Relativistic (a)causality in hydrodynamics and its effect on Bayesian analyses

Matthew Luzum

References:
T.S.Domingues, R.Krupczak, J.Noronha, T.N.da Silva, J-F.Paquet, ML; Phys.Rev.C 110 (2024) 6, 064904; arXiv:2409.17127
Arthur Lopez, ML; work in progress

University of São Paulo

Hot Jets: Advancing the Understanding of High Temperature QCD with Jets
January 9, 2025

Hydrodynamics and Causality

Hydrodynamic validity

- Hydrodynamics central to simulations
- Validity of fluid description not always clear
- Typically derived as expansion around equilibrium
- Sometimes system is far from equilibrium (early times, near jets)
- Not definitive: hydro can be valid far from equilibrium
- Relativistic causality: definitive test
- How important is this issue? Quantify with Bayesian analysis

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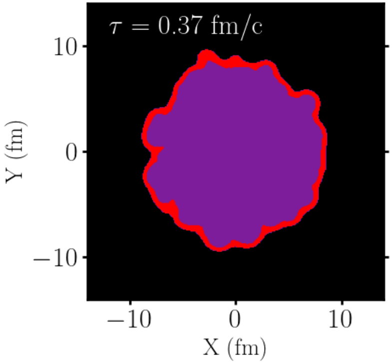

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Phys.Rev.C 109 (2024) 3, 034908

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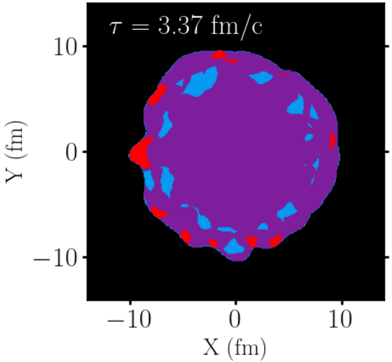

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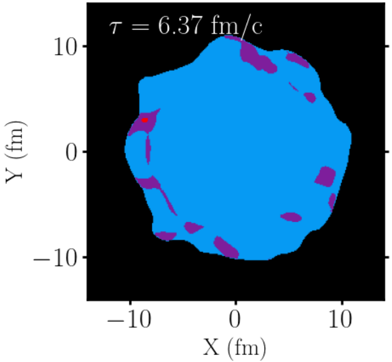

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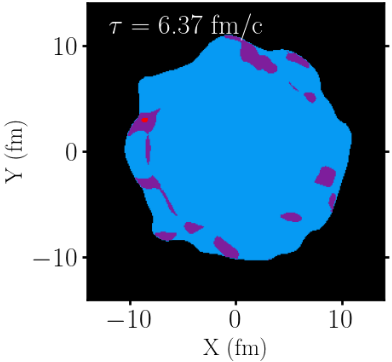

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Bayesian Analyses

- Bayesian inference: used to extract physical properties from data with systematic treatment of uncertainty
- JETSCAPE performed large-scale analysis of soft sector
  Phys.Rev.C 103 (2021) 5, 054904
- How are these results affected if we don't allow hydro to be used in acausal regime?

Norm. Pb-Pb 2.76 TeV	N[2.76 TeV]	[10, 20]
Norm. Au-Au 200 GeV	N[0.2 TeV]	[3, 10]
generalized mean	p	[–0.7, 0.7]
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temperature of (ζ/s) peak	T_ζ	[0.12, 0.3] GeV
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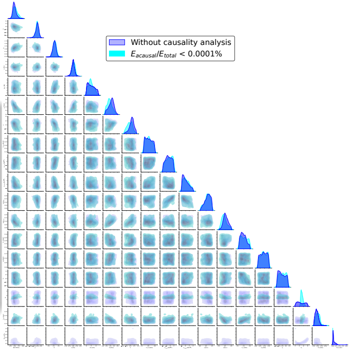

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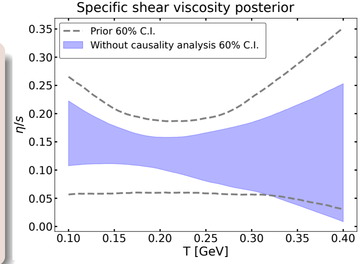

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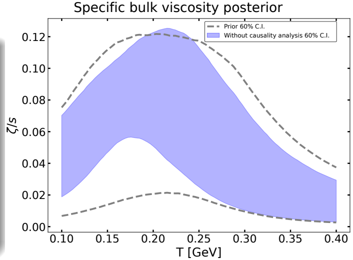

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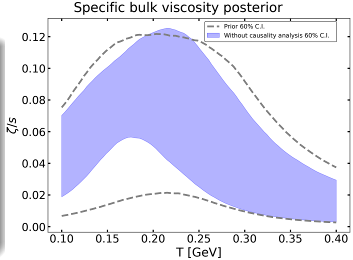

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Israel-Stewart hydrodynamics

- Modern hydrodynamic theory used in simulations:

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - (P+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \\
\partial_\mu T^{\mu\nu} &= 0, \\
\tau_\Pi \dot{\Pi} + \Pi &= -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \\
\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}
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- Common parameterizations:

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\begin{aligned}
\tau_\Pi &= b_\Pi \frac{\zeta}{\left(\frac{1}{3} - c_s^2\right)^2 (\epsilon + p)} \\
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Linear causality

- Simplest situation: infinitessimal perturbation around static global equilibrium
- Linearize equations of motion. Demanding signal propagation $v < c$ gives condition

$$n_{\text{static}} \equiv c_s^2 + \frac{4}{3}\frac{\eta}{\tau_\pi(\epsilon + P)} + \frac{\zeta}{\tau_\Pi(\epsilon + P)} \leq 1$$

$$\tau_\pi = b_\pi \frac{\eta}{sT}$$

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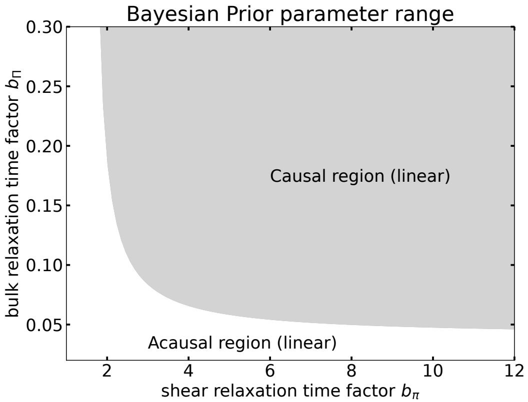
- Prior (and posterior) allows violation of linear causality!
- Small dependence of observables on τ_π and τ_Π gives flat posterior and no strong effect on conclusions about other parameters



Trajectum 1: *Phys. Rev. C 103, 054909 (2021)*

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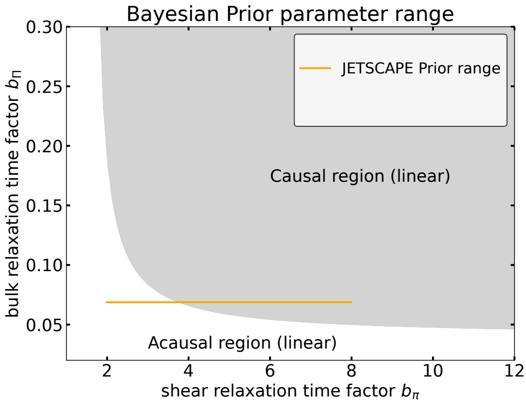

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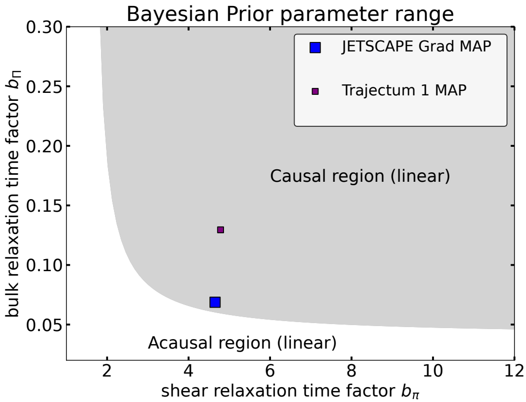

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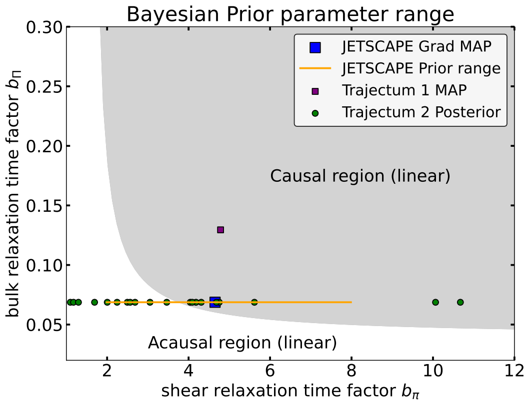

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Trajectum 2: *Phys. Rev. C 106, 044903 (2022)*

Nonlinear Causality

- Recently-derived (necessary) conditions for general, nonlinear case:

  Phys. Rev. Lett. 126, 222301 (2021)

$$n_1 \equiv \frac{2}{b_\pi} + \frac{\lambda_{\pi\Pi}}{\tau_\pi}\frac{\Pi}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{2\tau_\pi}\frac{|\Lambda_1|}{\varepsilon+P} \geq 0,$$

$$n_2 \equiv 1 - \frac{1}{b_\pi} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_3}{\varepsilon+P} \geq 0,$$

$$n_3 \equiv \frac{1}{b_\pi} + \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\frac{\Pi}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_3}{\varepsilon+P} \geq 0,$$

$$n_4 \equiv 1 - \frac{1}{b_\pi} + \left(1 - \frac{\lambda_{\pi\Pi}}{2\tau_\pi}\right)\frac{\Pi}{\varepsilon+P} + \left(1 - \frac{\tau_{\pi\pi}}{4\tau_\pi}\right)\frac{\Lambda_a}{\varepsilon+P} - \frac{\tau_{\pi\pi}}{4\tau_\pi}\frac{\Lambda_d}{\varepsilon+P} \geq 0,$$

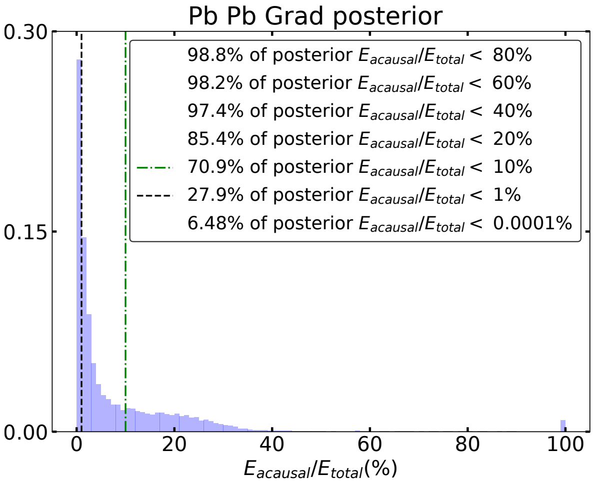
$$n_5 \equiv c_s^2 + \frac{4}{3}\frac{1}{b_\pi} + b_\Pi\left(\frac{1}{3} - c_s^2\right)^2 + \left(\frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} + \frac{\delta_{\Pi\Pi}}{\tau_\Pi} + c_s^2\right)\frac{\Pi}{\varepsilon+P}\left(\frac{3\delta_{\pi\pi}+\tau_{\pi\pi}}{3\tau_\pi} + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2\right)\frac{\Lambda_1}{\varepsilon+P} \geq 0,$$

$$n_6 \equiv 1 - \left(c_s^2 + \frac{4}{3}\frac{1}{b_\pi} + b_\Pi\left(\frac{1}{3} - c_s^2\right)^2\right) + \left(1 - \frac{2}{3}\frac{\lambda_{\pi\Pi}}{\tau_\pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi} - c_s^2\right)\frac{\Pi}{\varepsilon+P} + \left(1 - \frac{3\delta_{\pi\pi}+\tau_{\pi\pi}}{3\tau_\pi} - \frac{\lambda_{\Pi\pi}}{\tau_\Pi} - c_s^2\right)\frac{\Lambda_3}{\varepsilon+P} \geq 0.$$

- In practice, n_6 is the most stringent condition

Quantifying acausality

- JETSCAPE model: Trento → Free Streaming → Hydrodynamics → Cooper-Frye
- We perform a $b = 0$ simulation and quantify the fraction of system (defined by totel energy) that is in an acausal regime at onset of hydrodynamics
- What happens if we make cuts on the posterior?



QUANTIFYING ACAUSALITY

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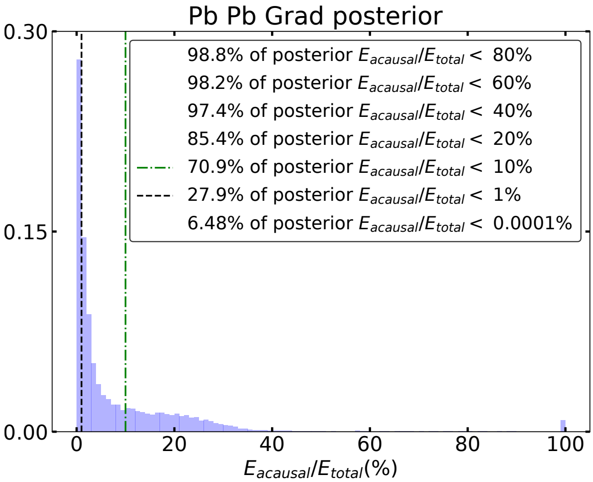
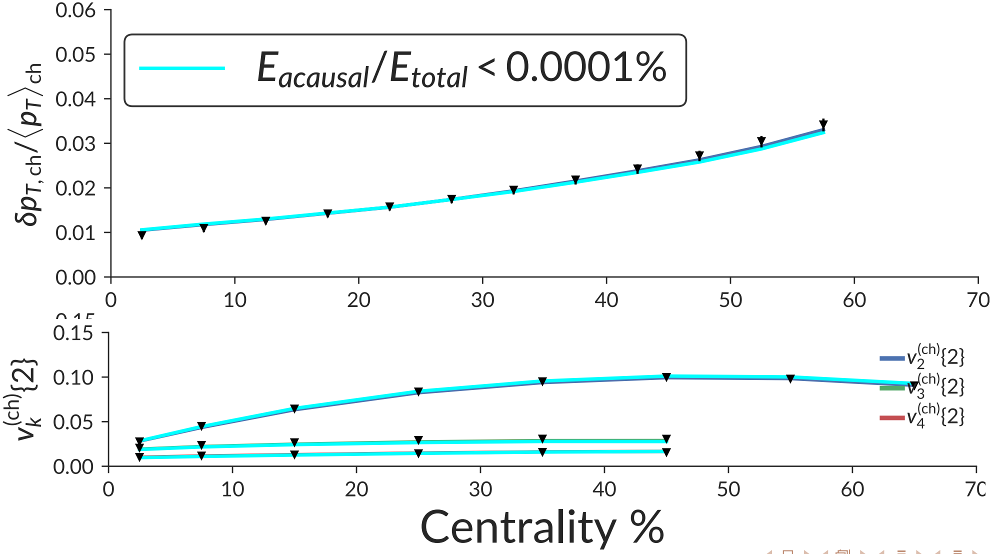


Observables (MAP with and without acausality cuts)



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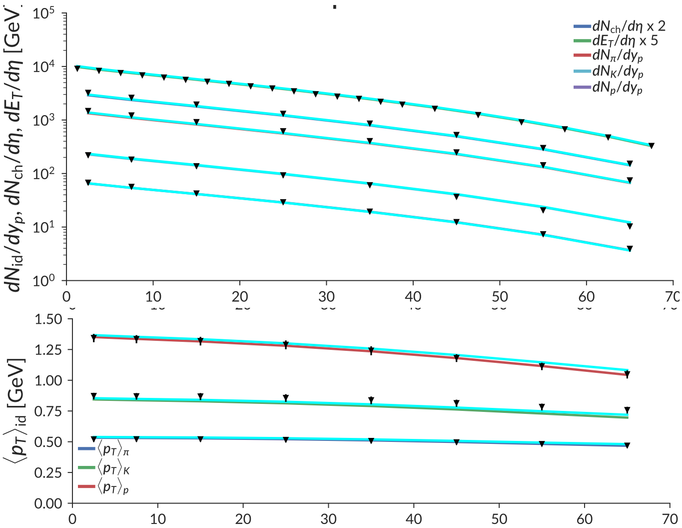
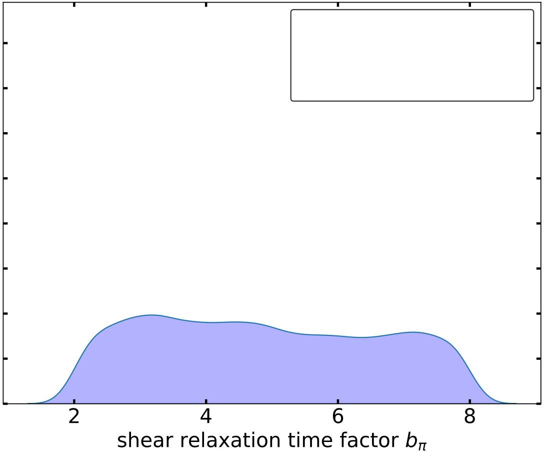


Observables

- Ability to fit data not destroyed by stringent causality demands
- Maximum probability (Maximum a Posteriori, MAP) of original posterior is $\sim$ 3 times as likely as best-fit after the strongest cut

- 1D marginalized posterior distributions
- Demanding causality alters posteriors



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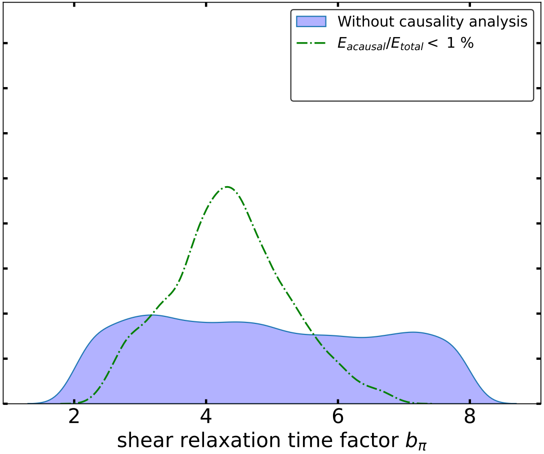

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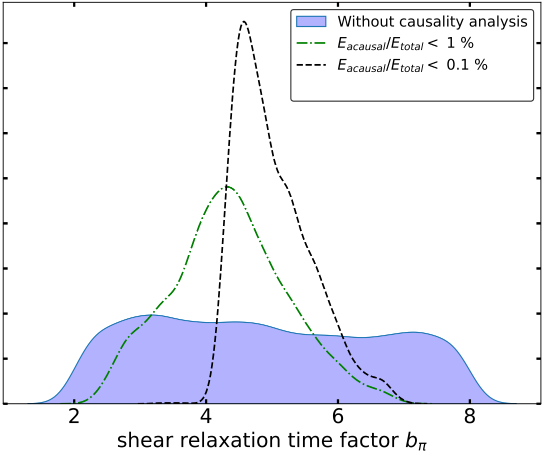

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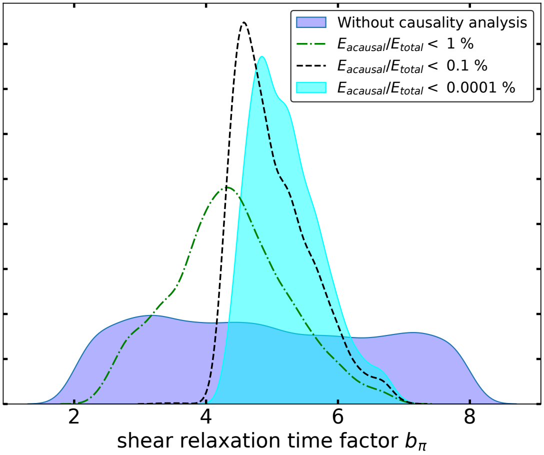

1D PARAMETER POSTERIORS

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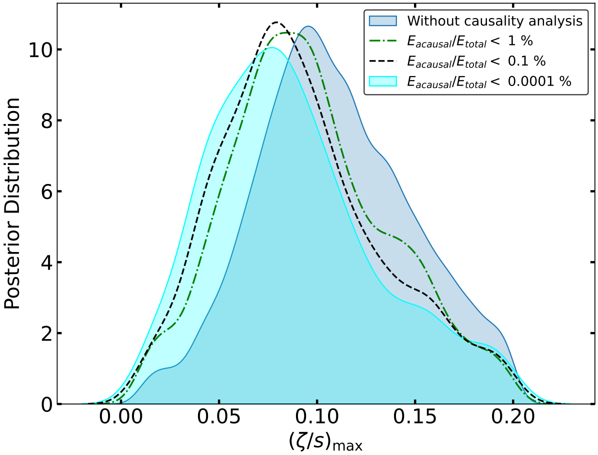
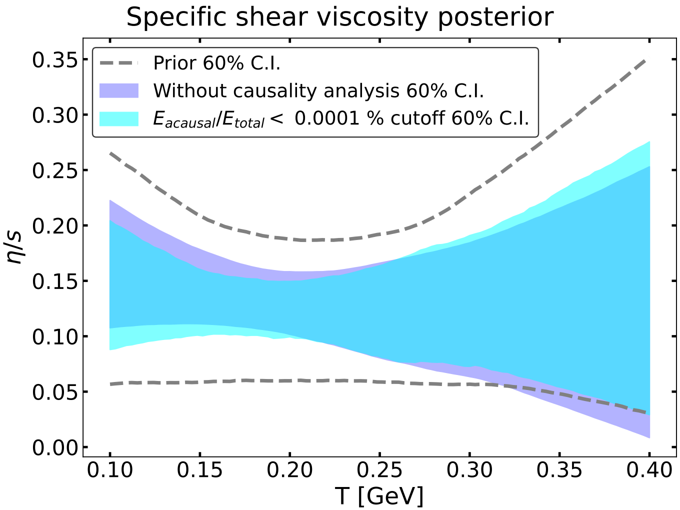


Viscosity posteriors

- Shear viscosity not signifantly affected
- Large bulk viscosity disfavored by causality cuts



Viscosity posteriors

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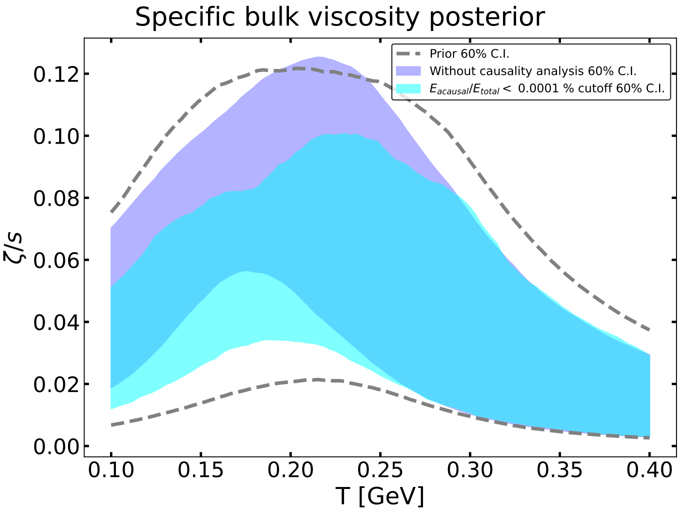
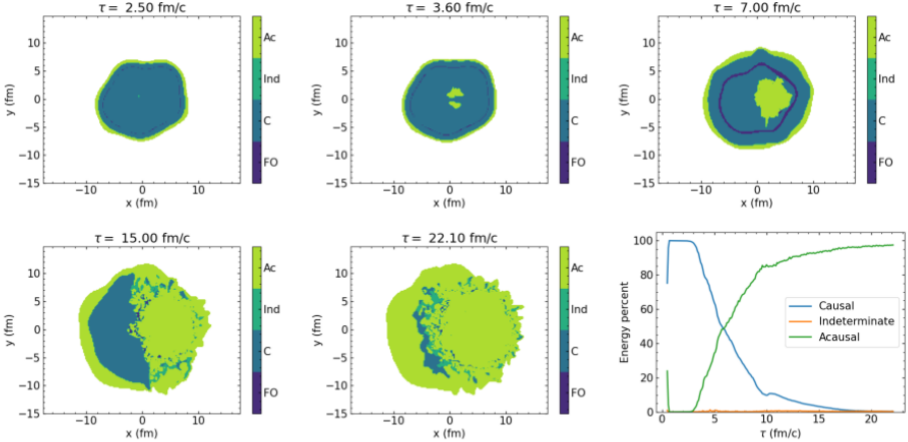


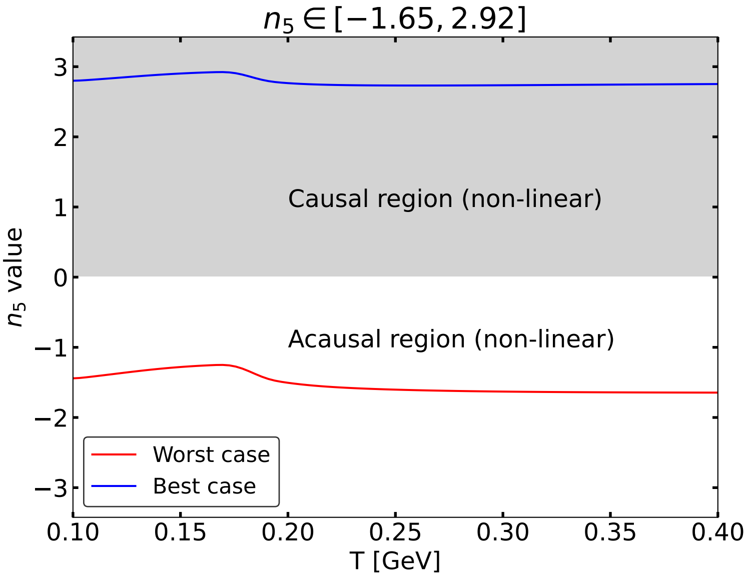
Conclusions

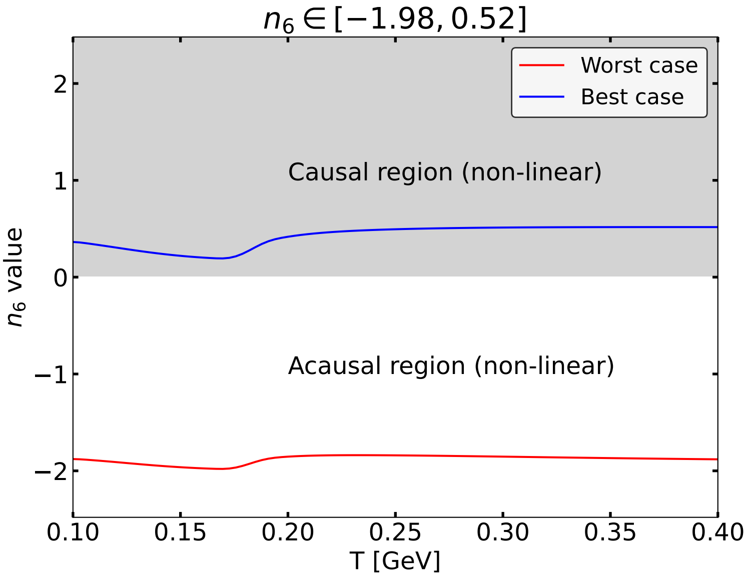
- Hydodynamic simulations typically enter acausal regimes, at least sometimes
- Demanding limits on acauasality has nonnegligible effects on existing Bayesian analyses
- In the era of precision heavy-ion physics, it is an issue that should be addressed
  - Improve pre-hydrodynamic description
  - Further developments in hydrodynamic theory

- Early times: system is far from equilibrium and must thermalize/hydrodynamize
- Same considerations near jets: energy lost by the jet must thermalize/hydrodynamize
- May have observable affects?

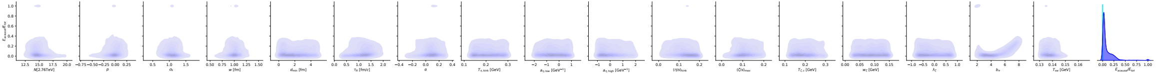


Extra Slides

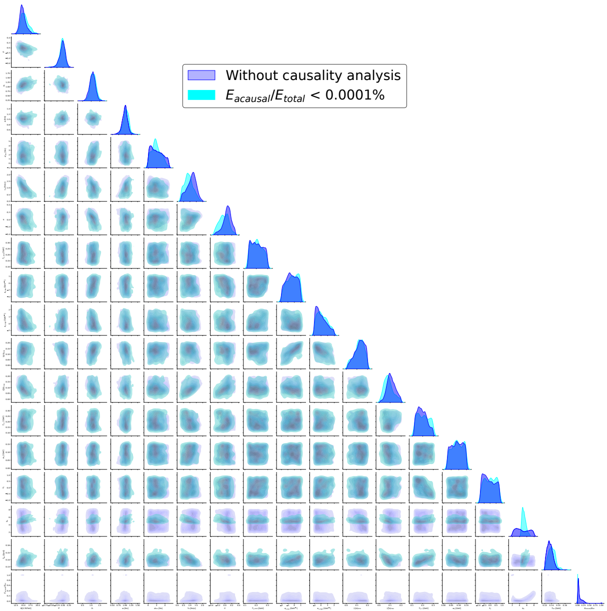

$n_5 \in [-1.65, 2.92]$
Causal region (non-linear)
Acausal region (non-linear)
Worst case
Best case
n_5 value
T [GeV]
3
2
1
0
−1
−2
−3
0.10
0.15
0.20
0.25
0.30
0.35
0.40



$n_6 \in [-1.98, 0.52]$
Worst case
Best case
Causal region (non-linear)
Acausal region (non-linear)
n_6 value
2
1
0
−1
−2
0.10
0.15
0.20
0.25
0.30
0.35
0.40
T [GeV]



$E_{acausal}/E_{tot}$
N[2.76TeV]
p
σ_k
w [fm]
d_{min} [fm]
τ_R [fm/c]
α
$T_{\eta,\mathrm{kink}}$ [GeV]
$a_{\eta,\mathrm{low}}$ [GeV^{-1}]
$a_{\eta,\mathrm{high}}$ [GeV^{-1}]
$(\eta/s)_{\mathrm{kink}}$
$(\zeta/s)_{\max}$
$T_{\zeta,c}$ [GeV]
w_ζ [GeV]
λ_ζ
b_π
T_{sw} [GeV]
$E_{acausal}/E_{tot}$



Without causality analysis
$E_{acausal}/E_{total}$ < 0.0001%

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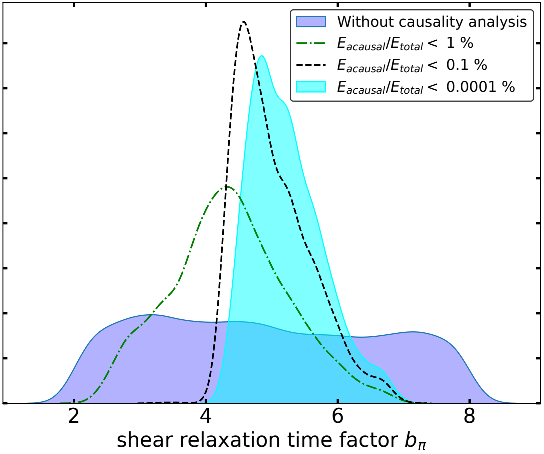

1D parameter posteriors

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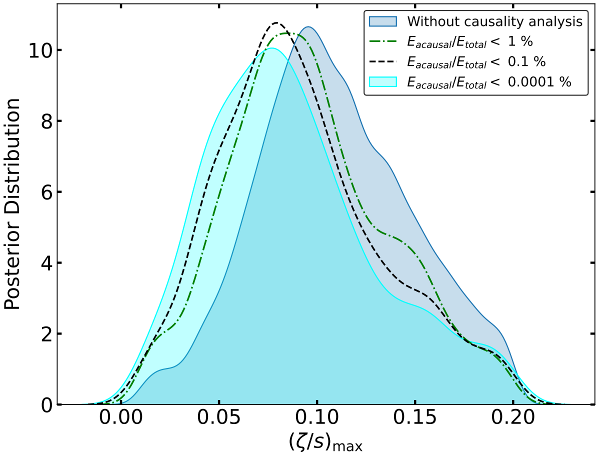

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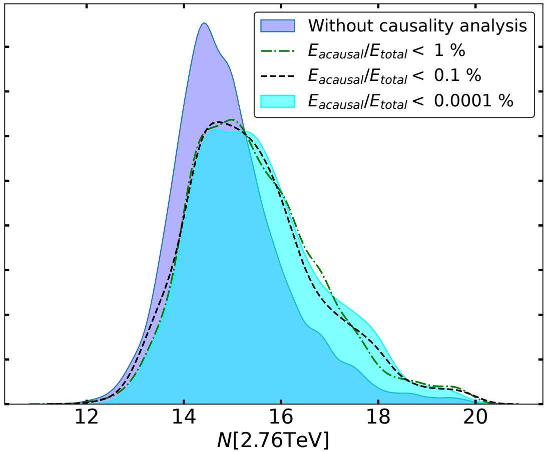

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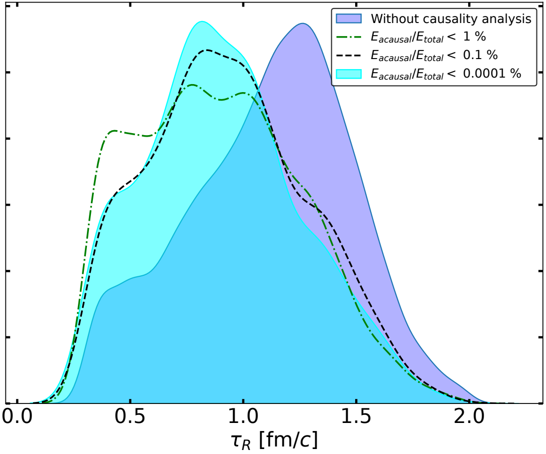

1D PARAMETER POSTERIORS

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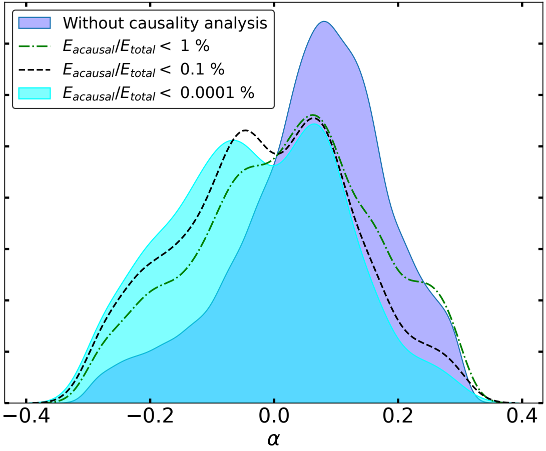

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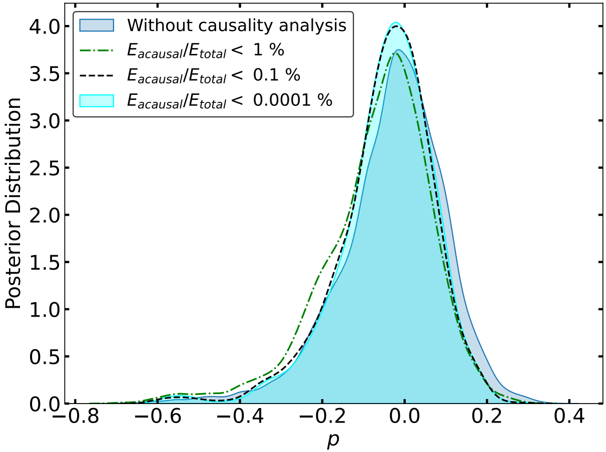

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