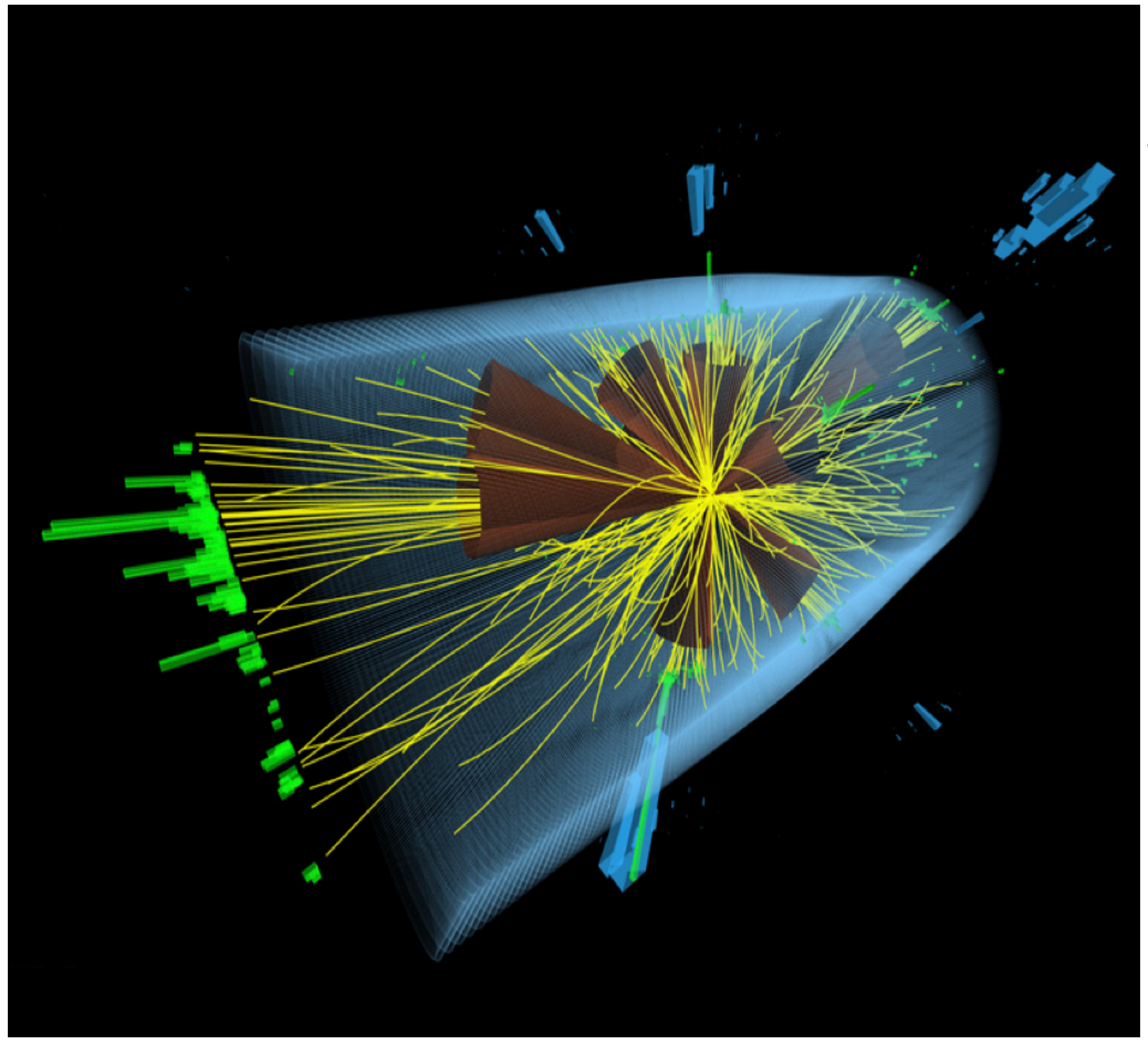
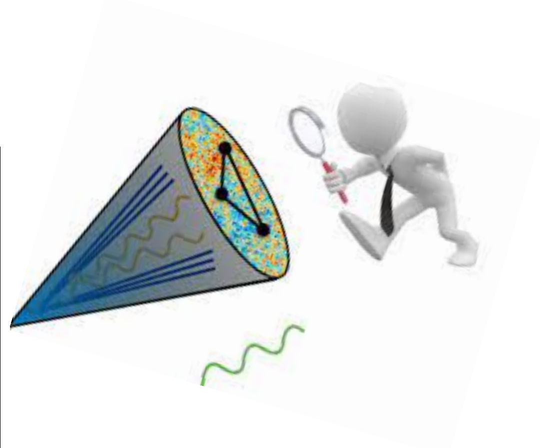
Factorized approach for jet production and substructure observables in HICs





Balbeer Singh

Department of Physics
University of South Dakota

Collaborators: Felix Ringer, Yacine Mehtar-Tani, Varun Vaidya, Ankita Budhraja

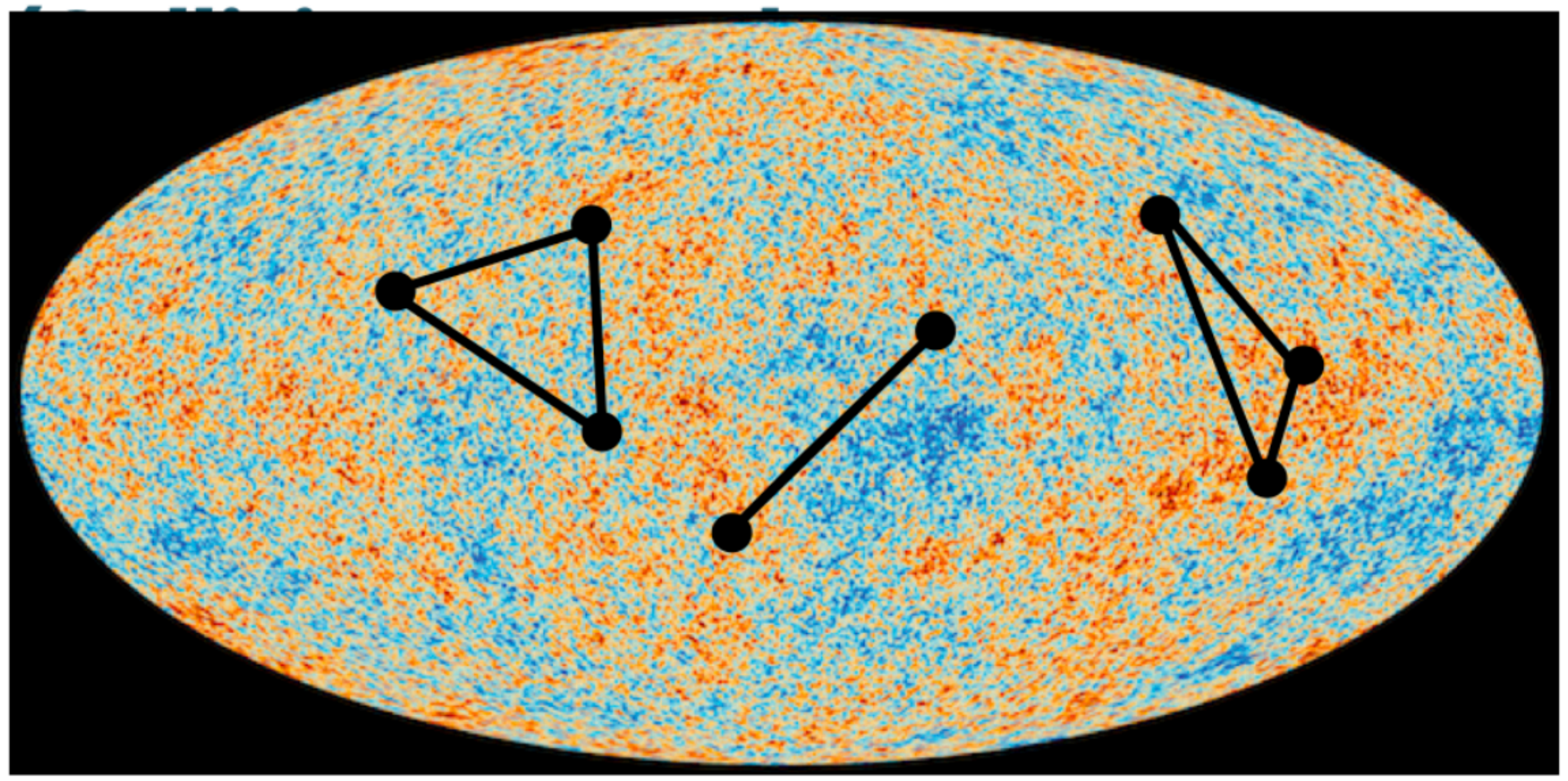
Hot Jets 2025

Based on works: 2412.18967, 2409.05957, 2408.02753, 25XX.XXXX, 25XX.XXXX

Energy flow operators

- Correlation functions are the fundamental objects that encode dynamics of underlying theory

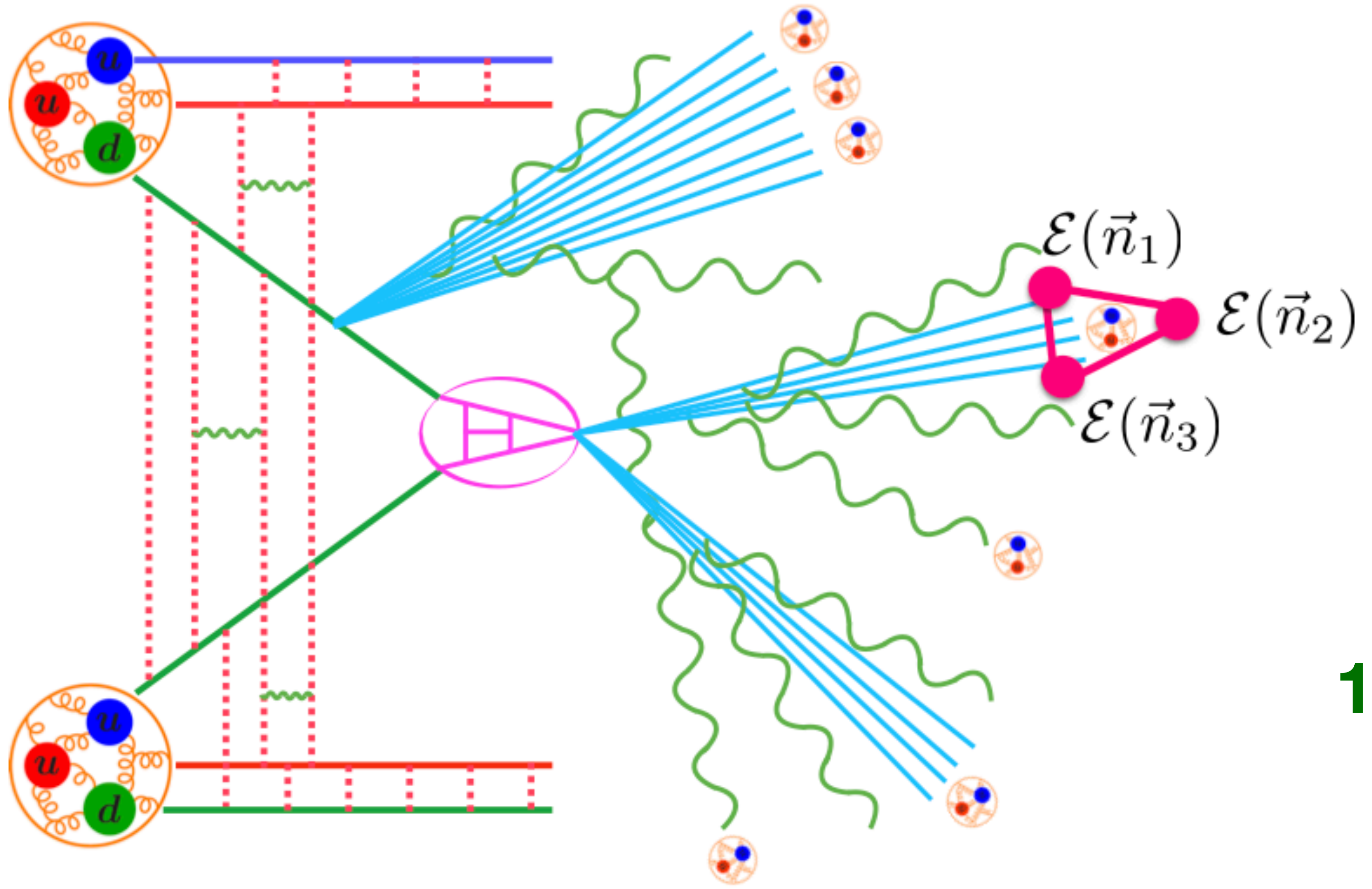
- In cosmology, correlations in tempertaure fluctuations relevant to study the structure formation in early universe



- In particle physics, correlations in energy flow in final state jets/particles are useful to understand the dynamics of QCD

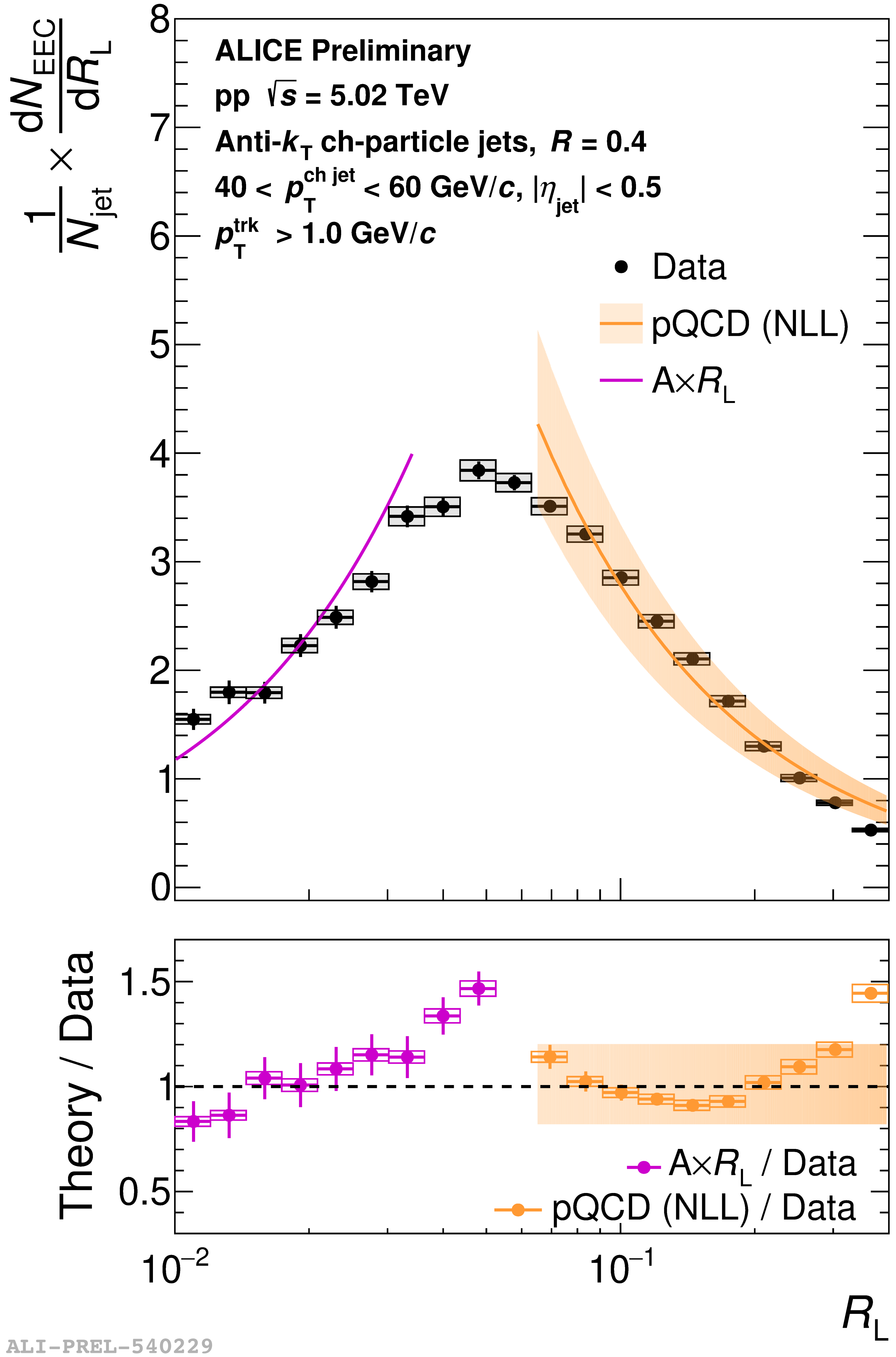
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\vec{n}_1\ldots d\vec{n}_N} = \frac{\langle \mathcal{O}\mathcal{E}(\vec{n}_1)\ldots\mathcal{E}(\vec{n}_N)\mathcal{O}^\dagger\rangle}{\langle \mathcal{O}\mathcal{O}^\dagger\rangle}$$

$$\mathcal{E}(\vec{n})\,|X\rangle = \sum_i k_i^0\,\delta^2(\Omega_{\vec{k}_i} - \Omega_{\vec{n}})\,|X\rangle$$



1905.01310

Energy-energy correlators



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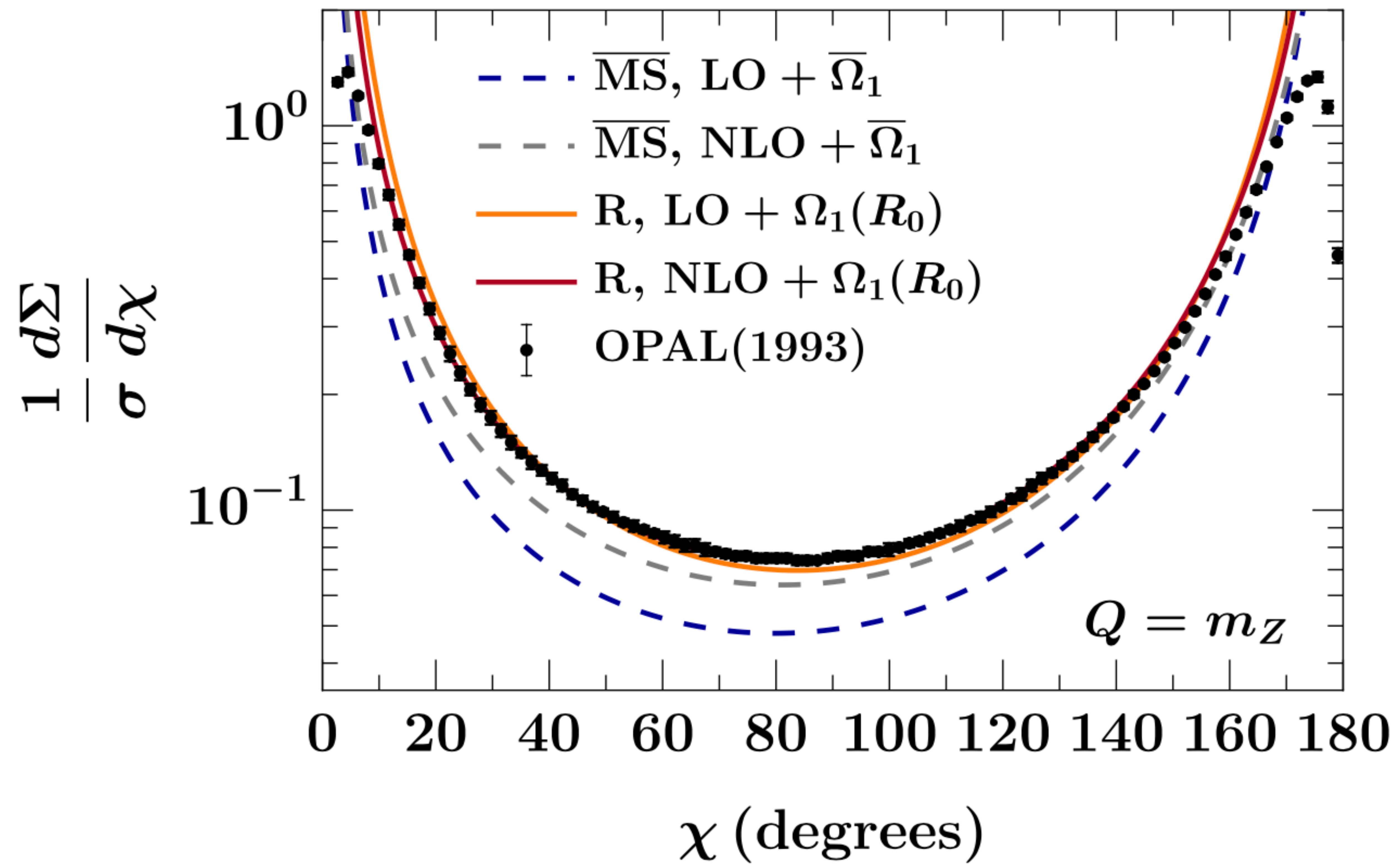
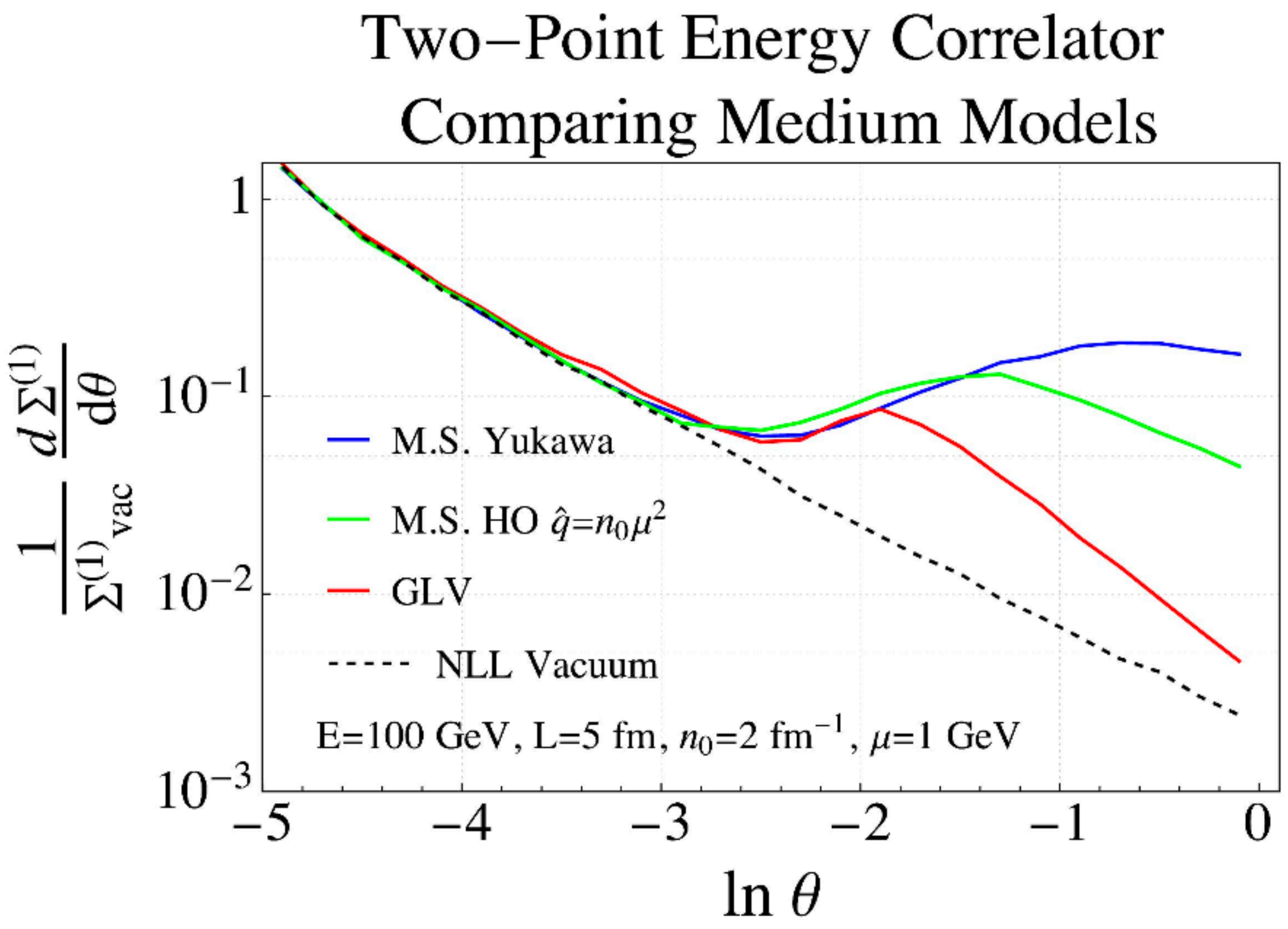
Two distinct scaling behaviours

Impressive agreement with data with leading non-perturbative effects



Schindler, Stewart, Sun '23

Some attempts for computing EEC in HIC



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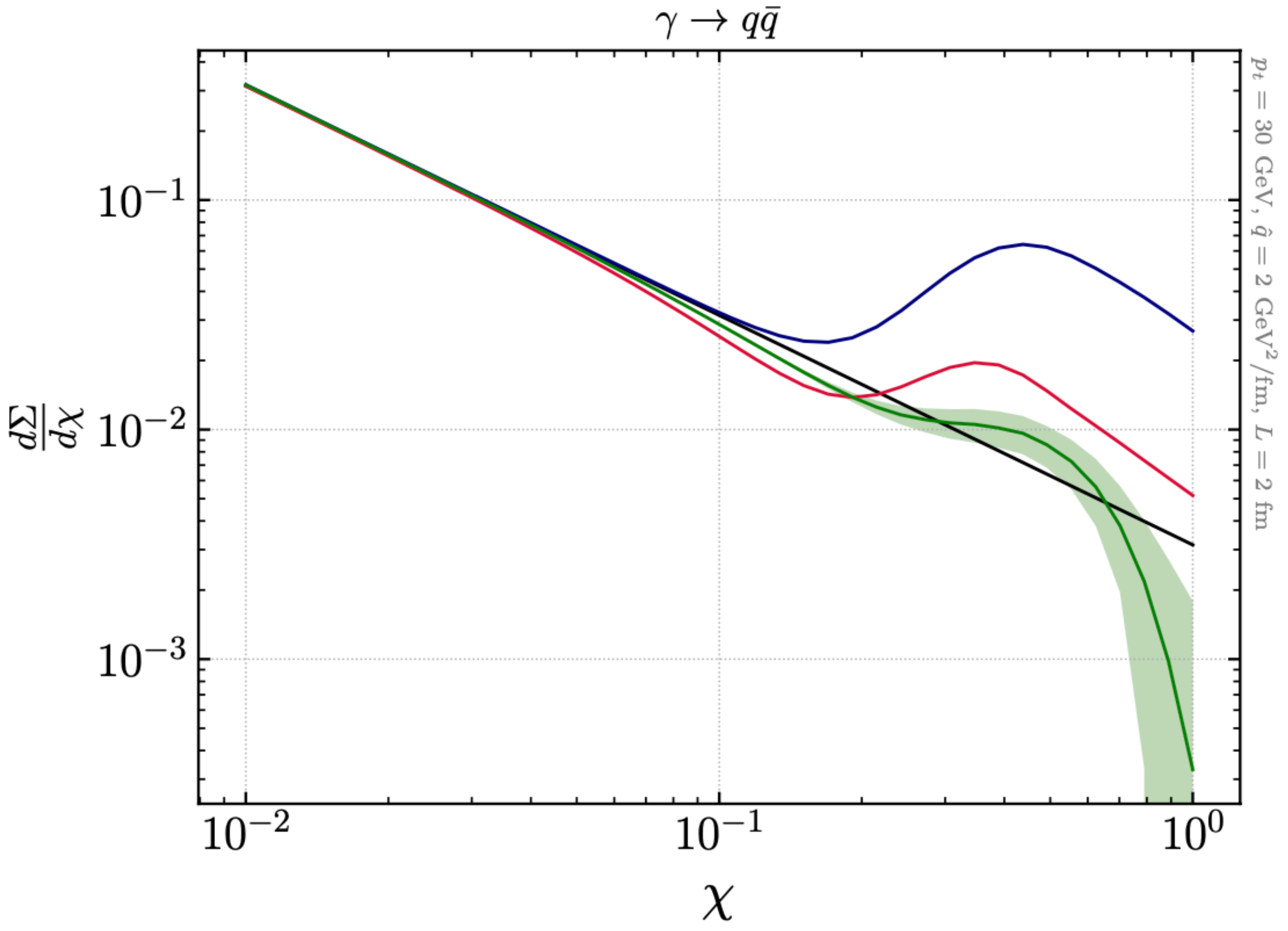
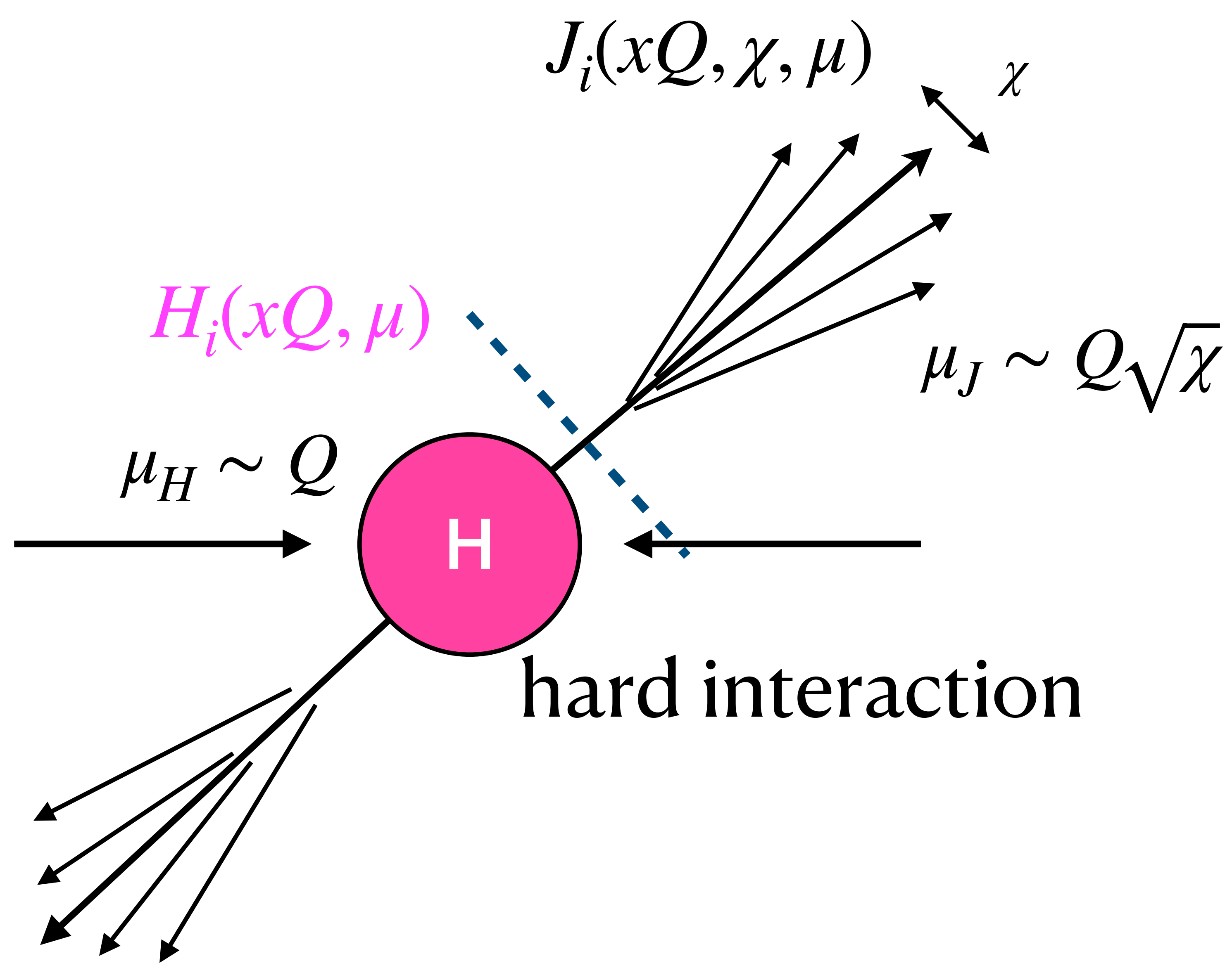
2312.12527 **CMS PAS HIN-23-004**



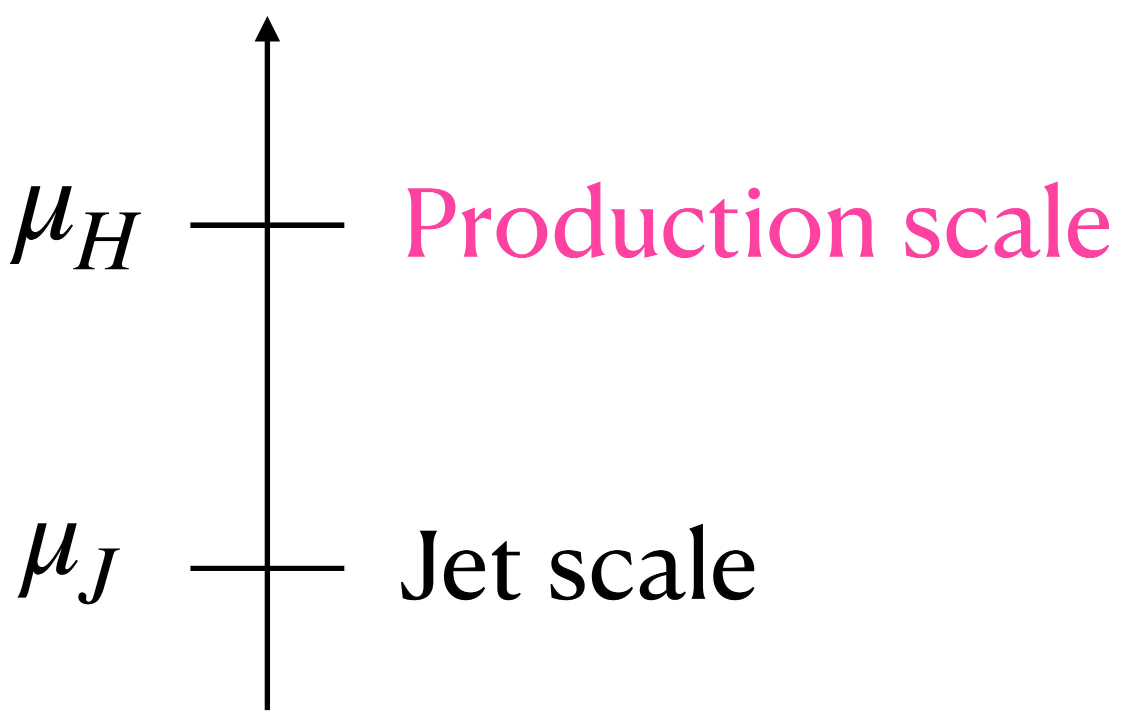
Full splitting function shows less enhancement in large angle region

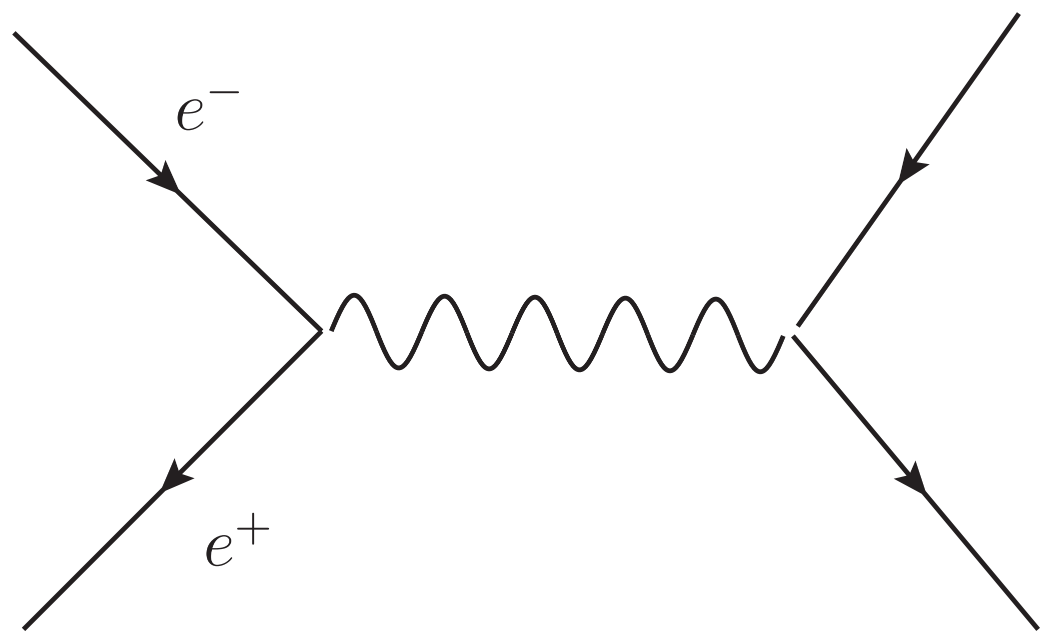
I will talk about factorization of this observable in HICs environment

Factorization in vacuum



Hard and jet scales are widely separated





Jet function describes subsequent evolution

Hard function describes the production of jet initiating parton

- Jets in vacuum: Only production and measurement scales

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\chi} = \sum_{i\in\{q,\bar{q},g\}}\int dx x^2 H_i(xQ,\mu)\, J_i(xQ,\chi,\mu)$$

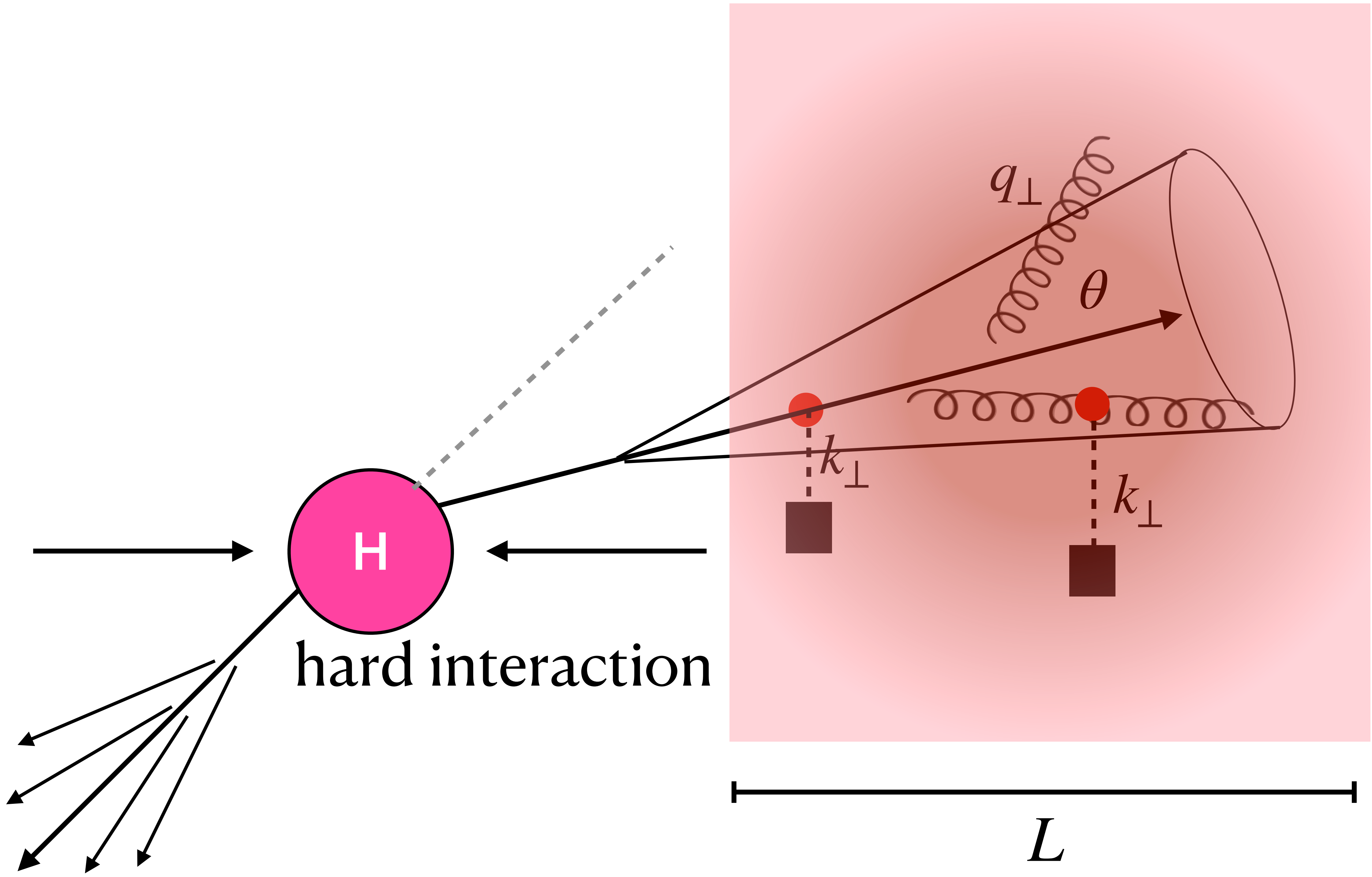
hard function μ_H — Jet function μ_J

$$J_q(\omega,\chi,\mu) = \frac{1}{2N_c}\sum_{X_n}\mathrm{Tr}\left[\frac{\bar{n}}{2}\langle 0|\chi_n(0)\mathscr{M}|X_n\rangle\langle X_n|\bar{\chi}_n(0)|0\rangle\right]$$

$\chi_n \rightarrow$ collinear quark operator

Jet propagation in medium

- Encounters multiple emergent and direct scales $T \sim m_D,\ L,\ t_f,\ \hat{q},\ \theta_c, \ldots$ Scale hierarchy: $\mu_H \gg T \sim m_D \gg \Lambda_{QCD}$



$T \sim m_D$ → medium temperature

$t_f \sim \dfrac{q_\perp^2}{\omega}$ → formation time

$\hat{q}$ → jet quenching parameter

$\theta_c \sim \dfrac{1}{\sqrt{\hat{q}L^3}}$ → critical angle **2411.11992**

Emergent scales should appear in the relevant phase space within the EFT set up

2412.18967

$k_\perp$ → Exchange momentum }

Single scattering: $k_\perp \sim m_D$

Multiple scattering: $k_\perp \sim \sqrt{\hat{q}L} \equiv Q_{\text{med}}$

Glauber interaction give small transverse kick to the jet parton and do not capture collisional energy loss

Wish list for jets in HIC

- Can we derive a similar factorization formula for any jet observable in HIC ?
- Can we separate out universal non-perturbative physics from the perturbative one ?
- Can we systematically improve computation/accuracy for jets in HIC ?
- Can we compute anomalous dimensions for jets and medium in HIC ?
- Can we relax model dependence ?

We attempt to answer some of these questions with energy correlators as jet substructure observable

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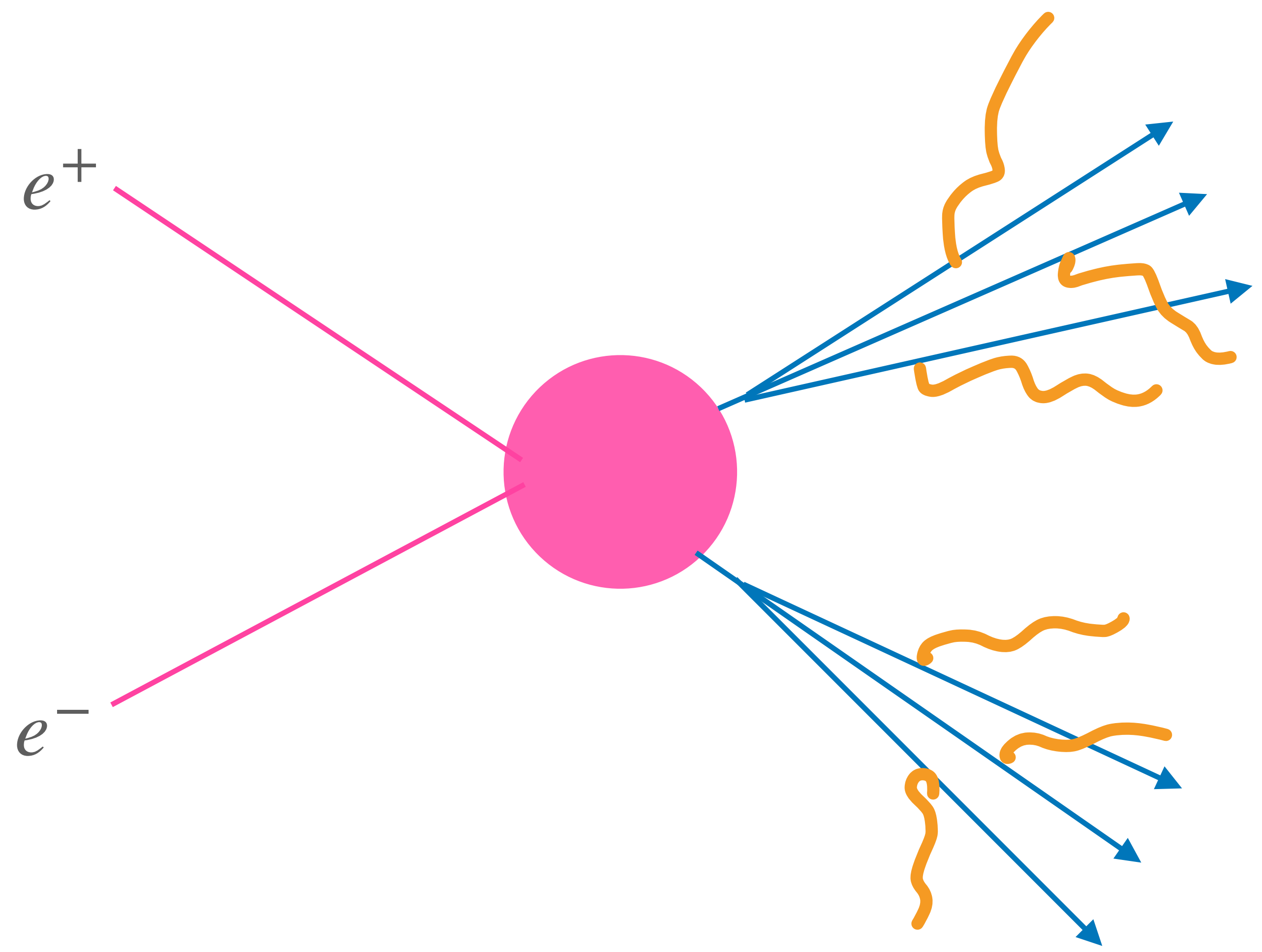
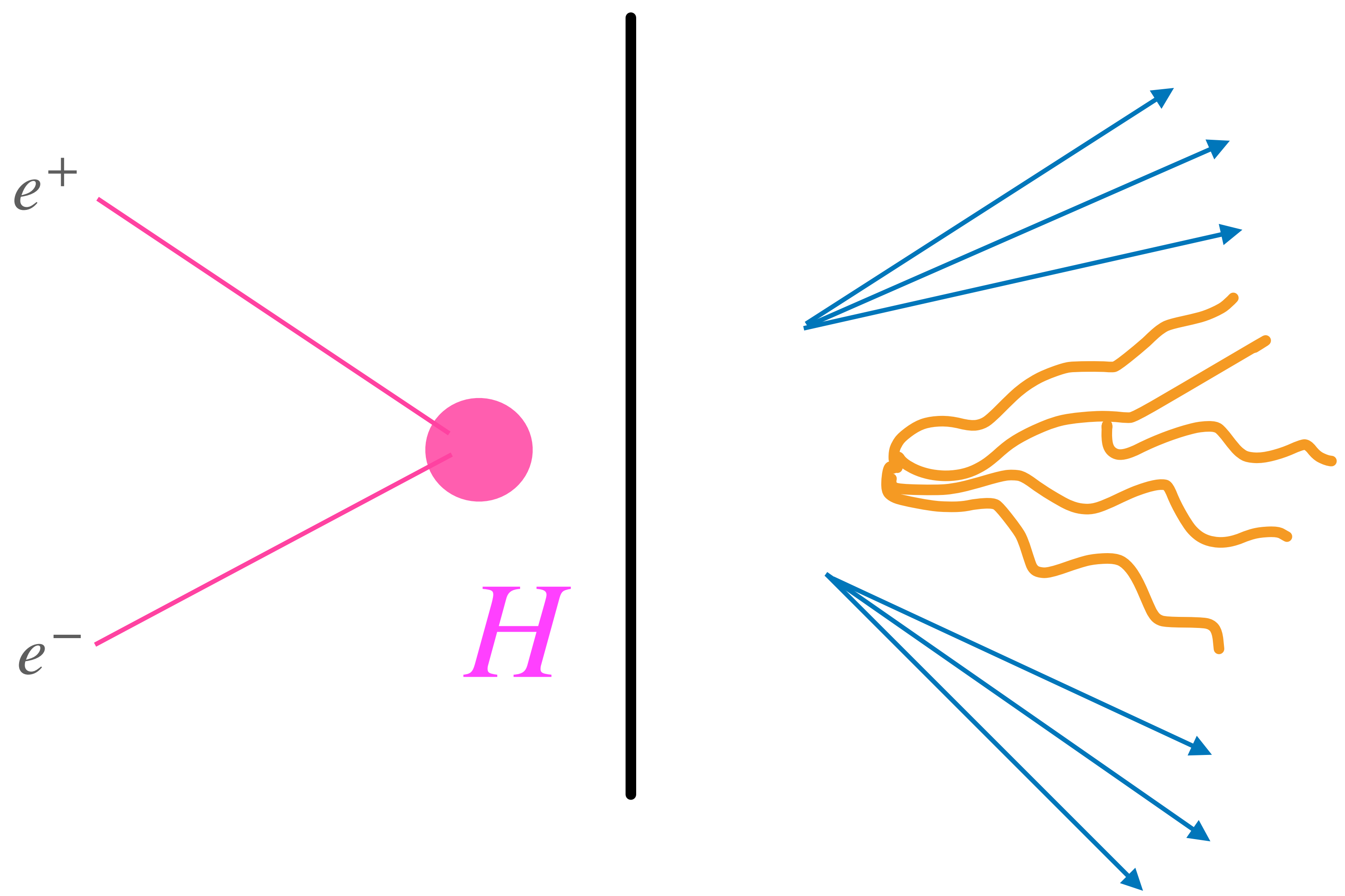
Soft Collinear Effective Theory (SCET)

- SCET is an EFT of QCD designed to study collinear and soft radiations
- Leading power Lagrangian for any process

$$\mathscr{L}_{\rm SCET}^{\rm hardscatter} = \sum_K C_K \otimes O_K(\xi_n, A_n, \psi_s, A_s) \qquad \mathscr{L}^0_{\rm SCET} = \mathscr{L}_s(\psi_s, A_s) + \sum_{n_i} \mathscr{L}^0_{n_i}(\xi_{n_i}, A_{n_i}) + \mathscr{L}_G(\xi_{n_i}, A_{n_i}, \psi_s, A_s)$$

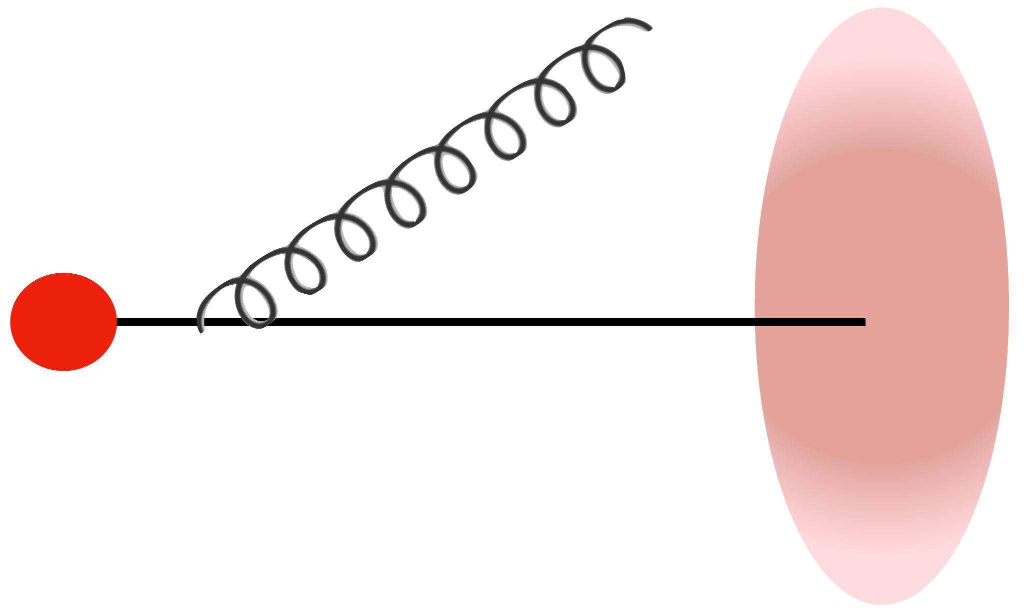
- Short distance physics of virtuality $p^2 \sim Q^2$ is integrated out
- Long distance physics is factored out into collinear and soft modes in the SCET lagrangian



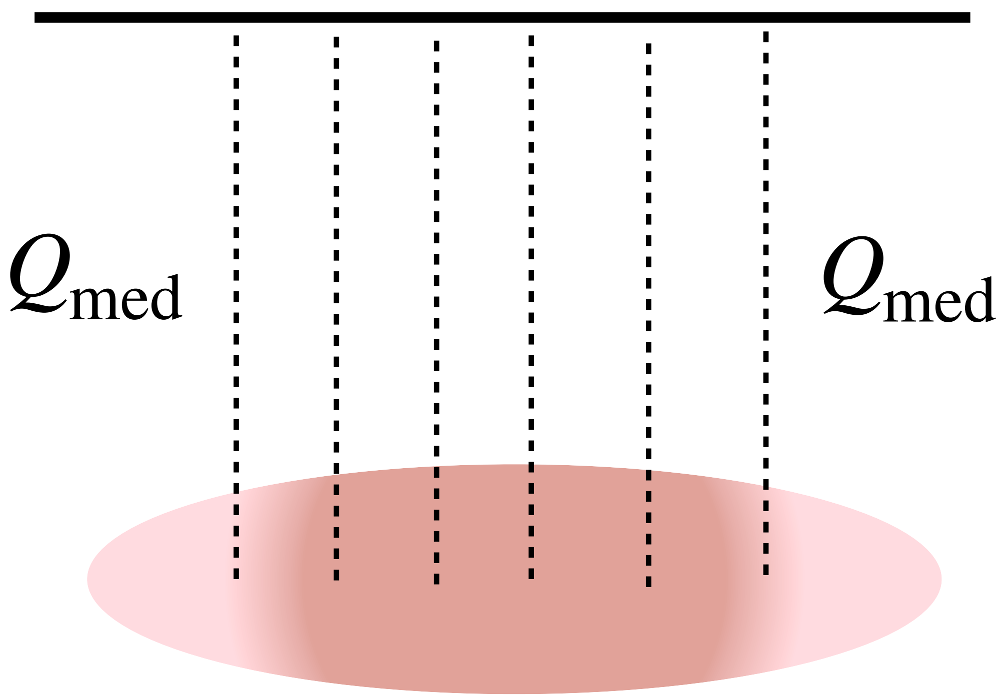


EFT modes

- Medium : soft mode $p_s \sim (Q_{\text{med}}, Q_{\text{med}}, Q_{\text{med}})$



$p = (p^-, p^+, p_\perp)$

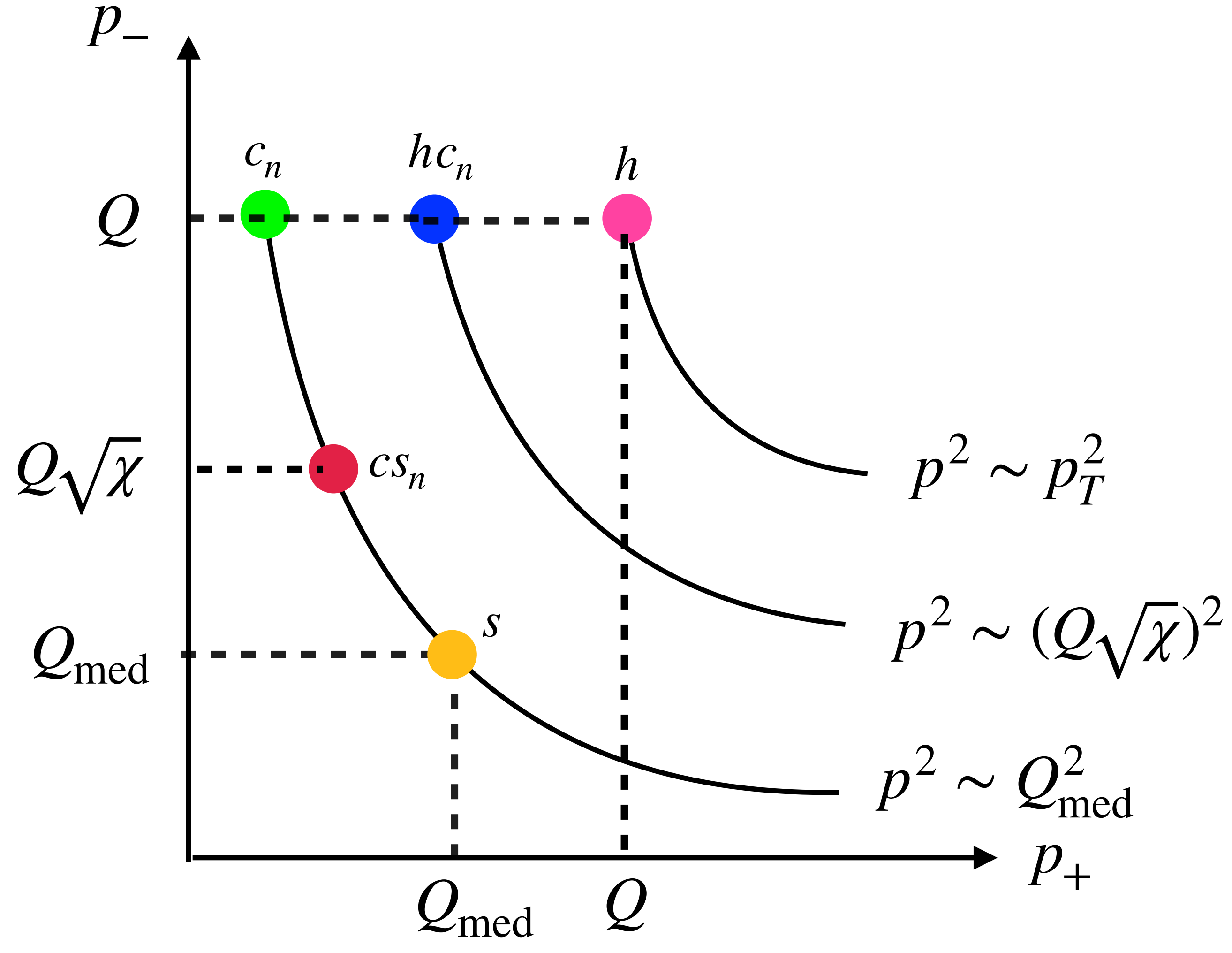


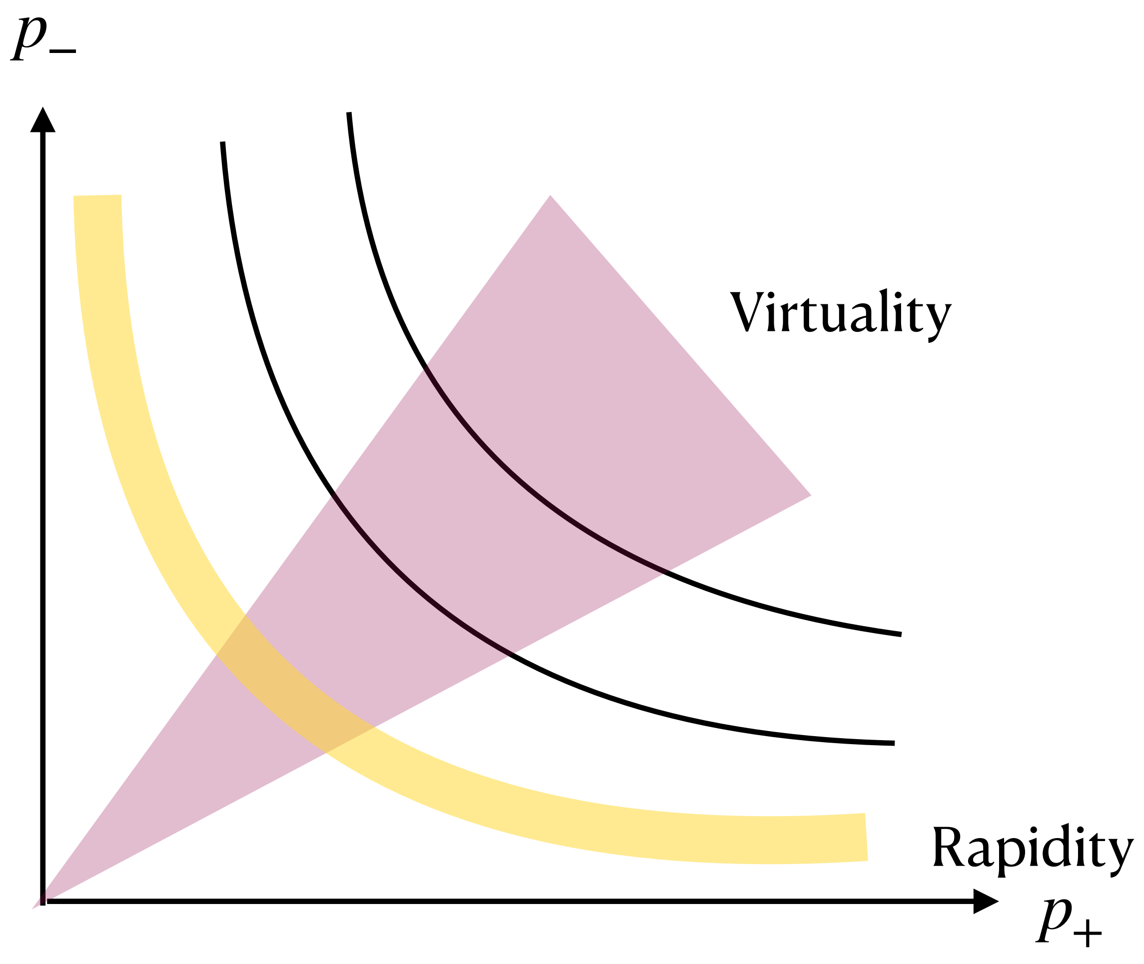
- Jets : hard collinear mode $p_c \sim Q(1,\lambda^2,\lambda)$

  Populate the jet at scale $p_c^2 \gg p_s^2$

- Energy modes : collinear soft $p_{cs} \sim \left(\frac{Q_{\text{med}}}{\sqrt{\chi}}, Q_{\text{med}}\sqrt{\chi}, Q_{\text{med}}\right)$

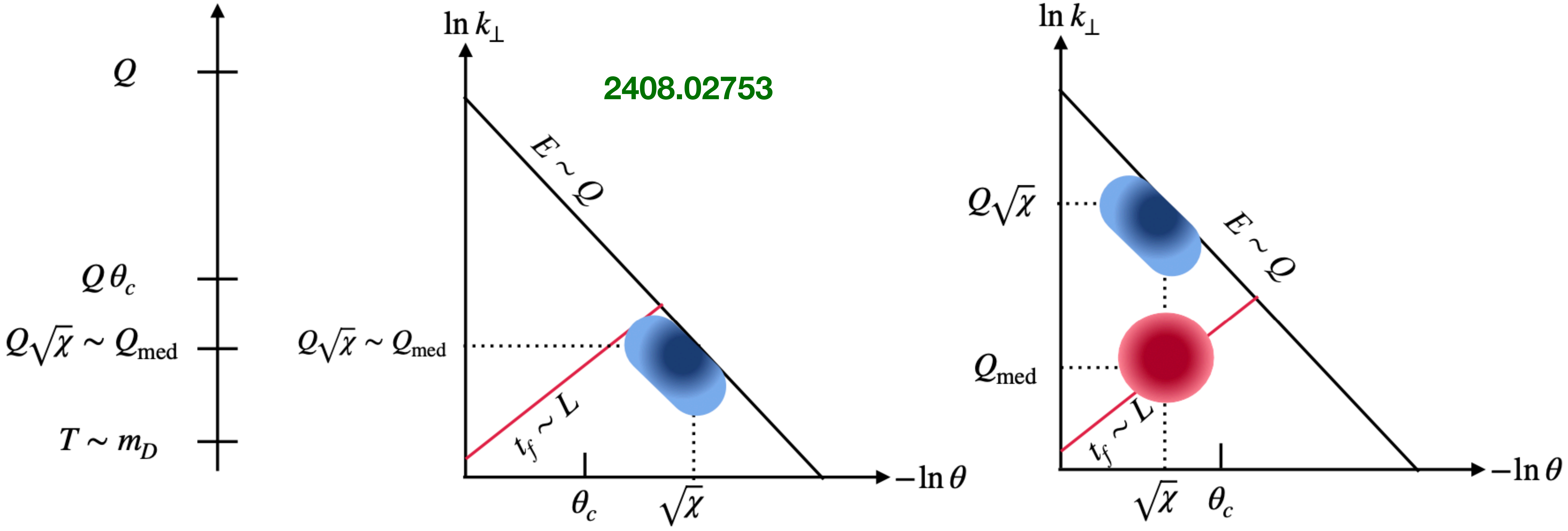
- Exchange : Scale such that the interaction should not change the off-shellness of collinear or soft modes:
  Glauber modes $p_G \sim Q(\lambda, \lambda^2, \lambda)$ Off-shell modes





Lund plane representation

- Intercepts of lines give relevant mode and its scaling
- Emissions with formation time larger than the medium length are not resolved by the medium



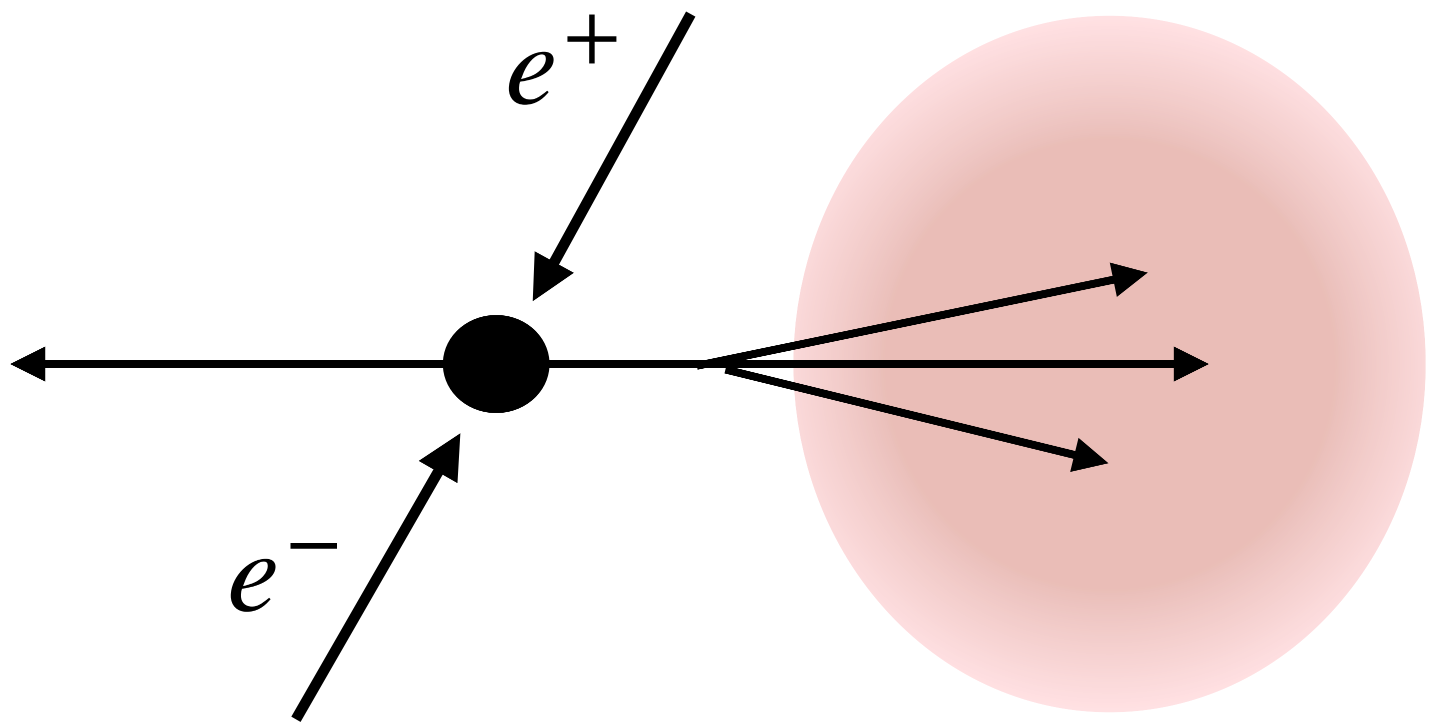
$k_\perp$ → transverse momentum of radiated gluon measured from jet axis

Emissions out the medium do not contribute to the measurement

Jet as an open quantum system

- Factorized total initial density matrix

$$\rho(0) = |e^+e^-\rangle\langle e^+e^-| \otimes \rho_M(0)$$



- Time evolution of the jet is defined through system density matrix evolution

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}$$

$$H = H_n + H_s + H_G + C(Q)l^\mu j_\mu \equiv H_S + \underbrace{\mathcal{O}_H}_{\text{Hard interaction}}$$

$$j^\mu = \bar{\chi}_n \gamma^\mu \chi_n$$

In realistic situations hard operator will also depend on initial state cold nuclear matter properties

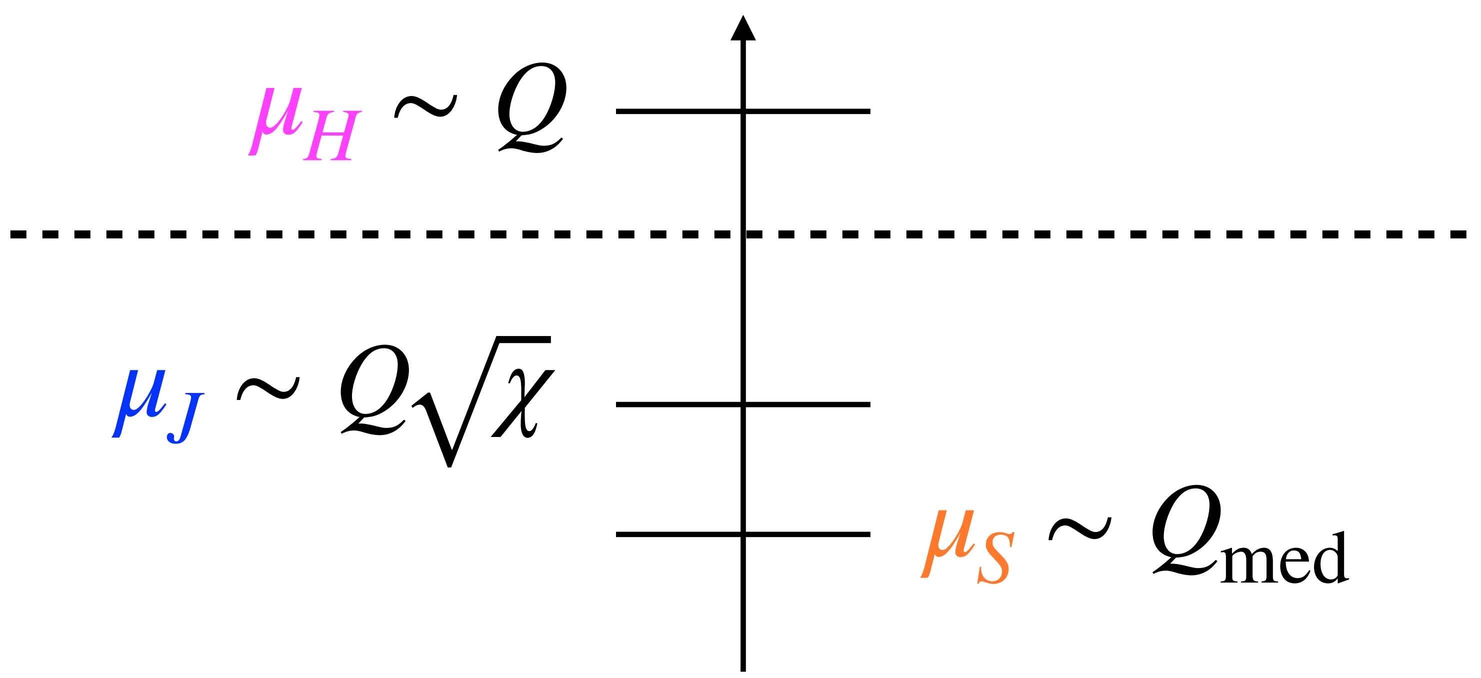
- Hard operator creates hard scattering event that produces the jet

$$\frac{d\sigma}{d\chi} = \lim_{t\to\infty} \mathrm{Tr}[\rho(t)\mathcal{M}] = \underbrace{|C(Q)|^2}_{\substack{\text{Hard matching}\\ \text{Wilson coefficient}}} L_{\mu\nu} \lim_{t\to\infty} \int d^4x d^4y e^{iq\cdot(x-y)} \mathrm{Tr}[e^{-iH_S t} j^\mu(x)\rho(0)\mathcal{M} j^\nu(y) e^{iH_S t}]$$

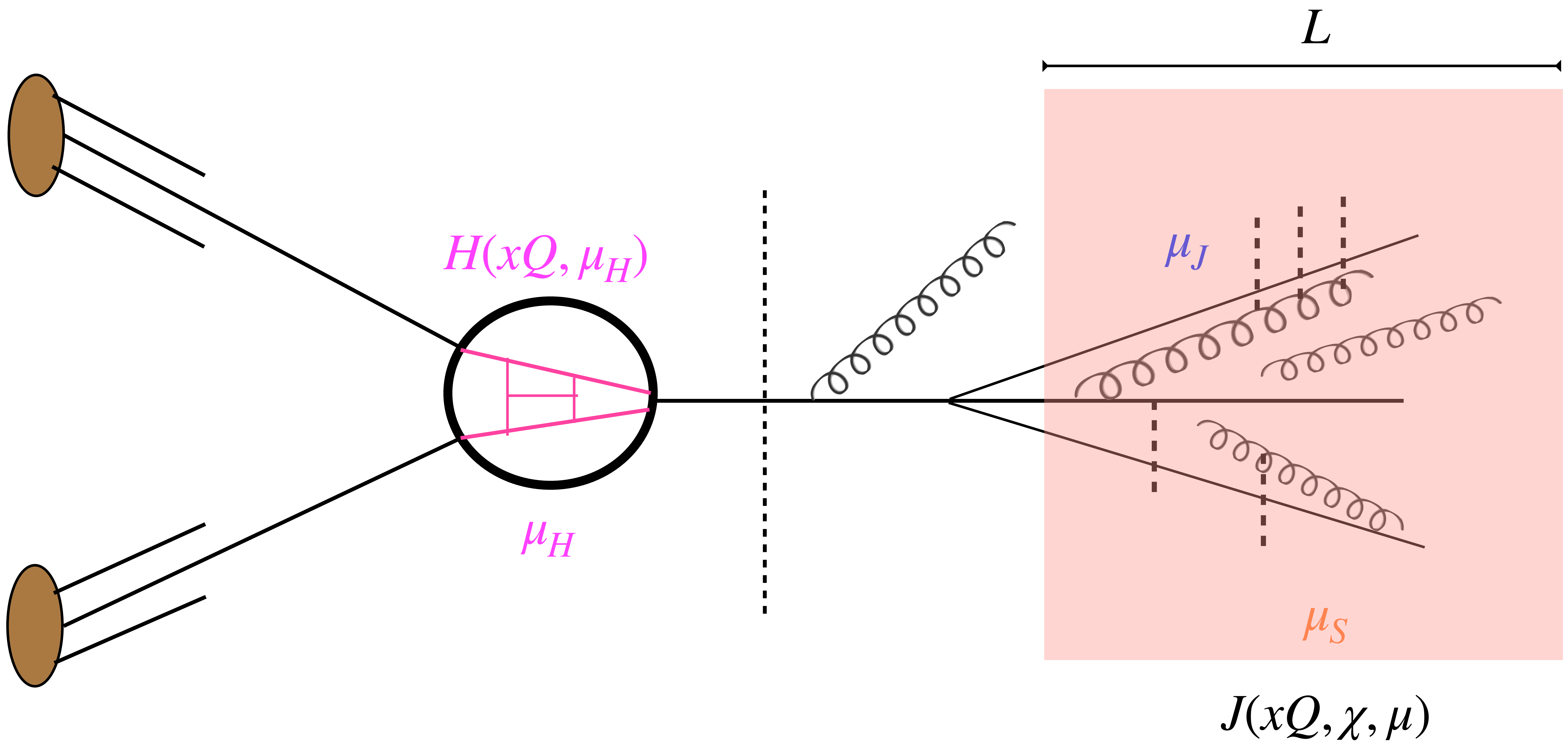
Factorizing differential cross-section

- OPE for factorizing hard scales

$$\frac{d\sigma^{[N]}}{d\chi} = \sum_{i\in\{q,\bar{q},g\}} \int dx x^N H_i(xQ,\mu)\, J_i^{[N]}(xQ,\chi,\mu)$$



- At this stage $J(xQ,\chi,\mu)$ contains both vacuum and medium physics



Initial state cold nuclear matter effects can be captured by incorporating nuclear PDFs which is similar to that of pp collision

Soft scale depends only on the Glauber momentum which is transferred to the jet by the medium

Factorizing the measurement function

$$J_q^{[N]}(\chi) = \frac{1}{2N_c}\sum_X \mathrm{Tr}\Big[\rho_M(0)\frac{\bar{n}}{2}e^{iH_{n+s}t}\bar{\mathbf{T}}\Big\{e^{-i\int_0^t dt_l H_{G,I}(t_l)}\chi_{n,I}(0)\Big\}\mathcal{M}^{[N]}|X\rangle\langle X|\mathbf{T}\Big\{\underbrace{e^{-i\int_0^t dt_r H_{G,I}(t_r)}}_{\text{Glauber insertion}}\bar{\chi}_{n,I}(0)\Big\}e^{-iH_{n+s}t}\Big]$$

- Factorizing measurement function

$$\mathcal{M} = \hat{E}(\chi)|X\rangle = \frac{1}{Q}\sum_{i\in\{X_n,X_s\}}\Big(E_{i,n}\Theta(\chi-\theta_{n,i}) + E_{i,s}\Theta(\chi-\theta_{s,i})\Big)|X_n\rangle|X_s\rangle$$

With order by order expansion in Glauber Hamiltonian we can now separate pure vacuum emissions from the medium induced ones

- Soft contributions to the measurement are power suppressed
- Glaubers being off-shell modes do not contribute to the measurement
- Energy weights are polynomial and can be analytically continued to non-integer values

$$\mathcal{M}^{[\nu]} = \sum_{a=1,2}\mathcal{W}^{[\nu]}(i_a)\delta(\chi) + \sum_{i_1<i_2}\mathcal{W}^{[\nu]}(i_1,i_2)\delta(\chi-\theta_{i_1i_2}) \qquad \mathcal{W}^{[\nu]}(i_1) = \frac{E_{i_1}^{\nu}}{Q^{\nu}} \qquad \mathcal{W}^{[\nu]}(i_1,i_2) = \frac{(E_{i_1}+E_{i_2})^{\nu}}{Q^{\nu}} - \sum_{a=1,2}\mathcal{W}^{[\nu]}(i_a)$$

2004.11381

Leading order : vacuum jet function

- To recover the vacuum jet function we expand Glauber Hamiltonian

$$J_q^{[\nu]}(\omega,\chi,\mu) = \sum_{i=0}^{\infty} J_{qi}^{[\nu]}(\omega,\chi,\mu)$$

$i = 0,$ vacuum
$i = 1,$ single scattering
$i \geq 2,$ multiple scattering

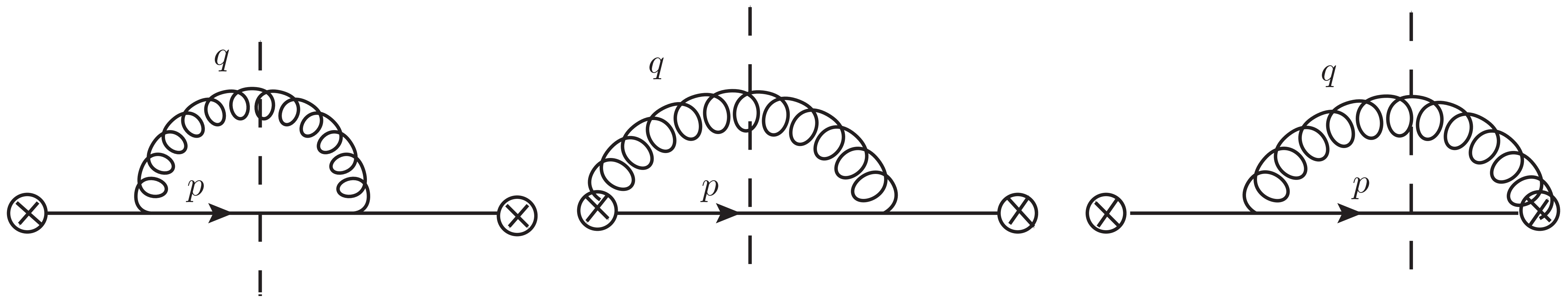
- Leading order $\quad J_q^{[\nu]}(\omega,\chi,\mu) = J_{q0}^{[\nu]}(\omega,\chi,\mu)$

- Soft function does not depend on the measurement and becomes identity

$$J_{q0}^{[\nu]}(\omega,\chi,\mu) = \frac{1}{2N_c}\sum_{X_n} \mathrm{Tr}\left[\frac{\bar{n}}{2}\langle 0|\chi_n(0)\mathscr{M}^{[\nu]}|X_n\rangle\langle X_n|\bar{\chi}_n(0)|0\rangle\right]$$

Virtual emissions do not contribute to the measurement

Jet function obeys DGLAP evolution equation



$$J_{q0}^{[2]}(\omega = xQ,\chi) = \delta(\chi) + \frac{\alpha_s C_F}{\pi}\left(-\frac{3}{2\epsilon}\delta(\chi) + \frac{3}{2}\left[\frac{1}{\chi}\right]_+ - \frac{3}{2}\delta(\chi)\ln\left(\frac{\mu^2}{\omega^2}\right) - \frac{19}{3}\delta(\chi) + \mathcal{O}(\epsilon)\right)$$

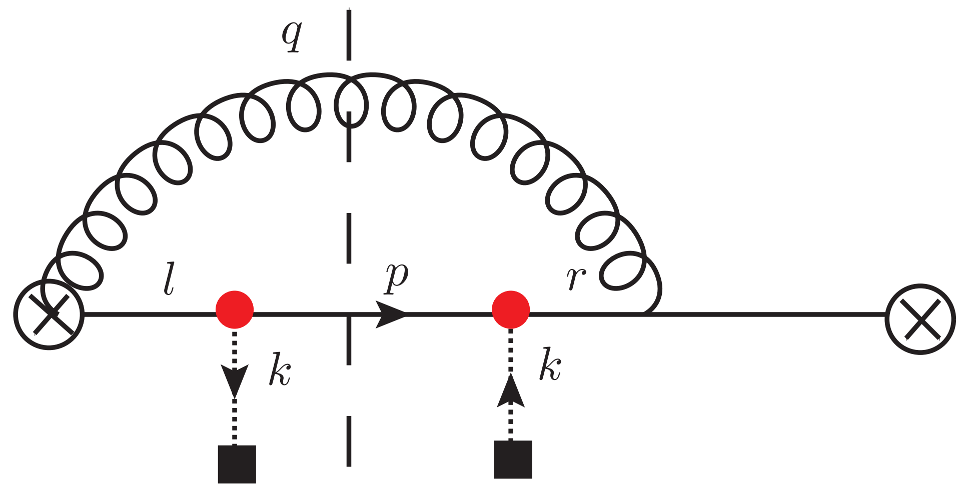
$$J_{\mathrm{LL}}^{[\nu]} = (1,1)\cdot\left(\frac{\alpha_s(\sqrt{\chi}Q)}{\alpha_s(Q)}\right)^{-\frac{\hat{\gamma}^{(0)}(\nu+1)}{\beta_0}}$$

Single scattering : Glauber insertions on opposite side

- Real contribution with Glauber insertions

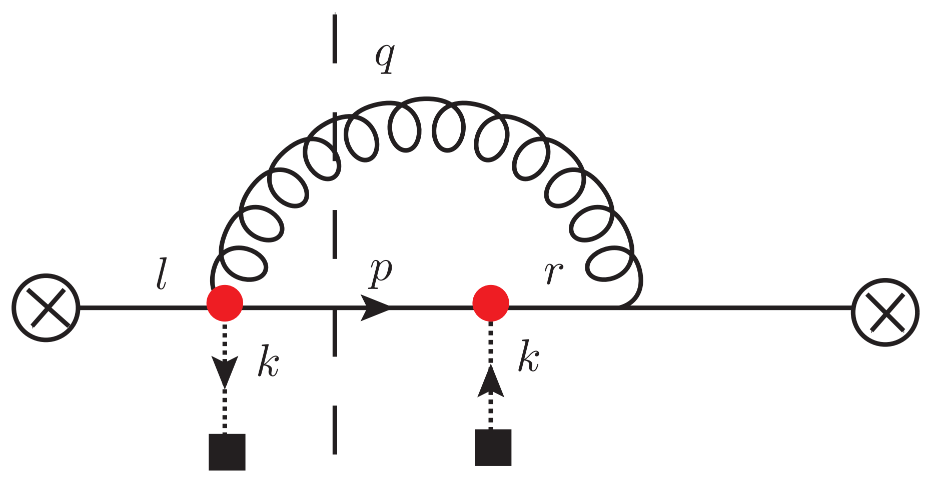
$$J_{q,o}^{[\nu]}(\chi, k_\perp; L) = \frac{1}{2N_c}\sum_X \int d^4x\Theta(L-x^-)\int d^4y\Theta(L-y^-)\mathrm{Tr}\Big[e^{iH_{n+s}t}\bar{\mathbf{T}}\Big\{H_{G,I}(x)\chi_{n,I}(0)\Big\}\rho_M(0)\frac{\bar{n}}{2}\mathbf{T}\Big\{H_{G,I}(y)\bar{\chi}_{n,I}(0)\Big\}e^{-iH_{n+s}t}\mathscr{M}^{[\nu]}\Big] + \mathcal{O}(H_G^4)$$

Θ function constraint Medium and jet interaction to finite extent of the medium L

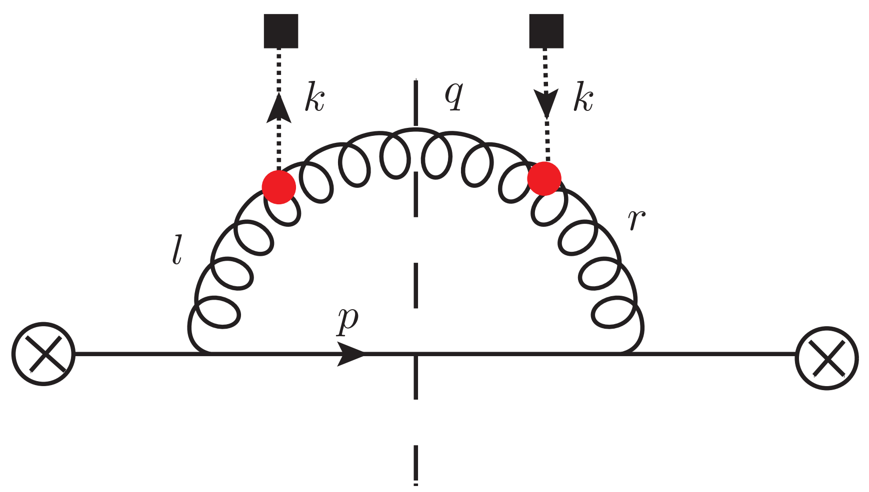


$$H_G(x) = \sum_{ij} C_{ij} O_{ns}^{ij}(x)$$

$$\mathcal{O}_{ns}^{qg} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{gA} \qquad \mathcal{O}_{ns}^{qq} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{qA}$$



$\mathbb{P}_\perp^2$ pulls out Glauber momentum from soft operators in the medium. In principle Glauber interactions break factorization but can be achieved with expansion Glauber Hamiltonian

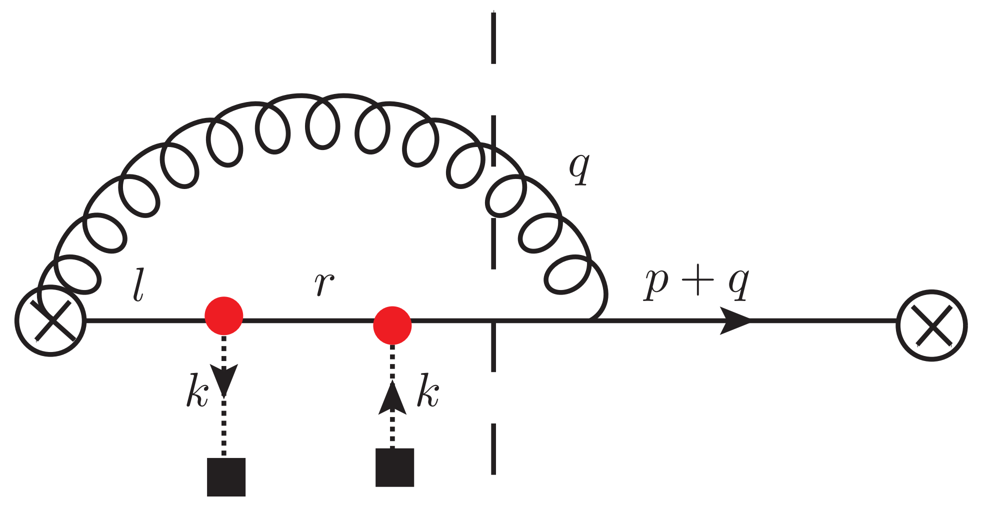


- Order by order factorization for jet and medium functions

$$J_{q,o}^{[\nu]}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2} J_{q,R}^{[\nu]}(\omega, \chi, k_\perp, \mu, \nu; L) \otimes \mathbf{B}(k_\perp, \mu, \nu)$$

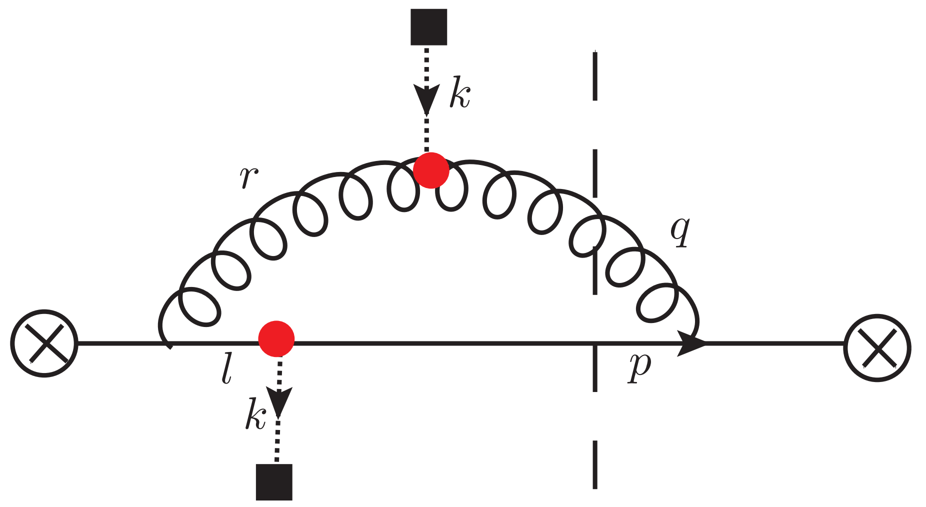
Single scattering : Glauber insertions on same side

- $\mathcal{O}(H_G^2)$ expansion for the same side Glauber insertions



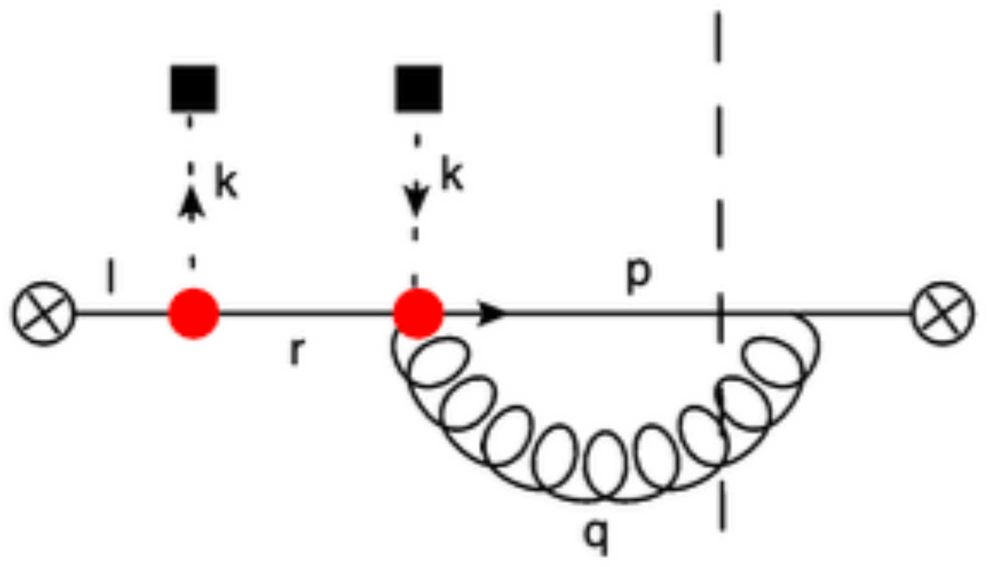
$$J_{q,s}^{[\nu]}(\chi, k_\perp; L) = \frac{1}{2N_c}\int d^4x\Theta(x^- - L)\int d^4y\Theta(y^- - L)\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\bar{\mathbf{T}}\Big\{e^{-i\int dt H_n(t)}\chi_n(0)\mathscr{M}^{[\nu]}|X\rangle\langle X|$$

$$\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\{H_{G,I}(x)H_{G,I}(y)\}\Big\}\bar{\chi}_n(0)\Big\}|0\rangle\Big] + \text{c.c} + \mathcal{O}(H_G^4)$$



- Similar to opposite side insertions Soft/Medium function explicitly factors out and does not depend on the measurement imposed on the final state partons

$$J_{q,s}^{[\nu]}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2} J_{q,V}^{[\nu]}(\omega, \chi, k_\perp, \mu, \nu; L) \otimes \mathbf{B}(k_\perp, \mu, \nu)$$



- To get the total medium induced jet function add all real and virtual contributions

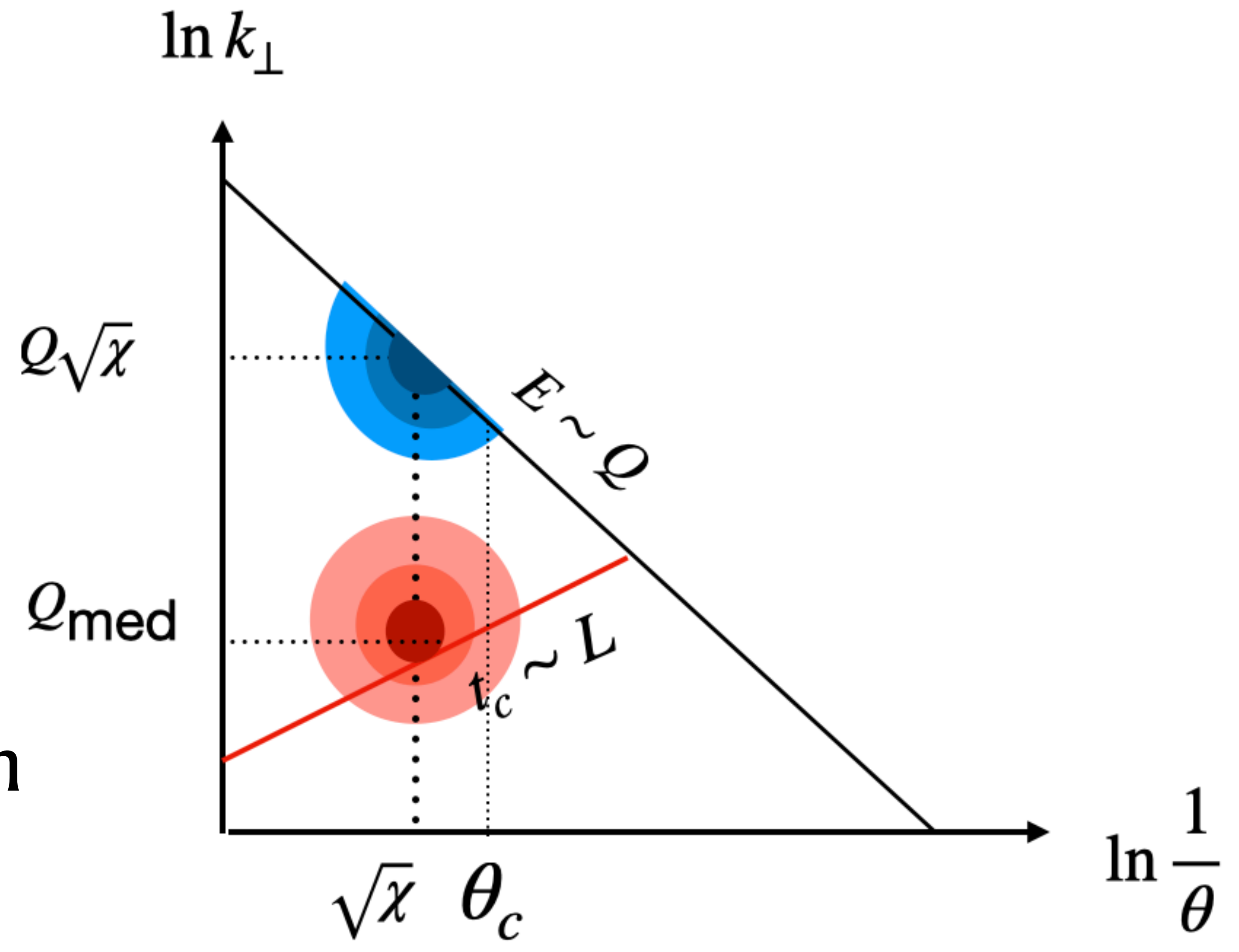
$$J_{q1}^{[\nu]}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2}\left[J_{q,R}^{[\nu]}(\omega, \chi, k_\perp, \mu, \nu; L) - J_{q,V}^{[\nu]}(\omega, \chi, k_\perp, \mu, \nu; L)\right] \otimes \mathbf{B}(k_\perp, \mu, \nu) + \mathcal{O}(H_G^4)$$

- For total jet function add vacuum contribution $\quad J_q^{[\nu]}(\omega, \chi) = J_{q0}^{[\nu]}(\omega, \chi) + J_{q1}^{[\nu]}(\omega, \chi; L)$ **2408.02753**

Case II : $Q \gg Q\sqrt{\chi} \gg Q_{\text{med}}$

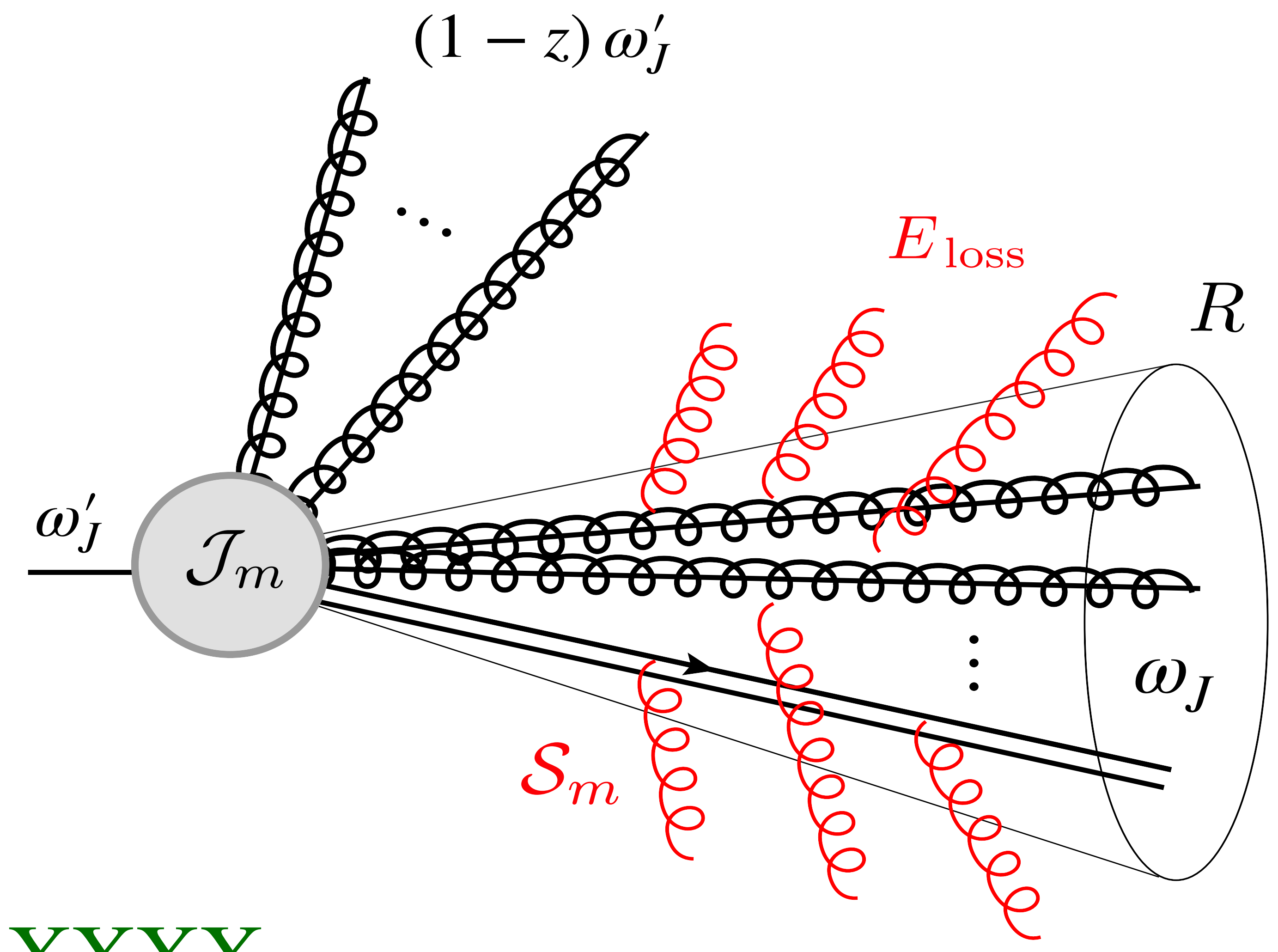
- Refactorize the jet function

$$J_i^{[\nu]} = J_{i0}^{[\nu]}(\omega,\chi,\mu) + \sum_{m=1}^{\infty}\sum_{j=1}^{m} \underbrace{\mathcal{J}_{i\to m}^{j[\nu]}(\{\underline{m}\},\theta_c,\omega,\mu)}_{\text{Matching function}} \otimes_\theta \underbrace{\mathcal{S}_{m,j}^{[\nu]}(\{\underline{m}\},\chi,\mu)}_{\text{Collinear-soft function}}$$



- Matching function describes the production of m resolved hard partons from initial parton i

- Collinear soft function describes the production of medium induced radiation



$$\mathcal{S}_m^{[\nu]}(\{\underline{m}\},\epsilon) \equiv \text{Tr}\Big[U_m(n_m)\ldots U_1(n_1)U_0(\bar{n})\rho_M U_0^\dagger(\bar{n})U_1^\dagger(n_1)\ldots U_m^\dagger(n_m)\mathcal{M}^{[\nu]}\Big]$$

$$U_n(x) = \text{P}\exp\left[ig\int_x^L ds O_{cs-s}^{j,B}T^B(ns)\right]U_{cs}(x)$$

B.S., Y.M-Tani, F. Ringer and V.Vaidya, 2409.05957, 25XX . XXXX

Jet function : NLO

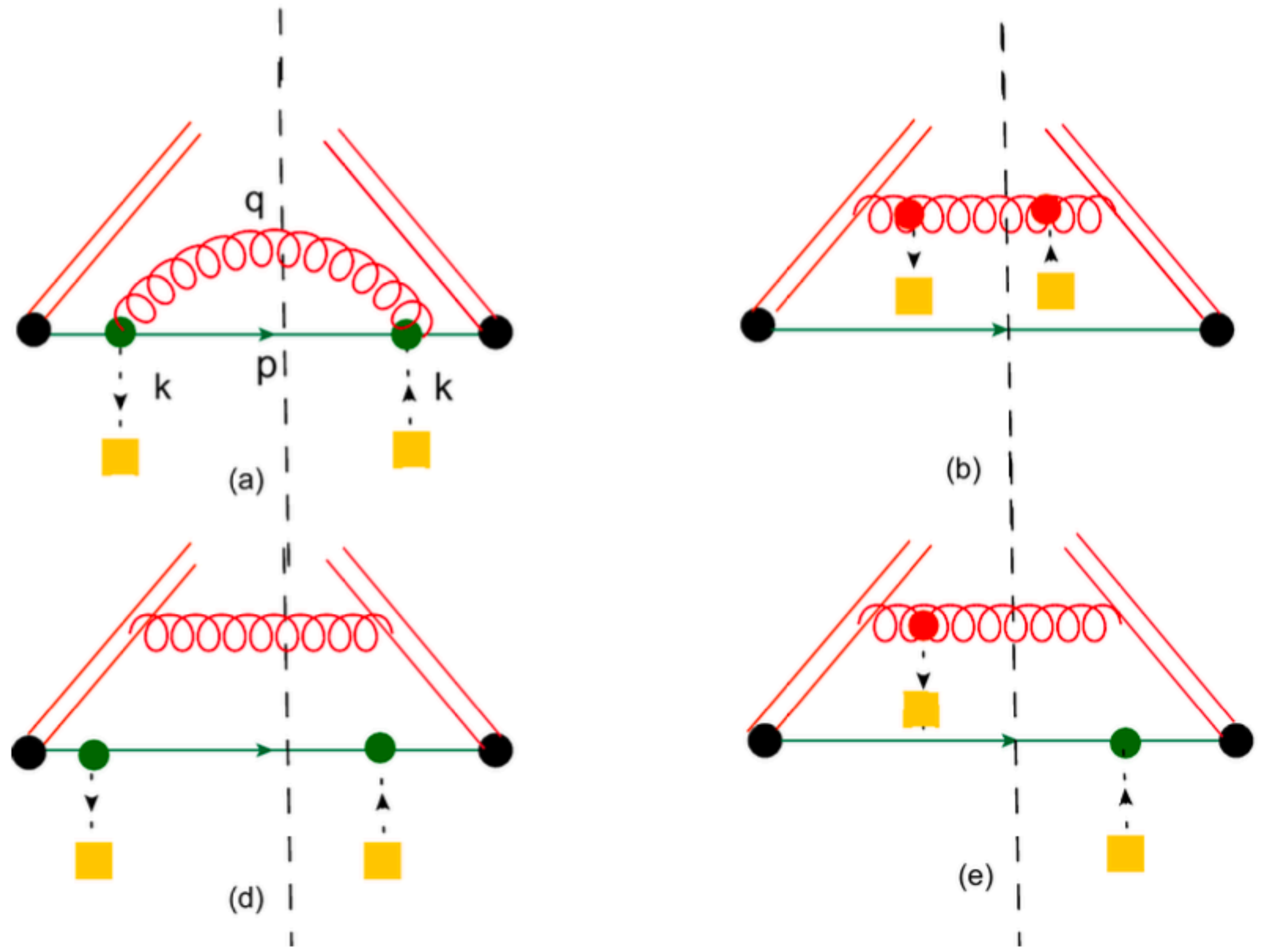
- Only one emission is resolved $\theta_c \sim R$

- First term for medium induced radiation

$$J_{q1}^{[\nu]} = \mathcal{J}_{i\to 1}(\theta_c, \omega, \mu)\, \mathcal{S}_1^{[\nu]}(\chi, \mu)$$

Matching function obtained by expanding the full jet function in $\frac{k_\perp}{Q}$ and match it to $\mathcal{S}_1$

$$\mathcal{J}_{i\to 1}(\theta_c, \omega, \mu) = 1$$



- Medium function is contained inside collinear-soft function

$$\mathcal{S}_1^{[\nu]}(\chi) = L \int \frac{d^2k_\perp}{(2\pi)^3} \mathbf{S}_1^{[\nu]}(\chi, k_\perp; L) \otimes \mathbf{B}(k_\perp)$$

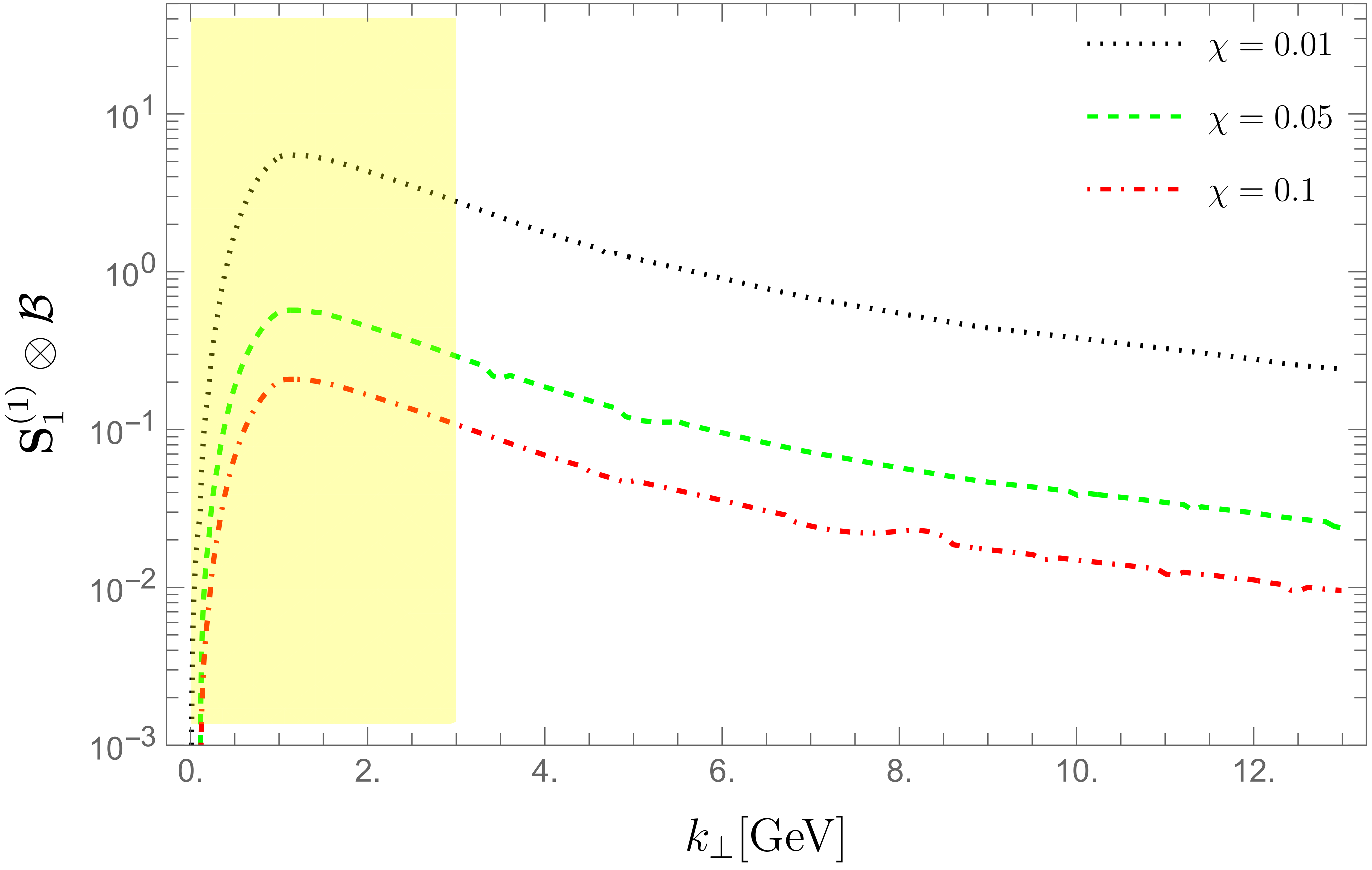
$$\mathbf{S}_1^{(1)} \sim \int \frac{dq^-}{(2\pi)^3} \int d^2q_\perp \frac{\vec{q}_\perp \cdot \vec{k}_\perp}{q_\perp^2 \kappa_\perp^2 q^-} \left(1 - \frac{q^-}{\kappa_\perp^2 L} \sin\left[\frac{L\kappa_\perp^2}{q^-}\right]\right) \mathcal{M}^{[2]}$$

$$\mathcal{M}^{[2]} = \frac{2q^-}{\omega}\left[\delta\left(\chi - \frac{q_\perp^2}{z^2\omega^2}\right) - \delta(\chi)\right]$$

While the energy of collinear soft function suppressed medium induced jet function is enhance by the length of the medium

Result I : Fixed order computation

- For single scattering the dominant contribution appears to be in the non-perturbative regime



$m_D = 0.8$ GeV, $L = 5$ fm

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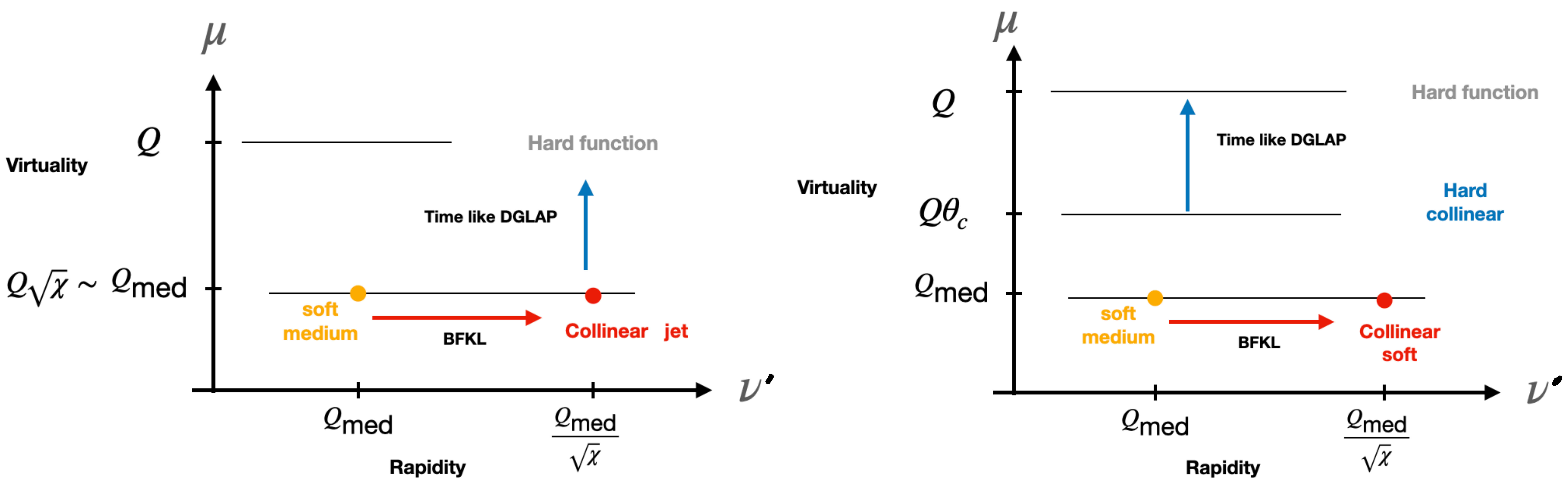
Beyond fixed order : RG flow

- Medium function obeys BFKL, therefore, from RG consistency collinear-soft jet function also obeys BFKL evolution equation **2107.00029**



- Natural scales for each functions are provided by the logs appearing in the functions

For precise knowledge of relevant scales for resummation higher order calculations of the jet function are required

Result II : BFKL resummation

- Resums $\sim \alpha_s \log \sqrt{\chi}$ terms which are relevant in small χ limit

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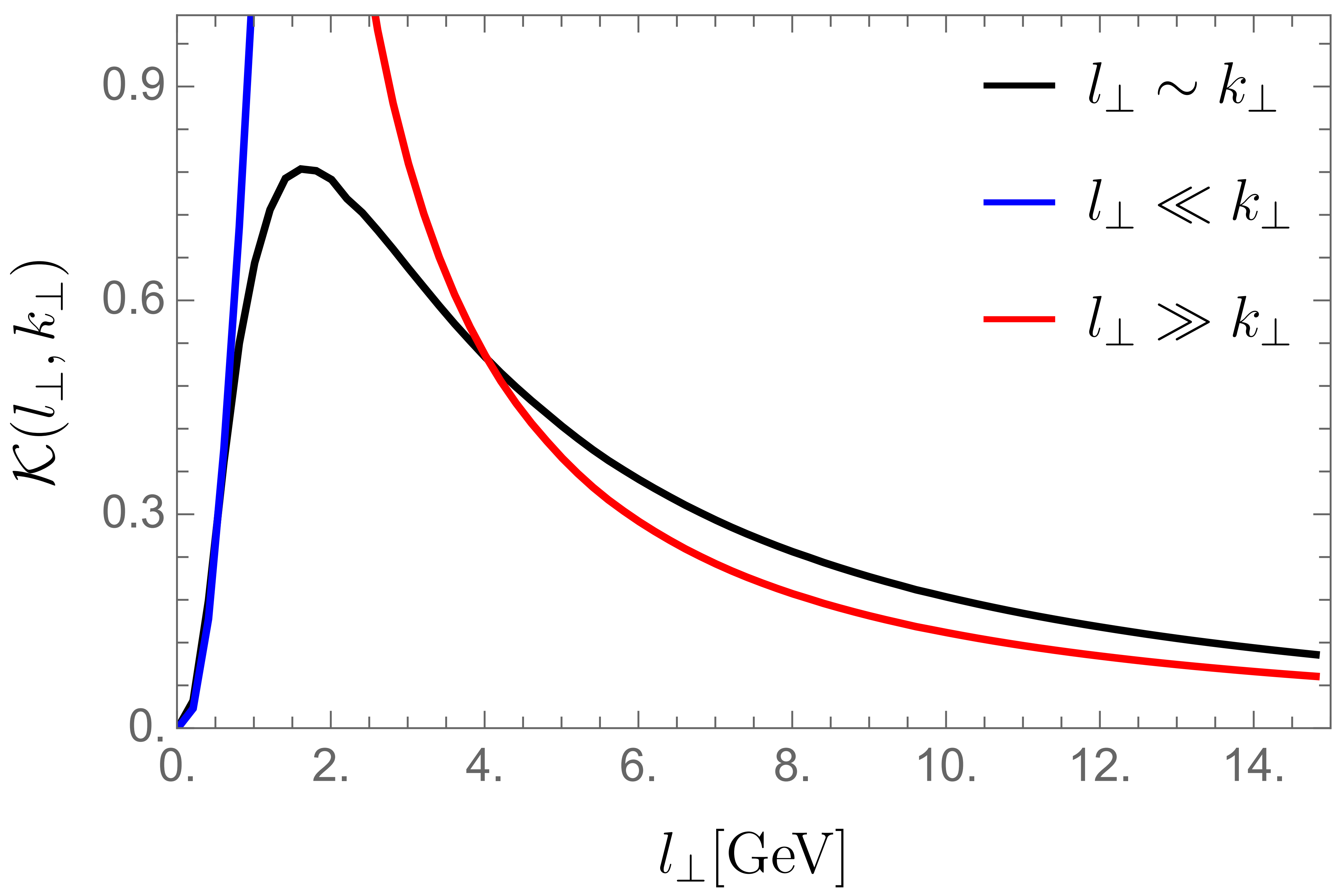
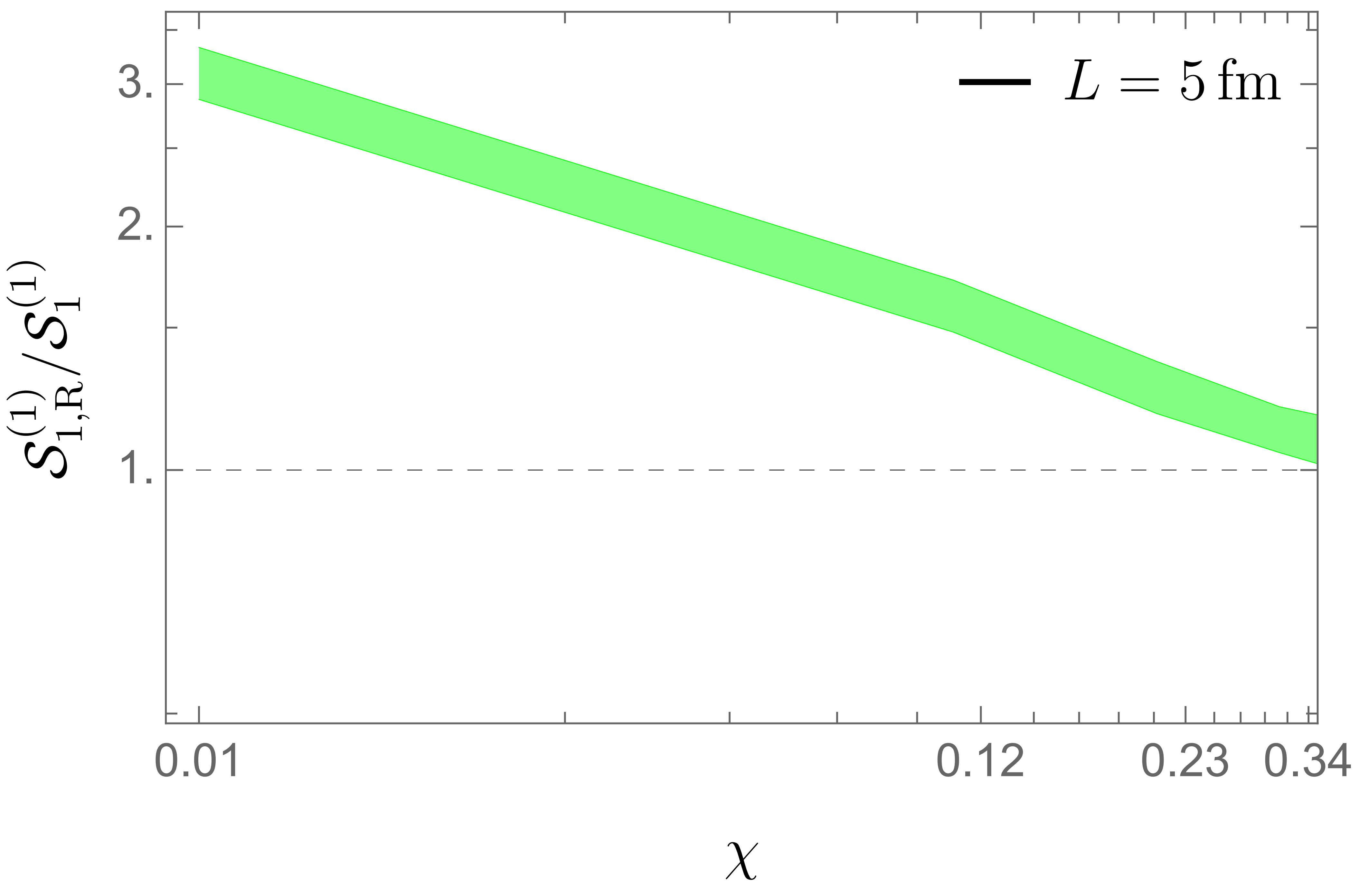




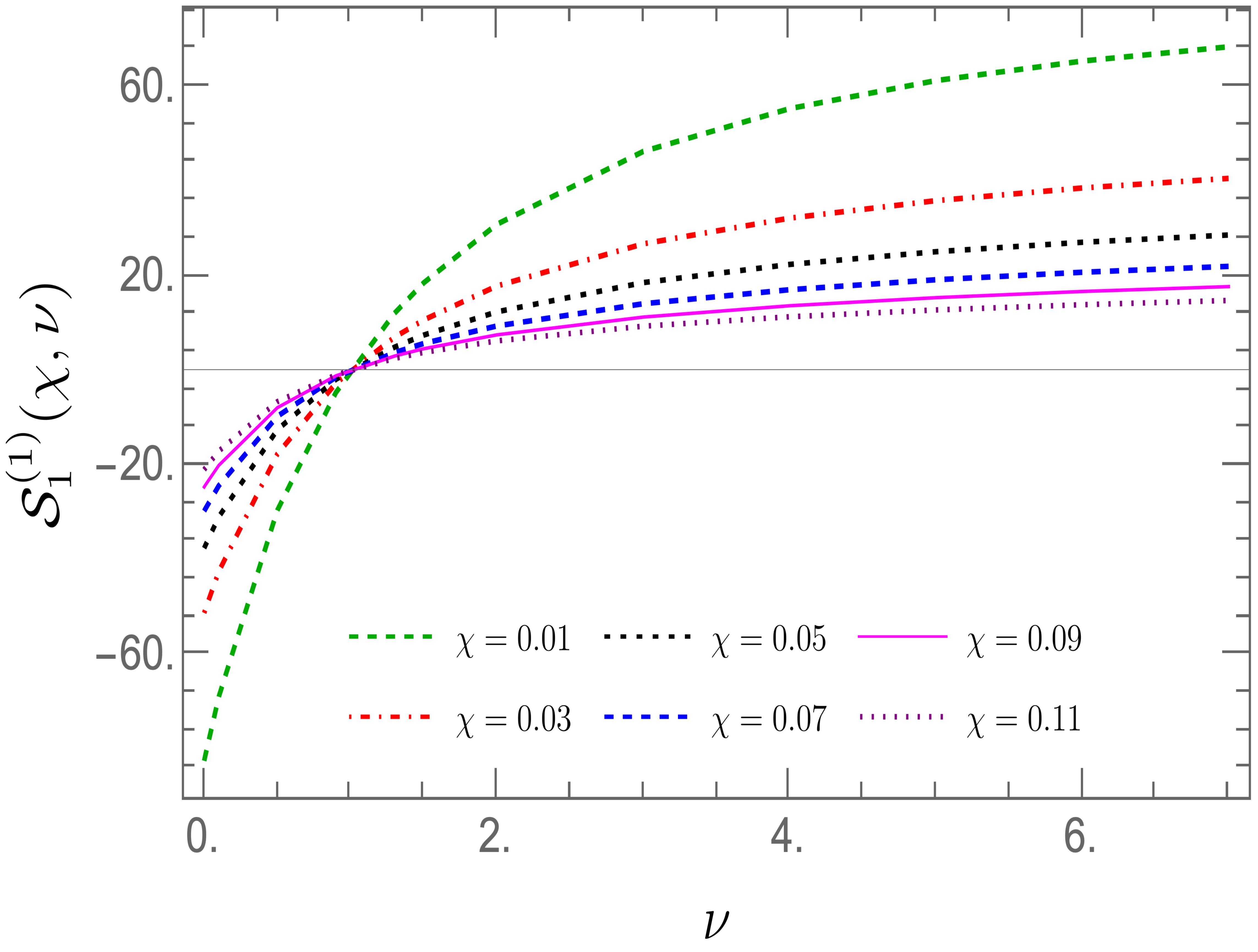
$$\alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 + \frac{\alpha_s(m_z)\beta_0}{2\pi}\log\left[\frac{\mu}{m_Z}\right]}$$

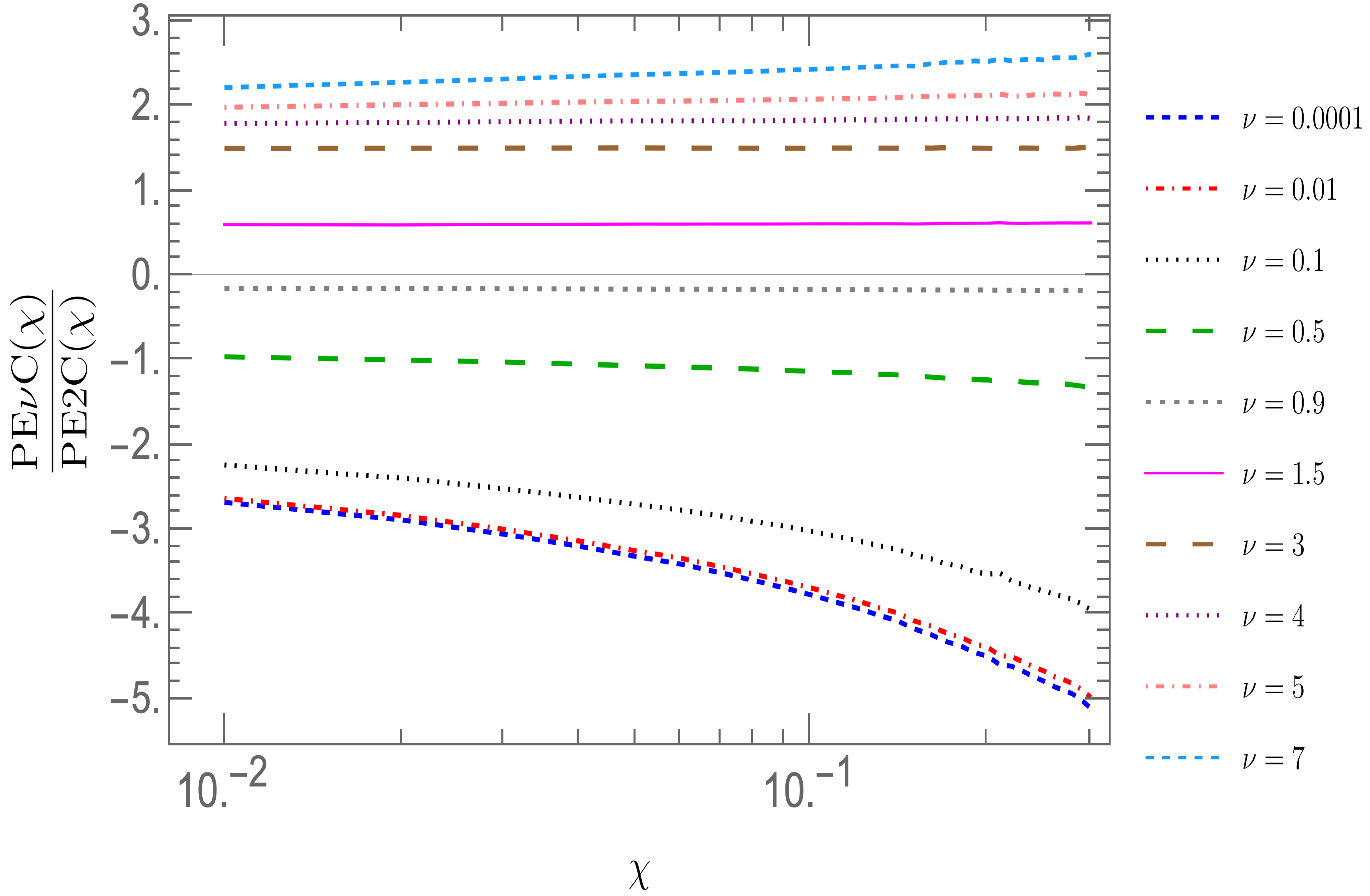
$\mu = Q_{\mathrm{med}} \sim \sqrt{\hat{q}L}$, $m_Z = 90$ GeV $Q_{\mathrm{med}} \in (2-3)$ GeV

Resummation of logarithm of the measurement has significant contributions in the small angle region

Results III: Preliminary

Larger ν point energy correlators are more sensitive to large angle radiations





For $\nu < 1$, a distinct scaling behaviour

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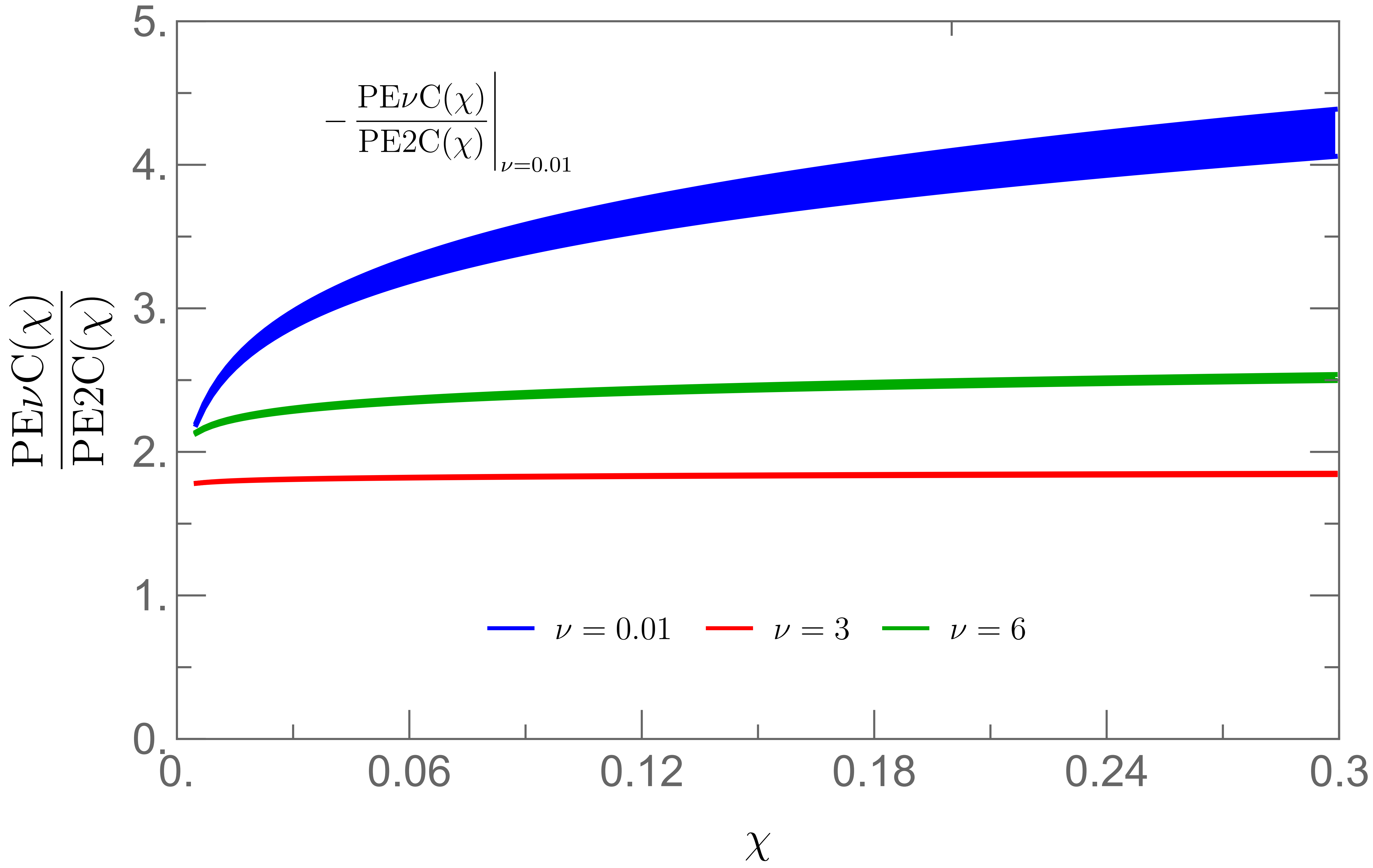
Results IV: Preliminary



For L=4 fm, T=0.5 GeV

$$\frac{\text{PE}\nu\text{C}}{\text{PE2C}}\Big|_{\nu=0.01} \propto \chi^{0.18}$$

$$\frac{\text{PE}\nu\text{C}}{\text{PE2C}}\Big|_{\nu=3} \propto \chi^{0.0095}$$

$$\frac{\text{PE}\nu\text{C}}{\text{PE2C}}\Big|_{\nu=6} \propto \chi^{0.042}$$

- $\nu < 1$ has stronger scaling behaviour compared to $\nu > 1$

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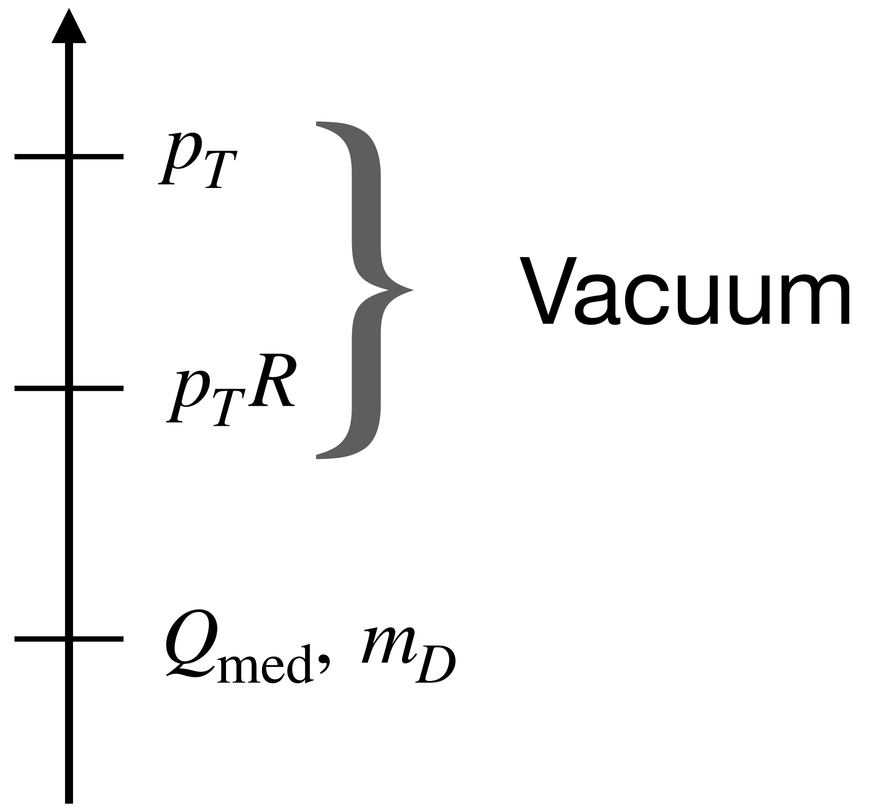
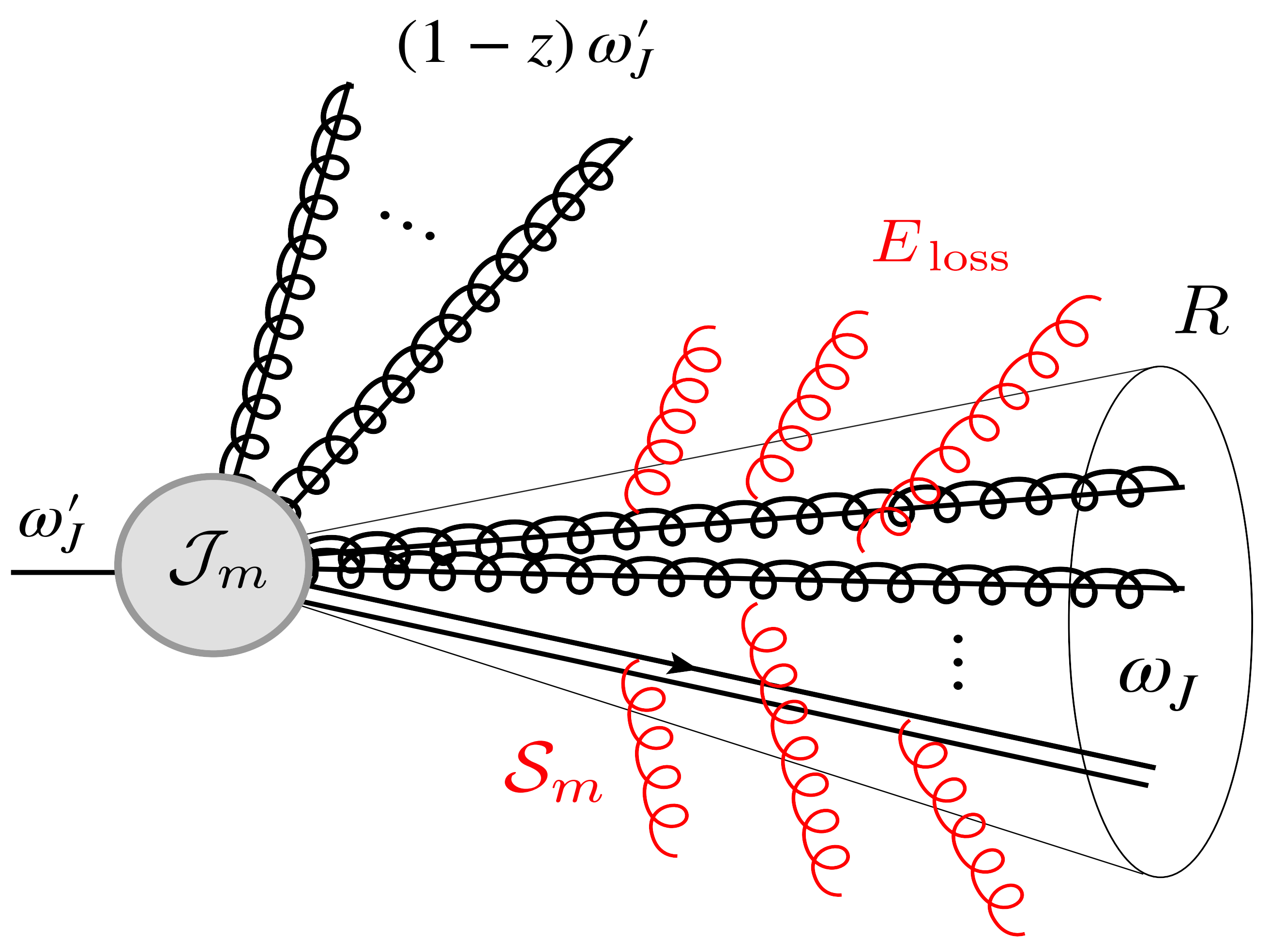
Towards an all order factorization with multiple scatterings

- For a dense medium multiple scatterings are important and can lead to non trivial evolution equations

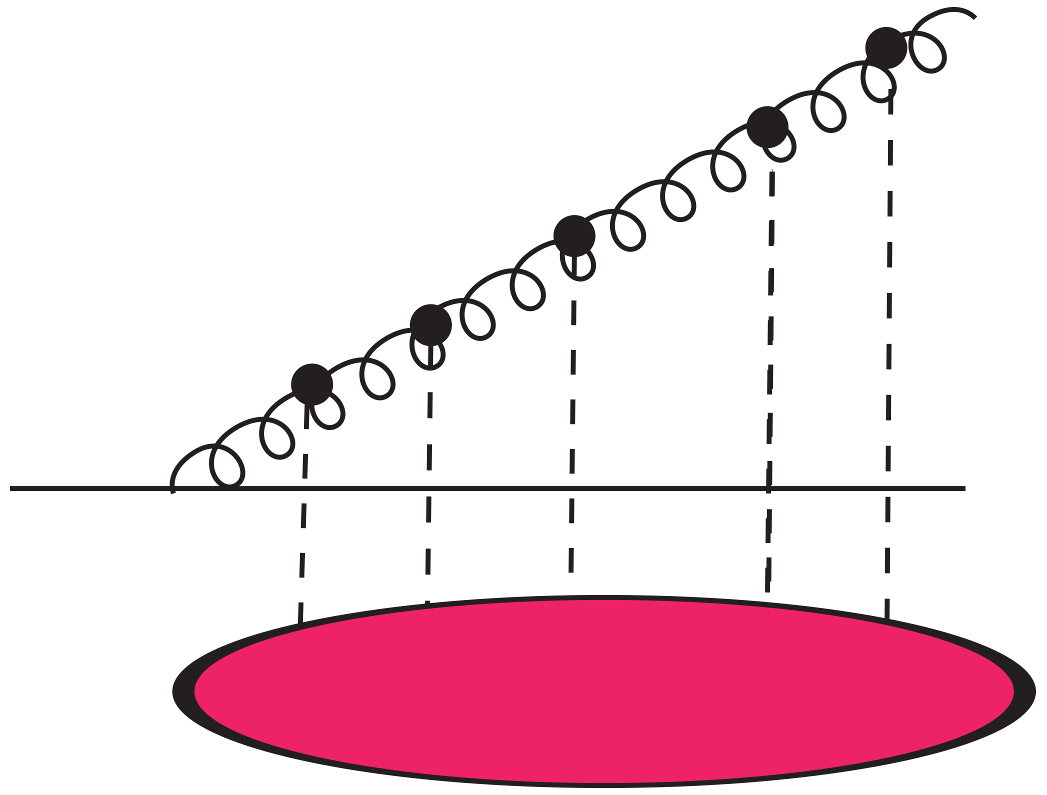




$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T \mathrm{d}\eta} = \sum_{i\in q,\bar{q},g} \int_0^1 \frac{\mathrm{d}z}{z} H_i\left(\omega = \frac{\omega_J}{z}, \mu\right) J_i(z, \omega_J, \mu)$$

$$J_i(z,\omega_J,\mu) = \int_0^1 dz' \int_0^\infty d\epsilon_L\, \delta(\omega'_J - \omega_J - \epsilon_L) \sum_m \prod_{j=2}^m \int \frac{d\Omega(n_j)}{4\pi} \mathcal{J}_{i\to m}\left(\{\underline{n}\}, z', \omega'_J = \frac{z'\omega_J}{z}, \mu, \mu_{cs}\right) \mathcal{S}_m(\{\underline{n}\}, \epsilon_L, \mu_{cs})$$

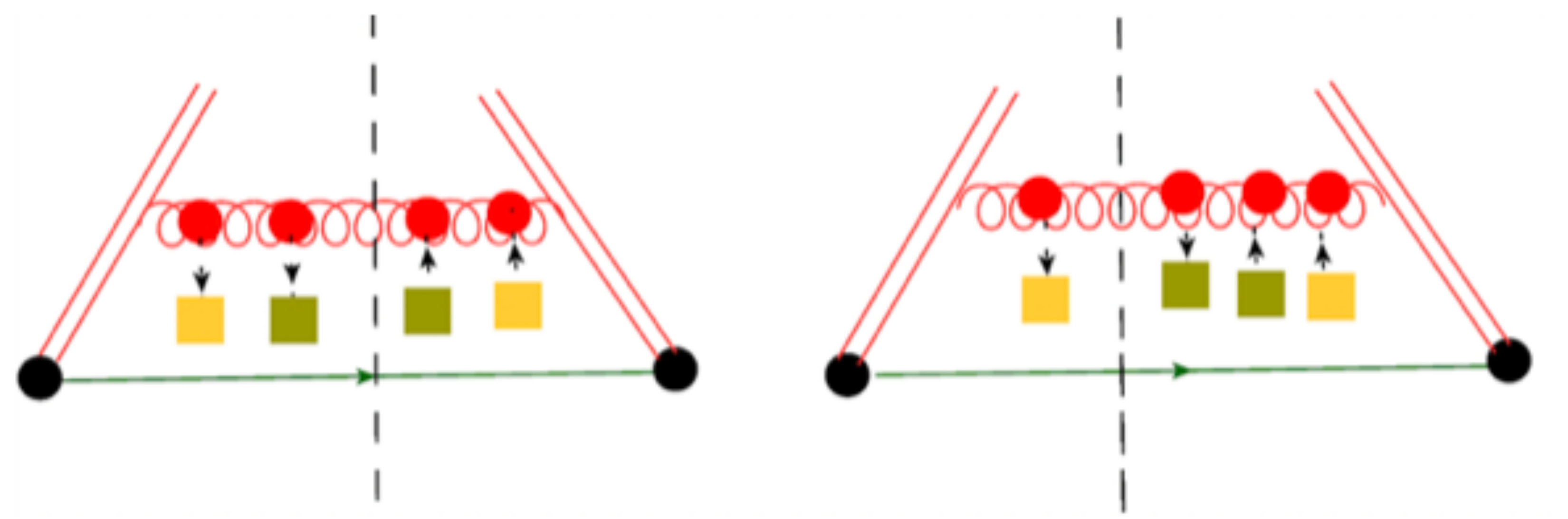
$$\mathcal{S}_1^{(n)}(\epsilon_L,\mu) = |C_G|^{2n} \left[\prod_{i=1}^n \int_0^L \mathrm{d}x_i^- \Theta(x_i^- - x_{i+1}^-) \int \frac{\mathrm{d}^2 k_i}{(2\pi)^3} \mathbf{B}(k_{i\perp}, \mu, \nu', x_i^-)\right] \mathbf{S}_1^{(n)}(\epsilon_L; k_{1\perp}, \ldots, k_{n\perp}; x_1^-, \ldots x_n^-; \nu')$$



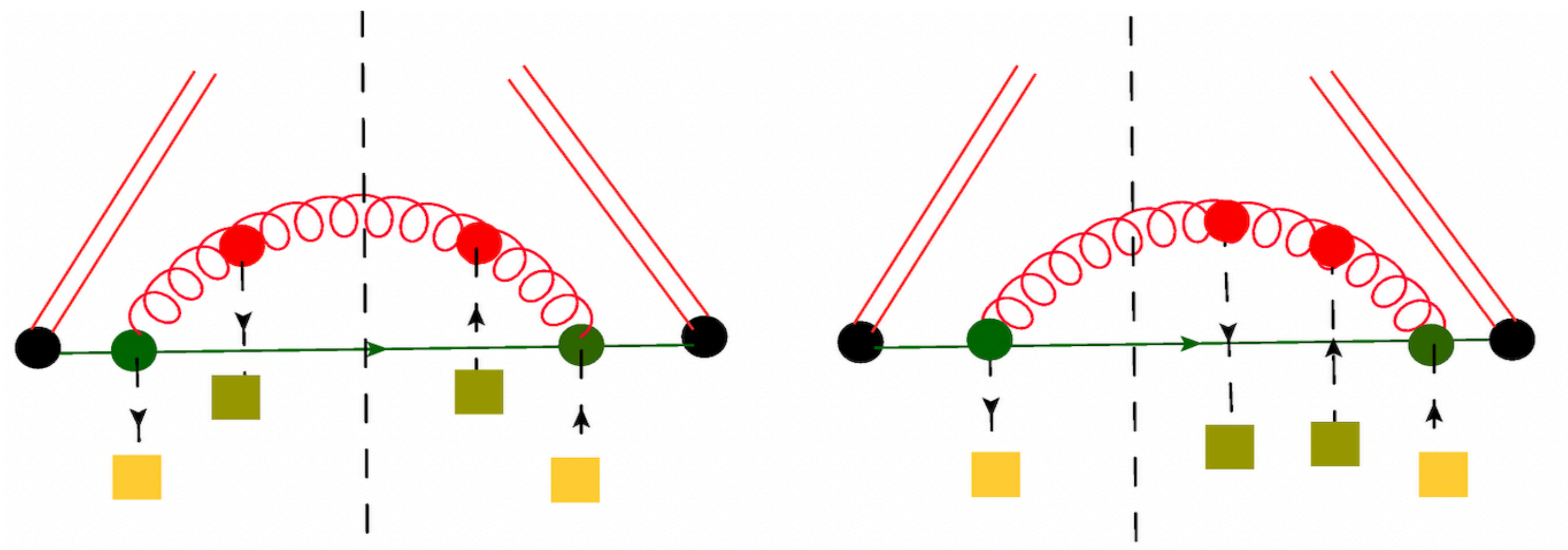
The factorization formula assumes that medium correlators are not correlated beyond the scattering length

Jet function with multiple scatterings

Broadening of vacuum emission



Broadening of medium induced emission



- In the Markovian limit and $L \to \infty$ limit, i.e., $t_f \ll L$

Phase space constraint for the measurement is $q_{f\perp}/\omega \sim R$

$$\mathbf{S}_B^{(n)}(\epsilon_L, k_{1\perp}, k_{2\perp}, \ldots, k_{n\perp}) = \frac{\alpha_s(N_c^2-1)}{4\pi^2}\int \mathrm{d}^2 q_\perp \int \frac{\mathrm{d}q^-}{q^-}\Theta\left(q_\perp - \frac{q^- R}{2}\right)\left[\delta(\epsilon_L) - \delta(q^- - \epsilon_L)\right]\int \frac{d^2 p_\perp}{(2\pi)^2}\int \frac{d^2 b e^{ip_\perp \cdot b}}{(p_\perp + q_\perp)^2}\prod_{i=1}^{n}\frac{\left(e^{-ik_{i\perp}\cdot b} - 1\right)}{k_{i\perp}^2}$$

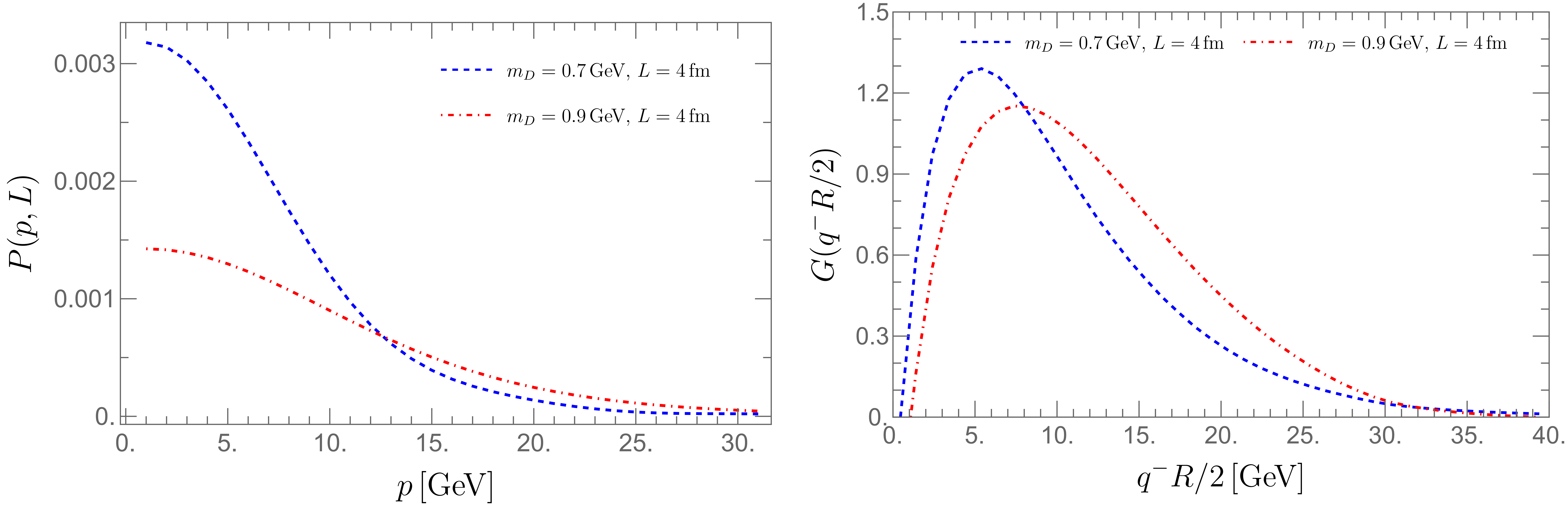
- The jet function can be written as a distribution in the impact parameter space

$$\mathcal{S}_{1,B}(\epsilon_L, \mu) \approx \frac{\alpha_s(N_c^2-1)}{4\pi^2}\int \frac{\mathrm{d}q^-}{q^-}\left[\delta(\epsilon_L) - \delta(q^- - \epsilon_L)\right]\int \frac{d^2 p_\perp}{(2\pi)^2}\int \frac{\mathrm{d}^2 q}{(q_\perp + p_\perp)^2}\Theta\left(q_\perp - \frac{q^- R}{2}\right)\left(P(p_\perp, L) - \delta^2(p_\perp)\right)$$

Results V : emergent perturbative scale

Multiple scatterings lead to larger momentum transfer from medium to jet parton

B.S. and V.Vaidya, 2412.18967



Peak in the distribution provides an estimate for the emergent scale Q_{med} through multiple scattering

- Exact value of the emergent scale depends on medium properties and parameters

Further matching

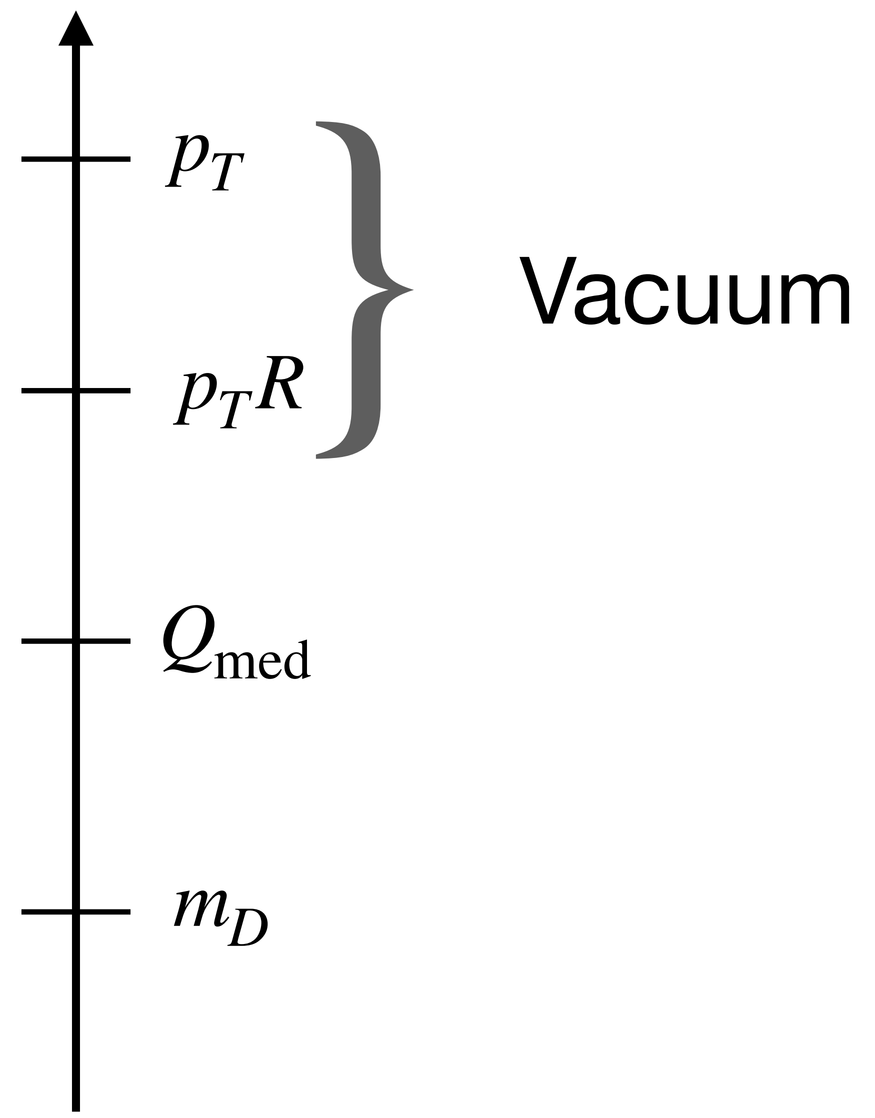
- $Q_{\text{med}} \gg m_D$ requires further matching from the scale Q_{med} to medium scale m_D

$$p_{cs} \sim \frac{Q_{\text{med}}}{R}\left(1, R^2, R\right)$$

Modes contributing to the measurement

$$p_{ucs} \sim \frac{Q_{\text{med}}}{R}\left(1, \frac{m_D^2 R^4}{Q_{\text{med}}^2}, \frac{m_D R}{Q_{\text{med}}}\right)$$

Modes contributing to the non-perturbative physics of the medium



- $k_\perp \sim m_D$

$$P_{\text{soft}}(p, L; \mu) = P(p, L) \xrightarrow[k \sim m_D]{} \int d^2 b\, e^{ip\cdot b}\left[\exp\left\{-2\pi L\, b^2 \int d^2 k_\perp \mathbf{B}_R(k_\perp, \nu_{cs}\right\}\right]$$

- $k_\perp \sim Q_{\text{med}}$, in this limit we can expect

2412.18967

$$P_{\text{hard}}(p, L; \mu) = P(p, L) \xrightarrow[k \sim 1/b]{} \int d^2 b\, e^{ip\cdot b}\left[\exp\left\{-\int \frac{d^2 k_\perp}{k_\perp^2}(1 - e^{-ik_\perp \cdot b}) \int_0^L dx^- \rho(x^-) \int_0^1 \frac{d\xi}{\xi} C(k_\perp, x/\xi; \mu) Y(\xi)\right\}\right]$$

Summary

- Factorization allows us to separate universal non-perturbative physics from perturbative one
- Factorization allows us to go beyond leading order computations by resummation techniques
- EFT allows to systematically improve predictions by doing higher order computations for factorized functions
- For EEC, we resum $\log(\sqrt{\chi})$ by BFKL resummation relevant in small χ region
- Projected correlators can be pivotal for separating the dynamics of large and small angle radiations. Similar to vacuum anomalous dimension may probe medium evolution
- Multiple scatterings lead to an emergent scale which requires refactorization to completely separate non-perturbative physics of the medium
- We should focus on improving theoretical calculations and compute higher order perturbative contributions to understand jet medium interaction dynamics

Thank you
for your attention

Jet function

- Soft/Medium function explicitly factors out

$$\mathbf{B}_{AB}(x,y) = \mathrm{Tr}\Big[\mathbf{T}\Big\{e^{-i\int dt_l H_{s,I}(t_l)}\Big(\frac{1}{\mathbb{P}_\perp^2}\mathcal{O}_s^A(x)\Big)\Big\}\rho_M(0)\bar{\mathbf{T}}\Big\{e^{-i\int dt_r H_{s,I}(t_r)}\Big(\frac{1}{\mathbb{P}_\perp^2}\mathcal{O}_s^B(y)\Big)\Big\}\Big]$$

SCET operators

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A \frac{\bar{n}}{2}\chi_n$$

$$\mathcal{O}_s^{qA} = \bar{\chi}_s T^A \frac{n}{2}\chi_s$$

- sinc function leads to LPM terms

$$J_{q,R}(\chi,k_\perp;L) = \frac{e^{-i\frac{L}{2}(\mathbb{P}_+^A-\mathbb{P}_+^B)}}{2N_c}\mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A-\mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\bar{\mathbf{T}}\Big\{e^{-i\int dt H_n(t)}\Big[O_n^{qB}(0)\Big]\chi_n(0)\Big\}\mathcal{M}\,|X\rangle\langle X|$$
$$\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\Big[O_n^{qB}(0)\Big]\Big[\bar{\chi}_n(0)\Big]\Big\}\Big] + \mathcal{O}(H_G^4)$$

- Sinc function leads to LPM terms

$$J_{q,V}(\omega,\chi,k_\perp;L) = \frac{1}{2N_c}\frac{1}{2}e^{-i\frac{L}{2}(\mathbb{P}_+^A+\mathbb{P}_+^B)}\mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A+\mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\,\bar{\mathbf{T}}\Big\{e^{-i\int dt H_{n,I}(t)}\chi_n(0)\Big\}\mathcal{M}\,|X\rangle\langle X|\,\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\Big[O_n^A(0)\Big]$$
$$\times\Big[O_n^B(0)\Big]\Big[\bar{\chi}_n(0)\Big]\Big\}\,|0\rangle\Big]\delta^{AB} + c.c + \mathcal{O}(H_G^4)$$

BFKL evolution equation

- From RG consistency the jet function obeys BFKL evolution equation

$$\frac{d\sigma}{d\chi} \sim H \otimes (J^0(\chi) + \mathcal{J} \otimes \mathbf{S}_1^{(1)}(\chi) \otimes \mathbf{B})$$

$$\nu \frac{d\mathbf{S}_1^{(1)}(k_\perp, \nu)}{d\nu} = -\frac{\alpha_s(\mu) N_c}{\pi^2} \int d^2 l_\perp \left[\frac{\mathbf{S}_1^{(1)}(l_\perp, \nu)}{(\vec{l}_\perp - \vec{k}_\perp)^2} - \frac{k_\perp^2 \mathbf{S}_1^{(1)}(k_\perp, \nu)}{2 l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \right]$$

Scale for jet function

$$\nu_0 \sim \frac{Q_{\text{med}}}{\sqrt{\chi}}$$

- Fixed order NLO jet function sets the boundary condition
- Running from jet scale to medium scale

$$\mathbf{S}_{1,\mathrm{R}}^{(1)}(k_\perp, \mu, \nu_f) = \int d^2 l_\perp \mathbf{S}_1^{(1)}(l_\perp, \mu, \nu_0) \int \frac{d\nu}{2\pi} k_\perp^{-1+2i\nu} l_\perp^{-1-2i\nu} e^{in(\phi_k - \phi_l)} e^{-\frac{\alpha_s(\mu) N_c}{\pi} \chi(n,r) \log \frac{\nu_f}{\nu_0}}$$

Medium scale

$$\nu_f \sim Q_{\text{med}}$$

- Resums $(a_p - 1)\log(\nu_0 / k_\perp)$
- Solution for $k_\perp \sim l_\perp$

$$\mathbf{S}_{1,R}^{(1)}(\chi, k_\perp; L) = \frac{1}{\pi k_\perp} \sqrt{\frac{\pi}{14\zeta(3)\bar{\alpha}Y}} e^{(a_p - 1)Y} \int d^2 l_\perp \frac{\mathbf{S}_1^{(1)}(\chi, l_\perp; L)}{l_\perp} e^{-\frac{\log^2(k_\perp / l_\perp)}{14\zeta(3)\bar{\alpha}Y}}$$

Medium function

- Medium function can be obtained from spectral function which can computed in perturbatively and can also be evaluated on lattice

$D_>(k_\perp)$ is Weightman correlator in a thermal medium and depends on the properties of the medium

$$\mathbf{B}(k_\perp) = D^g_>(k) + D^q_>(k) \qquad D_>(k) = (1 + f(k_0))\rho(k)$$

- In SCET framework spectral function is obtained from soft operators in the medium and also depends on the local properties of the plasma through soft operators

SCET operators

$$\mathcal{O}^{qA}_n = \bar{\chi}_n T^A \frac{\bar{n}}{2}\chi_n$$

$$\mathcal{O}^{qA}_s = \bar{\chi}_s T^A \frac{n}{2}\chi_s$$

$$\mathcal{O}^{gA}_s = \frac{i}{2} f^{ACD}\mathcal{B}^C_{S\perp\mu}\frac{n}{2}\cdot(\mathcal{P} + \mathcal{P}^\dagger)\mathcal{B}^{D\mu}_{S\perp}$$

$$D^{AB}_E(K) = \int_0^\beta d\tau \int d^3x\, e^{iK\cdot X}\langle \frac{1}{\mathbb{P}^2_\perp} O^{g_n A}_s(X) \frac{1}{\mathbb{P}^2_\perp} O^{g_n B}_s(0)\rangle \propto \delta^{AB}[\ldots]$$

- Leading order medium function

$$\mathbf{B}_{\text{LO}}(k_\perp) = (8\pi\alpha_s)^2\left(\frac{2\pi N_c^2}{16 k_\perp^4}\mathcal{J}^g(k_\perp) + \frac{2\pi N_f}{k_\perp^4}\mathcal{J}^q(k_\perp)\right)$$

At leading order medium function contains a Coulomb tail

Medium function

- At leading order medium function has somewhat weak dependence on Glauber momentum