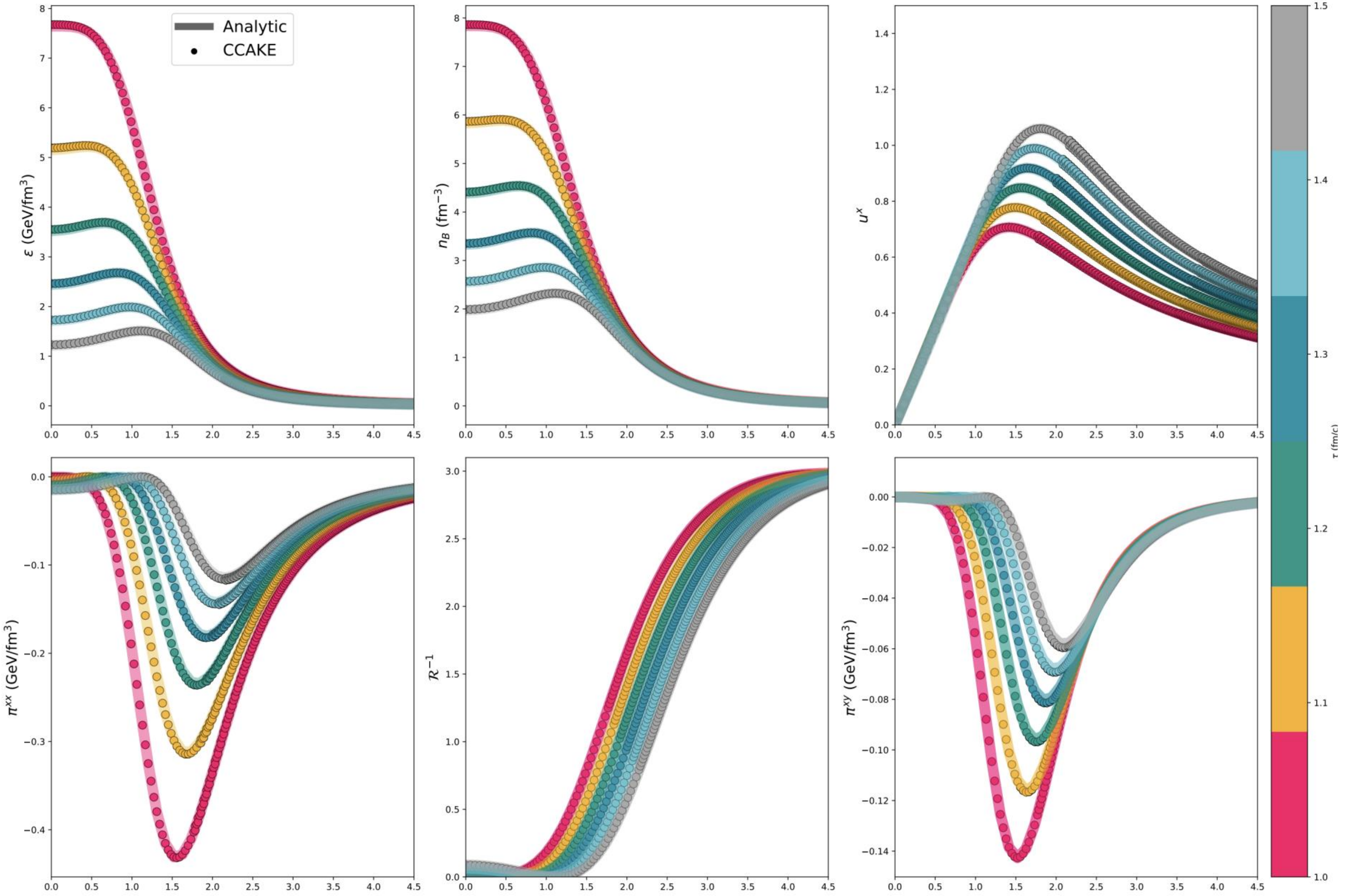
$$\tan(\theta) = \frac{2qr}{1 + q^2\tau^2 - q^2r^2}$$

- $(2+1)D$ transversal - Gubser
- Conformal EOS
- Shear viscosity

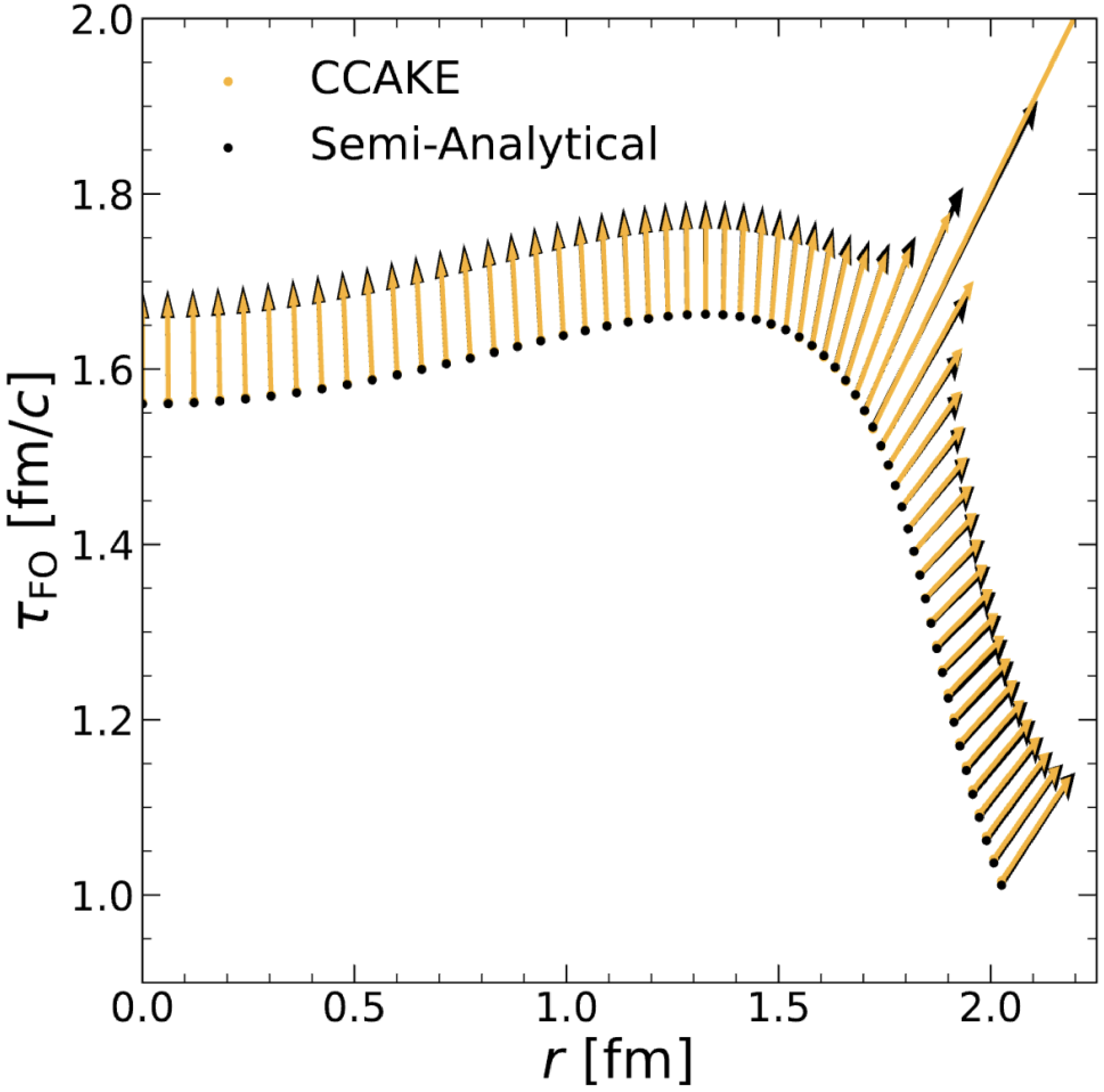
$$\frac{1}{\hat{T}}\frac{d\hat{T}}{d\rho} = \left[\frac{1}{3}(\hat{\pi} - 2) + \hat{\pi}\sum_Y \left(\frac{\mu_Y}{T}\right)^2\right]\tanh(\rho)$$

$$\frac{1}{\hat{\mu}_Y}\frac{d\hat{\mu}_Y}{d\rho} = -\frac{2}{3}(1 + \hat{\pi})\tanh(\rho)$$

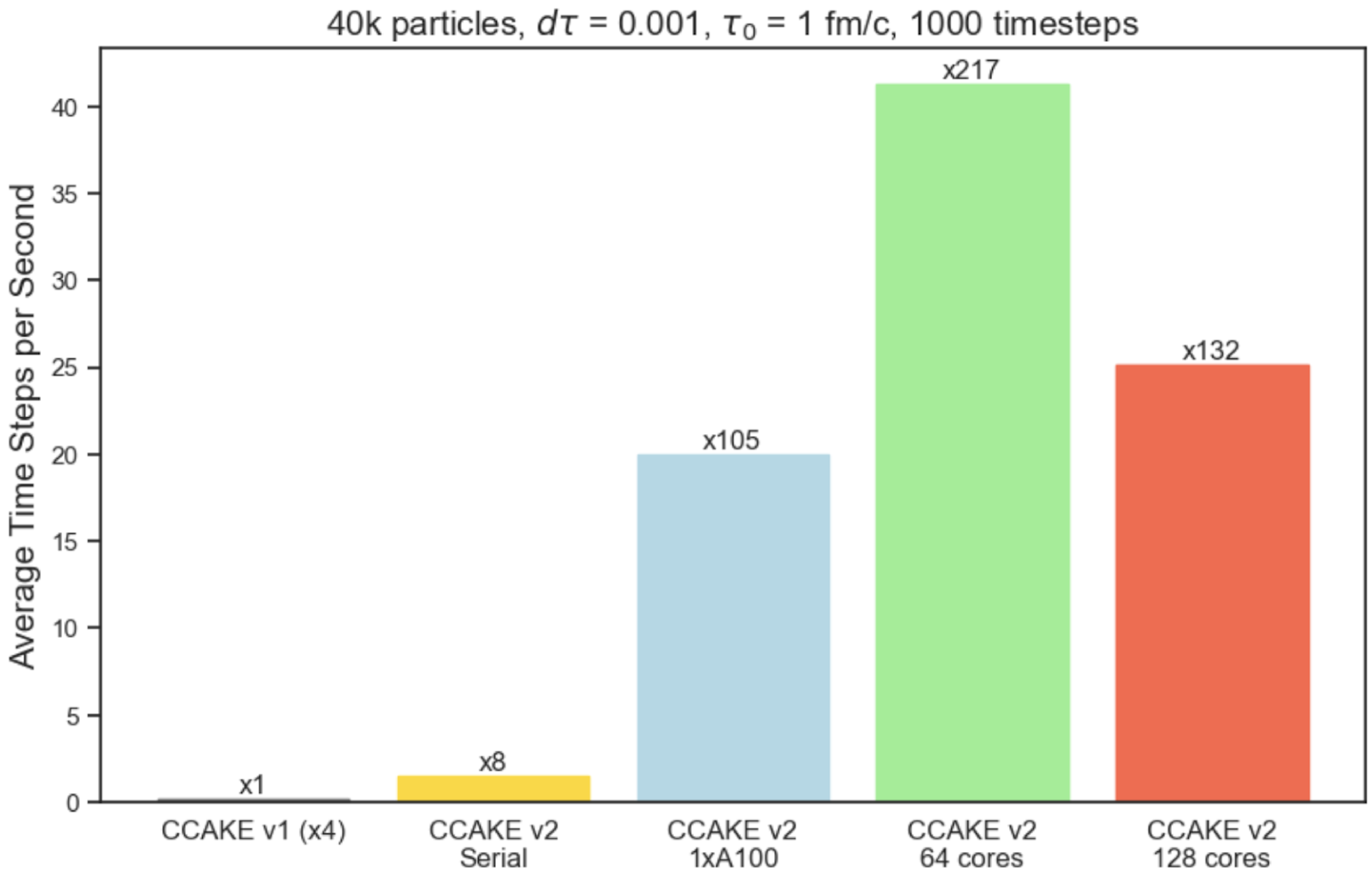
$$\frac{d\hat{\pi}}{d\rho} = \frac{4}{12}\tanh(\rho) - \frac{\hat{\pi}}{\tau_R} - \frac{4}{3}\hat{\pi}^2\tanh(\rho)$$


Analytic
CCAKE
ε (GeV/fm³)
$n_B$ (fm⁻³)
$u^x$
$\pi^{xx}$ (GeV/fm³)
$\mathcal{R}^{-1}$
$\pi^{xy}$ (GeV/fm³)
τ (fm/c)

# (2+1)D ANALYTICAL



# PERFORMANCE



# (2+1)D EBE SIMULATIONS

arXiv:2405.09648

- $(2+1)D$ with shear viscosity
- Full EOS

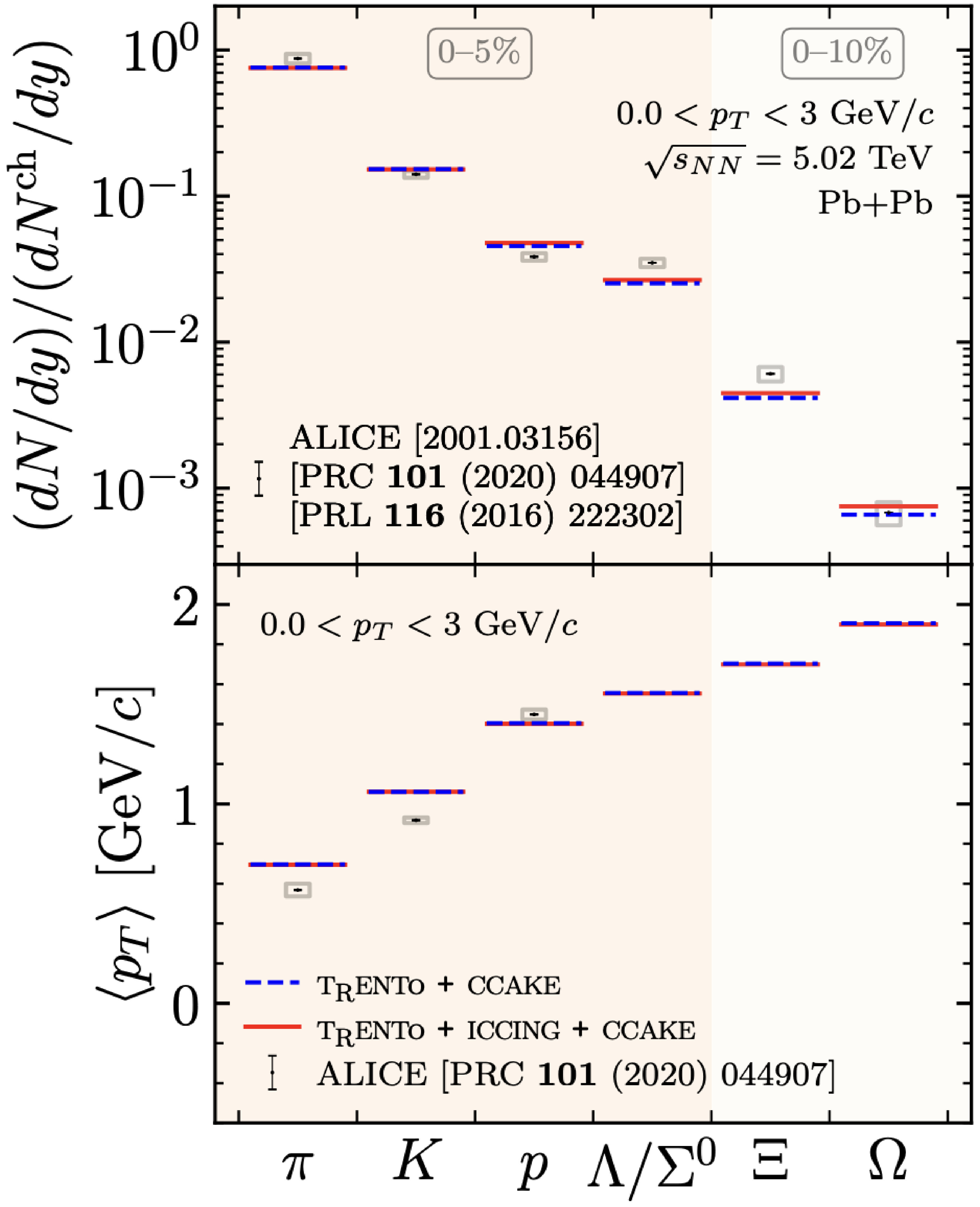
$T_R ENTo$ — PRC 92 (2015) 011901

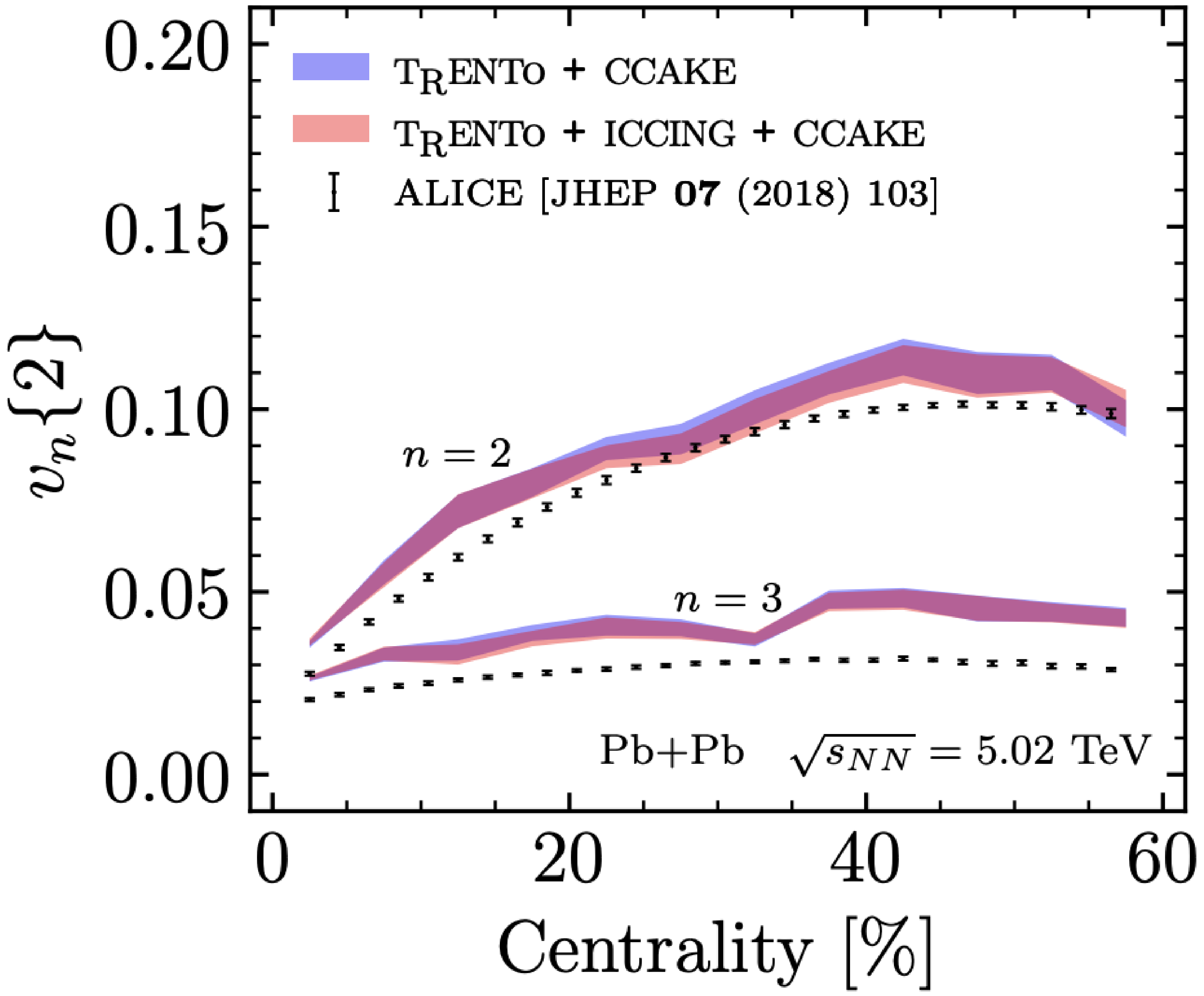
↓

ICCING — arXiv:1911.10272, PRC 105 (2022) 034908

↓

CCAKE V1 → FINAL SPECTRA



# SOURCE TERMS

PRC 90 (2014) 2, 024914
PRL 97 (2006) 062301

- Di-jet in the transverse plane
- Moving at the speed of light

$$\vec{r}_{jet} = \vec{r}_{jet,0} + (\tau - \tau_0)\vec{v}$$

$$j^\nu(\tau, \vec{r}) = \frac{dE}{dl}\delta^3(\vec{r} - \vec{r}_{jet})\frac{u^\nu}{\gamma}$$

# SOURCE TERMS

PRC 90 (2014) 2, 024914
PRL 97 (2006) 062301

- Di-jet in the transverse plane $\vec{r}_{jet} = \vec{r}_{jet,0} + (\tau - \tau_0)\vec{v}$
- Moving at the speed of light

Normalization $\int \frac{W}{\tau} \tau dx dy d\eta = \Delta\eta$

$$W(\vec{r} - \vec{r}_{jet}, h)\frac{1}{\tau}$$

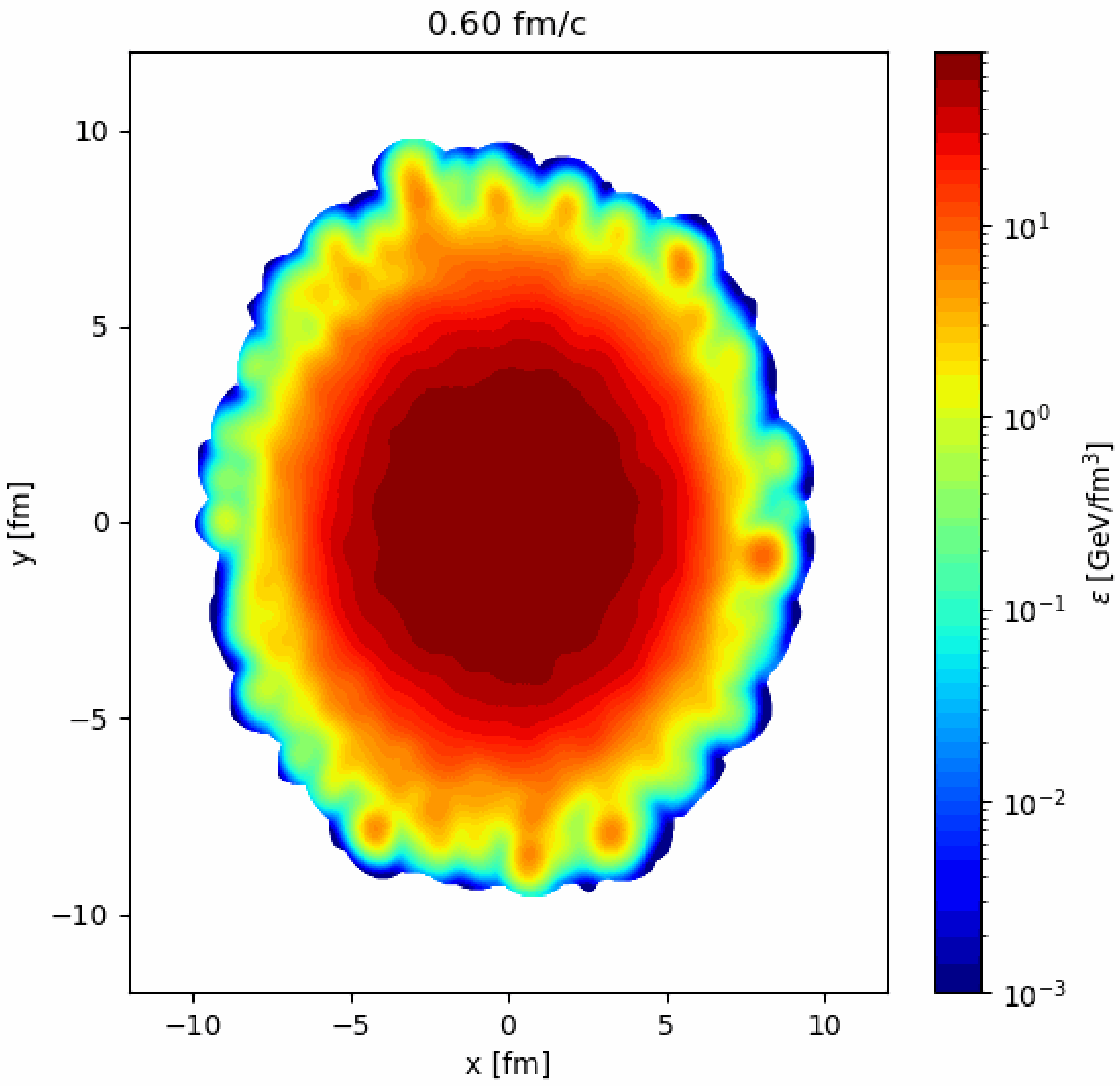
$$j^\nu(\tau, \vec{r}) = \frac{dE}{dl}\delta^3(\vec{r} - \vec{r}_{jet})\frac{u^\nu}{\gamma}$$

$$\frac{dE}{dl} = \frac{s(\vec{r}_{jet})}{s_0}\frac{dE}{dl}|_0$$

# SOURCE TERMS

- 300 ICCING events averaged
- PbPb $\sqrt{s}_{NN} = 5.02TeV$ , 0-5%
- Toy model to explore effects in hydro
- Di-jet forming at (0,0) at $\tau_0$
- Moving in $\pm x$ direction

# SOURCE OFF

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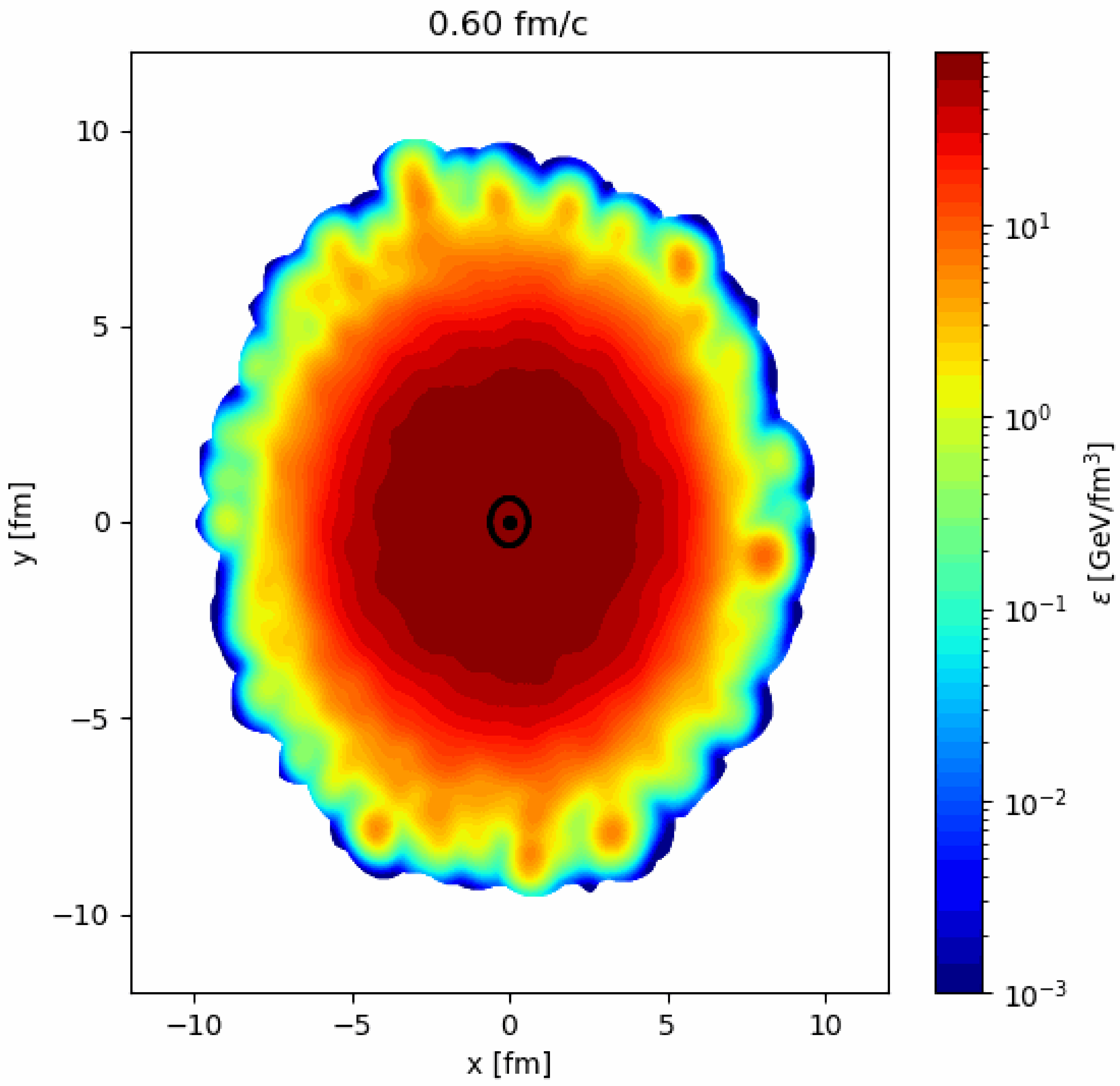

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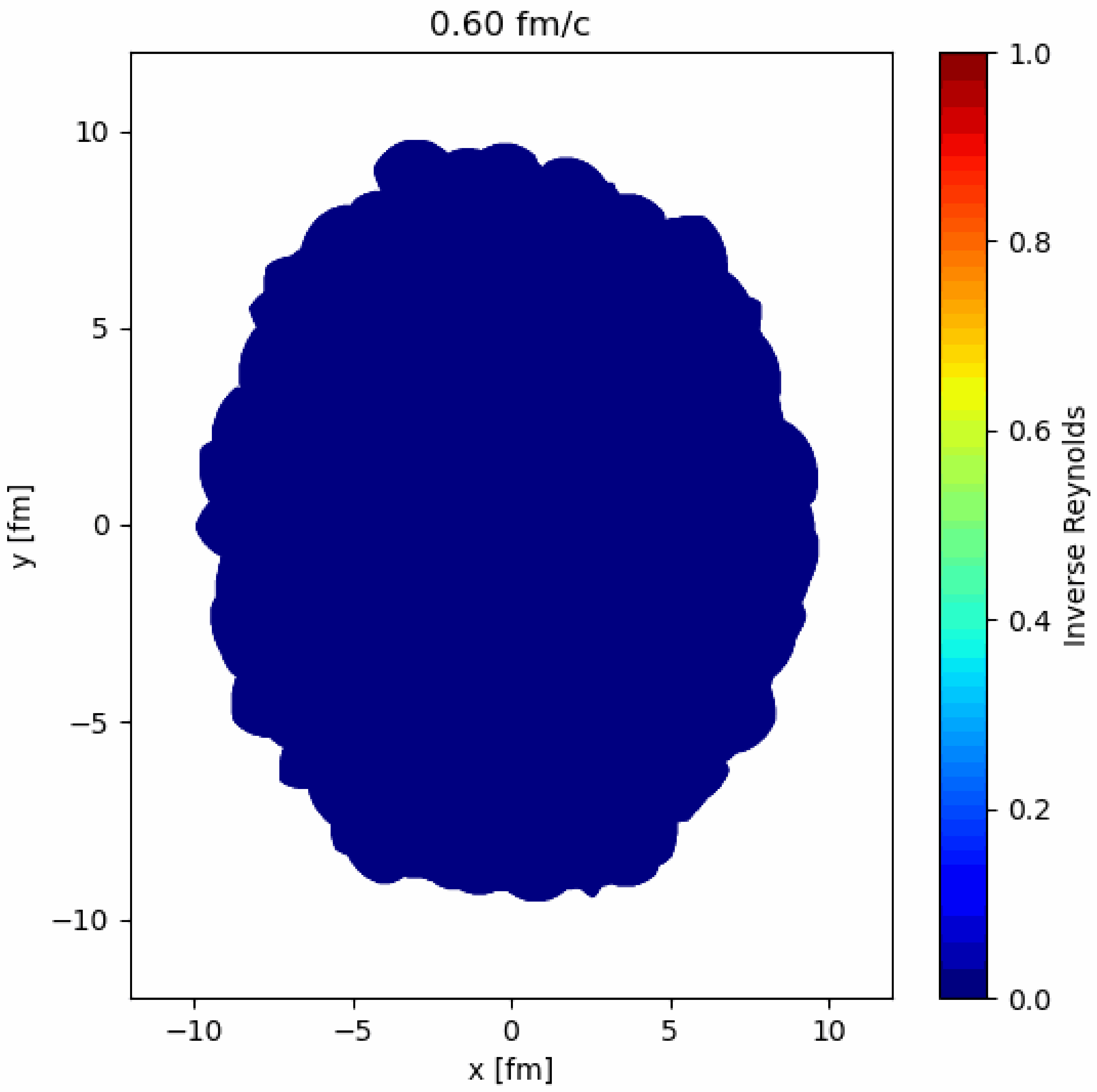


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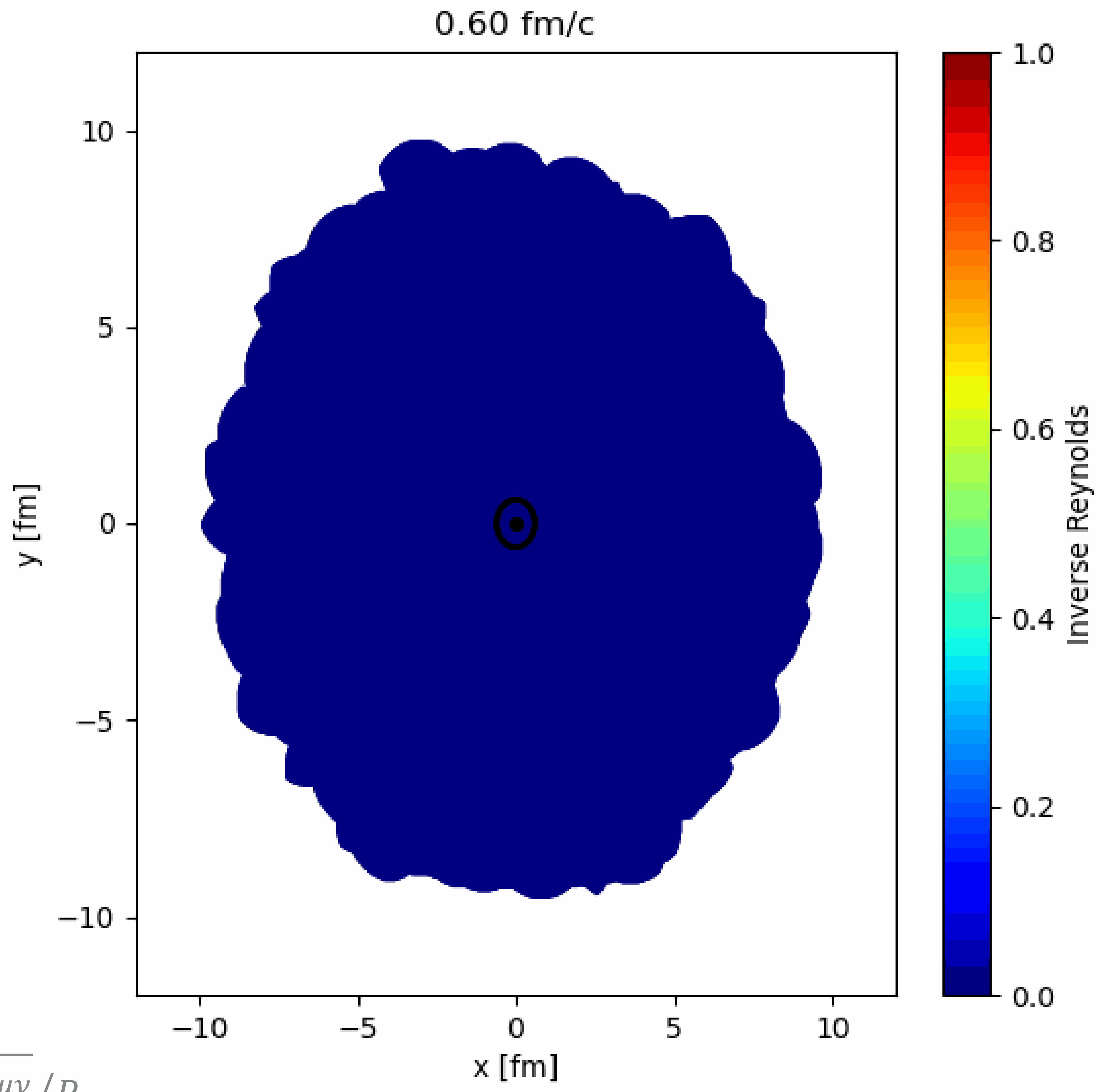


$\sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/P$

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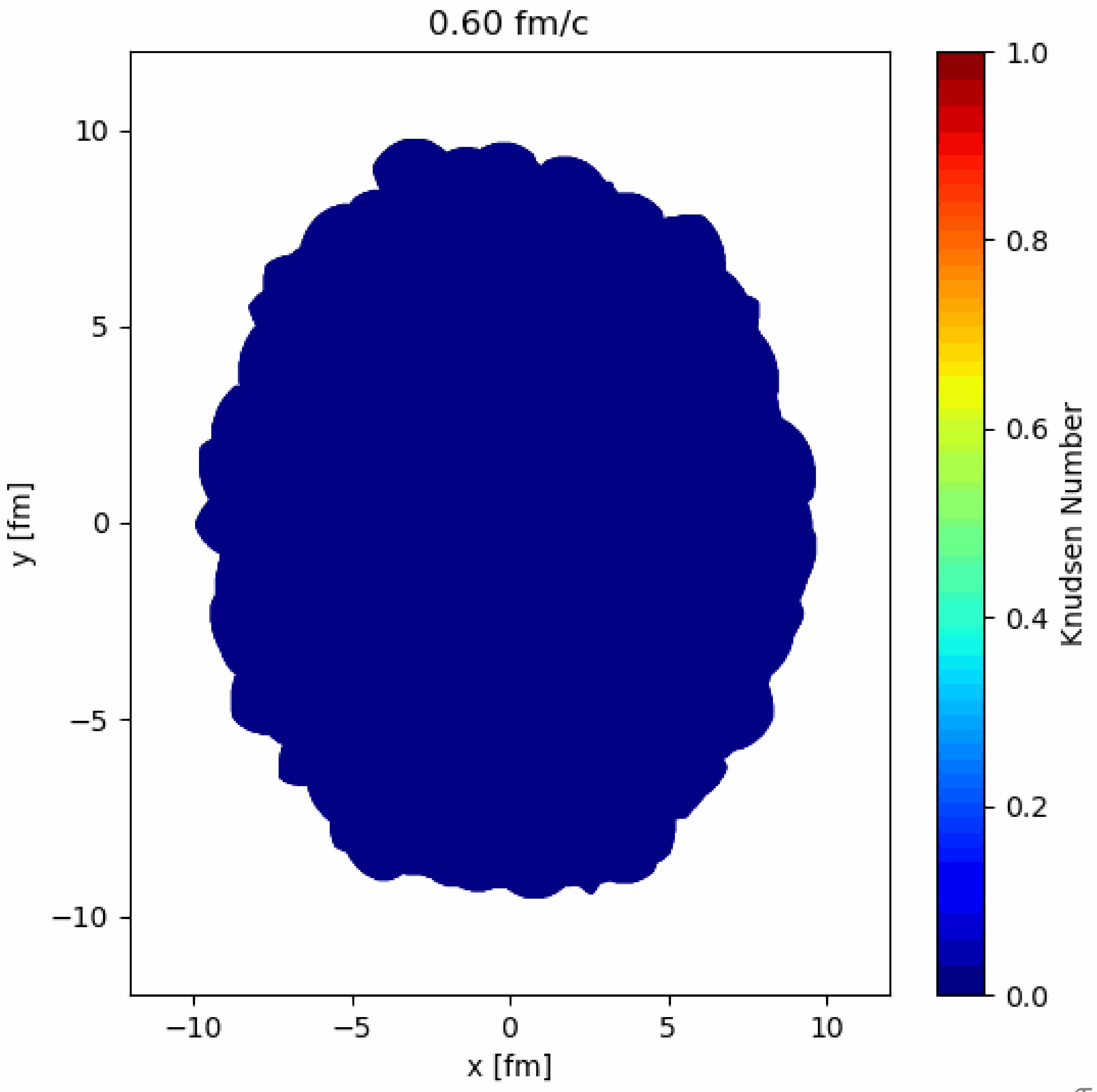


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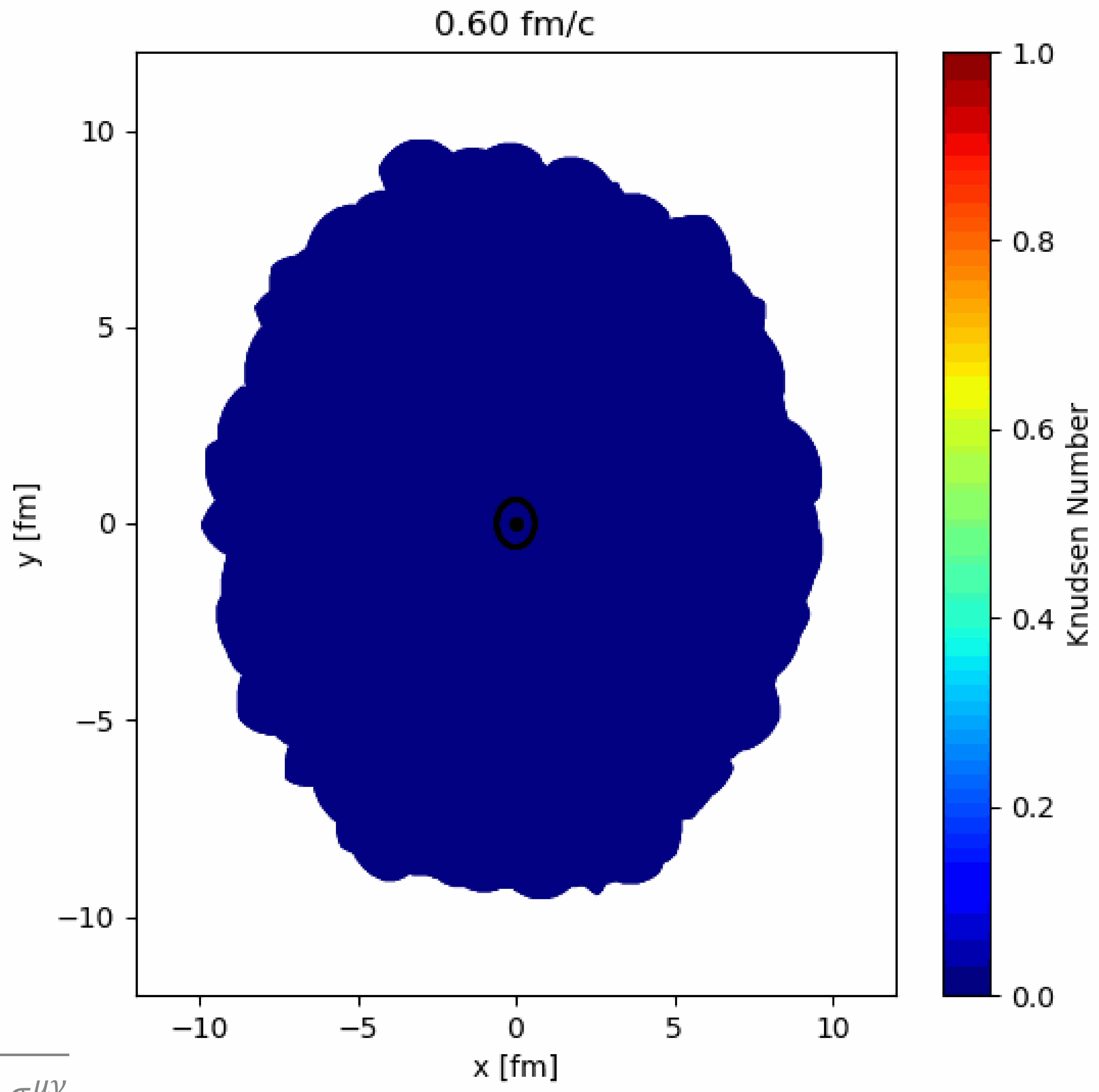


$\tau_\pi \sqrt{\sigma_{\mu\nu}\sigma^{\mu\nu}}$

# CONCLUSIONS AND OUTLOOK

- Performance portability (GPU/CPU)
- Excellent results for the (2+1)D and (1+1)D checks
- Reproduce data
- Hydrodynamics with sources terms
- Check causality near the jet
- Simulate with more physical parameters
- Couple with jet simulator

# BACKUP

# COORDINATE SYSTEM

- $c = \hbar = k_b = 1$
- $x^{\mu} == \{\tau, x, y, \eta_s\}$
- $\tau = \sqrt{t^2 - z^2}$
- $\eta_s = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$

# DERIVATIVES

- Partial derivatives

$$\partial_k q^* = \sum_{b=1}^{N} m_b \left[ q_b^* \frac{(\sigma_a^*)^{-1}}{(\sigma_b^*)^{n}} - q_a^* \frac{(\sigma_a^*)^{-1}}{(\sigma_b^*)^{n}} \right] \partial_k W_{ab}$$

- $n = 0$ usual
- $n = 1$ antisymmetric
- $n = -1$ symmetric

# DERIVATIVES

- Partial time derivatives depends on total derivatives

$$\mathrm{d}_0 A^\mu = \frac{\mathrm{d}A^\mu}{\mathrm{d}x_0} - v^i \partial_i A^\mu + \Gamma^\mu_{0\sigma} A^\sigma,$$

# $T^{\mu\nu}$ AND $N^{\mu}$

- Full

$$N^{\mu} = nu^{\mu} + q^{\mu}$$

$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^{\mu}u^{\nu} - g^{\mu\nu}(P + \Pi) + \pi^{\mu\nu}$$

$$\begin{aligned} u^{\mu}\mathrm{d}_{\mu} &= \mathrm{D}, \\ \Delta^{\mu\nu} &= g^{\mu\nu} - u^{\mu}u^{\nu} \\ \nabla_{\mu} &= \Delta_{\mu}^{\alpha} d_{\alpha} \end{aligned}$$

# CCAKE: FREEZE-OUT SURFACE

- Surface $\varepsilon_{fo} = \varepsilon(x = (x^0, x^1, x^2, x^3))$

$$n_\mu = \frac{N_\mu}{\sqrt{|N_\mu N^\mu|}} = -\frac{\partial_\mu \varepsilon_{fo}(x)}{\sqrt{|\partial_\mu \varepsilon_{fo}(x) \partial^\mu \varepsilon_{fo}(x)|}}$$

# EXAFLOP

- $10^{18}$ operations / second

# JET SIMULATION PARAMETERS

$$s_0 = 200fm^{-3}$$

$$\frac{dE}{dl}|_0 = 30GeV$$

$$h_{jet} = 0.3fm$$

$$\Delta\eta = 0.1$$

$$\Delta E \sim 500GeV$$

$$\tau_0 = 0.6fm/c$$

$$\frac{\eta}{s} = 0.2$$

$$\tau_\pi = \frac{\eta}{5w}$$