The Geometry of Particle Collisions: Hidden in Plain Sight

Jesse Thaler

Israeli Joint Seminar, Jerusalem — January 11, 2023

[Submitted on 13 Jun 2007 (v1), last revised 21 Nov 2007 (this version, v2)]

Probing Minimal Flavor Violation at the LHC

Yuval Grossman, Yosef Nir, Jesse Thaler, Tomer Volansky, Jure Zupan

If the LHC experiments discover new particles that couple to the Standard Model fermions, then measurements by ATLAS and CMS can contribute to our understanding of the flavor puzzles. We demonstrate this statement by investigating a scenario where extra SU(2)-singlet down-type quarks are within the LHC reach. By measuring masses, production cross sections and relative decay rates, minimal flavor violation (MFV) can in principle be excluded. Conversely, these measurements can probe the way in which MFV applies to the new degrees of freedom. Many of our conclusions are valid in a much more general context than this specific extension of the Standard Model.

Comments:	18 pages, 1 figure, appendix added, journal version
Subjects:	**High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex)
Cite as:	arXiv:0706.1845 **[hep-ph]**
	(or arXiv:0706.1845v2 **[hep-ph]** for this version)
	https://doi.org/10.48550/arXiv.0706.1845
Journal reference:	Phys.Rev.D76:096006,2007
Related DOI:	https://doi.org/10.1103/PhysRevD.76.096006

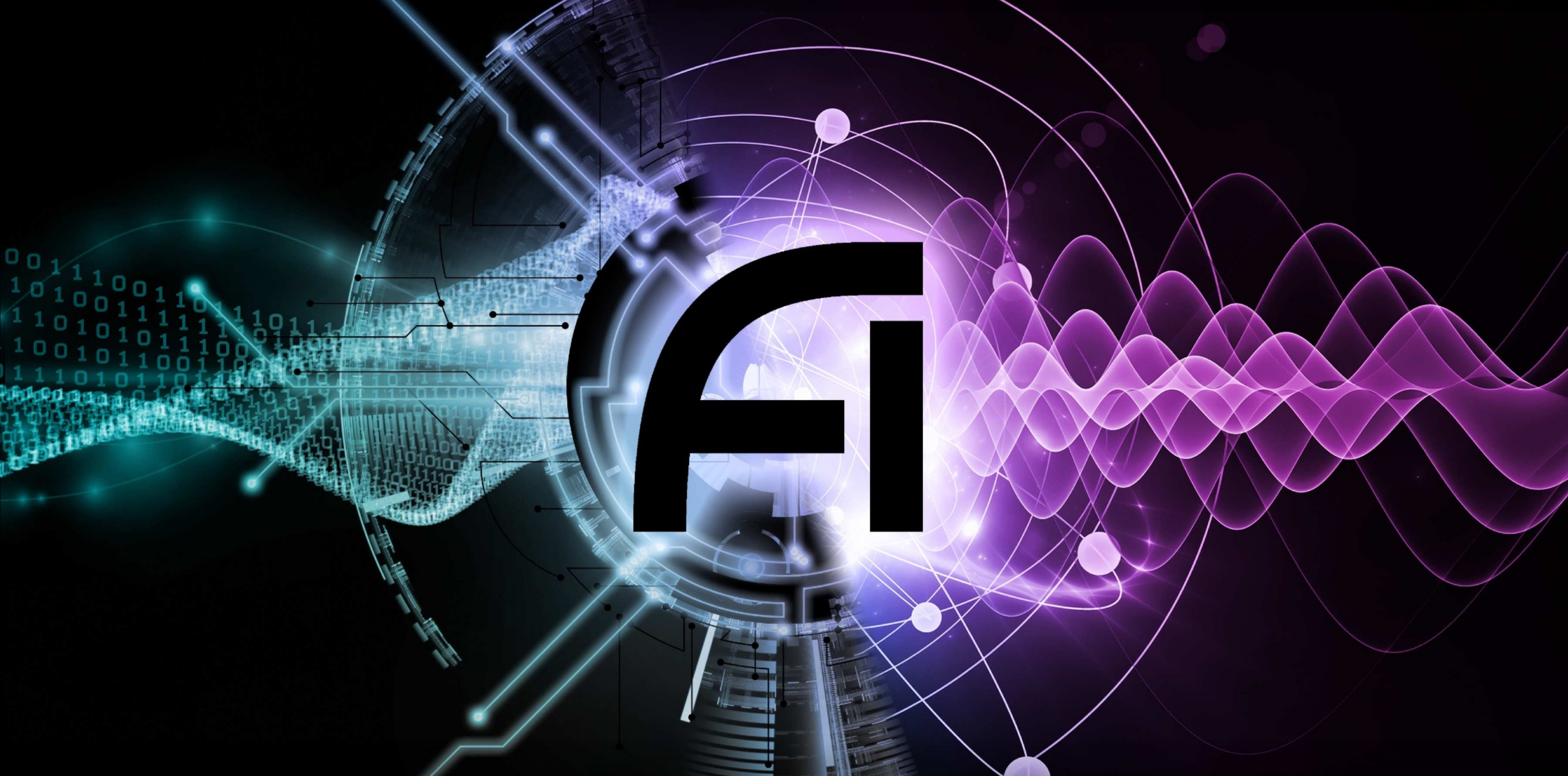
Submission history

From: Yosef Nir [view email]
[v1] Wed, 13 Jun 2007 08:25:01 UTC (24 KB)
[v2] Wed, 21 Nov 2007 19:45:23 UTC (25 KB)

(a more detailed account of this procedure will be given in subsequent work [11])

[11] Y. Grossman, Y. Nir, J. Thaler, T. Volansky, and J. Zupan, work in progress.

The NSF AI Institute for Artificial Intelligence and Fundamental Interactions (IAIFI /aɪ-faɪ/ iaifi.org)

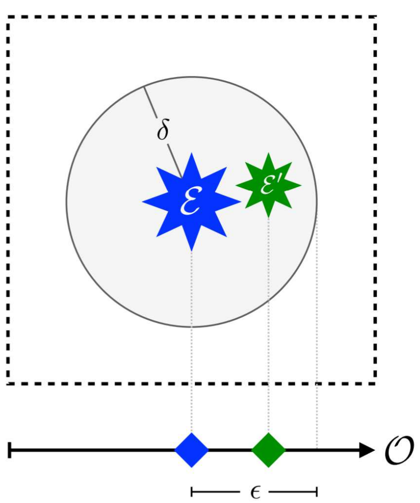


Advance physics knowledge — from the smallest building blocks of nature to the largest structures in the universe — and galvanize AI research innovation

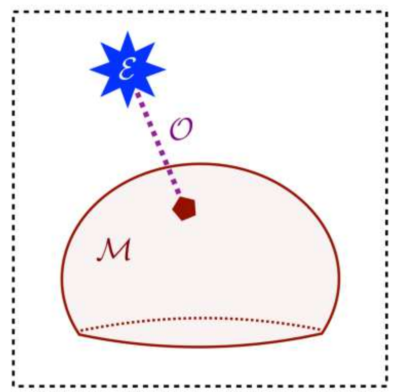
Today: Hidden in "Plane" Sight

Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events

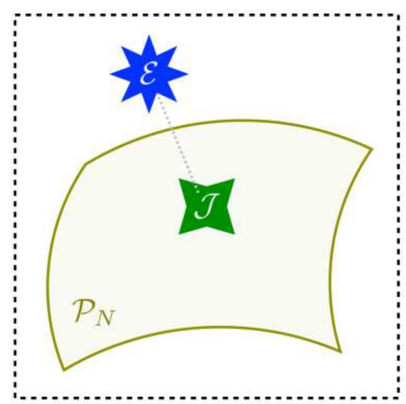


Event shapes are distances from events to manifolds.



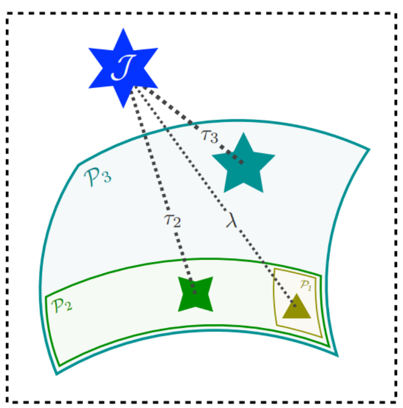
$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$

Jets are projections to few-particle manifolds.



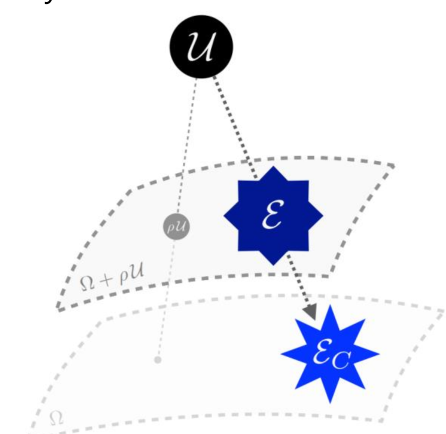
$\mathcal{J} = \text{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$

Substructure resolves emissions within the jet.



$\tau(\mathcal{J}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_\beta(\mathcal{J}, \mathcal{E}').$

Pileup mitigation moves away from uniform radiation.



$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \text{EMD}(\mathcal{E}, \mathcal{E}' + \rho\,\mathcal{U})\,.$

Taming infinities | Event Shapes | Jet Algorithms | Jet Substructure | Pileup

1960 — 2020

1962-1964
Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977
Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993
k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998
C/A jet clustering
[Wobisch, Wengler, 1998]
[Doskhitzer, Leder, Moretti,Webber, JHEP 1997]

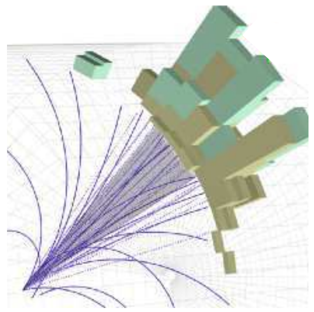
2010-2015
N-(sub)jettiness, XCone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019
Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

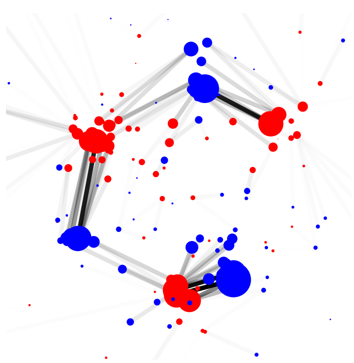
And many more!

[Komiske, Metodiev, JDT, JHEP 2020; timeline from Metodiev]

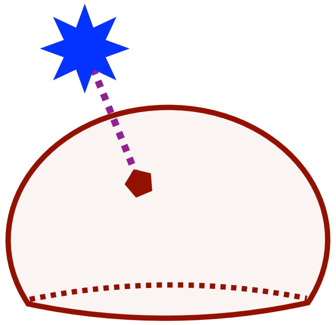
Outline



Going with the (Energy) Flow

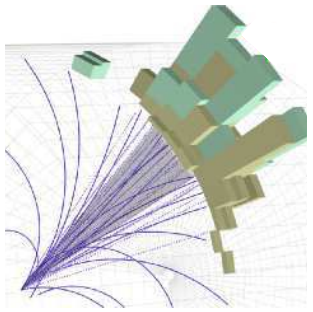


The Energy Mover's Distance



Revealing a Hidden Geometry

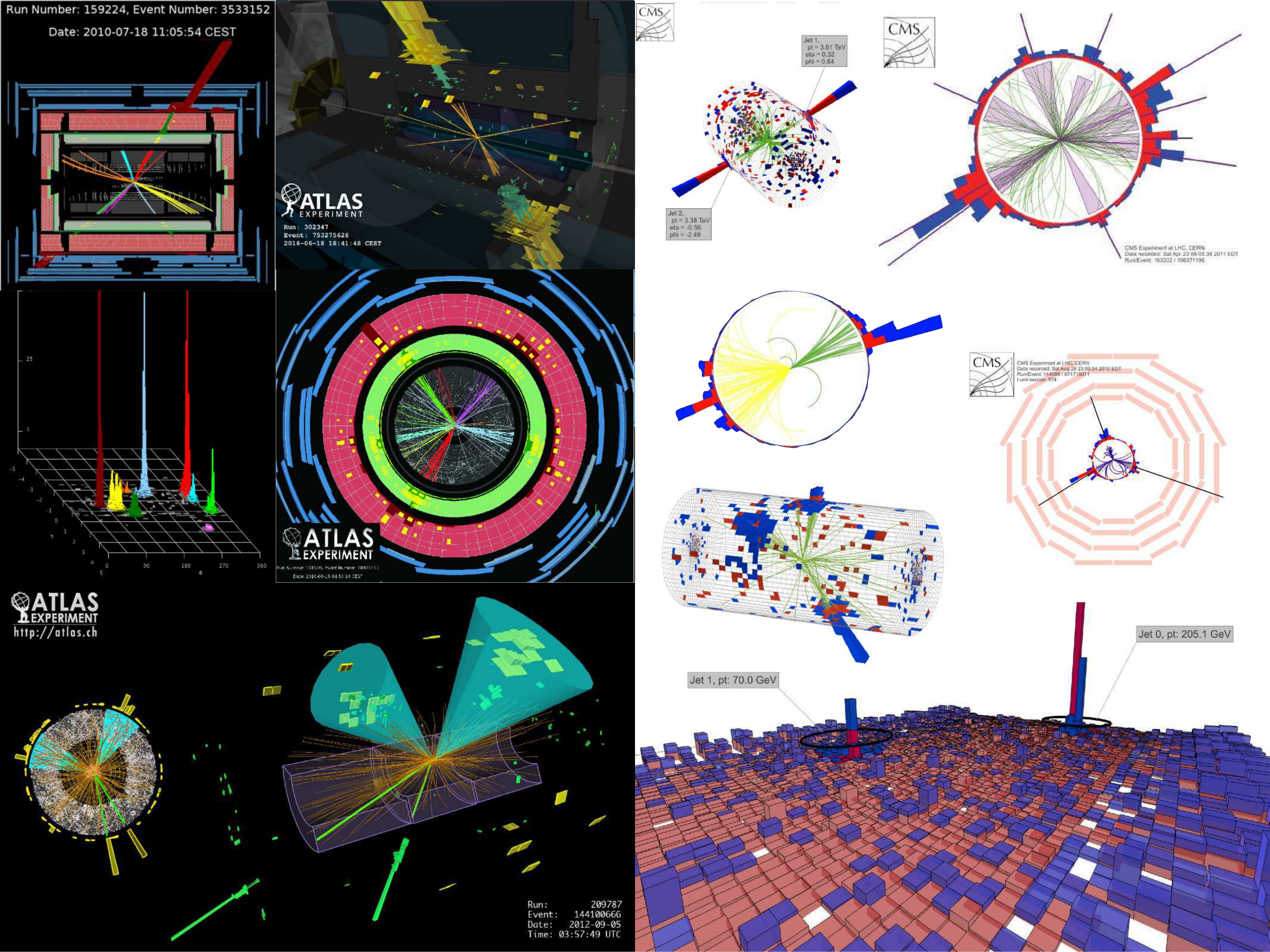
Extended review of my virtual Israel Physics Colloquium from November 2020



Going with the (Energy) Flow

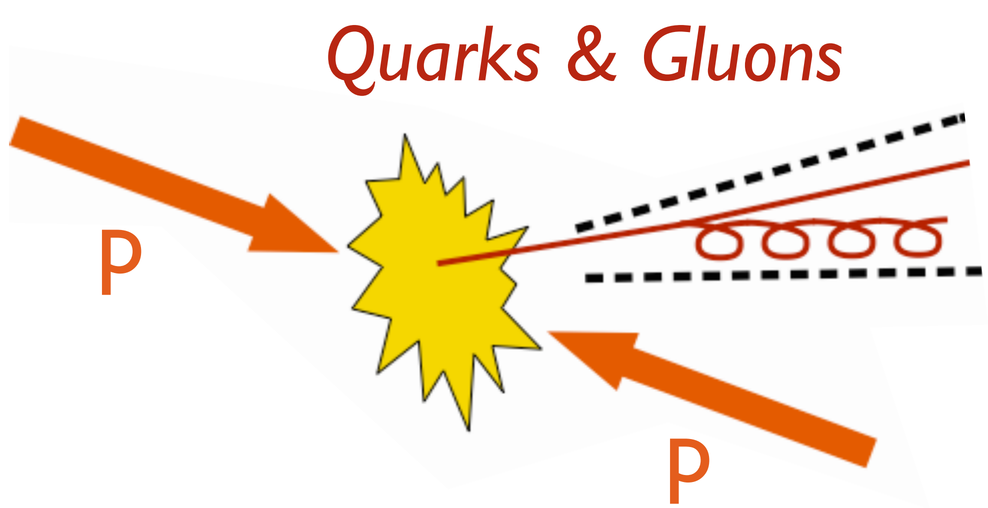
The Energy Mover’s Distance

Revealing a Hidden Geometry


Run Number: 159224, Event Number: 3533152
Date: 2010-07-18 11:05:54 CEST
ATLAS EXPERIMENT
Run: 302347
Event: 753275626
2016-06-18 18:41:48 CEST
CMS
Jet 1, pt = 3.61 TeV eta = 0.32 phi = 0.64
Jet 2, pt = 3.38 TeV eta = -0.56 phi = -2.49
CMS
CMS Experiment at LHC, CERN
Data recorded: Sat Apr 23 08:05:38 2011 EDT
Run/Event: 163332 / 196371106
ATLAS EXPERIMENT
CMS
CMS Experiment at LHC, CERN
Lumi section: 574
ATLAS EXPERIMENT
http://atlas.ch
Jet 0, pt: 205.1 GeV
Jet 1, pt: 70.0 GeV
Run: 209787
Event: 144100666
Date: 2012-09-05
Time: 03:57:49 UTC

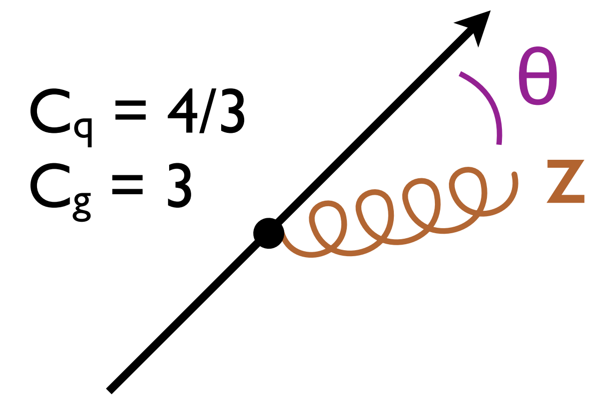
Energy Flow Representation

Emphasizes infrared and collinear safety



Altarelli-Parisi Splitting

Core prediction of QCD

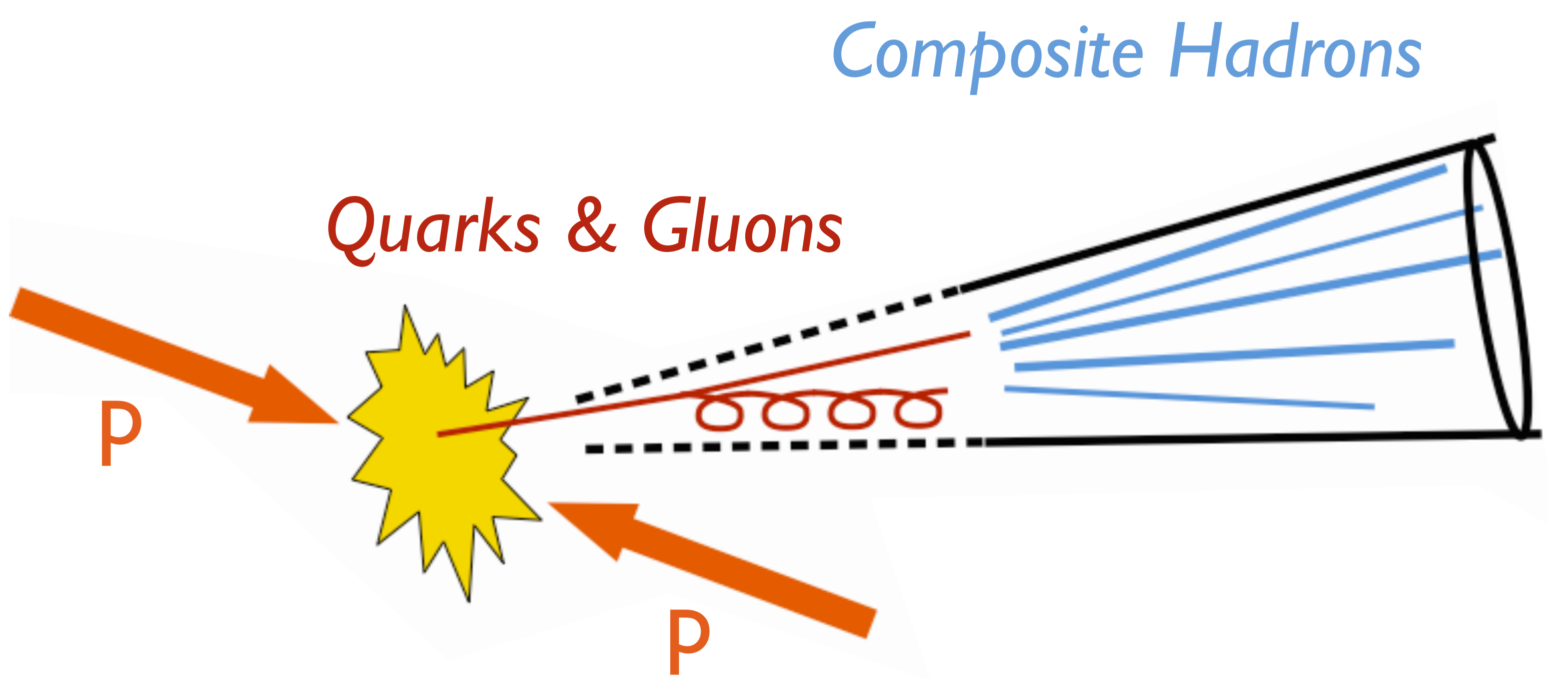


$$\mathrm{d}P_{i\to ig} \simeq \frac{2\alpha_s}{\pi} C_i \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}z}{z}$$

Collinear Soft

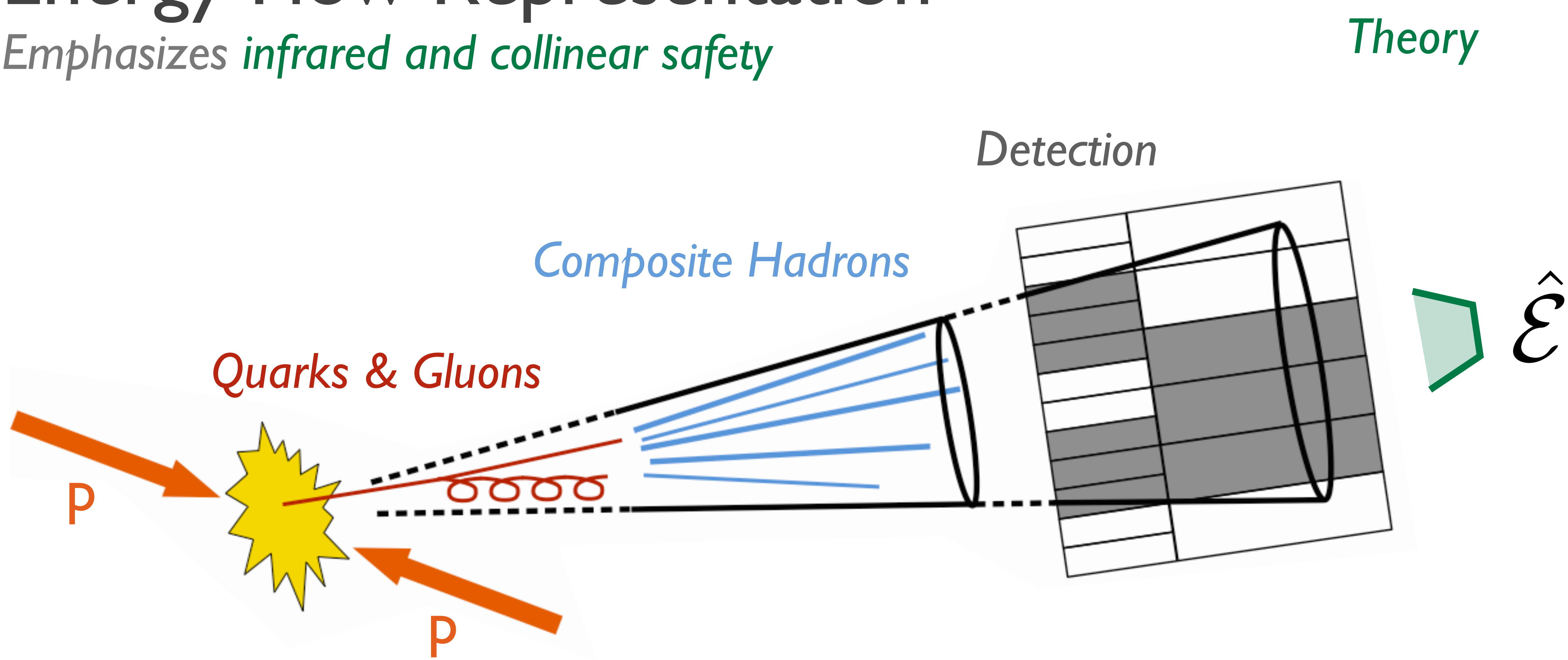
Energy Flow Representation

Emphasizes *infrared and collinear safety*



Energy Flow Representation

Emphasizes infrared and collinear safety



Energy Flow:

Robust to hadronization and detector effects

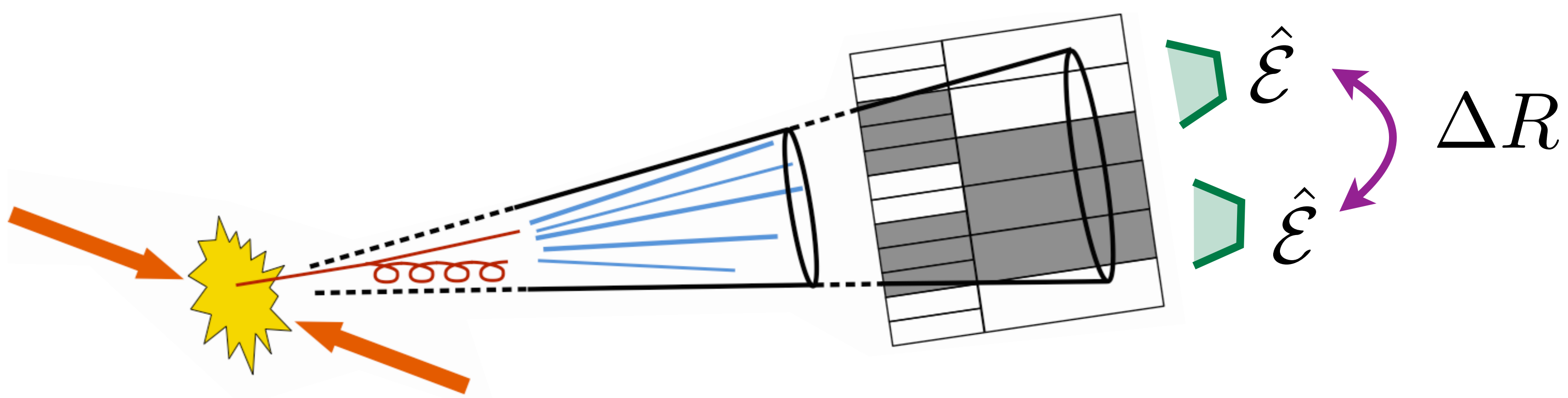
$$\hat{\mathcal{E}} \simeq \lim_{t \to \infty} \hat{n}_i T^{0i}(t, vt\hat{n})$$

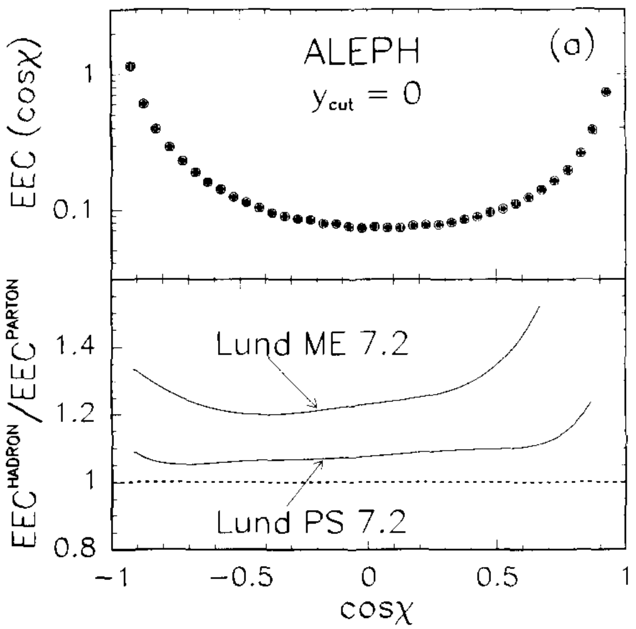
[see e.g. Sveshnikov, Tkachov, PLB 1996; Hofman, Maldacena, JHEP 2008; Mateu, Stewart, JDT, PRD 2013; Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, PRL 2014; Chen, Moult, Zhang, Zhu, PRD 2020]

(

Energy-Energy Correlators

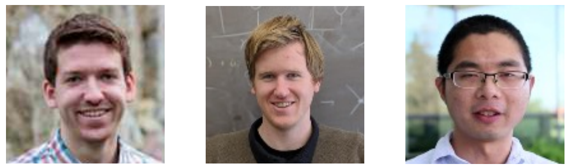
A long history probing the collinear dynamics of QCD:





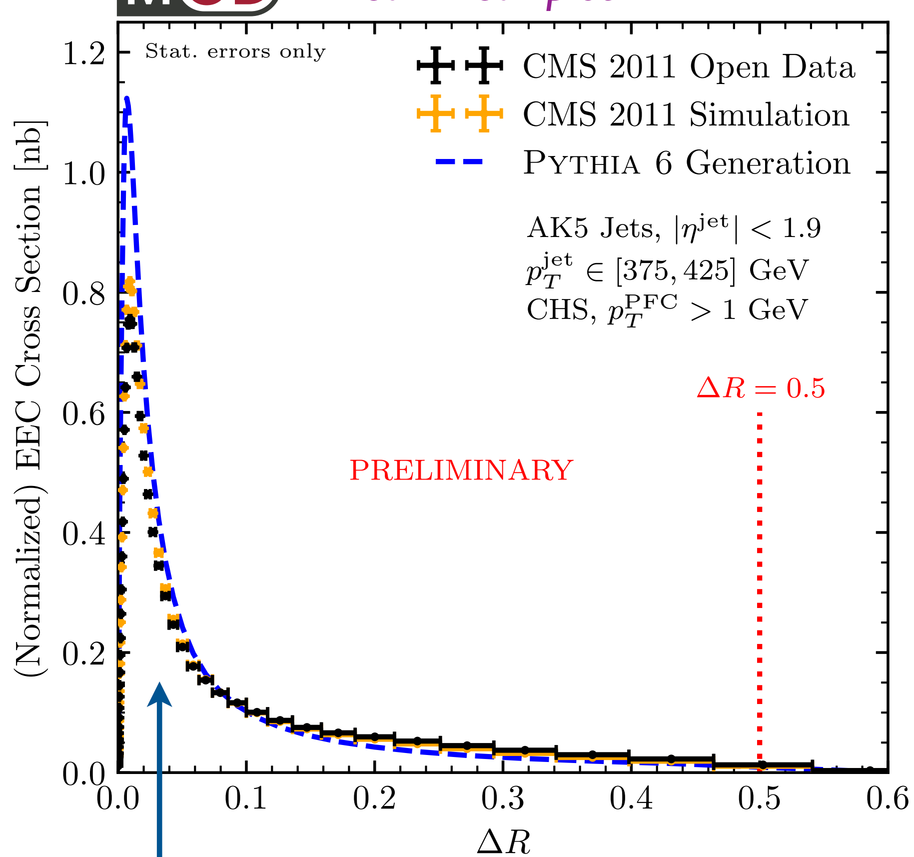
A new chapter leveraging insights from the conformal limit!

[Basham, Brown, Ellis, Love, PRL 1978; ALEPH, PLB 1991]
[cf. Ian Moult's Inspire page; Komiske, Moult, JDT, Zhu, arXiv 2022]



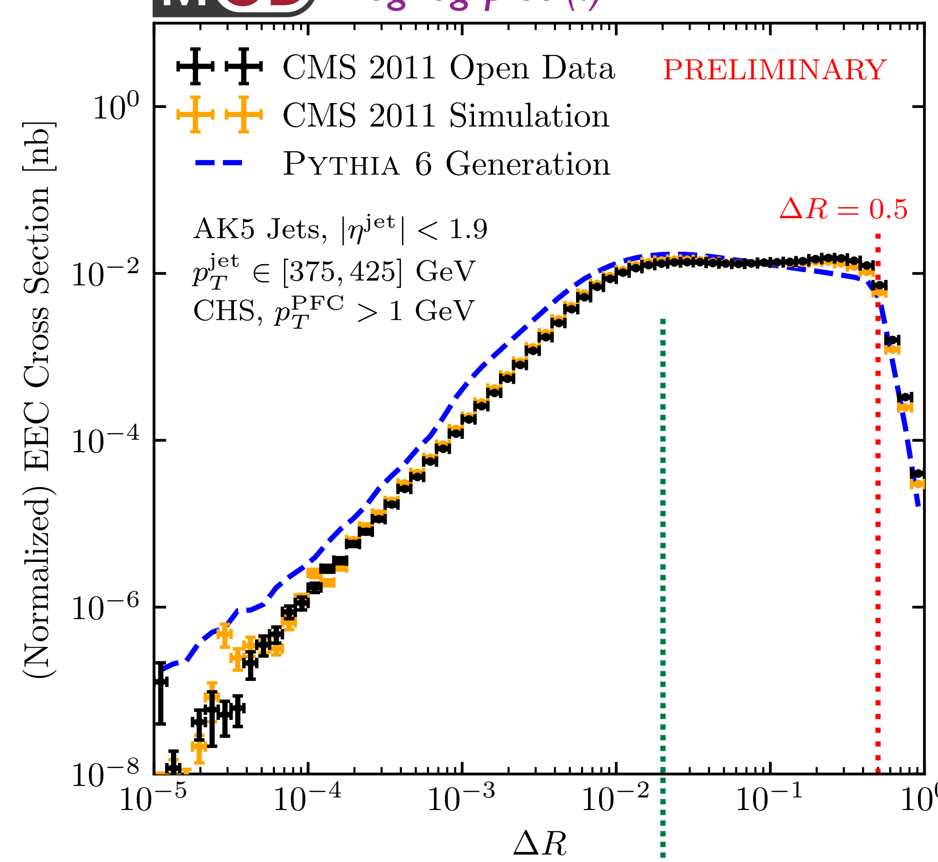
QCD Phase Transition in Jets?

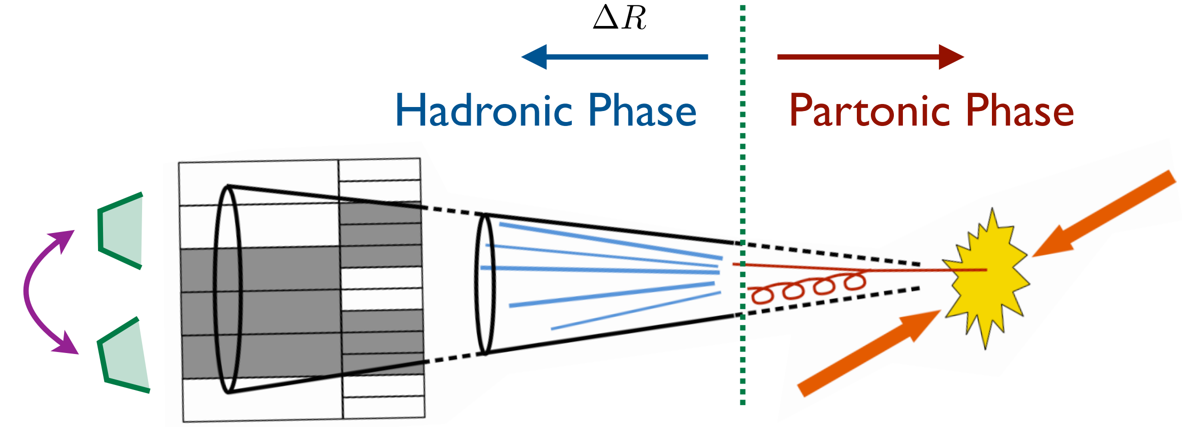
MOD *Linear-linear plot*



Are we learning something about small angle limit of QCD?

MOD *Log-log plot (!)*

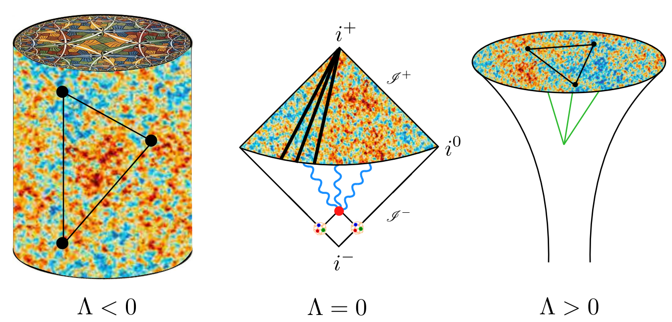


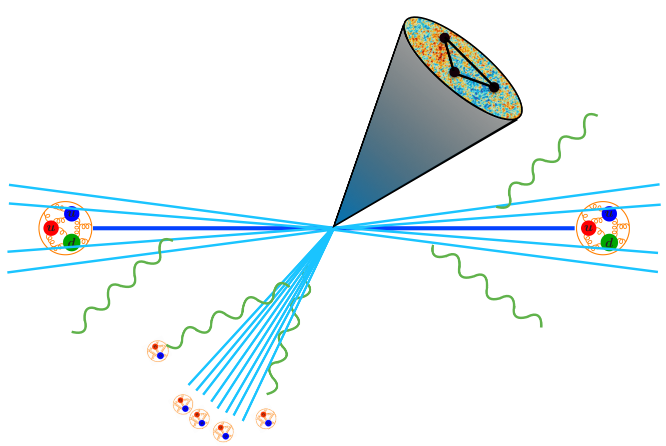


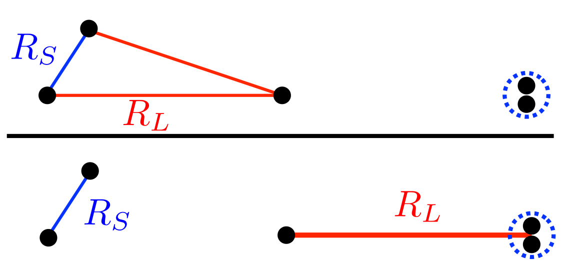
First Jet EEC Plot from the LHC (!)

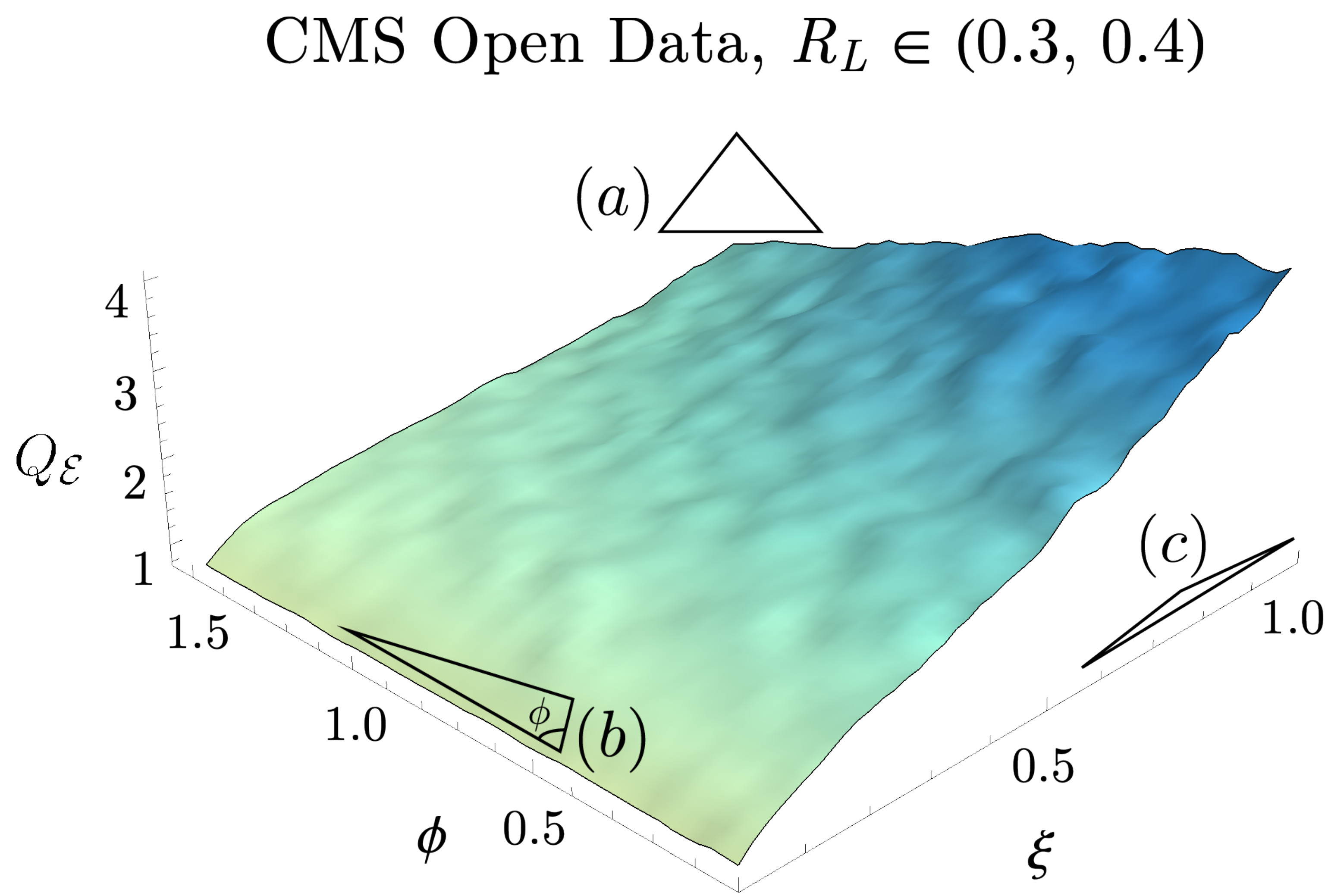
[Komiske, Moult, JDT, Zhu, arXiv 2022; see talks by Moult, BOOST 2019, BOOST 2020]

"Non-Gaussianities" in Collider Energy Flux

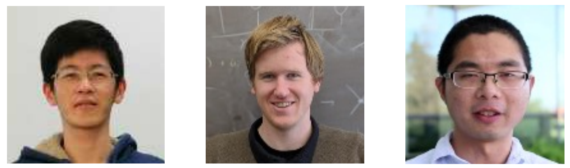








[Chen, Moult, JDT, Zhu, JHEP 2022]



)

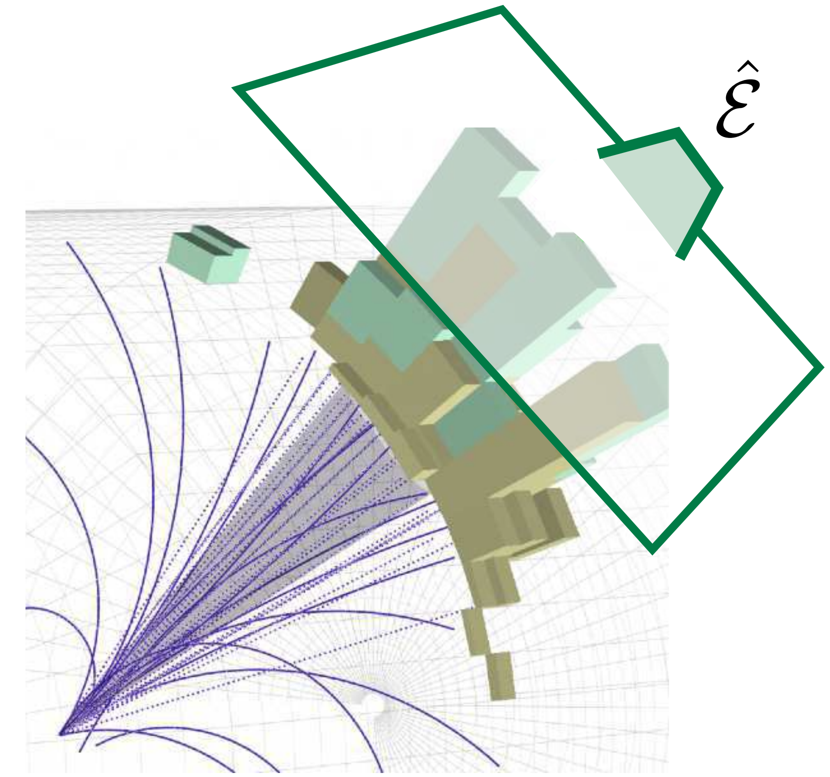
Jets as Weighted Point Clouds

- Energy-Weighted Directions

$$\vec{p} = \{E, \hat{n}_x, \hat{n}_y, \hat{n}_z\}$$

Energy — Direction

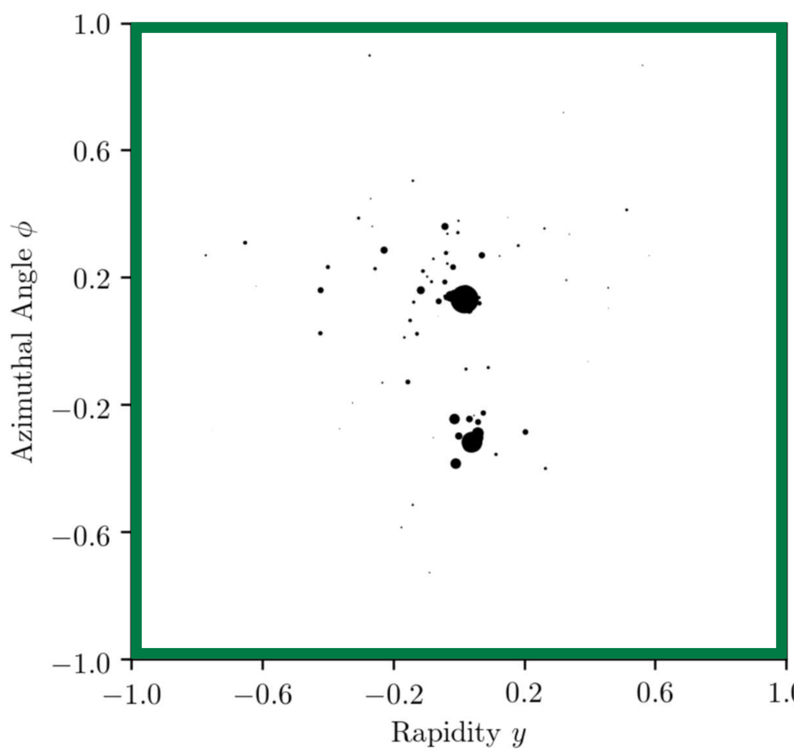
(suppressing "unsafe" charge/flavor information)

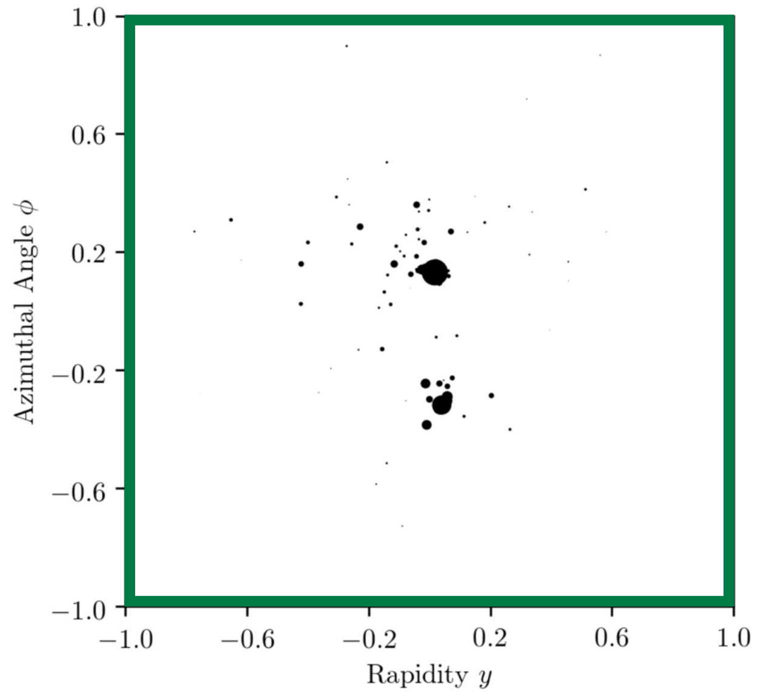


- Equivalently: Energy Density

$$\rho(\hat{n}) = \sum_{i \in \mathcal{J}} E_i \, \delta^{(2)}(\hat{n} - \hat{n}_i)$$

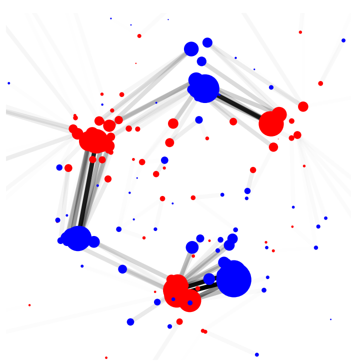
Energy — Direction





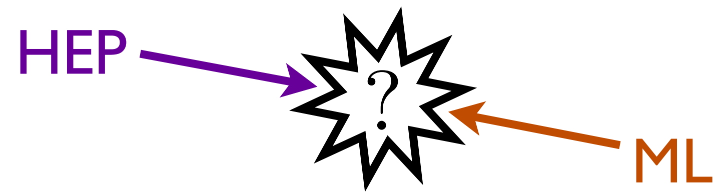
When restricted to IRC safe information, jets/events are naturally represented as energy densities

Going with the (Energy) Flow



The Energy Mover’s Distance

Revealing a Hidden Geometry



If you ask your local computational geometry expert how to process densities…

The Earth Mover's Distance

Optimal Transport:

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Minimum "work" (stuff x distance) to make one distribution look like another distribution

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

The Earth Mover’s Distance

Optimal Transport: Minimum “work” (stuff x distance) to make one distribution look like another distribution

[Peleg, Werman, Rom, IEEE 1989;
Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000;
Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

Additive Weight:

Ground Distance:

$\Rightarrow$ Distance Between Distributions

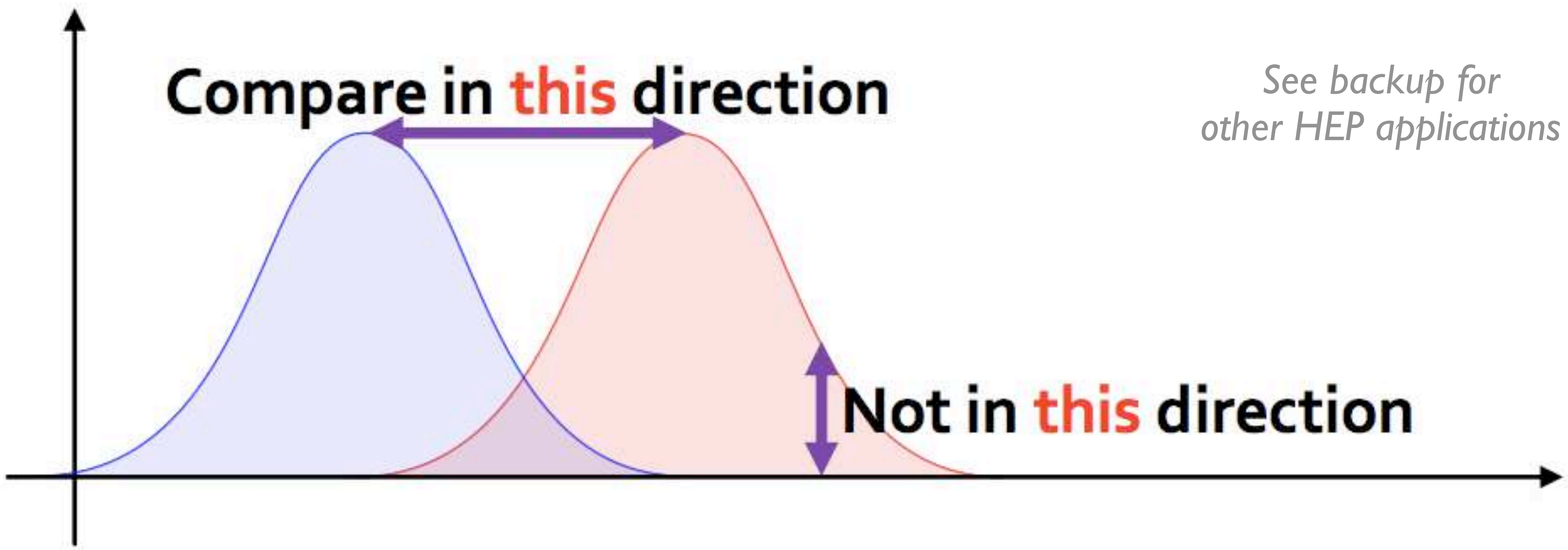
[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

The Earth Mover's Distance

Optimal Transport: Minimum "work" (stuff x distance) to make one distribution look like another distribution

[Peleg, Werman, Rom, IEEE 1989; Rubner, Tomasi, Guibas, ICCV 1998, ICJV 2000; Pele, Werman, ECCV 2008; Pele Taskar, GSI 2013]

"Horizontal" comparison (EMD) yields better dynamic range than "vertical" comparison (e.g. KL)



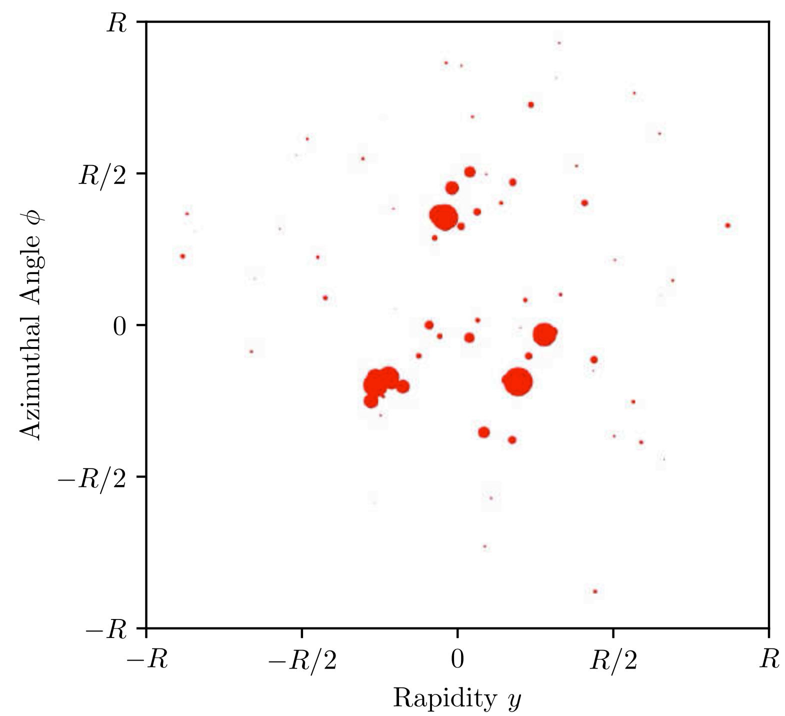
See backup for other HEP applications

[figure from Kun, Math ∩ Programming]

[h/t Niles-Weed, ML4Jets 2020; Monge, 1781; Kantorovich, 1939; Vaseršteĭn, 1969; Wikipedia]

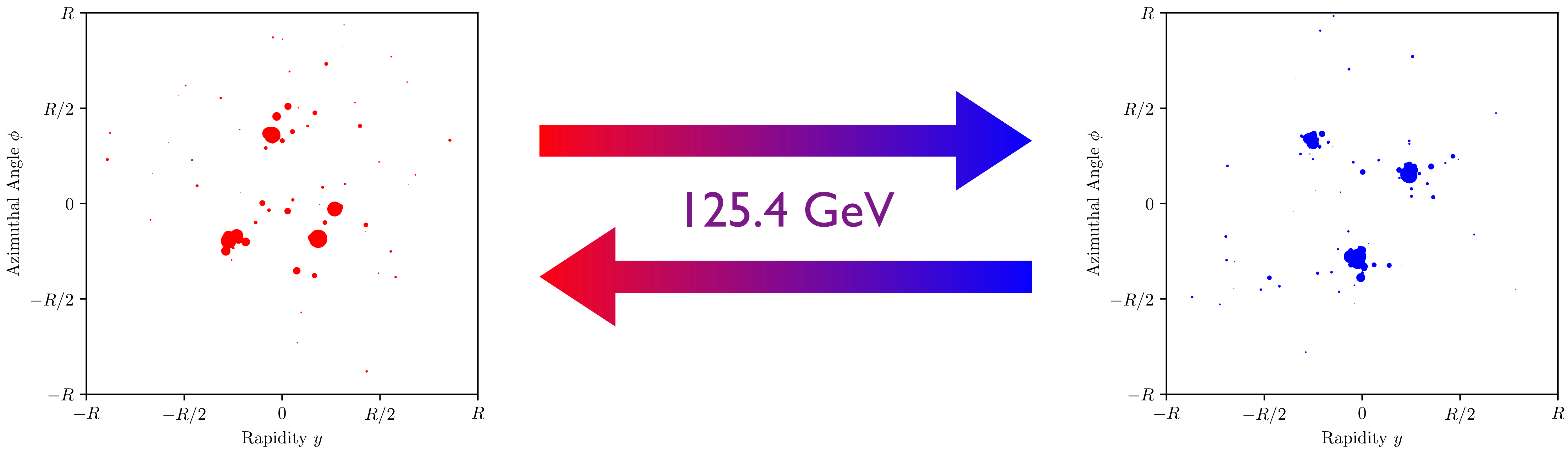
Similarity of Two Energy Flows

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$



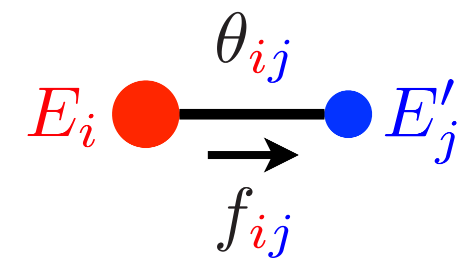
Optimal Transport:

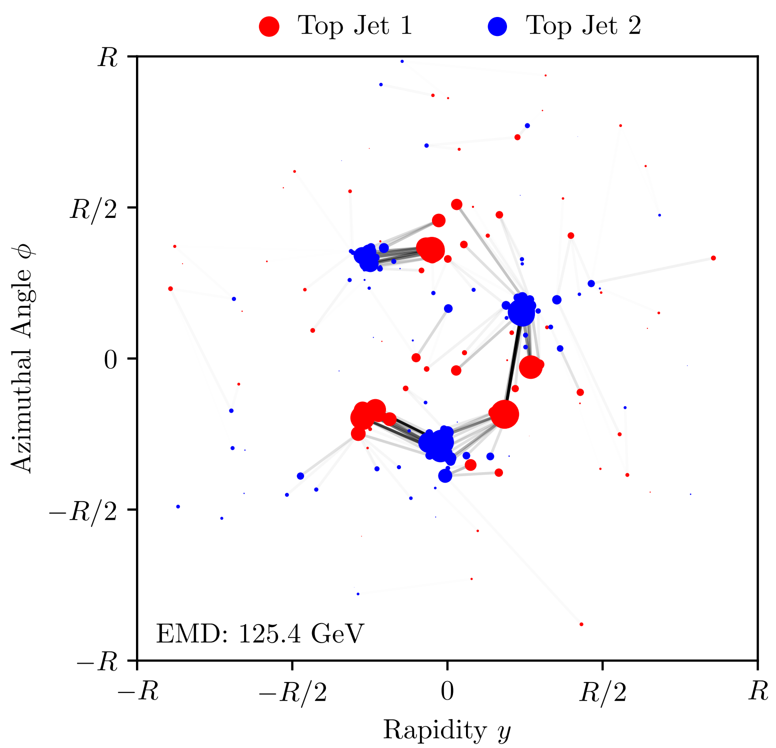
Earth Mover's Distance
a.k.a. 1-Wasserstein metric



[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network]

The Energy Mover's Distance





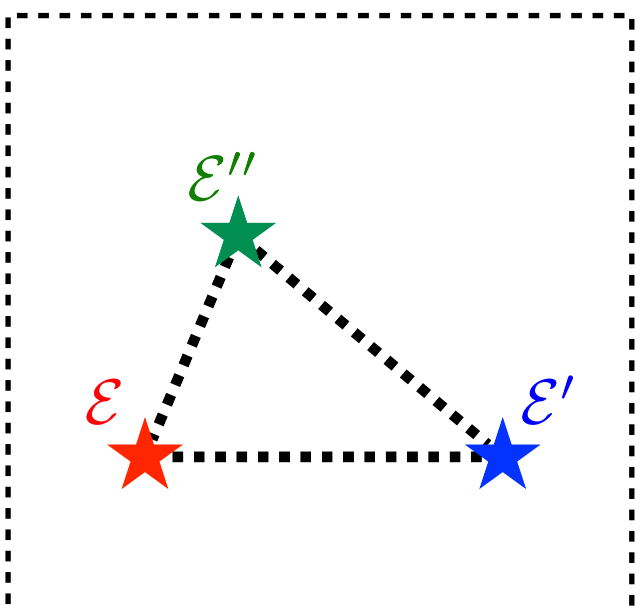
Optimal transport between energy flows…

$$\mathrm{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_i E_i - \sum_j E'_j \right|$$

in GeV

Cost to move energy

Cost to create energy



…defines a metric on the space of events

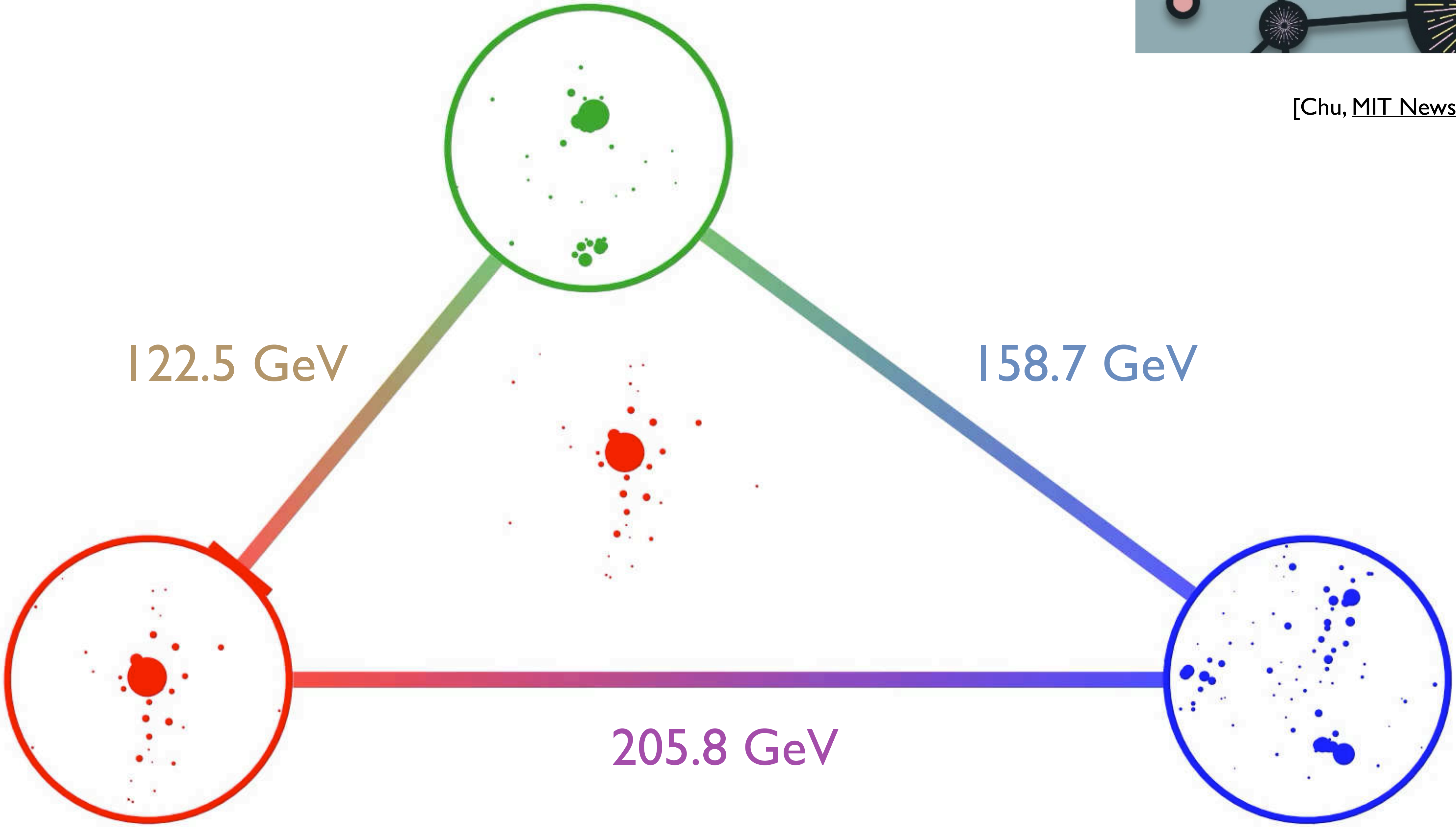
$$0 \leq \mathrm{EMD}(\mathcal{E}, \mathcal{E}') \leq \mathrm{EMD}(\mathcal{E}, \mathcal{E}'') + \mathrm{EMD}(\mathcal{E}', \mathcal{E}'')$$

(assuming R ≥ θ_{max}/2, i.e. R ≥ jet radius for conical jets)

[Komiske, Metodiev, JDT, PRL 2019;
see also Pele, Werman, ECCV 2008; Pele, Taskar, GSI 2013;
[see flavored variant in Crispim Romão, Castro, Milhano, Pedro, Vale, EPJC 2021]
[see linearized and unbalanced transport in Cai, Cheng, Craig, Craig, PRD 2020, arXiv 2021]

Similarity of Three Energy Flows

[Chu, MIT News July 2019]



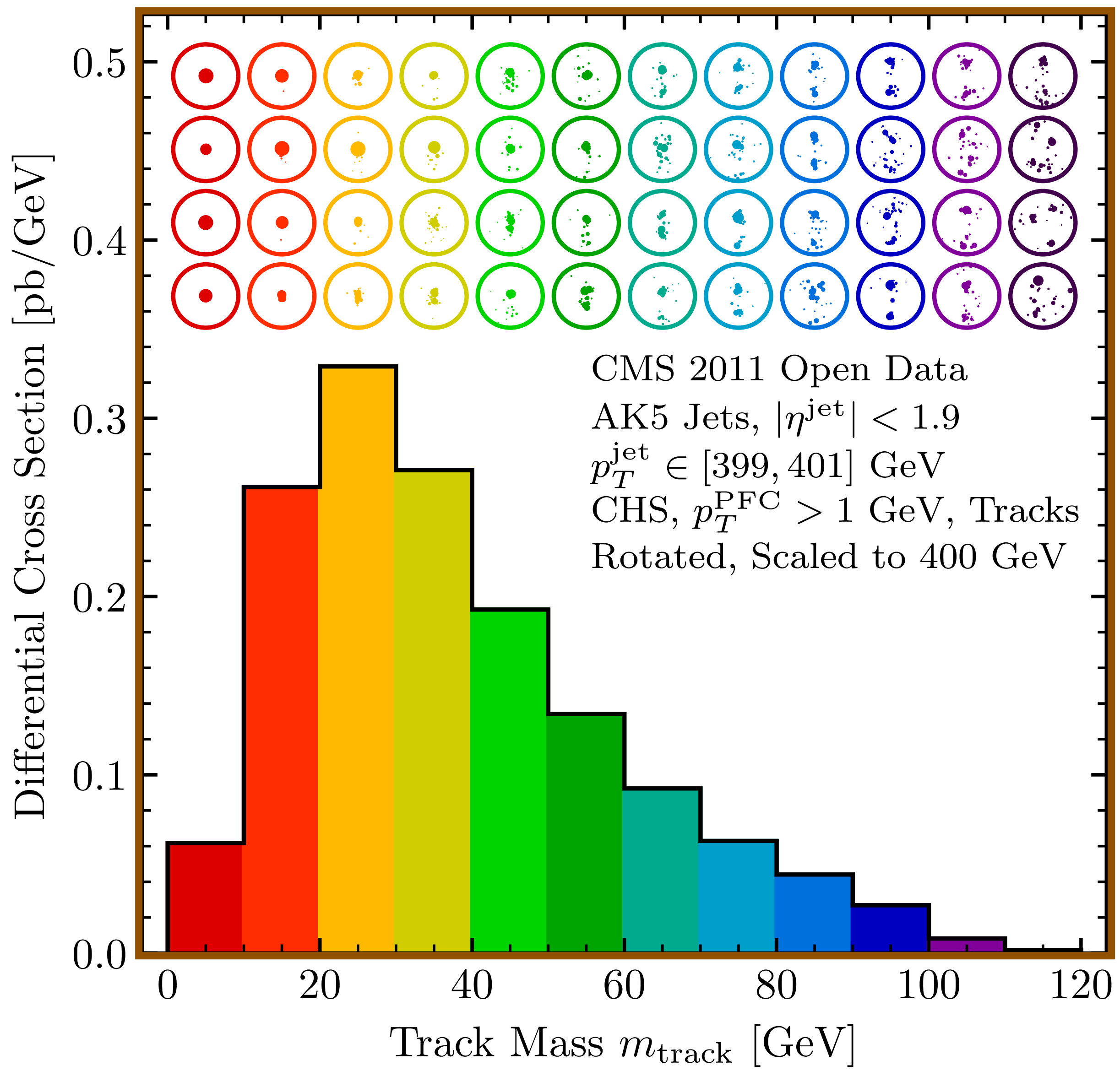
[Komiske, Metodiev, JDT, PRL 2019; code at Komiske, Metodiev, JDT, energyflow.network;
see alternative graph network approach in Mullin, Pacey, Parker, White, Williams, JHEP 2021]

“Cool graphics, Jesse, but this choice of metric seems a bit arbitrary…”

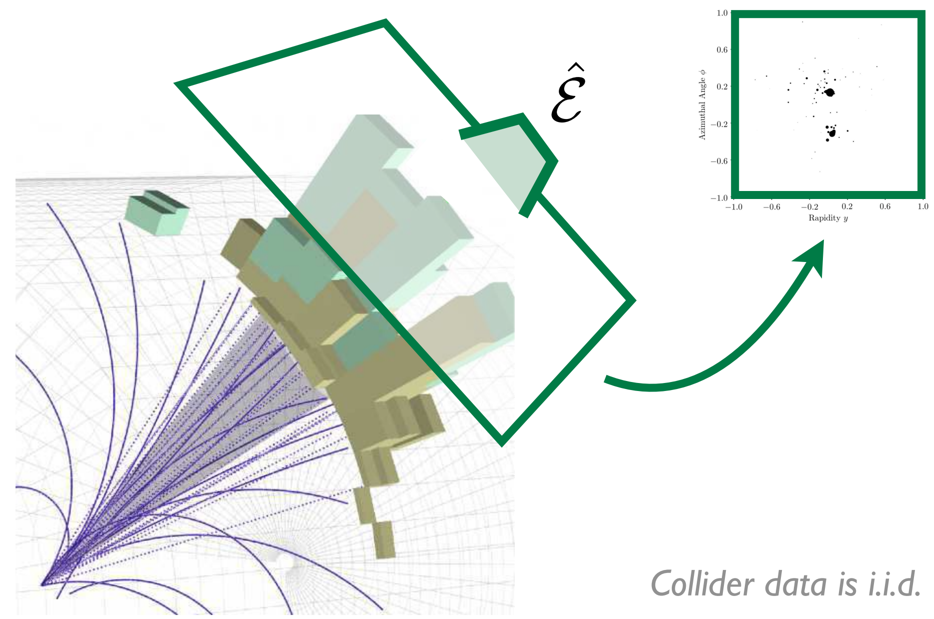
My answer c. 2020:	*Yes, this is a choice, but it is a very nice choice, trust me*
My answer c. 2023:	*Yes, this is a choice, but it is the only “faithful” choice*

[Ba, Dogra, Gambhir, Tasissa, JDT, in progress]

The Forest and the Trees



A Histogram of Observables

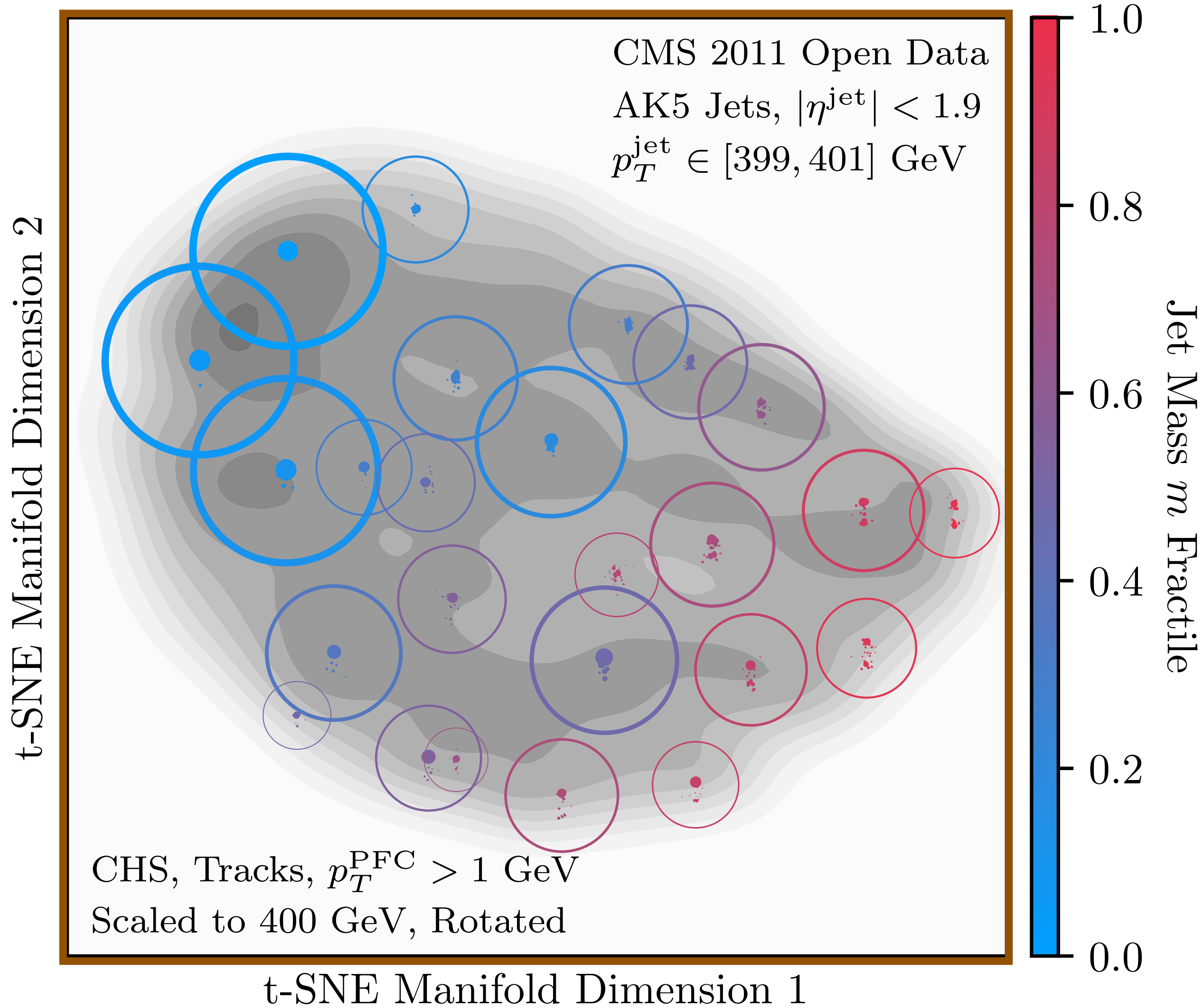


Collider data is i.i.d.

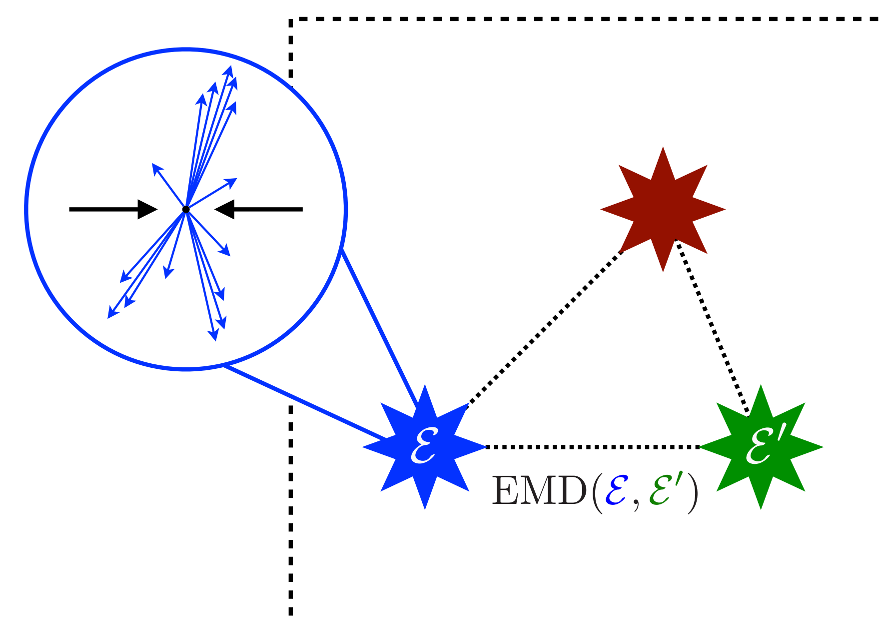
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using CMS Open Data]




Building the Forest from the Trees



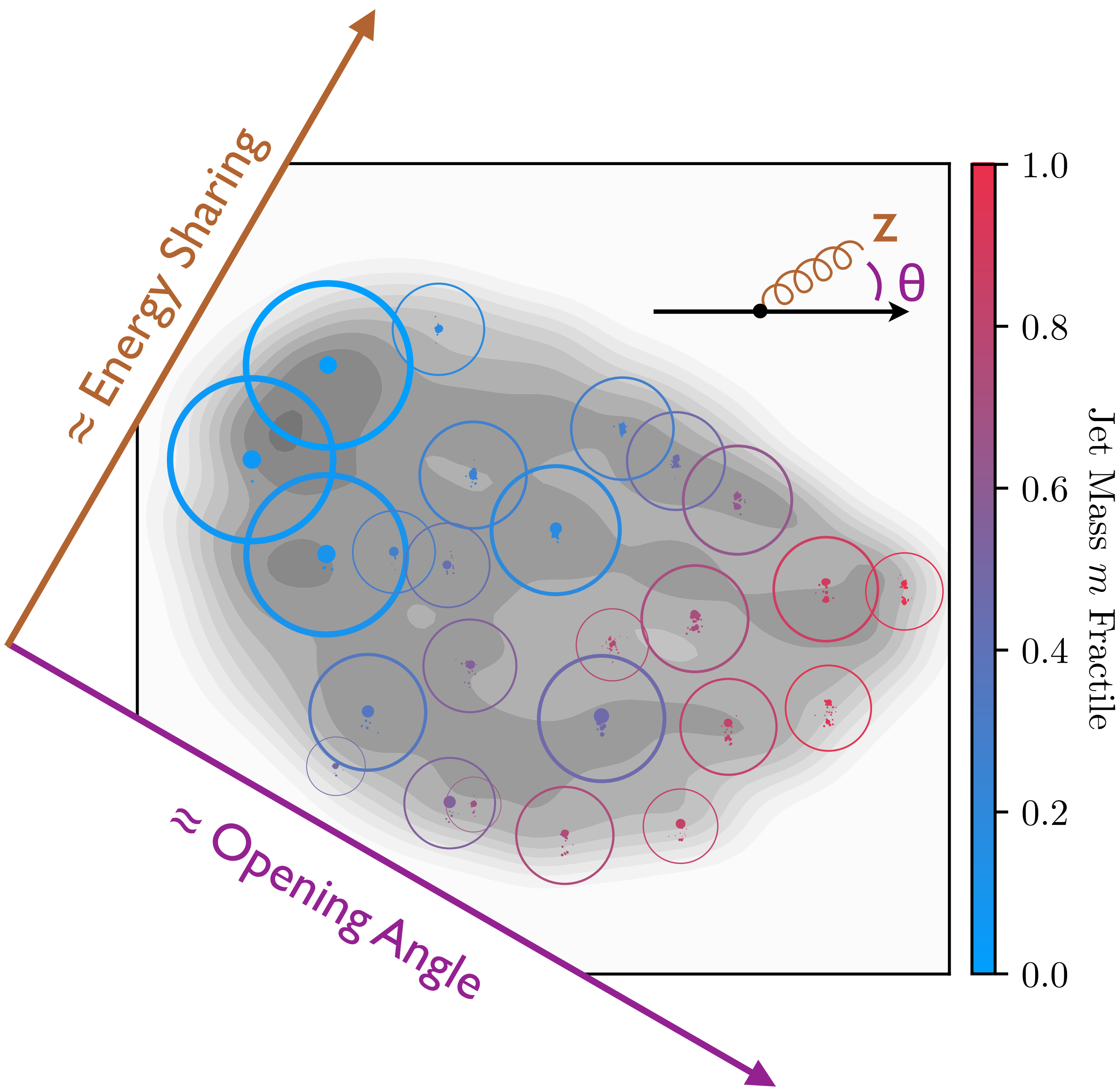
The Space of Energy Flows



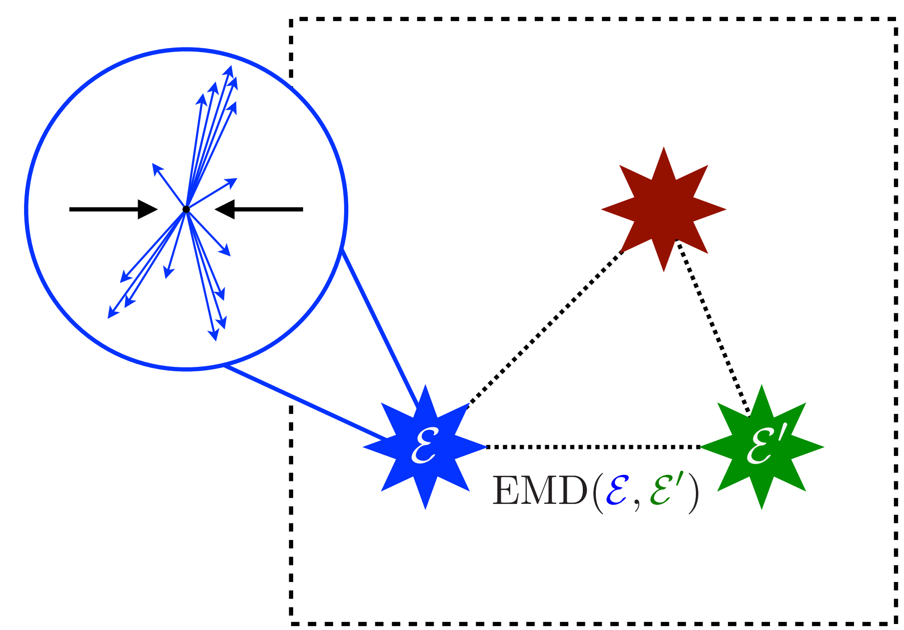
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using van der Maaten, Hinton, JMLR 2008; using CMS Open Data]

Building the Forest from the Trees

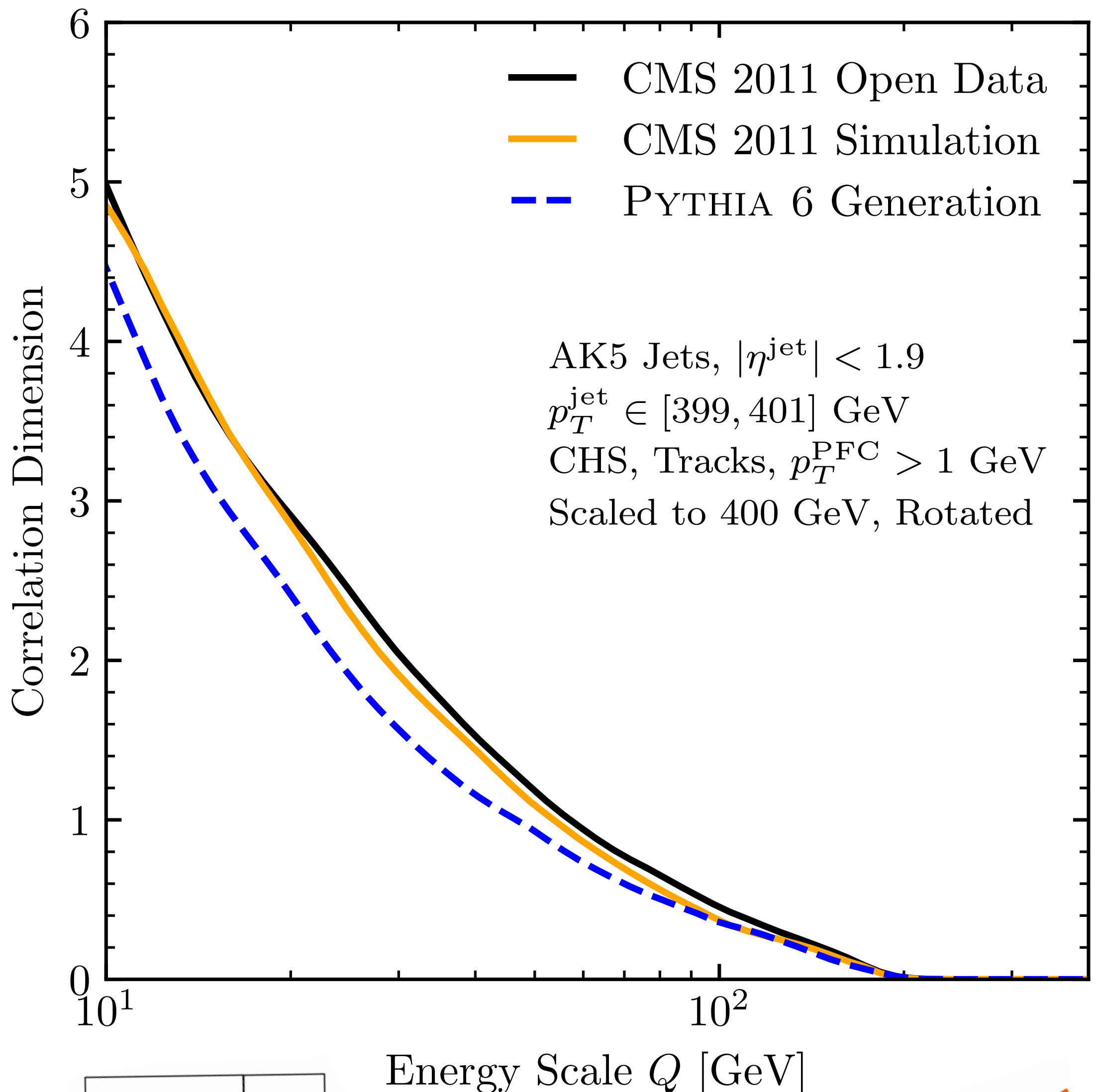


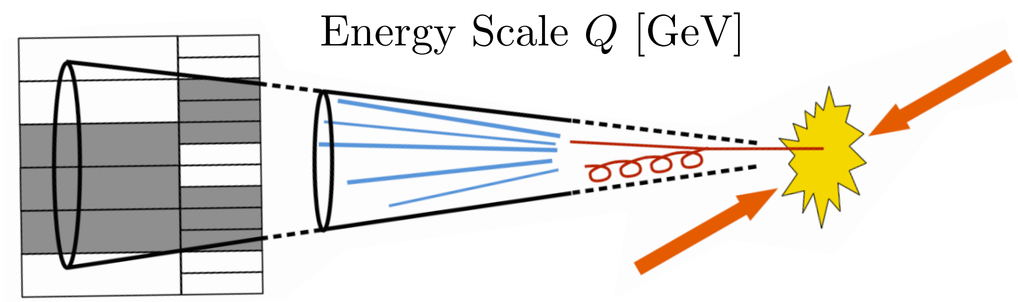
The Space of Energy Flows



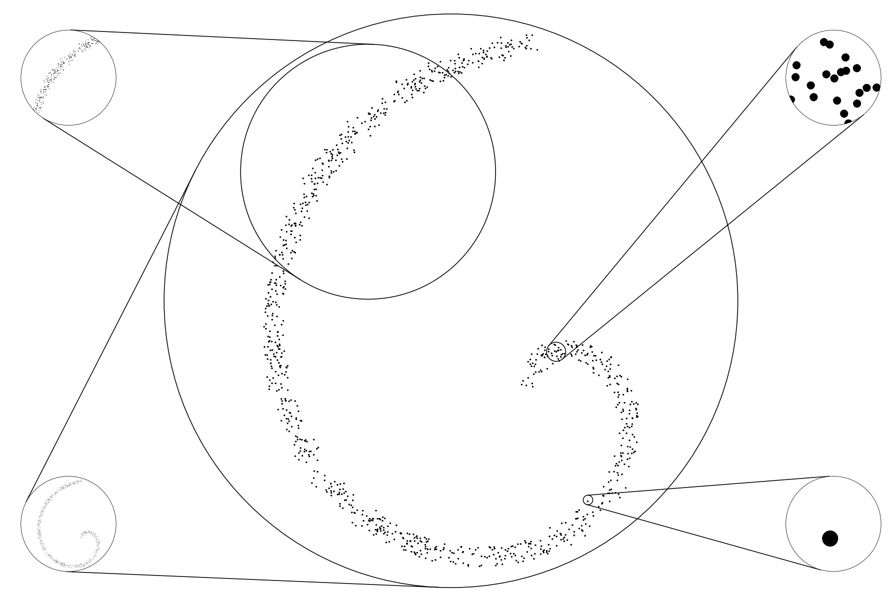
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using van der Maaten, Hinton, JMLR 2008; using CMS Open Data]

A Super-Fractal Forest made from Trees



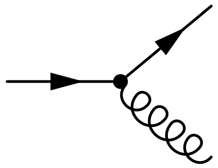


Dimension of Space of Energy Flows

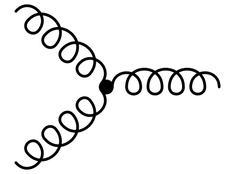


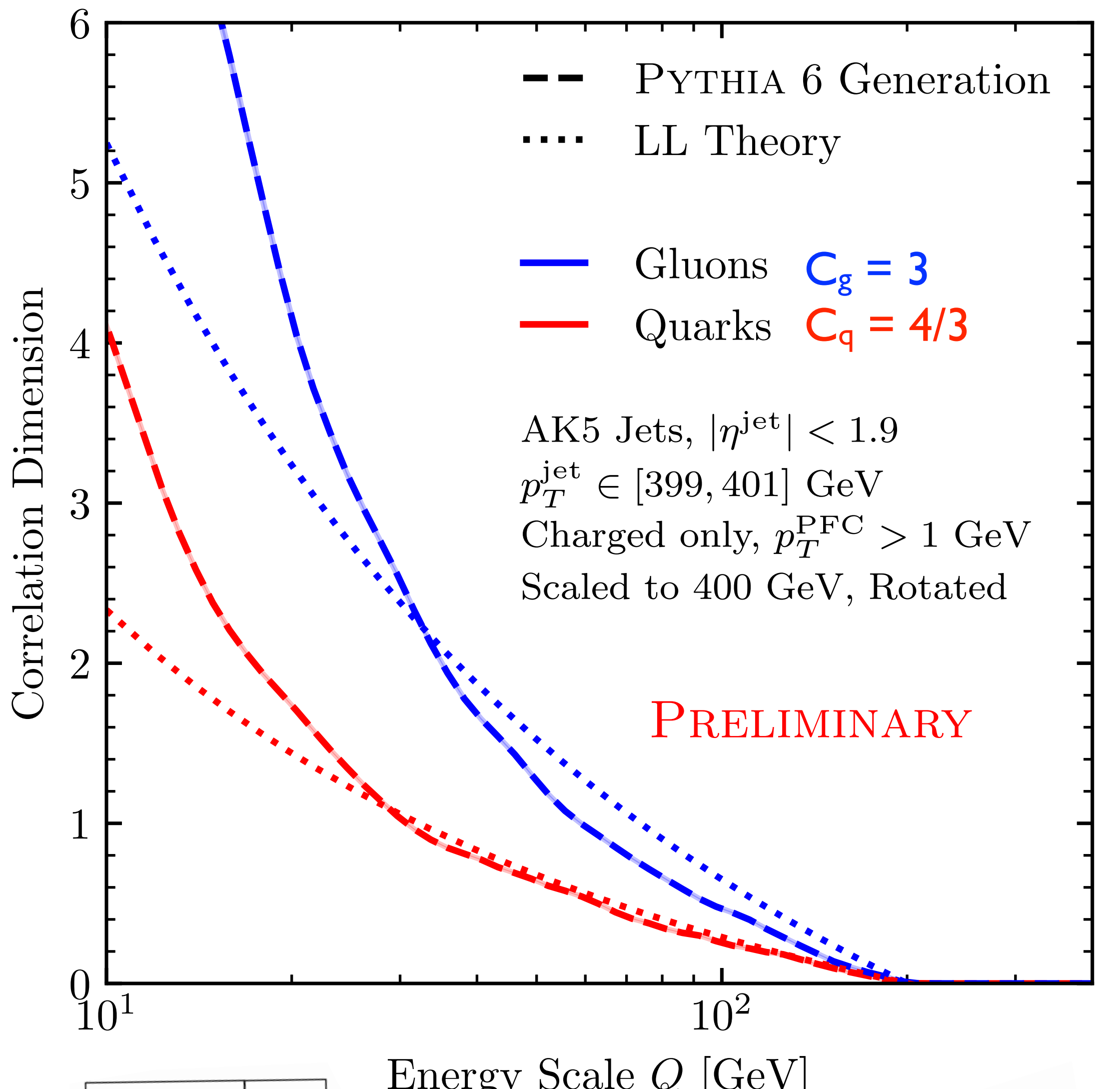
[Komiske, Mastandrea, Metodiev, Naik, JDT, PRD 2020;
using Grassberger, Procaccia, PRL 1983; Kégl, NIPS 2002; using CMS Open Data]

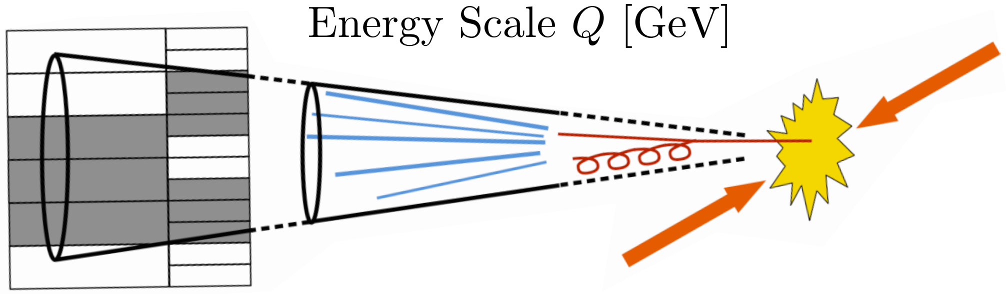
A Calculable Super-Fractal Forest of Trees



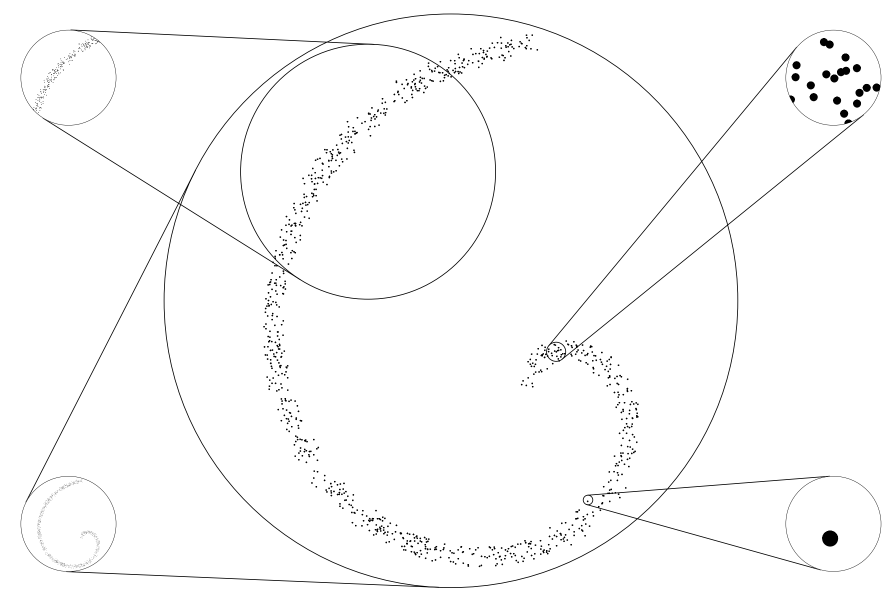
$$\dim_i(Q) \simeq -\frac{8\alpha_s}{\pi} C_i \ln \frac{Q}{p_T}$$





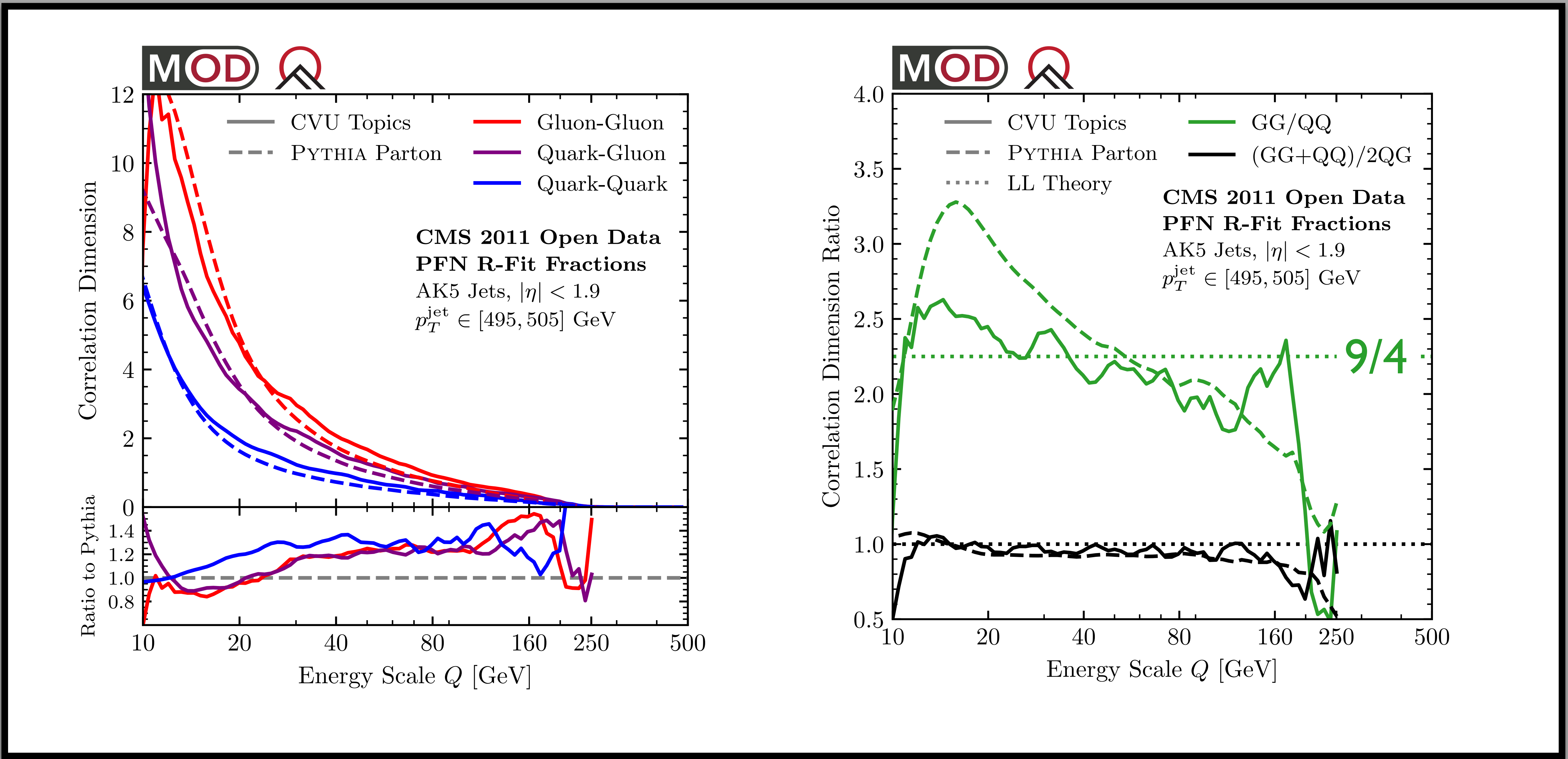


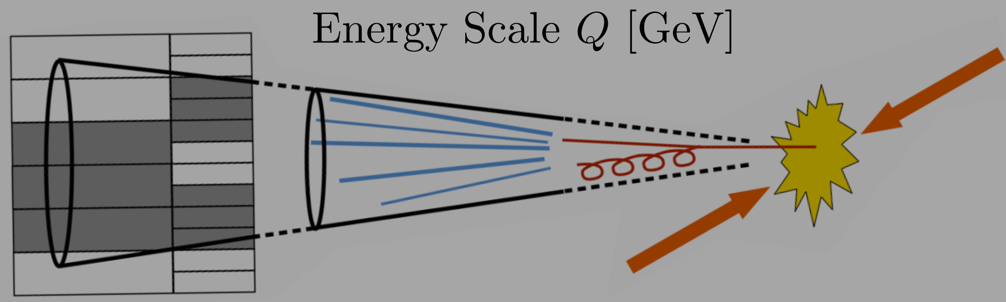
QCD Calculation of the Dimension of Space of Energy Flows



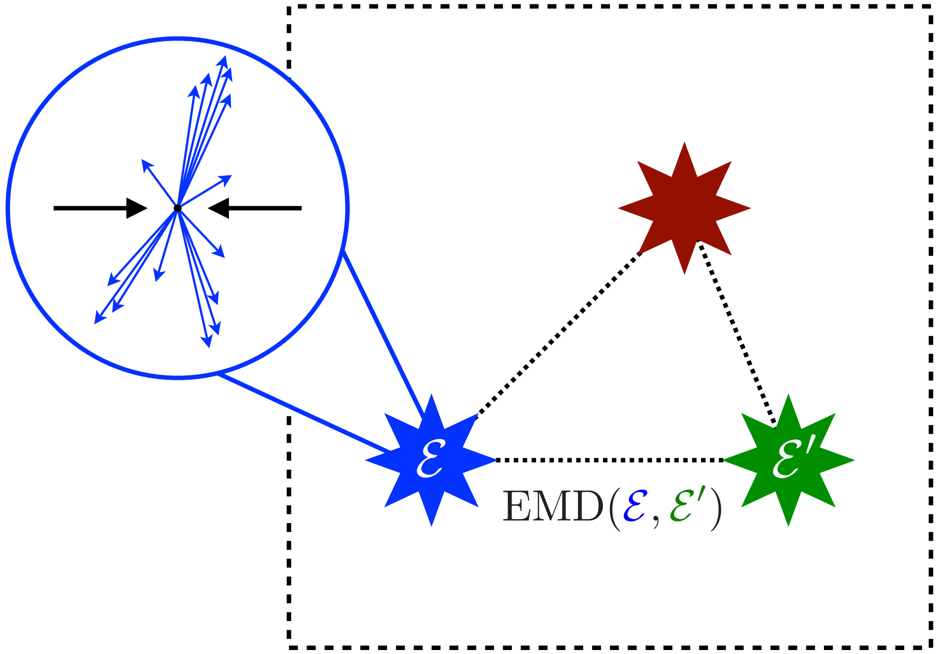
[Komiske, Kryhin, JDT, PRD 2022;
using Metodiev, JDT, PRL 2018; Komiske, Metodiev, JDT, JHEP 2018;
Andreassen, Komiske, Metodiev, Nachman, JDT, PRL 2020; + Suresh, ICLR SimDL 2021]

A Calculable Super-Fractal Forest of Trees





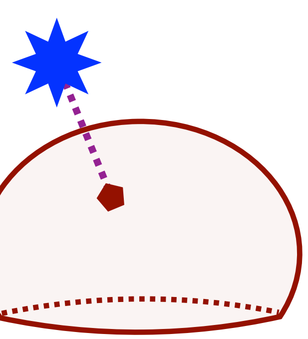
[Komiske, Kryhin, JDT, PRD 2022;
using Metodiev, JDT, PRL 2018; Komiske, Metodiev, JDT, JHEP 2018;
Andreassen, Komiske, Metodiev, Nachman, JDT, PRL 2020; + Suresh, ICLR SimDL 2021]



Viewed through the data science lens, the EMD unlocks a suite of geometric analysis strategies

Going with the (Energy) Flow

The Energy Mover’s Distance

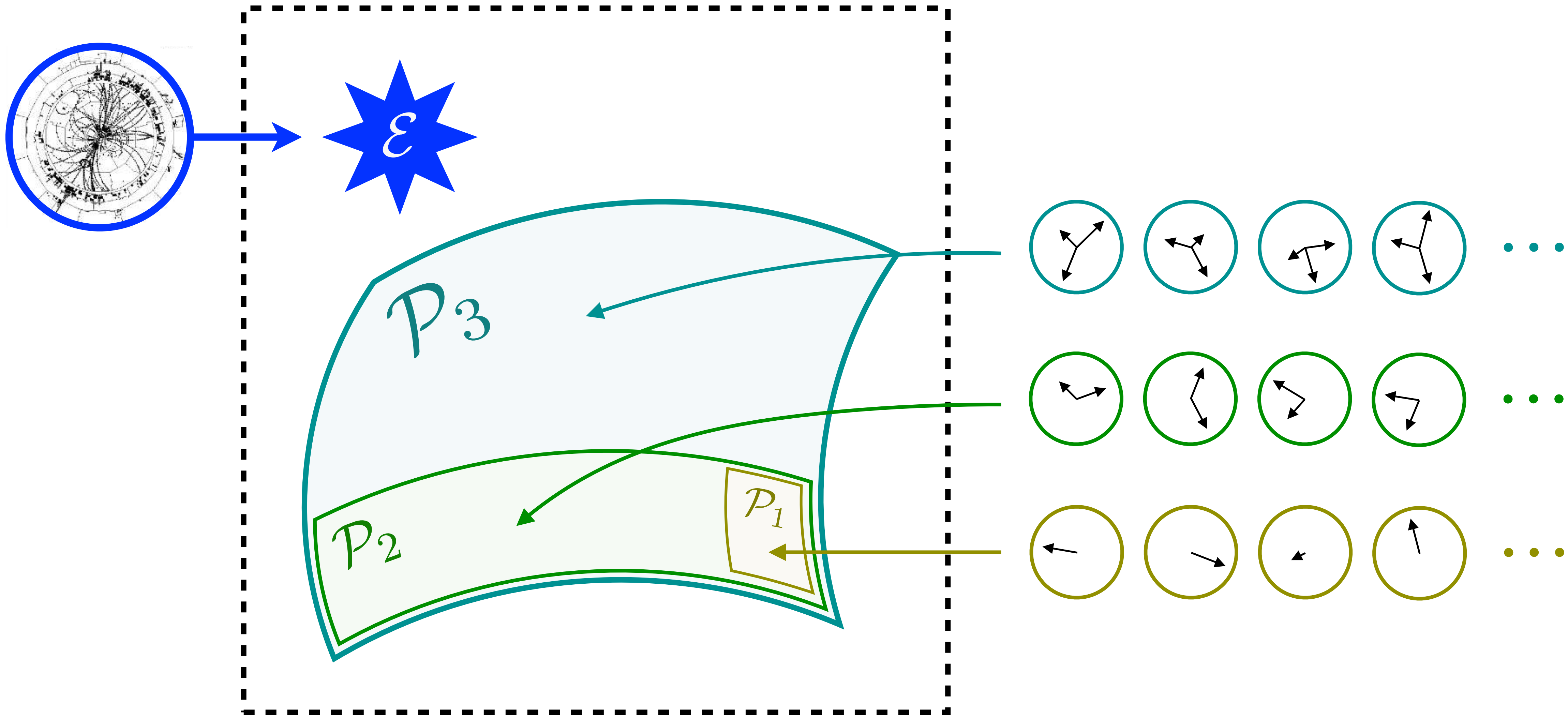


Revealing a Hidden Geometry

Given a metric space, the first geometric object you might think to construct is…

Introducing N-particle Manifolds

$\mathcal{P}_N$ = set of all N-particle configurations

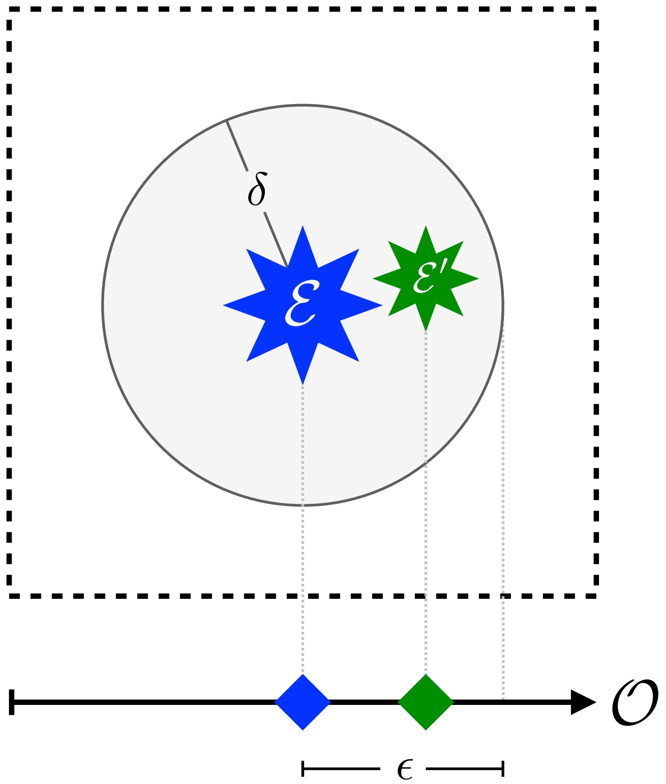


$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \ldots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

[see related discussion in Larkoski, Melia, PRD 2020]

Introducing N-particle Manifolds

$\mathcal{P}_N$ = set of all N-particle configurations



Infrared & Collinear Safety

≈ calculable in perturbative quantum field theory

*is**

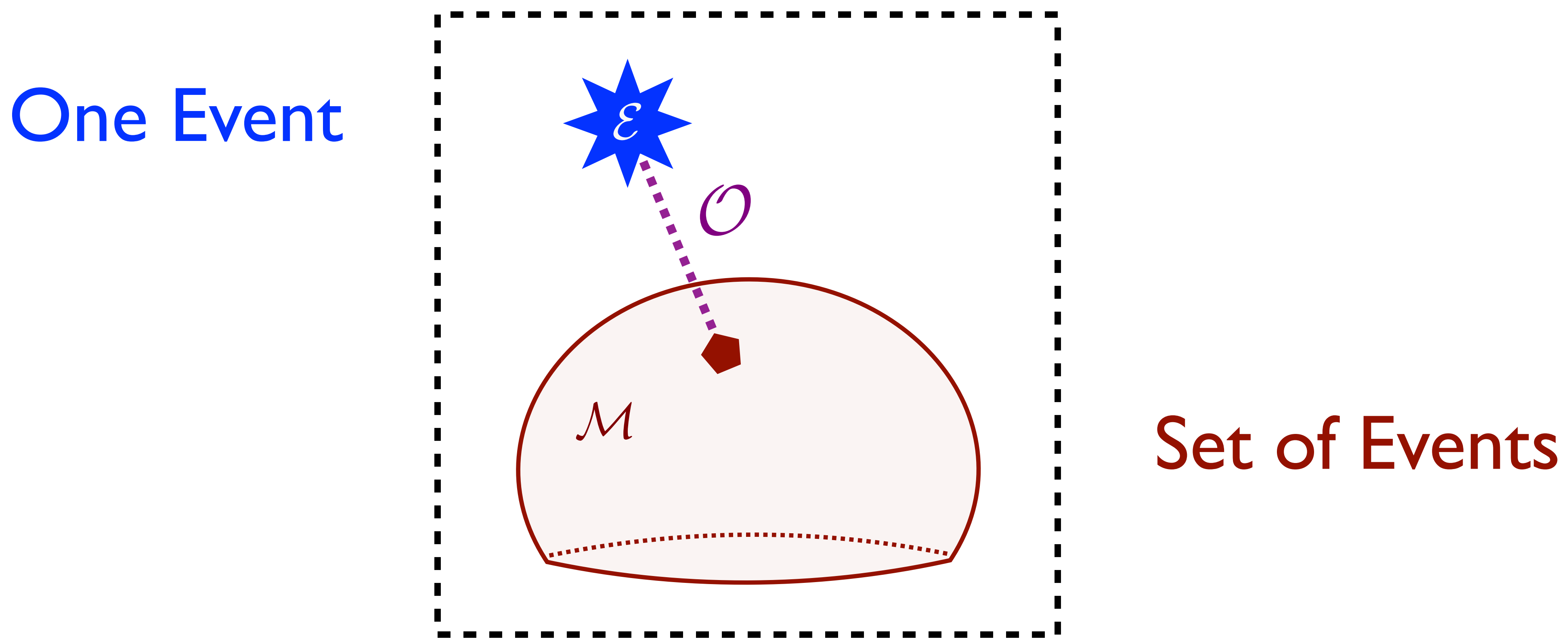
Continuity in EMD Space

[Komiske, Metodiev, JDT, JHEP 2020]
[Sterman, Weinberg, PRL 1977; Sterman, PRD 1979]
[see also Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

$\mathcal{P}_N \supset \mathcal{P}_{N-1} \supset \ldots \supset \mathcal{P}_2 \supset \mathcal{P}_1$ by soft/collinear limits

[see related discussion in Larkoski, Melia, PRD 2020]

Manifolds for Observables



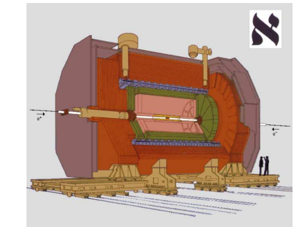
Distance of Closest Approach ⇒ Observable

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

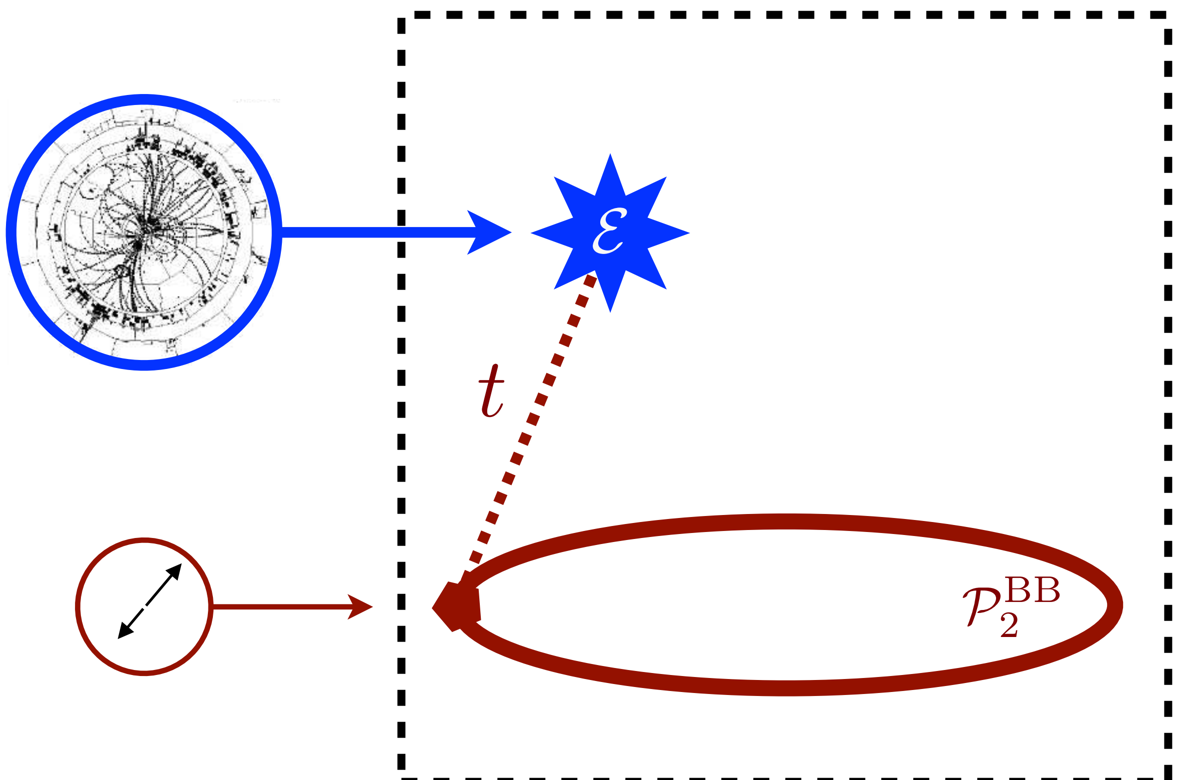
[Komiske, Metodiev, JDT, JHEP 2020]

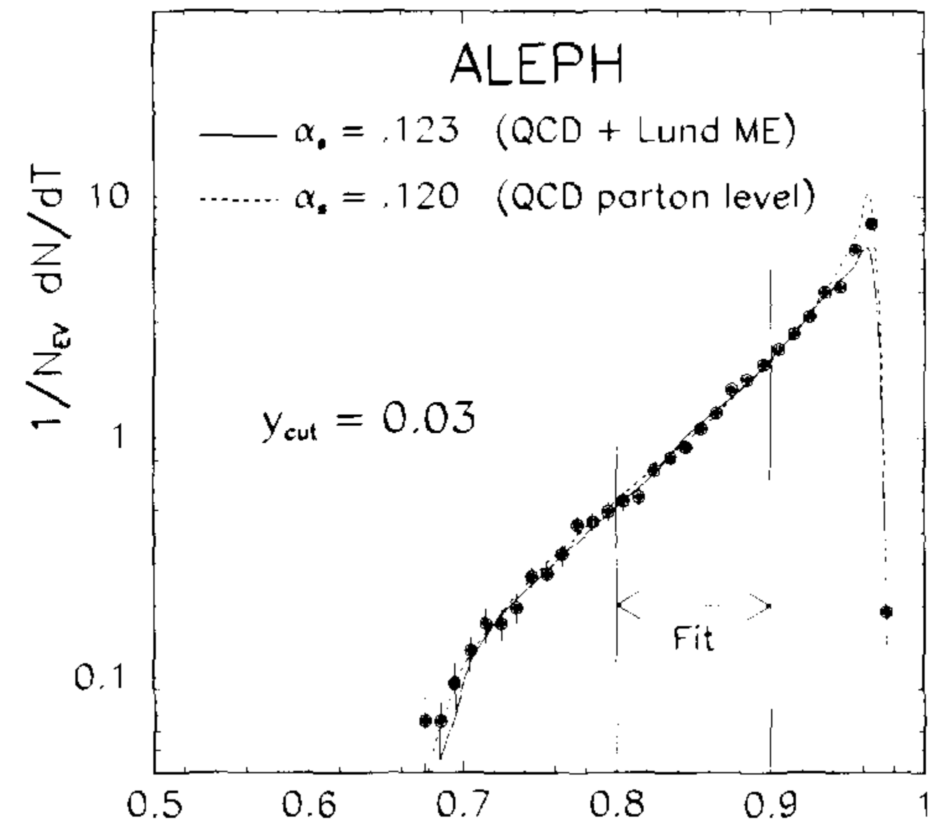
E.g. Thrust

How dijet-like is an event?



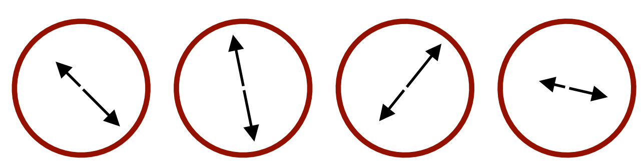
$$t(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{P}_2^{\mathrm{BB}}} \mathrm{EMD}_2(\mathcal{E}, \mathcal{E}')$$





$$1 - \frac{t}{2E_{\mathrm{CM}}}$$

All Back-to-Back Two Particle Configurations

$\mathcal{P}_2^{\mathrm{BB}} = \{$  $\cdots \}$

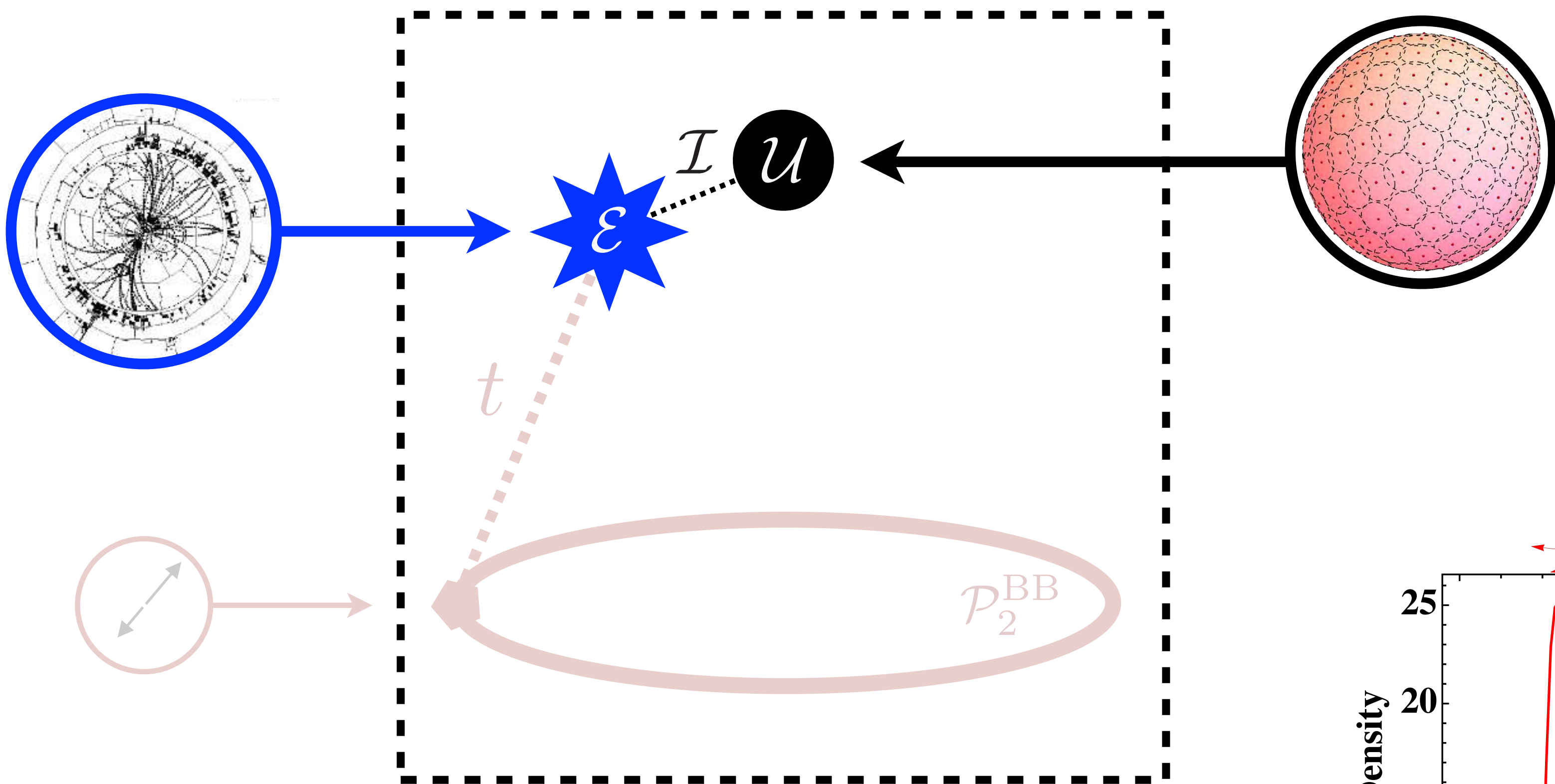
cf. $T(\mathcal{E}) = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_j |\vec{p}_j|}$

(using β=2 EMD variant)

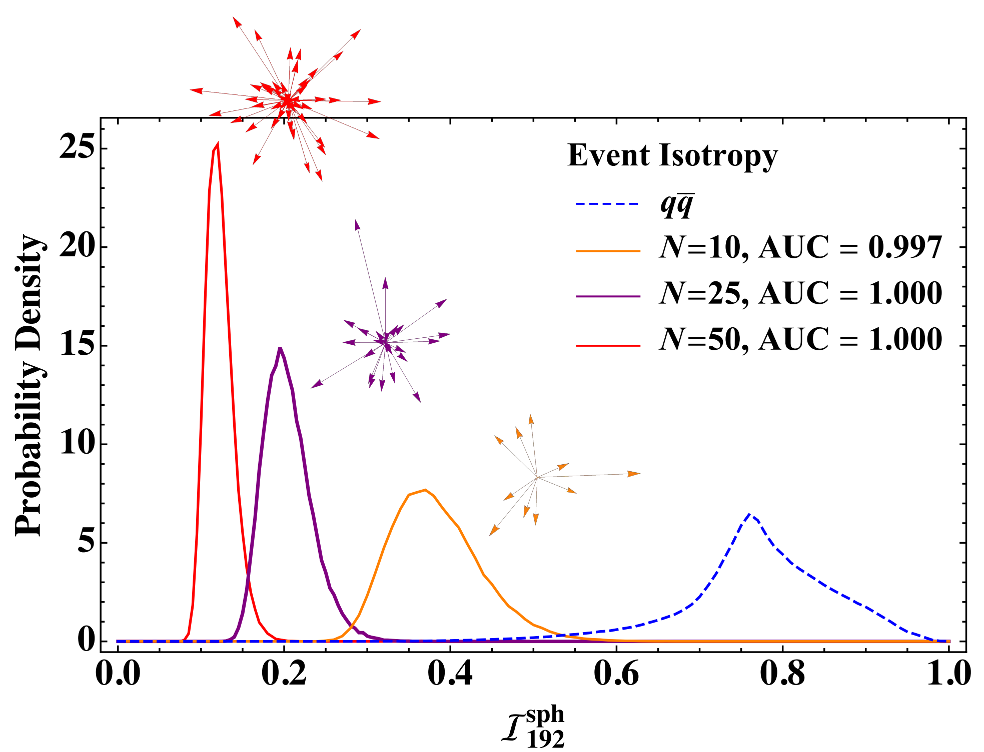
[Komiske, Metodiev, JDT, JHEP 2020]
[Brandt, Peyrou, Sosnowski, Wroblewski, PL 1964; Farhi, PRL 1977; ALEPH, PLB 1991]

New! Event Isotropy

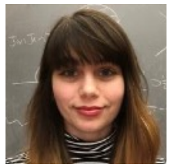
How isotropic is an event?



$$\mathcal{I}(\mathcal{E}) = \text{EMD}(\mathcal{E}, \mathcal{U})$$

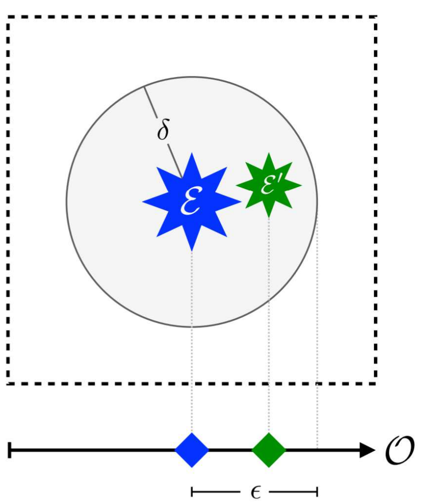


[Cesarotti, JDT, JHEP 2020;
see also Cesarotti, Reece, Strassler, JHEP 2021]

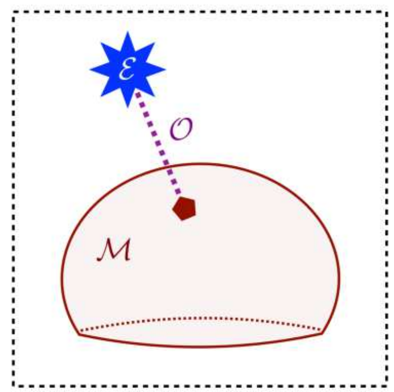


Six Decades of Collider Physics Translated into a New Geometric Language!

IRC Safety is smoothness in the space of events

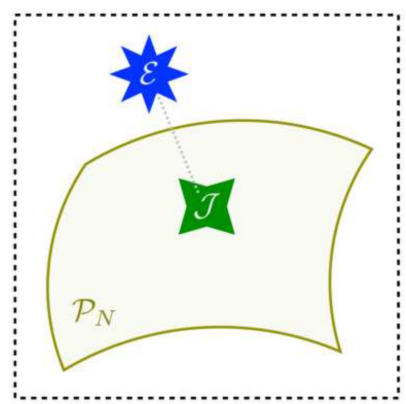


Event shapes are distances from events to manifolds.



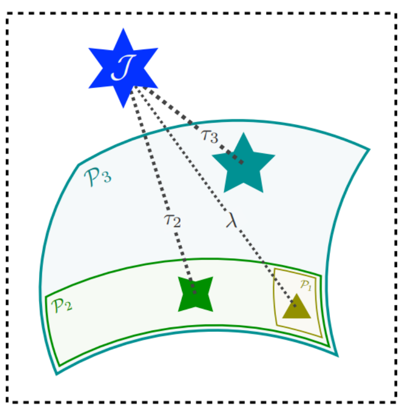
$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$

Jets are projections to few-particle manifolds.



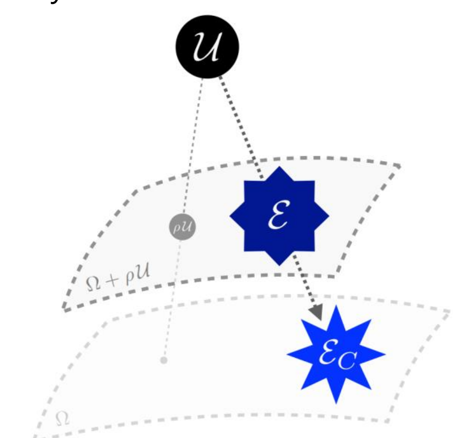
$\mathcal{J} = \operatorname{argmin}_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta,R}(\mathcal{E}, \mathcal{E}')$

Substructure resolves emissions within the jet.



$\tau(\mathcal{J}) = \min_{\mathcal{E}' \in \mathcal{P}_N} \text{EMD}_{\beta}(\mathcal{J}, \mathcal{E}').$

Pileup mitigation moves away from uniform radiation.



$\mathcal{E}_C = \arg\min_{\mathcal{E}'} \text{EMD}(\mathcal{E}, \mathcal{E}' + \rho\, \mathcal{U})\,.$

Taming infinities — Event Shapes — Jet Algorithms — Jet Substructure — Pileup

1960 → 2020

1962-1964 Infrared Safety
[Kinoshita, JMP 1962]
[Lee, Nauenberg, PR 1964]

1977 Thrust, Sphericity
[Farhi, PRL 1977]
[Georgi, Machacek, PRL 1977]

1993 k_T jet clustering
[Ellis, Soper, PRD 1993]
[Catani, Dokshitzer, Seymour, Webber, NPB 1993]

1997-1998 C/A jet clustering
[Wobisch, Wengler, 1998]
[Doskhitzer, Leder, Moretti,Webber, JHEP 1997]

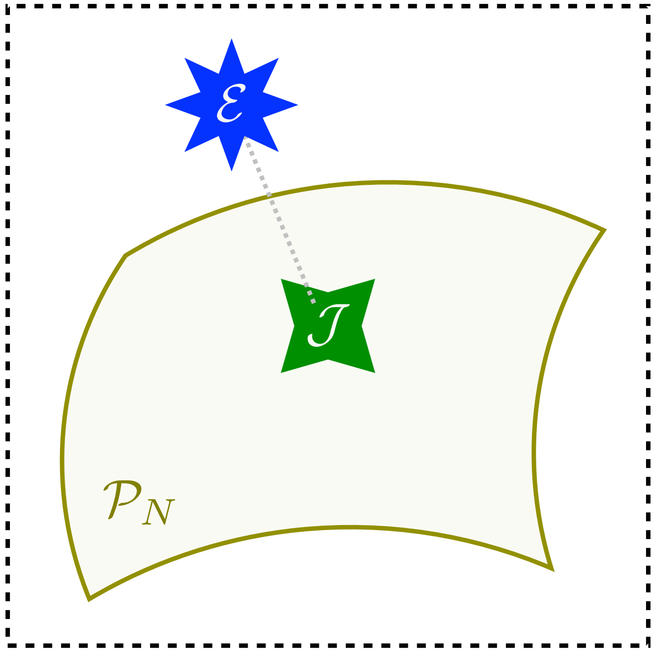
2010-2015 N-(sub)jettiness, XCone
[Stewart, Tackmann, Waalewijn, PRL 2010]
[Thaler, Van Tilburg, JHEP 2011]
[Stewart, Tackmann, Thaler, Vermilion, Wilkason, JHEP 2015]

2014-2019 Constituent Subtraction
[Berta, Spousta, Miller, Leitner, JHEP 2014]
[Berta, Masetti, Miller, Spousta, JHEP 2019]

And many more!

[Komiske, Metodiev, JDT, JHEP 2020; timeline from Metodiev]

More Fun with N-particle Manifolds



N-jettiness

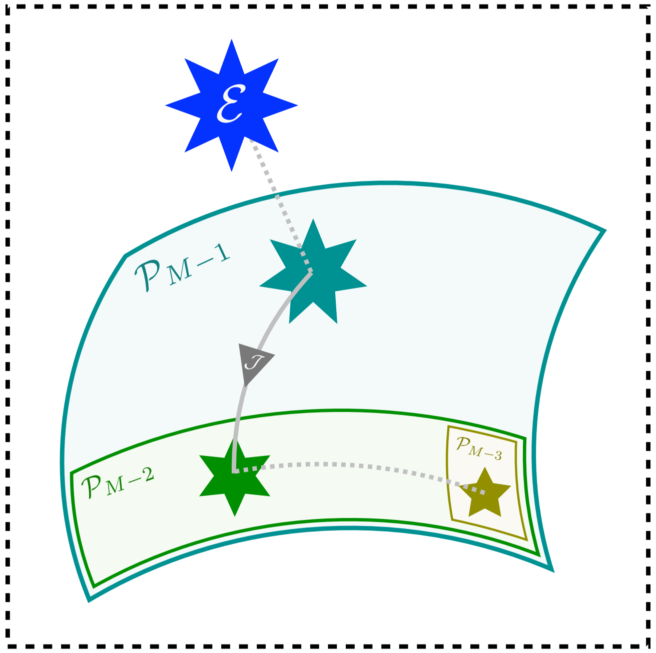
Distance of closest approach to N-particle manifold

[Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

Exclusive Cone Jet Finding

Point of closest approach on N-particle manifold

[Stewart, Tackmann, JDT, Vermilion, Wilkason, JHEP 2015]



Sequential Jet Recombination

Iteratively stepping between various N-particle manifolds

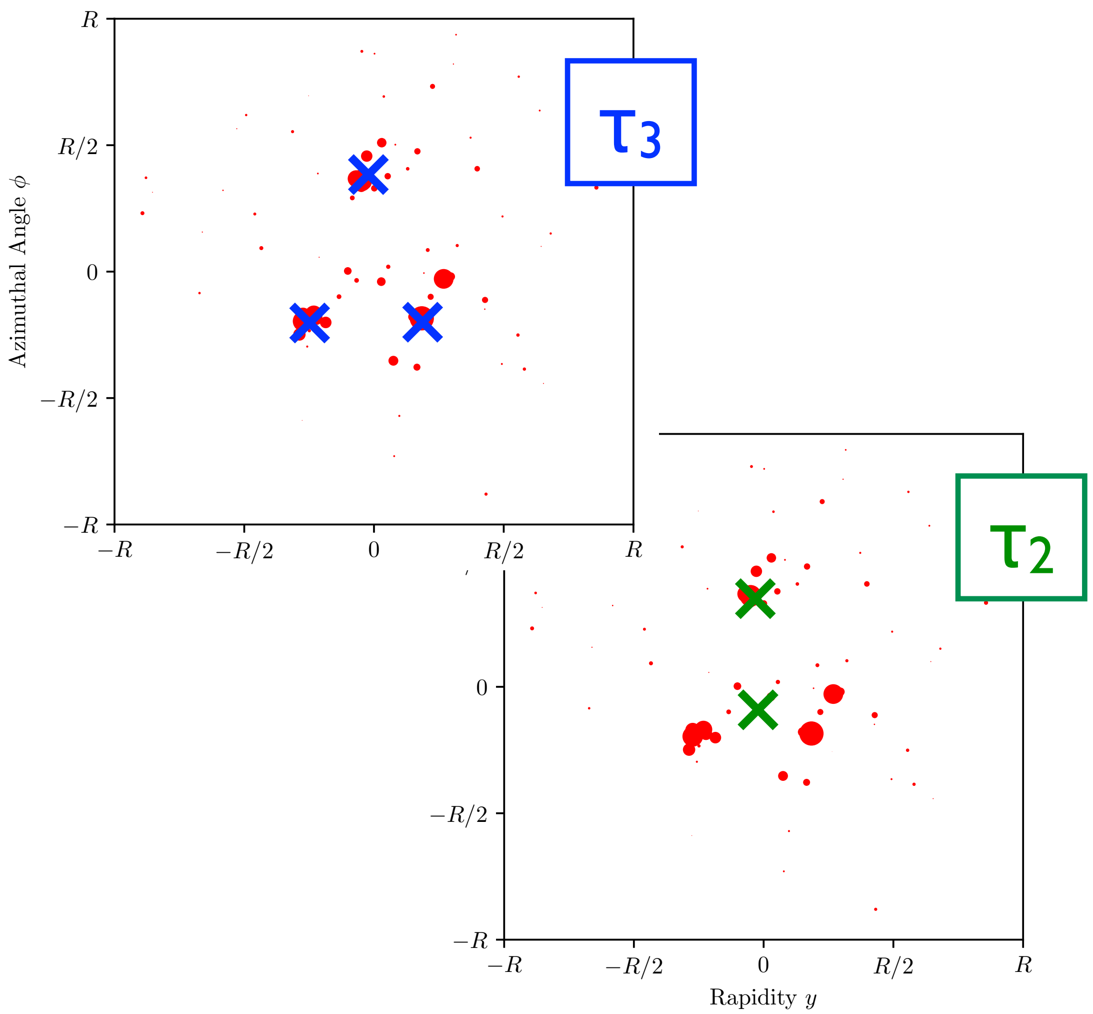
[Catani, Dokshitzer, Seymour, Webber, NPB 1993; Ellis, Soper, PRD 1993]
[Dokshitzer, Leder, Moretti, Webber, JHEP 1997; Wobisch, Wengler, arXiv 1999]
[Butterworth, Couchman, Cox, Waugh, CPC 2003; Larkoski, Neill, JDT, JHEP 2014]

[Komiske, Metodiev, JDT, JHEP 2020]

N-subjettiness

Ubiquitous jet substructure observable used for almost a decade…

$$\tau_N(\mathcal{J}) = \min_{N\ \text{axes}} \sum_i E_i \min\{\theta_{1,i}, \theta_{2,i}, \ldots, \theta_{N,i}\}$$

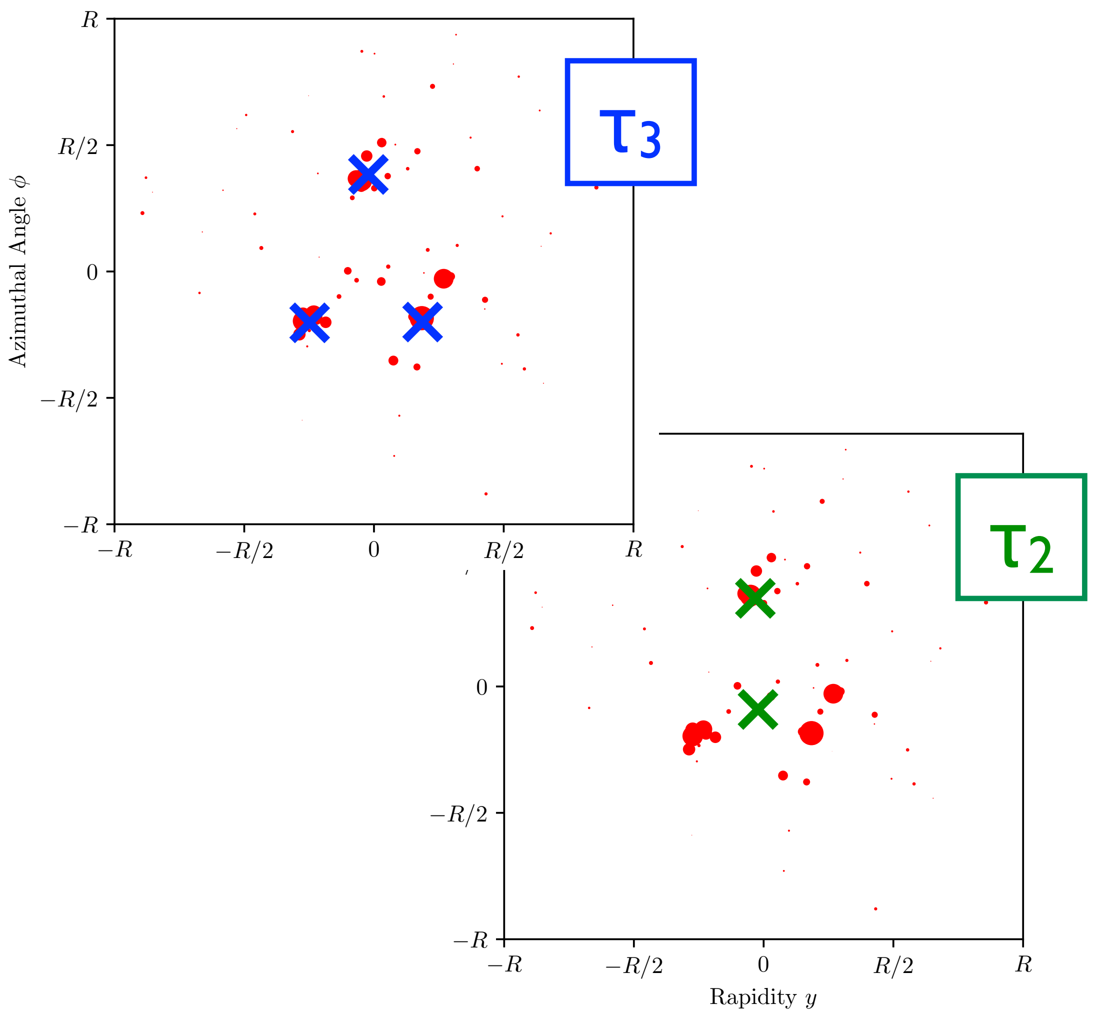


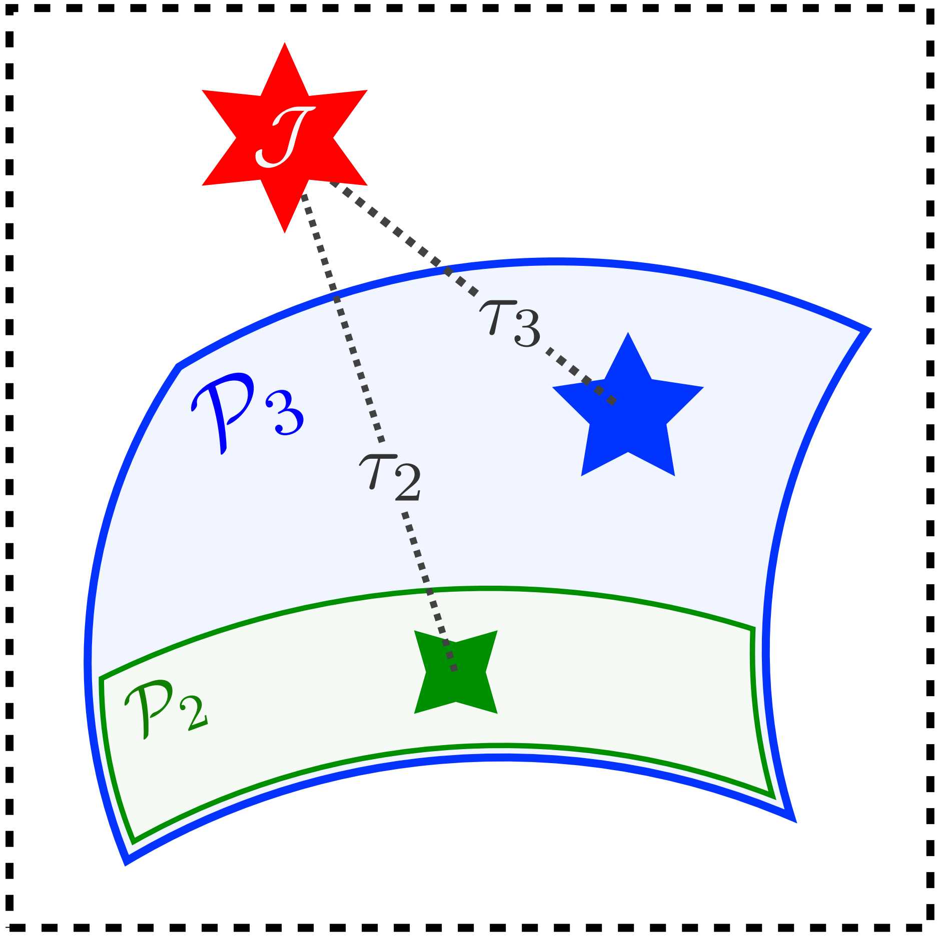
[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
based on Brandt, Dahmen, ZPC 1979; Stewart, Tackmann, Waalewijn, PRL 2010]

N-subjettiness = Point to Manifold EMD

…is secretly an optimal transport problem

$$\tau_N(\mathcal{J}) = \min_{\mathcal{J}' \in \mathcal{P}_N} \mathrm{EMD}(\mathcal{J}, \mathcal{J}')$$





[JDT, Van Tilburg, JHEP 2011, JHEP 2012;
rephrased in the language of Komiske, Metodiev, JDT, PRL 2019]

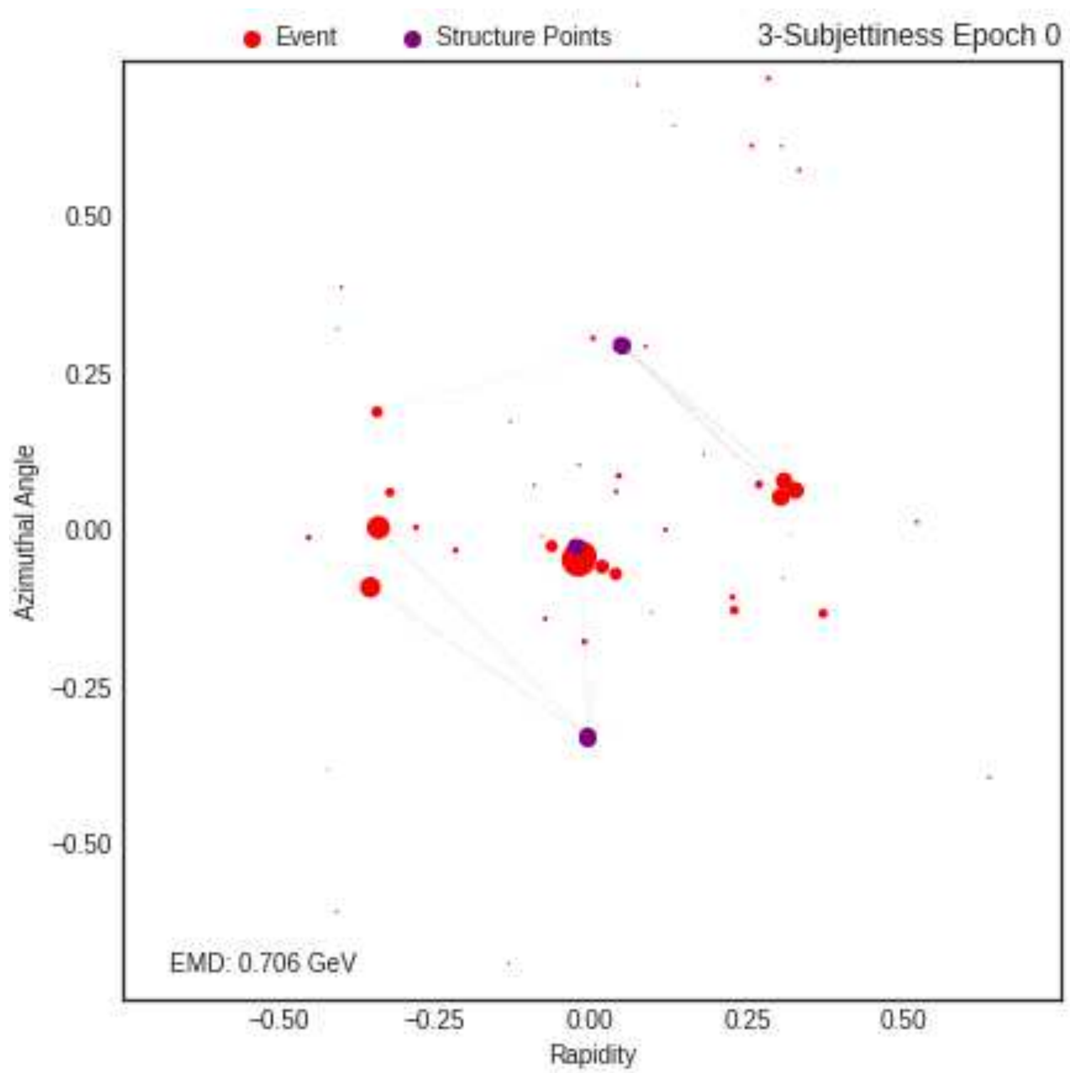


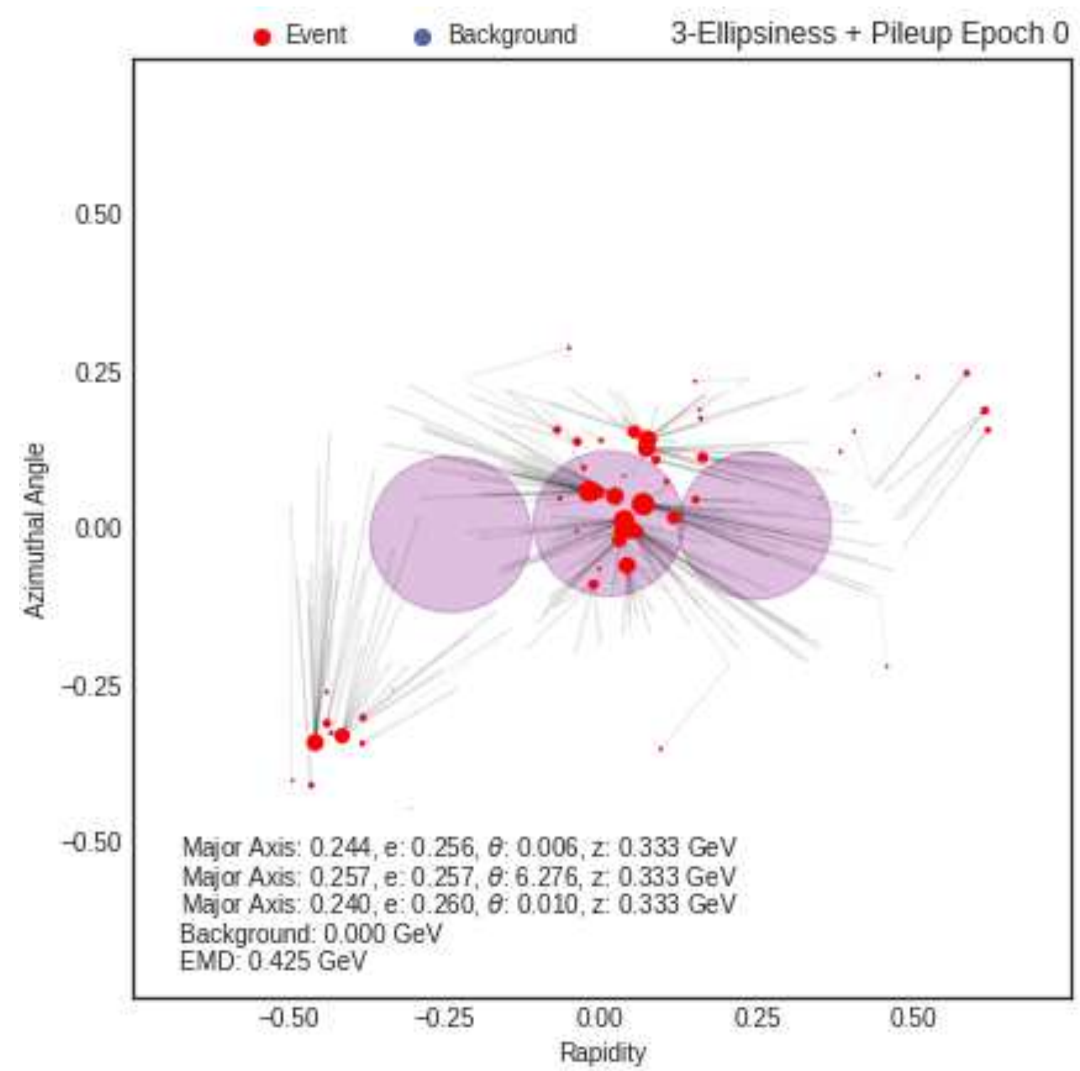
Deep Manifold Learning

SHAPER: Optimal transport meets gradient descent

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \text{EMD}(\mathcal{E}, \mathcal{E}')$$

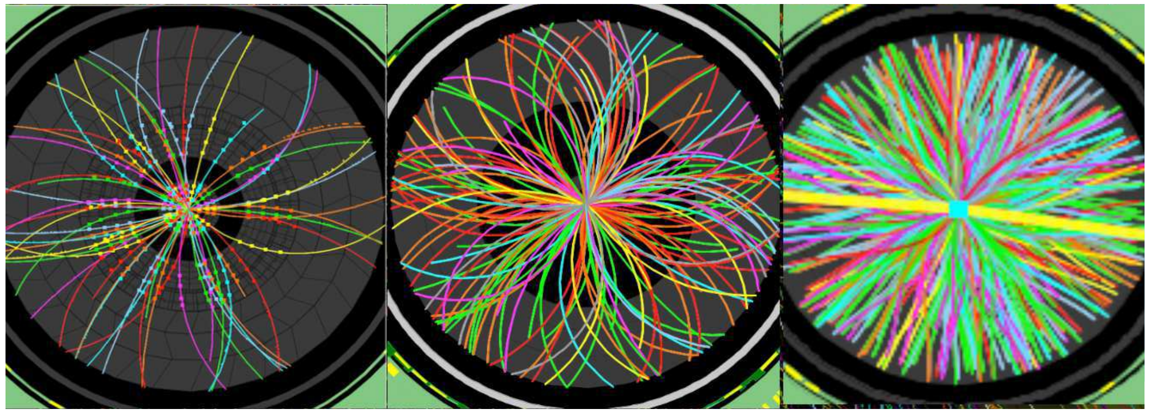
3-subjettiness vs. 3-ellipsiness + pileup





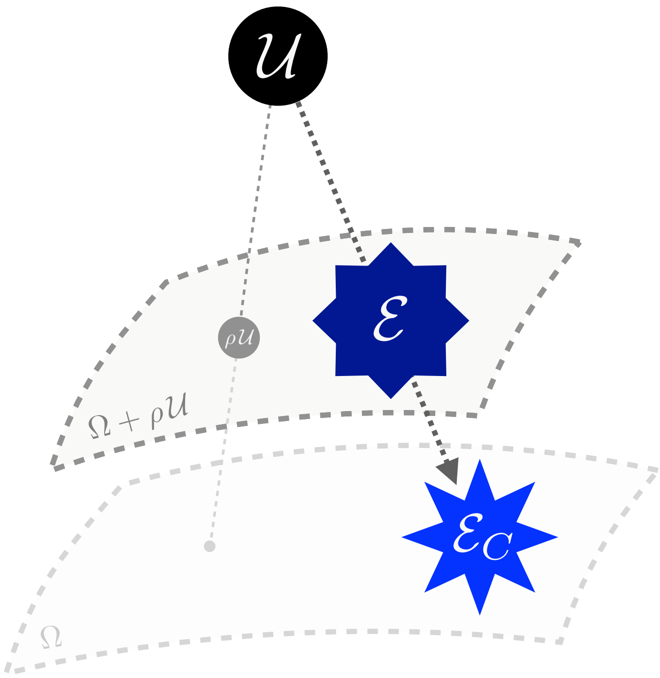
[Ba, Dogra, Gambhir, Tasissa, JDT, in progress;
inspired by Tankala, Tasissa, Murphy, Ba, arXiv 2020;
algorithmic progress in Kitouni, Nolte, Williams, arXiv 2022]

Pileup Mitigation



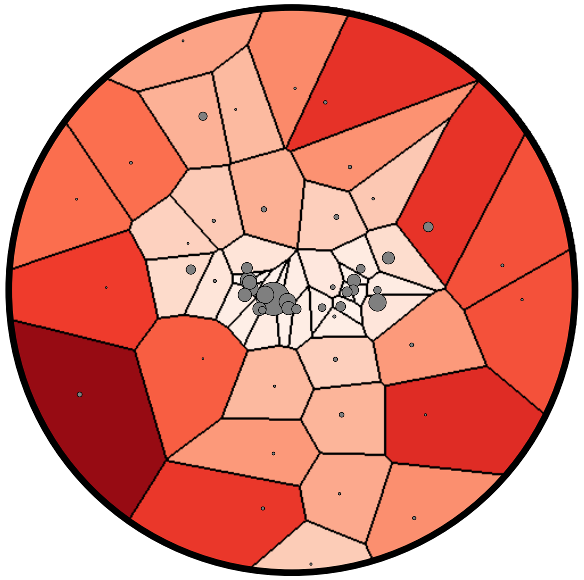
[see review in Soyez, PR 2019]

Uniform event contamination from overlapping proton-proton collisions



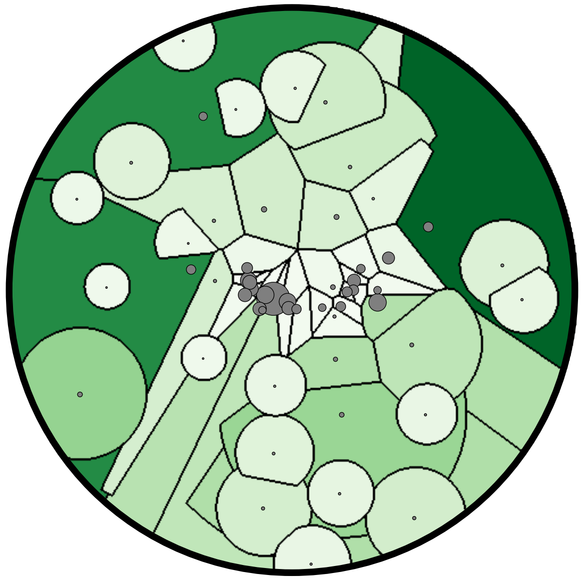
Pileup Mitigation:
"Move away" from uniform event

Voronoi



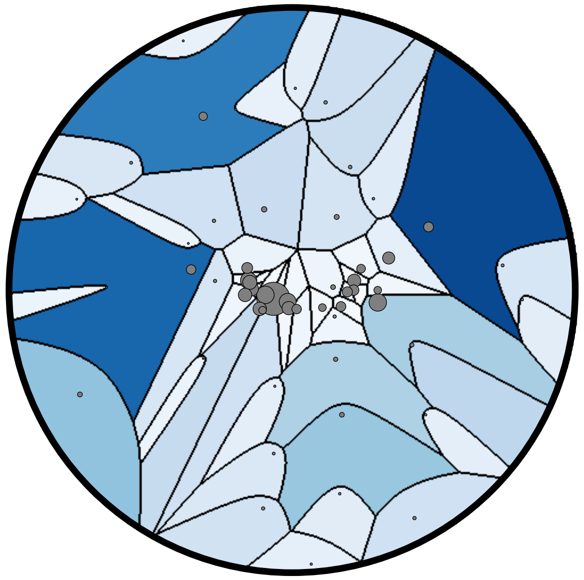
[Cacciari, Salam, Soyez, JHEP 2008]

Constituent Subtraction



[Berta, Spousta, Miller, Leitner, JHEP 2014]

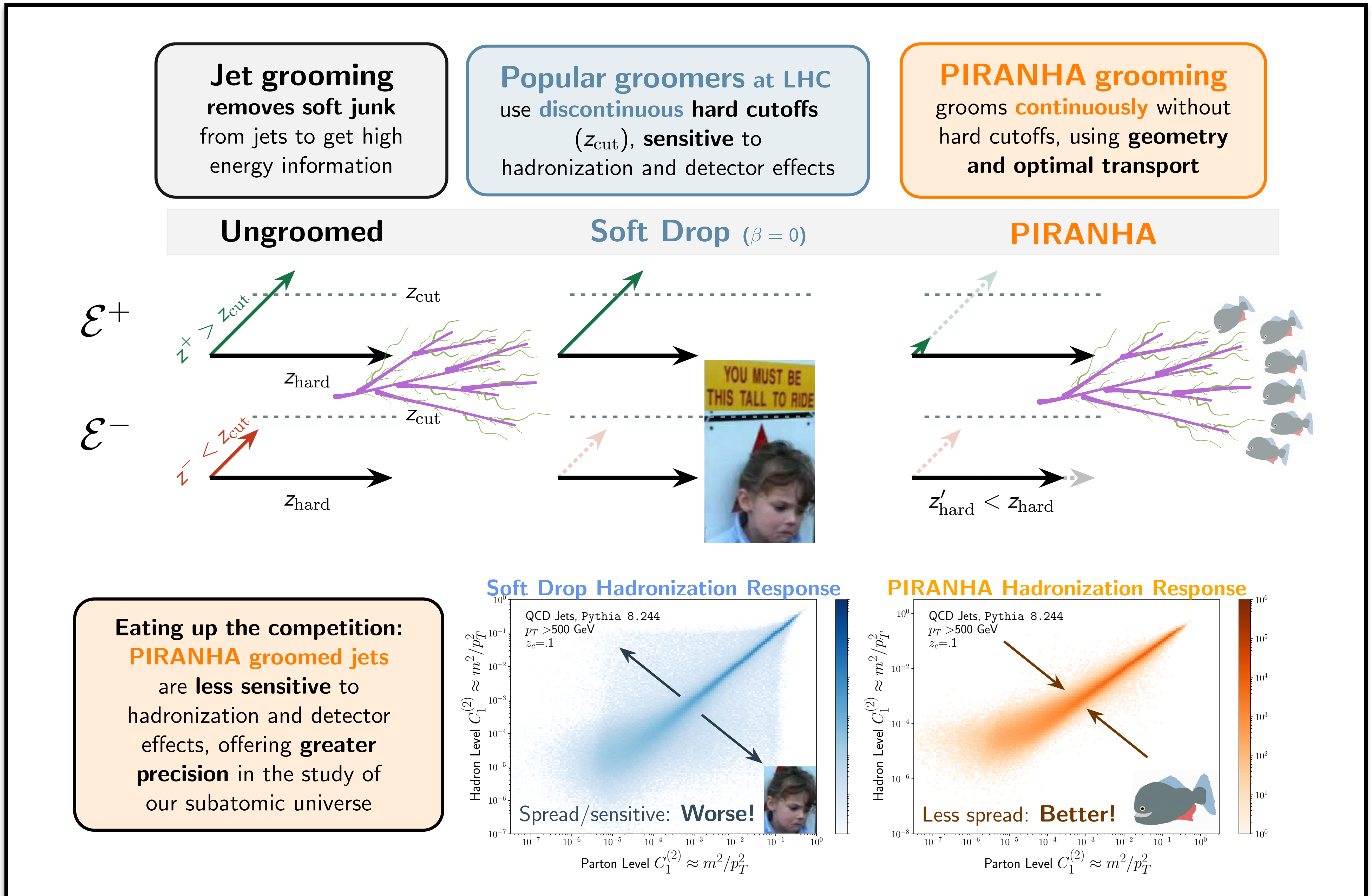
Apollonius



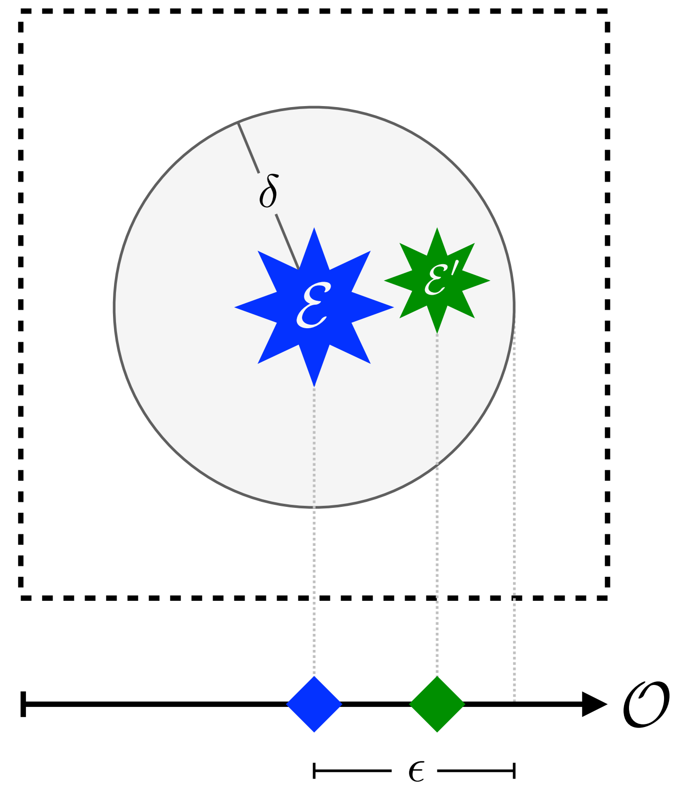
[Komiske, Metodiev, JDT, JHEP 2020]

Pileup and Infrared Radiation AnNiHilAtion

Recursive Safe Subtraction: tree-based approx. to optimal transport grooming

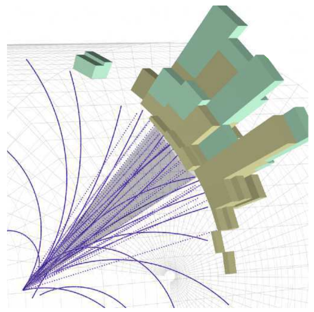


[Slides from Sam Alipour-fard]
[Alipour-fard, Komiske, Metodiev, JDT, in progress]



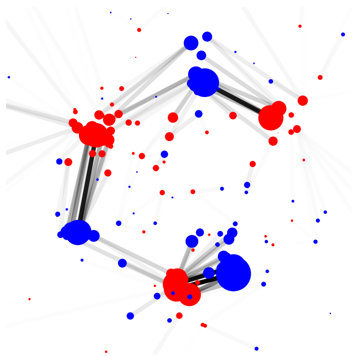
We are just beginning to leverage the conceptual richness of optimal transport for high-energy physics applications

Summary



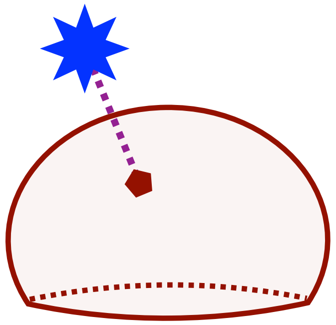
Going with the (Energy) Flow

Restricting our attention to IRC safe information
is a theoretically motivated data analysis strategy



The Energy Mover's Distance

Optimal transport allows us to triangulate the space
of collider events and define an emergent geometry

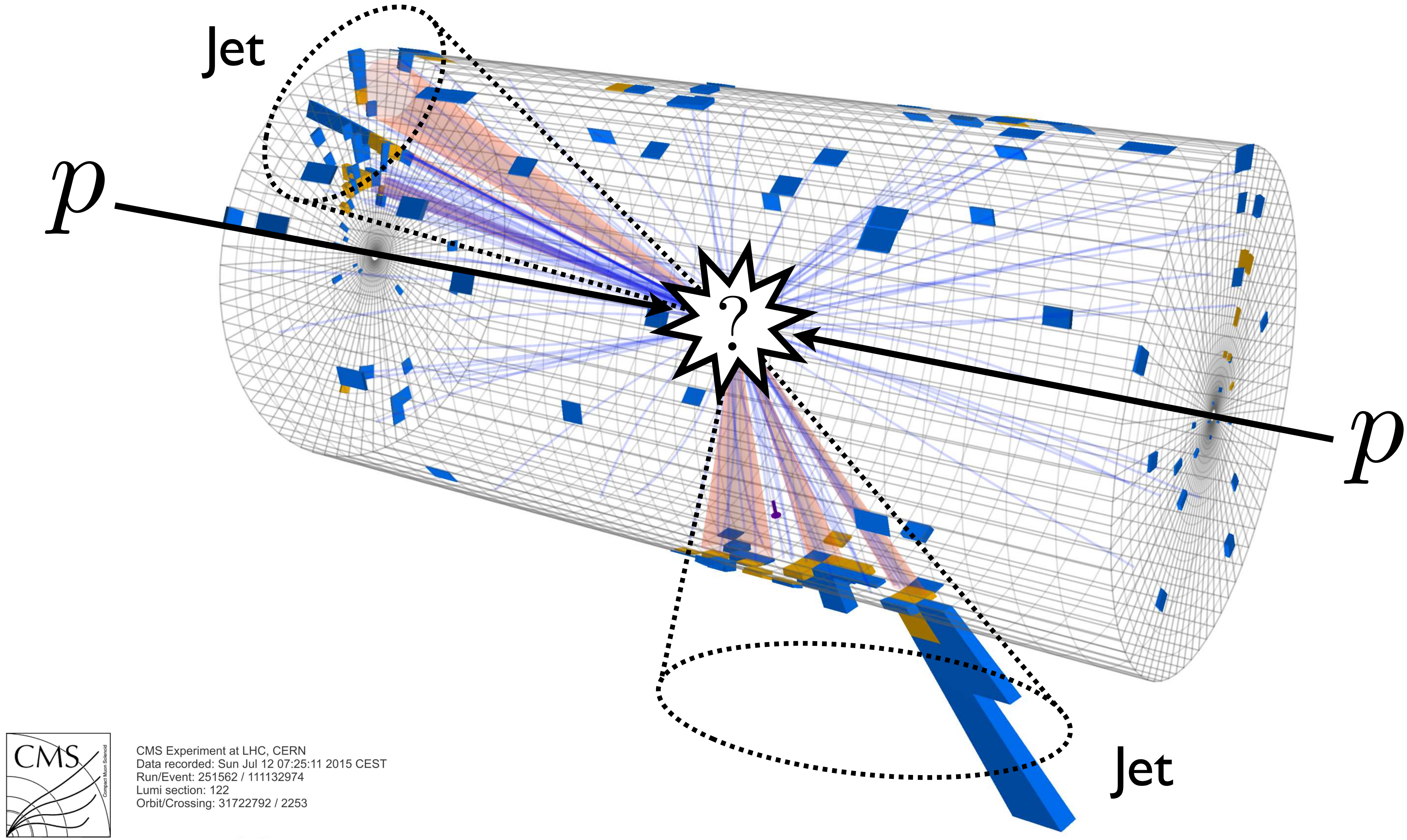


Revealing a Hidden Geometry

We can gain new perspectives on concepts/techniques
in QFT and collider physics from the last half century

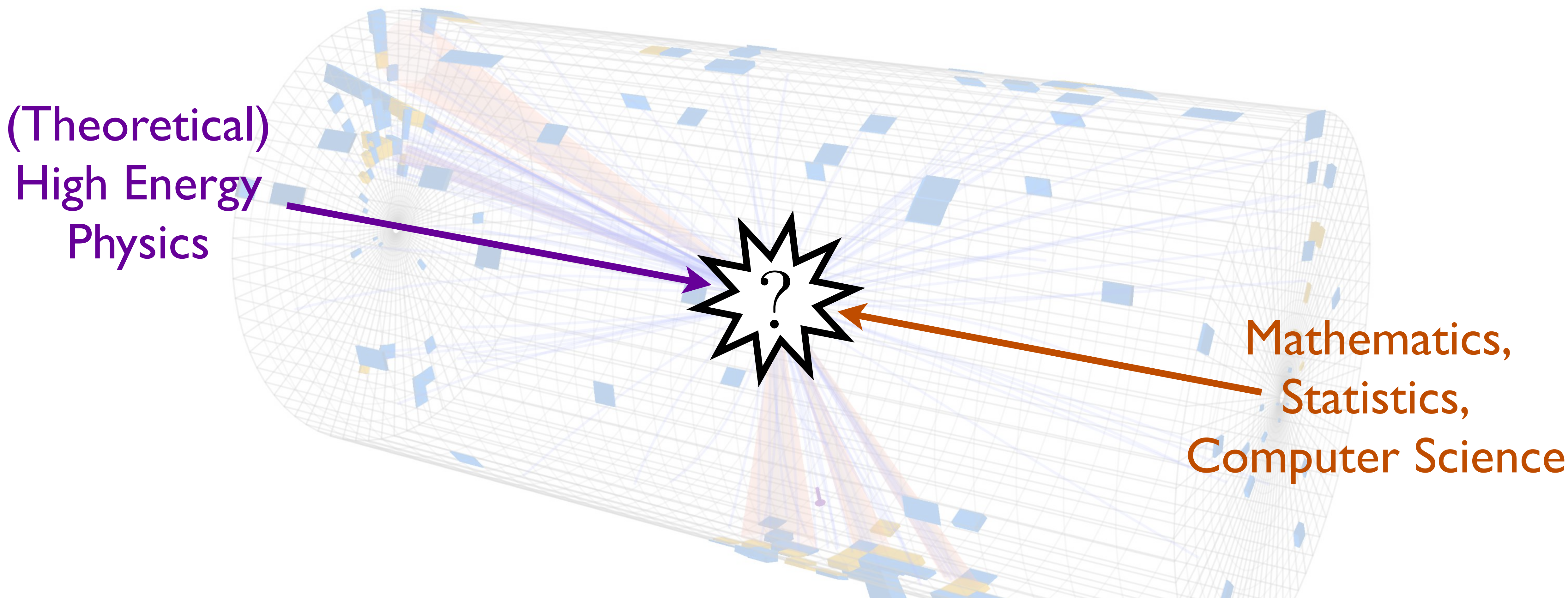
Backup Slides

“Collision Course”



"Collision Course"

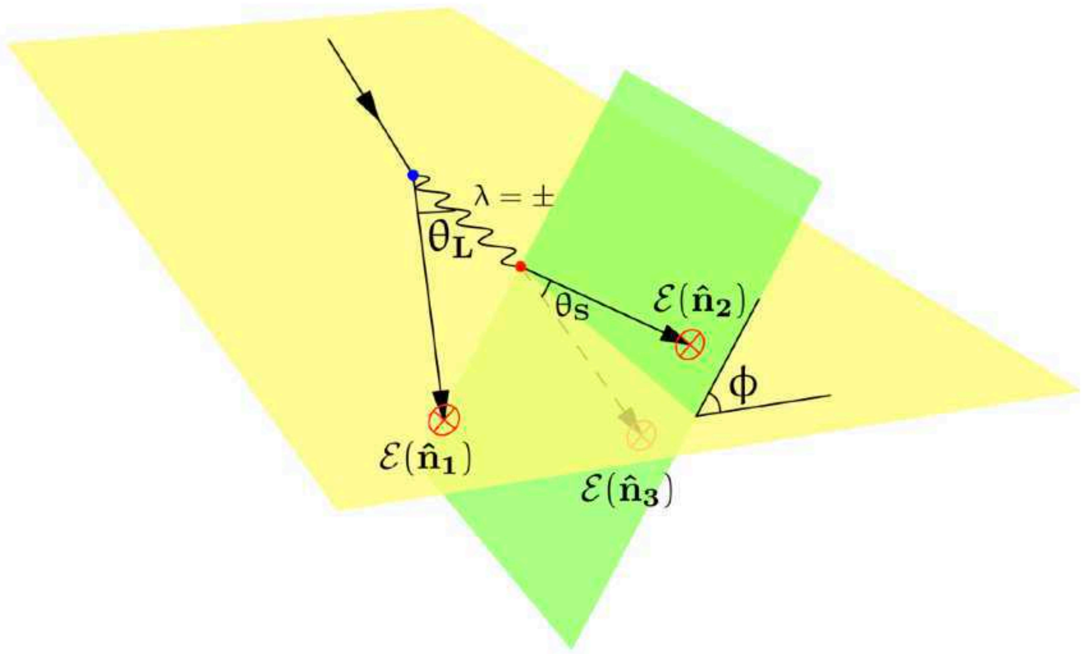
"Theoretical Physics for Machine Learning"
Aspen Center for Physics, January 2019

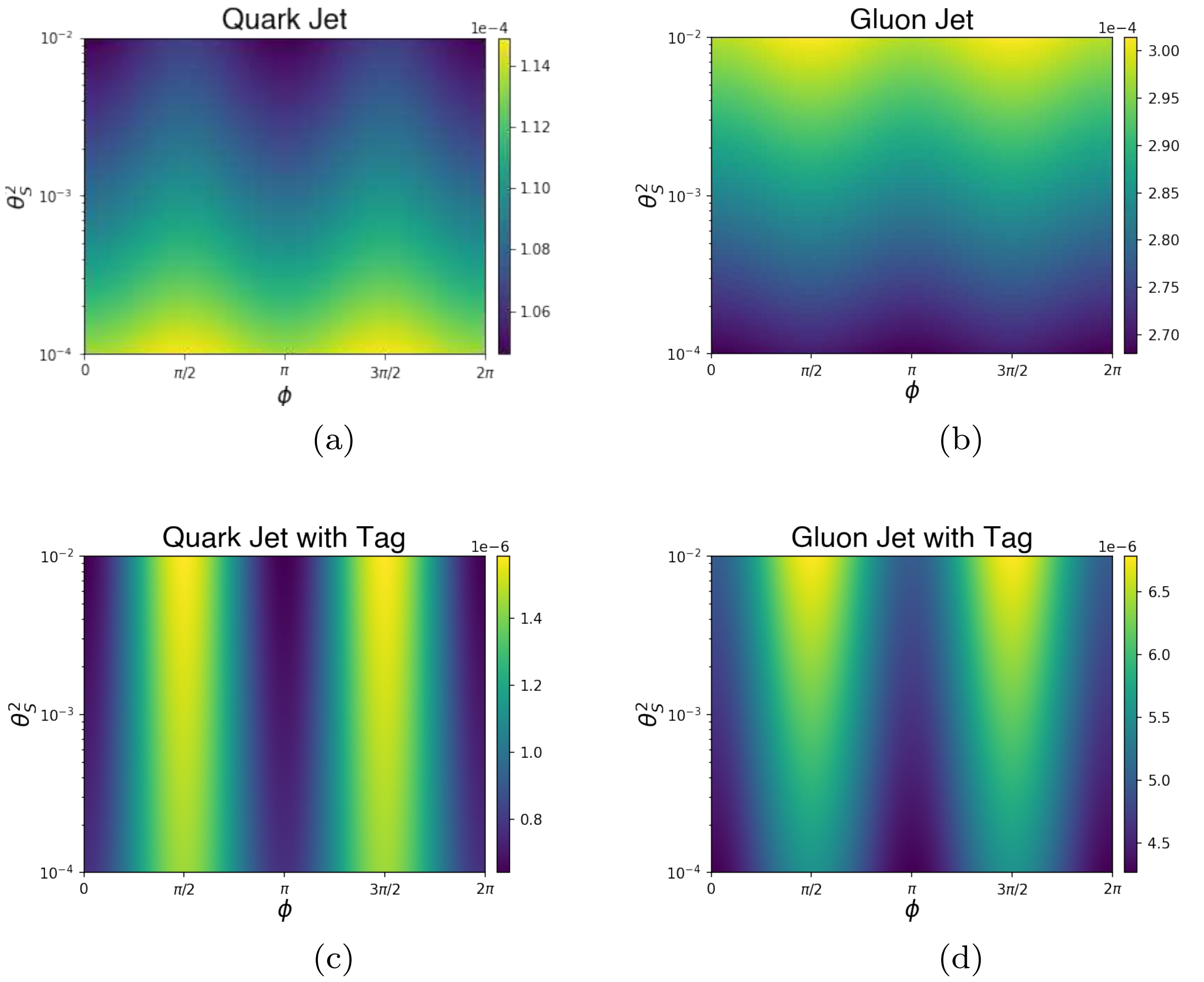


*New insights into particle physics**
*facilitated by advances in machine learning**

(and vice versa!)

Fun with Three Point Correlators





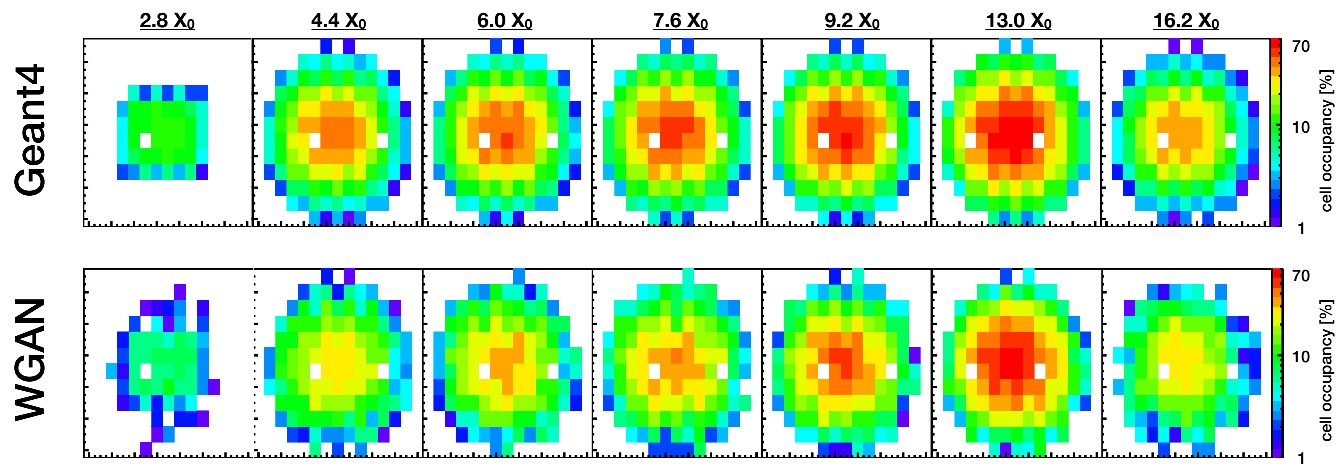
(with help from b-tagging)

Extracting quantum interference effects of spinning gluons!

[Chen, Moult, Zhu, PRL 2021]

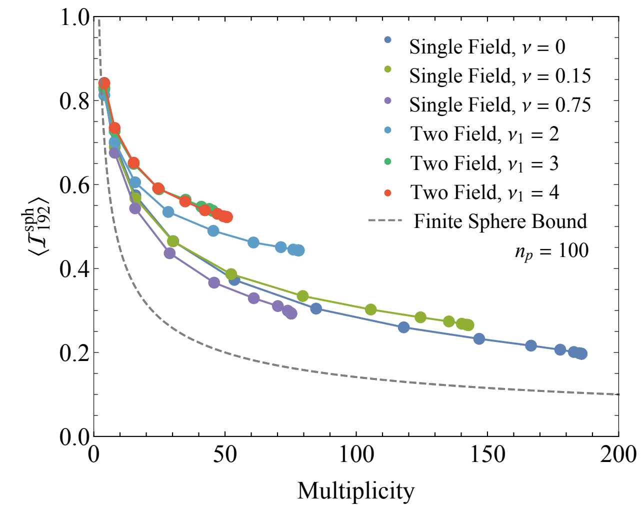
Wasserstein in HEP

Generative Modeling



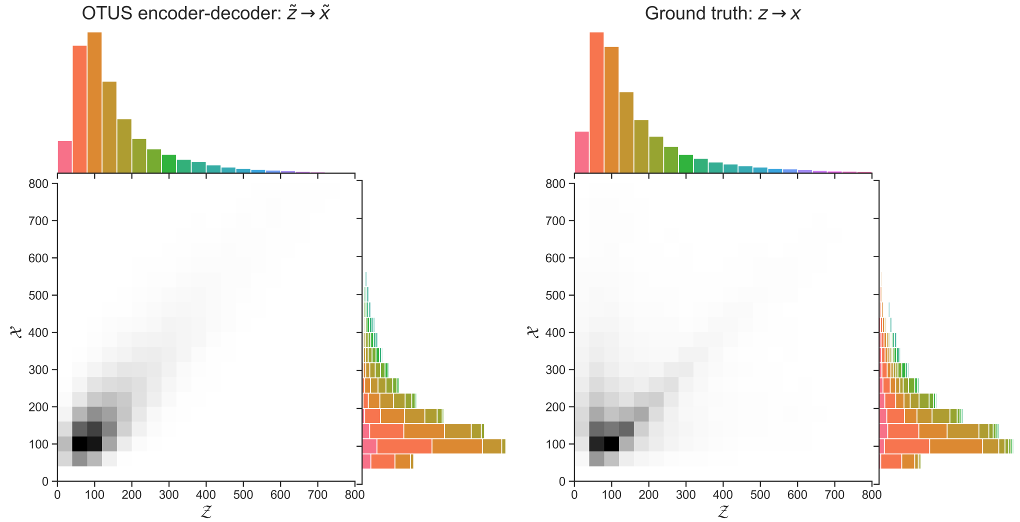
[Erdmann, Geiger, Glombitza, Schmidt, CSBS 2018; Erdmann, Glombitza, Quast, CSBS 2019; Chekalina, Orlova, Ratnikov, Ulyanov, Ustyuzhanin, Zakharov, CHEP 2018]

BSM Characterization



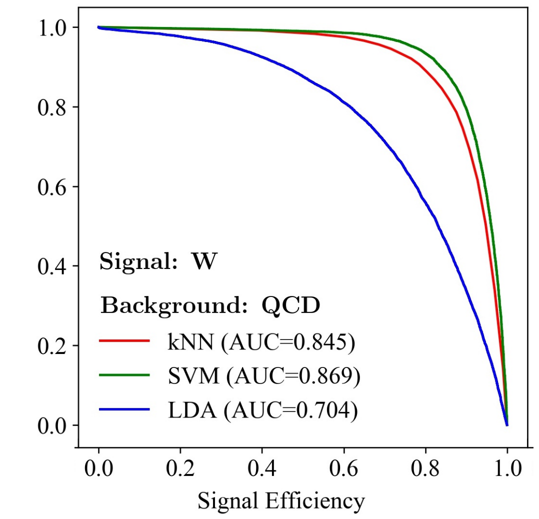
[Cesarotti, Reece, Strassler, JHEP 2021, arXiv 2020]

Estimated Simulation/Unfolding



[Howard, Mandt, Whiteson, Yang, arXiv 2021]

Jet Classification



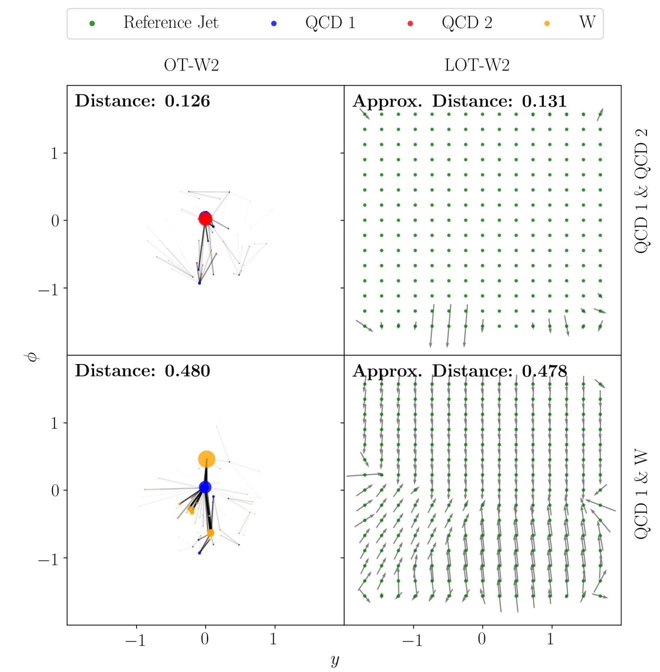
[Cai, Cheng, Craig, Craig, PRD 2020]

. . .

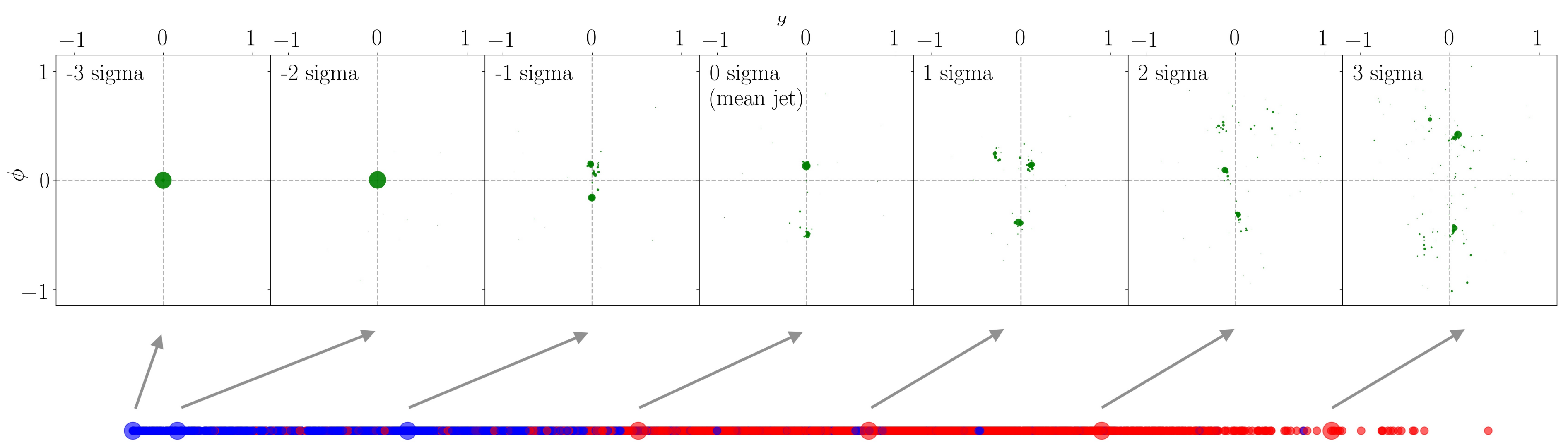
Linearized Optimal Transport

With the help of a reference event, transportation distances* can be efficiently mapped to Euclidean distances

* assuming the 2-Wasserstein measure



Enables coordinate-based techniques like Linear Discriminate Analysis

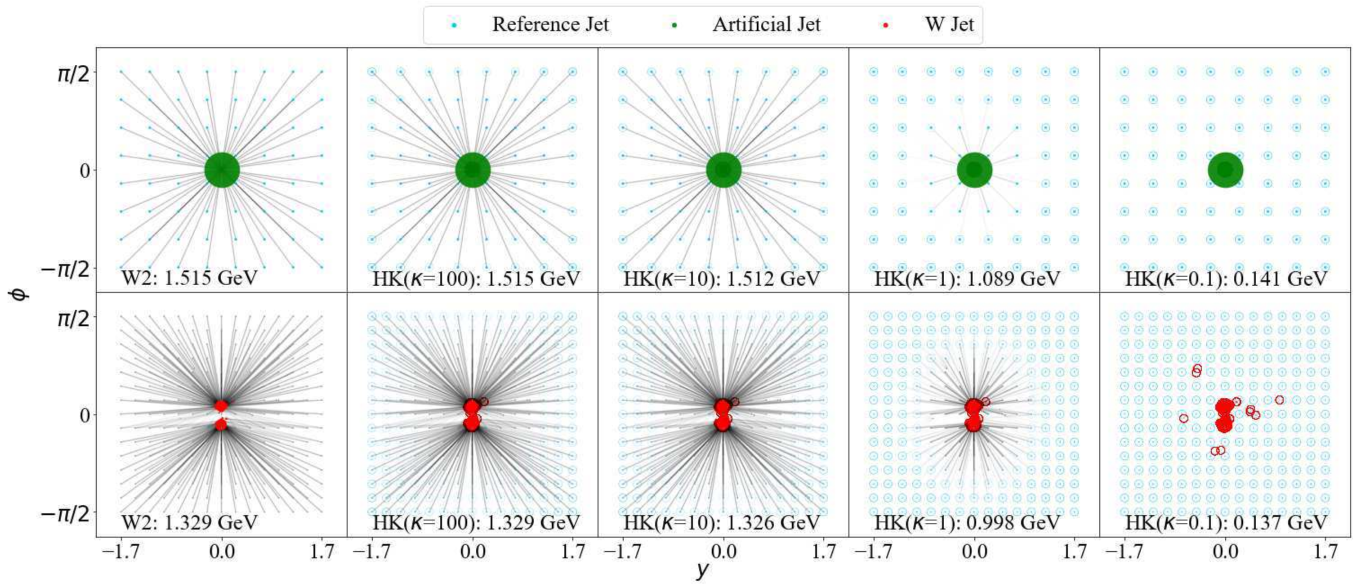


[Cai, Cheng, Craig, Craig, PRD 2020]

Opening a Dialogue Between Communities

HEP domain knowledge ⇔ *interdisciplinary insights*

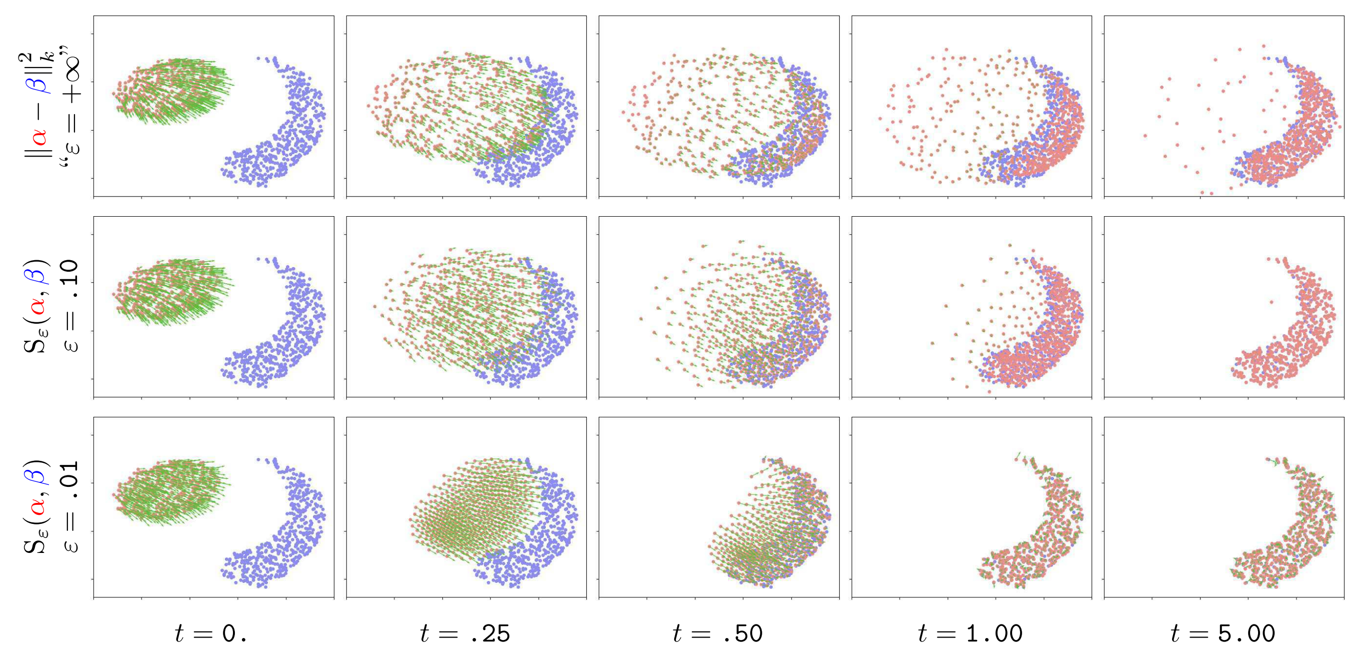
Analyzing Jets with Linearized Transport & Partial Transport



[Cai, Cheng, Craig, Craig, PRD 2020, arXiv 2021]

Interpolating between Optimal Transport & Kernel Methods

(see next slide to justify color coding)



[Feydy, Séjourné, Vialard, Amari, Trouvé, Peyré, arXiv 2018]

Siloing in the Scientific Community

$$\text{Kernel}_k(\alpha, \beta) \;=\; \tfrac{1}{2}\langle \alpha,\, k \star \alpha\rangle - \langle \alpha,\, k \star \beta\rangle + \tfrac{1}{2}\langle \beta,\, k \star \beta\rangle$$

$$= \tfrac{1}{2} \sum_{i=1}^{\text{N}} \sum_{j=1}^{\text{N}} \alpha_i \alpha_j \, k(x_i, x_j) \;-\; \sum_{i=1}^{\text{N}} \sum_{j=1}^{\text{M}} \alpha_i \beta_j \, k(x_i, y_j) \;+\; \tfrac{1}{2} \sum_{i=1}^{\text{M}} \sum_{j=1}^{\text{M}} \beta_i \beta_j \, k(y_i, y_j)$$

Kernel methods. Formulas in the mould of Eqs. (3.99-3.101) are **ubiquitous in applied sciences**: from physics to machine learning, applying a convolution is the simplest way of modelling spatial correlations and pair-wise interactions. Unfortunately though, few papers and textbooks take the time to draw explicit links between fields that have, at first glance, very little in common. Before going any further, we devote a few pages to a short panorama around the six major interpretations of Eq. (3.99). As we identify with each other the theories of:

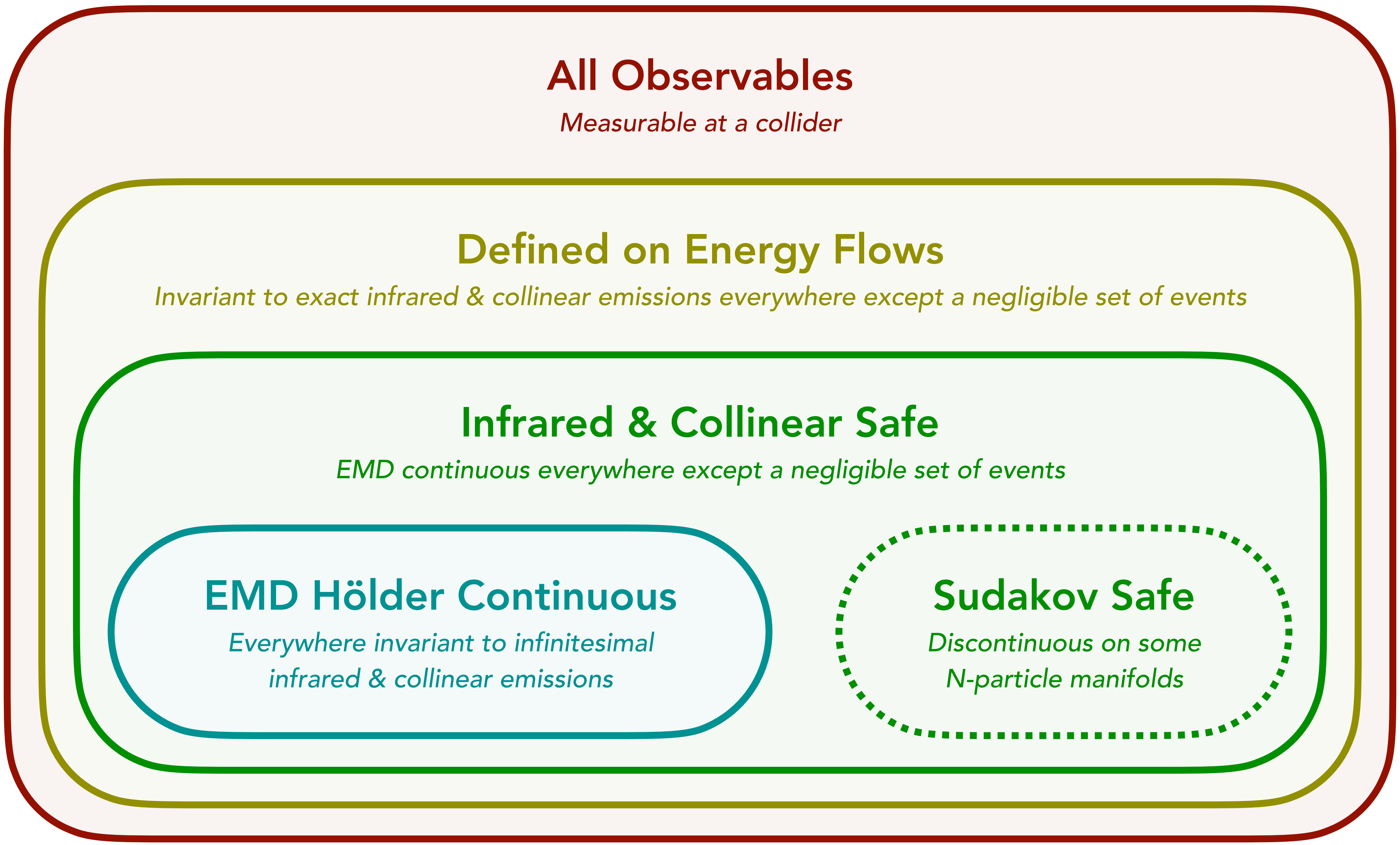
1. **Newtonian gravitation and electrostatics** in physics,
2. **blurred squared distances** in imaging sciences,
3. **Sobolev norms** in functional analysis,
4. **maximum mean discrepancies** in statistics,
5. **reproducing kernel Hilbert spaces** in machine learning and
6. **Kriging**, **splines** or **Gaussian processes** in geostatistics, imaging and probabilities,

we will hopefully help the reader to get a deeper understanding of a theory that is central to modern data sciences.

[Feydy, Geometric data analysis, beyond convolutions]

Observable Taxonomy

See backup for example observables



[Komiske, Metodiev, JDT, JHEP 2020; cf. Sterman, PRD 1979; Banfi, Salam, Zanderighi, JHEP 2005; Larkoski, Marzani, JDT, PRD 2015]

The Spectrum of Safety

All Observables	Comments
Multiplicity ($\sum_i 1$)	IR unsafe and C unsafe
Momentum Dispersion [65] ($\sum_i E_i^2$)	IR safe but C unsafe
Sphericity Tensor [66] ($\sum_i p_i^\mu p_i^\nu$)	IR safe but C unsafe
Number of Non-Zero Calorimeter Deposits	C safe but IR unsafe
Defined on Energy Flows	
Pseudo-Multiplicity ($\min\{N \mid \mathcal{T}_N = 0\}$)	Robust to exact IR or C emissions
Infrared & Collinear Safe	
Jet Energy ($\sum_i E_i$)	Disc. at jet boundary
Heavy Jet Mass [67]	Disc. at hemisphere boundary
Soft-Dropped Jet Mass [38, 68]	Disc. at grooming threshold
Calorimeter Activity [69] (N_{95})	Disc. at cell boundary
Sudakov Safe	
Groomed Momentum Fraction [39] (z_g)	Disc. on 1-particle manifold
Jet Angularity Ratios [37]	Disc. on 1-particle manifold
N-subjettiness Ratios [47, 48] (τ_{N+1}/τ_N)	Disc. on N-particle manifold
V parameter [36] (Eq. (2.11))	Hölder disc. on 3-particle manifold
EMD Hölder Continuous Everywhere	
Thrust [40, 41]	
Spherocity [42]	
Angularities [70]	
N-jettiness [44] ($\mathcal{T}_N$)	
C parameter [71–74]	Resummation beneficial at $C = \frac{3}{4}$
Linear Sphericity [72] ($\sum_i E_i n_i^\mu n_i^\nu$)	
Energy Correlators [36, 75–77]	
Energy Flow Polynomials [15, 17]	

On my reading list…

Renormalization Group Flow as Optimal Transport

Jordan Cotler[1,2,3] **and Semon Rezchikov**[4]

[1] *Harvard Society of Fellows, Cambridge, MA 02138 USA*
[2] *Black Hole Initiative, Harvard University, Cambridge, MA 02138 USA*
[3] *Center for Fundamental Laws of Nature, Harvard University, Cambridge, MA 02138 USA*
[4] *Department of Mathematics, Harvard University, Cambridge, MA 02138 USA*

Abstract

We establish that Polchinski's equation for exact renormalization group flow is equivalent to the optimal transport gradient flow of a field-theoretic relative entropy. This provides a compelling information-theoretic formulation of the exact renormalization group, expressed in the language of optimal transport. A striking consequence is that a regularization of the relative entropy is in fact an RG monotone. We compute this monotone in several examples. Our results apply more broadly to other exact renormalization group flow equations, including widely used specializations of Wegner-Morris flow. Moreover, our optimal transport framework for RG allows us to reformulate RG flow as a variational problem. This enables new numerical techniques and establishes a systematic connection between neural network methods and RG flows of conventional field theories.

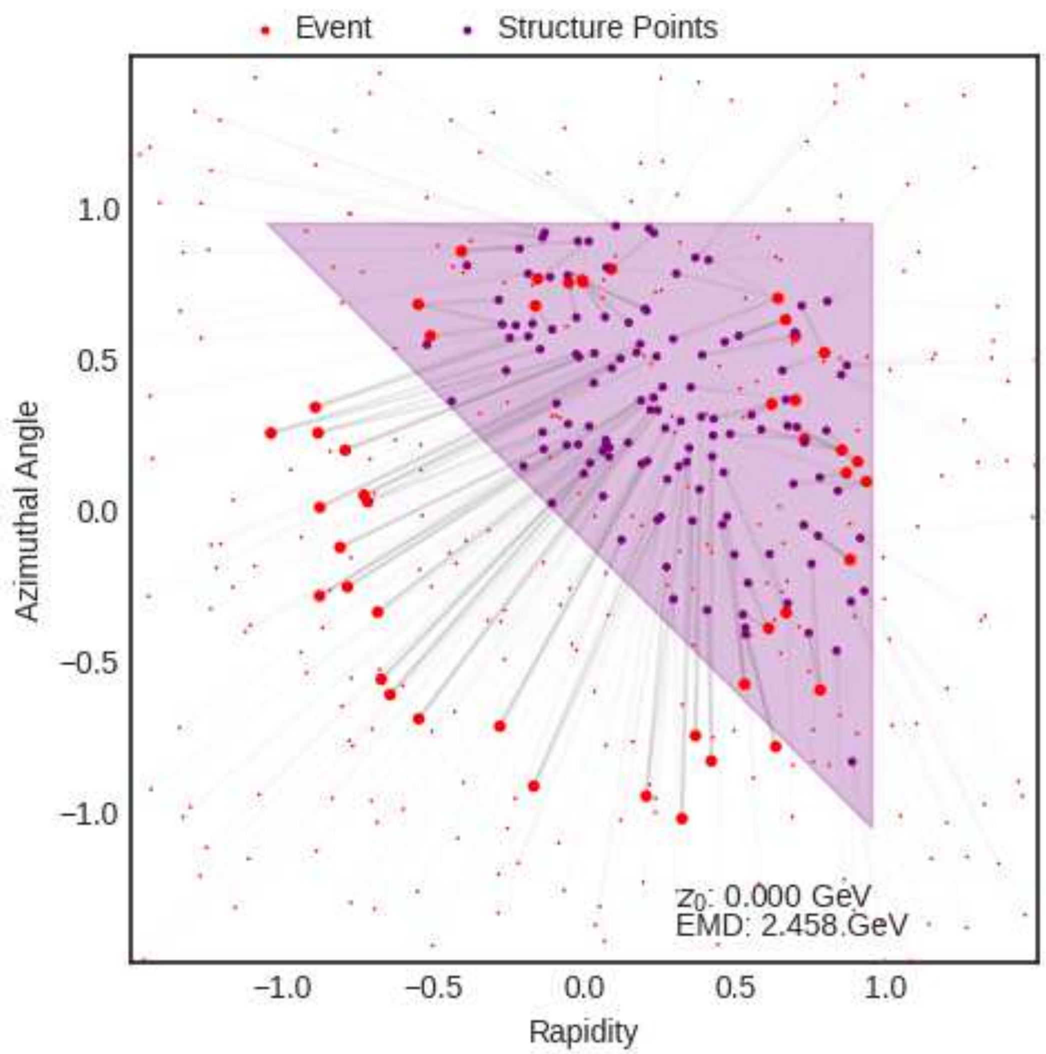
[Cotler, Rezchikov, arXiv 2022]

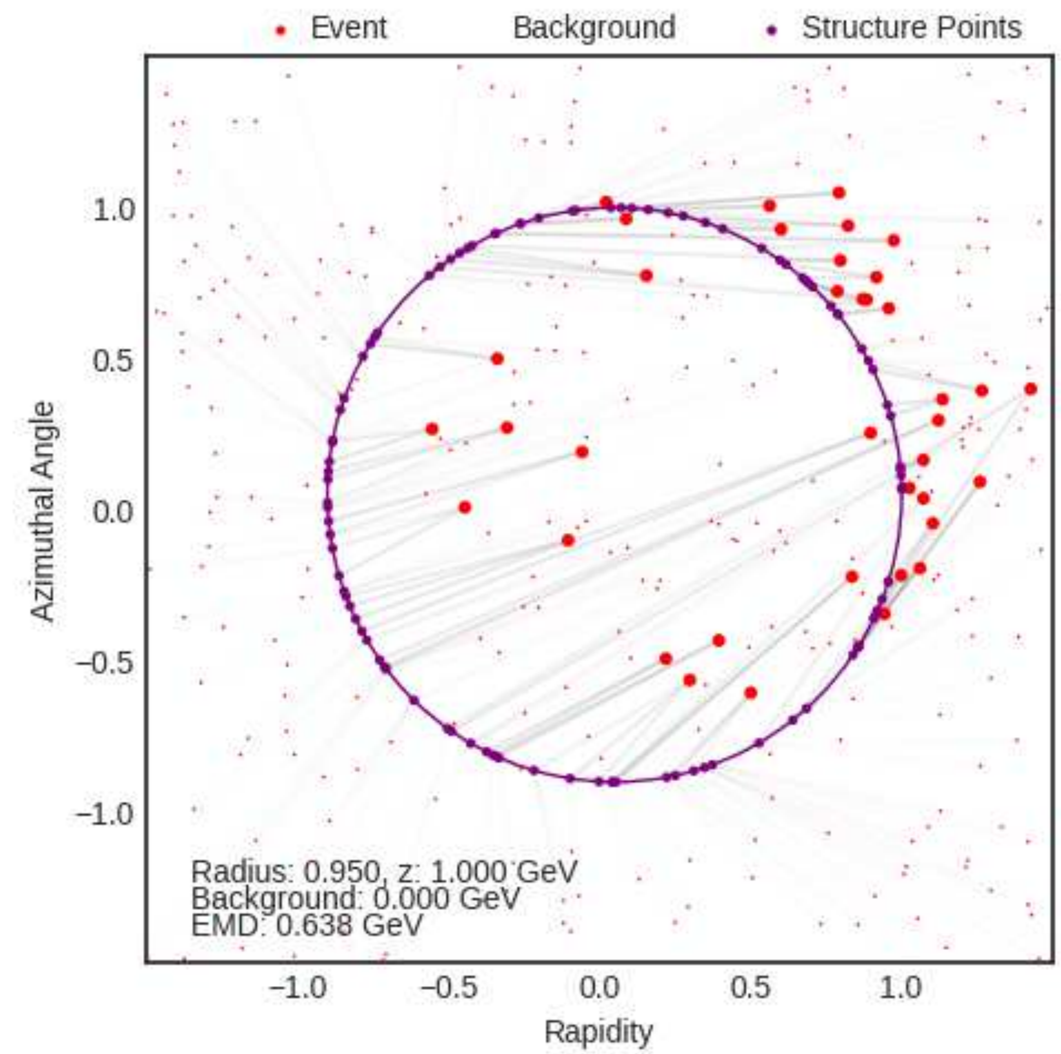
Deep Manifold Learning

SHAPER: Optimal transport meets gradient descent

$$\mathcal{O}(\mathcal{E}) = \min_{\mathcal{E}' \in \mathcal{M}} \mathrm{EMD}(\mathcal{E}, \mathcal{E}')$$

How triangle-like / ring-like is this jet?





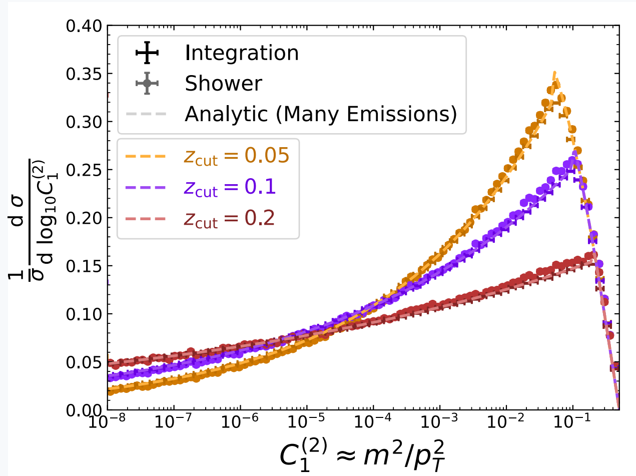
[Ba, Dogra, Gambhir, Tasissa, JDT, in progress;
inspired by Tankala, Tasissa, Murphy, Ba, arXiv 2020;
algorithmic progress in Kitouni, Nolte, Williams, arXiv 2022]

Pileup and Infrared Radiation AnNiHilAtion

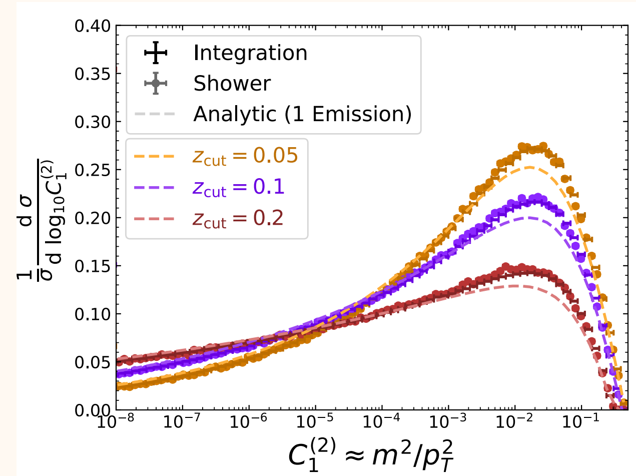
Recursive Safe Subtraction: tree-based approx. to optimal transport grooming

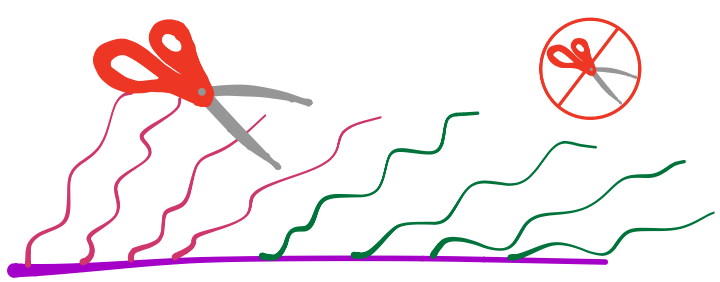
Fixed coupling, **multiple emission** calculations:

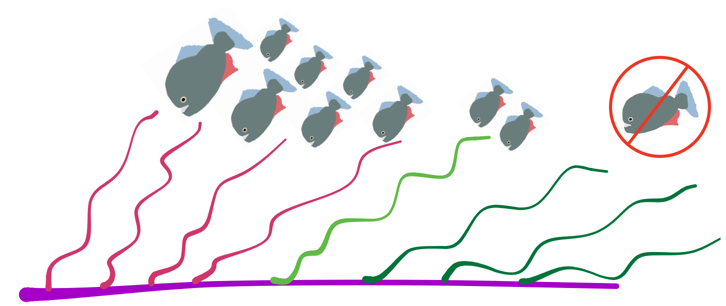
Soft Drop/mMDT



PIRANHA-RSS $(f = 1)$







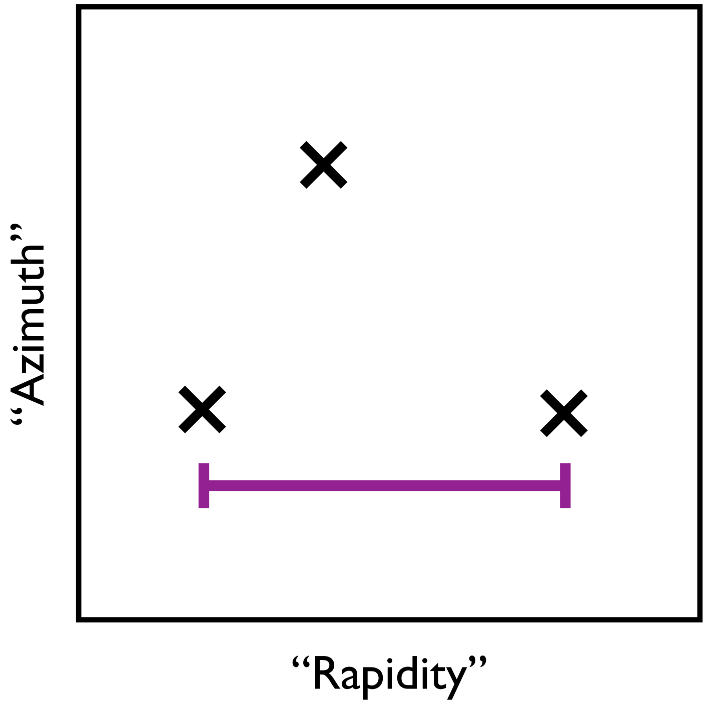
Sharp cutoff $\rightarrow$ kink

No sharp cutoff $\rightarrow$ smooth

[Slides from Sam Alipour-fard]
[Alipour-fard, Komiske, Metodiev, JDT, in progress]

Introducing Theory Space

Direction Space



× = Direction

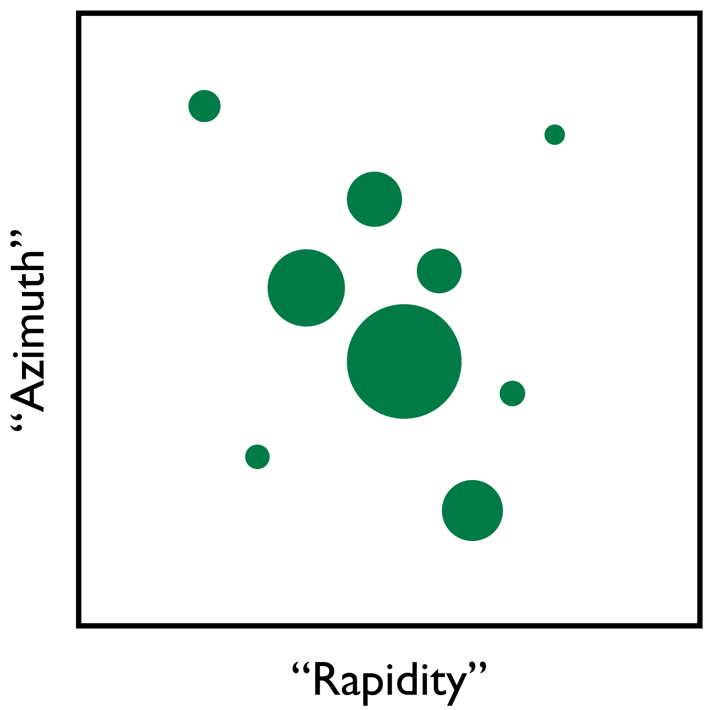
$$n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$$

⊢⊣ = Angular Distance

$$\theta_{ij} = \sqrt{2 n_i^\mu n_{j\mu}}$$

(for massless particles)

Direction Space Distribution

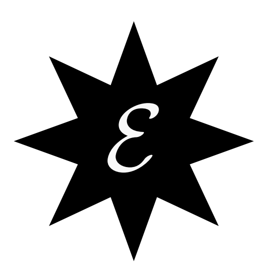


● = Weighted Direction $n_i^\mu = \frac{p_i^\mu}{E_i} = (1, \hat{n})^\mu$

$w_i = E_i$

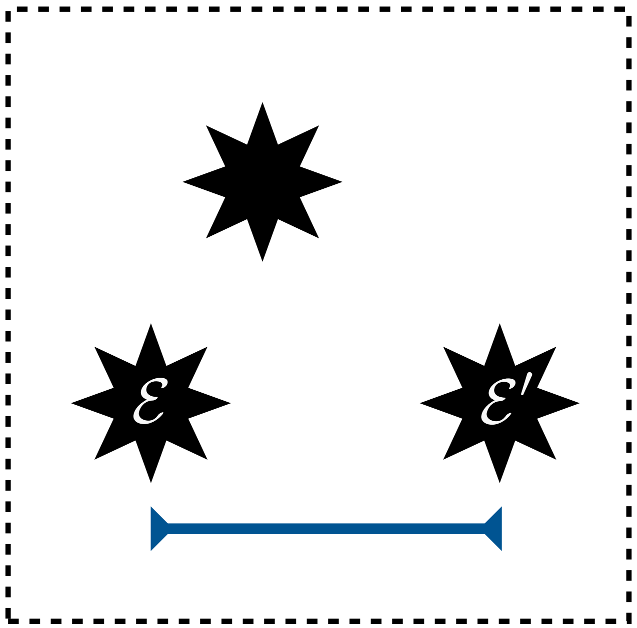
⟼ = Angular Distance $\theta_{ij} = \sqrt{2 n_i^\mu n_{j\mu}}$

(for massless particles)



= Event

Event Space



= Event

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$

= EMD

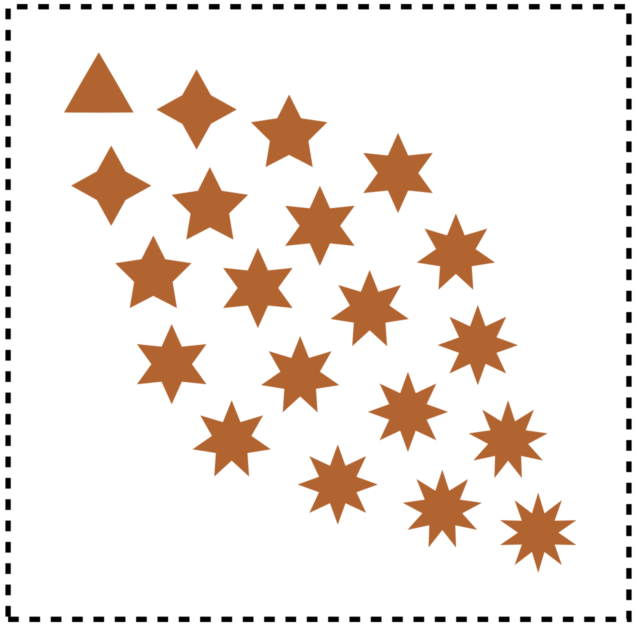
Energy
Mover's Distance

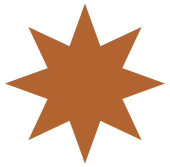
$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

(for equal total energy)

[Komiske, Metodiev, JDT, PRL 2019]

Event Space Distribution



 = Weighted Event

$$\mathcal{E}(\hat{n}) = \sum_i E_i \, \delta(\hat{n} - \hat{n}_i)$$

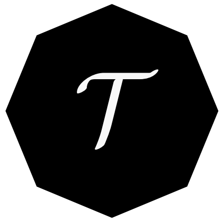
$$w_a = \sigma_a$$

 = EMD

Energy
Mover's Distance

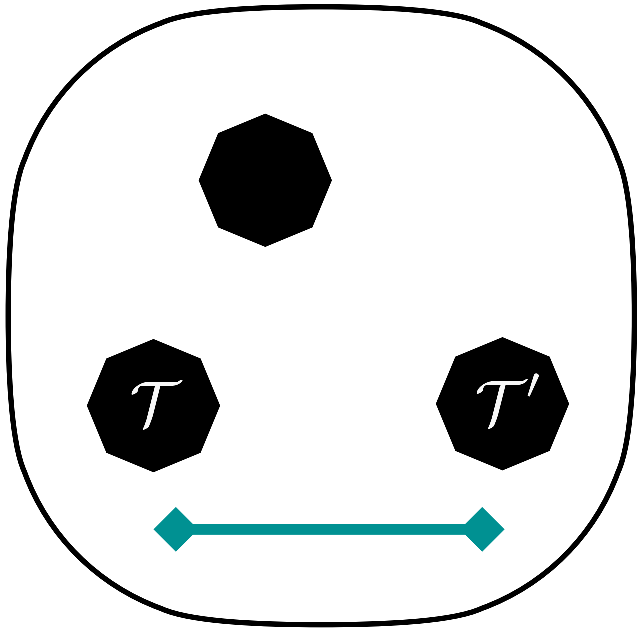
$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_i \sum_j f_{ij} \theta_{ij}$$

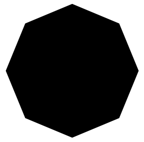
(for equal total energy)



= Theory

Theory Space



 = Theory

$$\mathcal{T}(\mathcal{E}) = \sum_a \sigma_a \, \delta(\mathcal{E} - \mathcal{E}_a)$$

 = ΣMD

Cross-Section
Mover's Distance

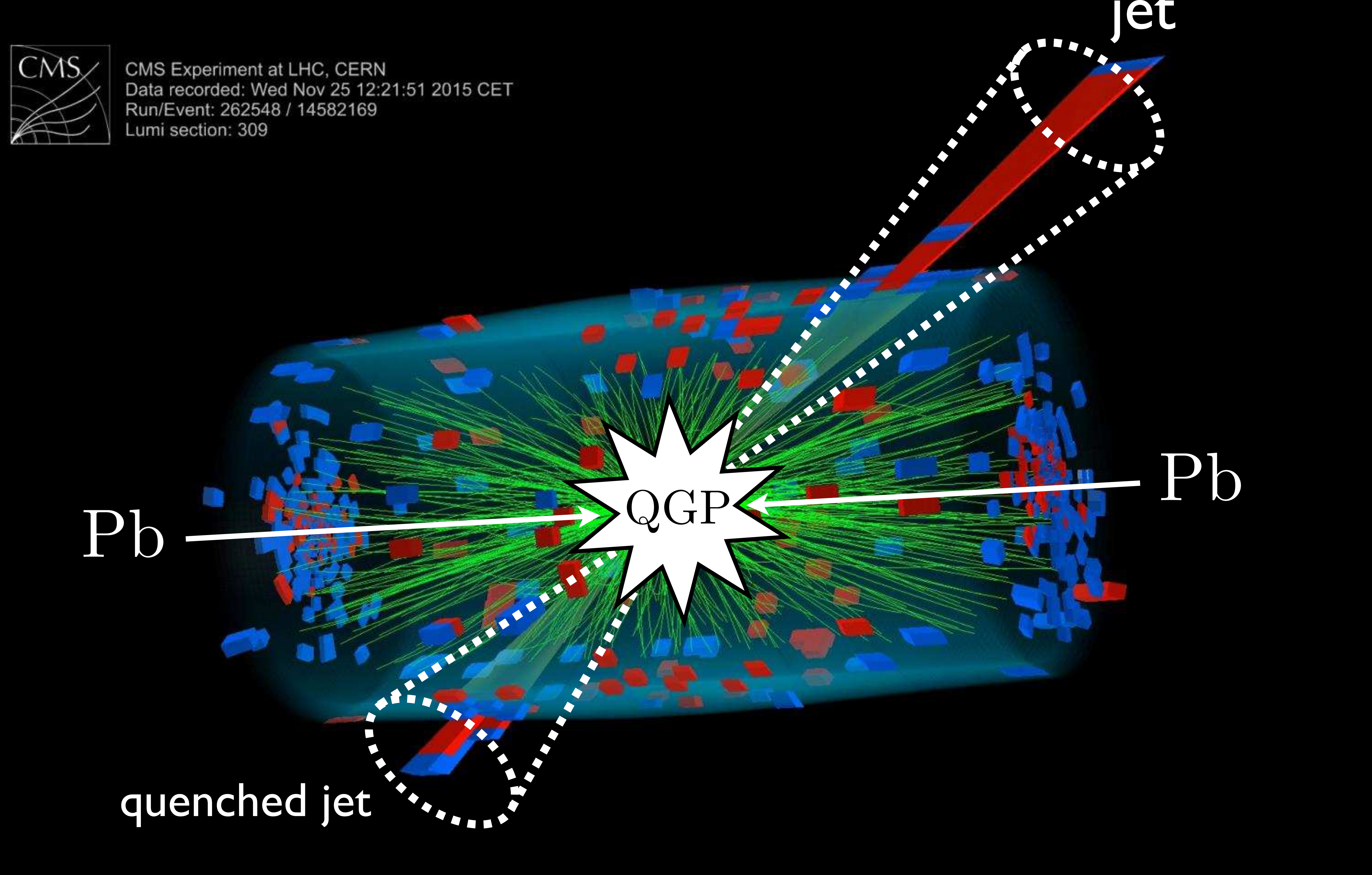
$$\Sigma\text{MD}(\mathcal{T}, \mathcal{T}') = \min_{\{\mathcal{F}\}} \sum_a \sum_b \mathcal{F}_{ab} \text{EMD}(\mathcal{E}_a, \mathcal{E}'_b)$$

(for equal total xsec)

A distance between theories!

(e.g. EMD : N-jettiness :: ΣMD : k-eventiness)

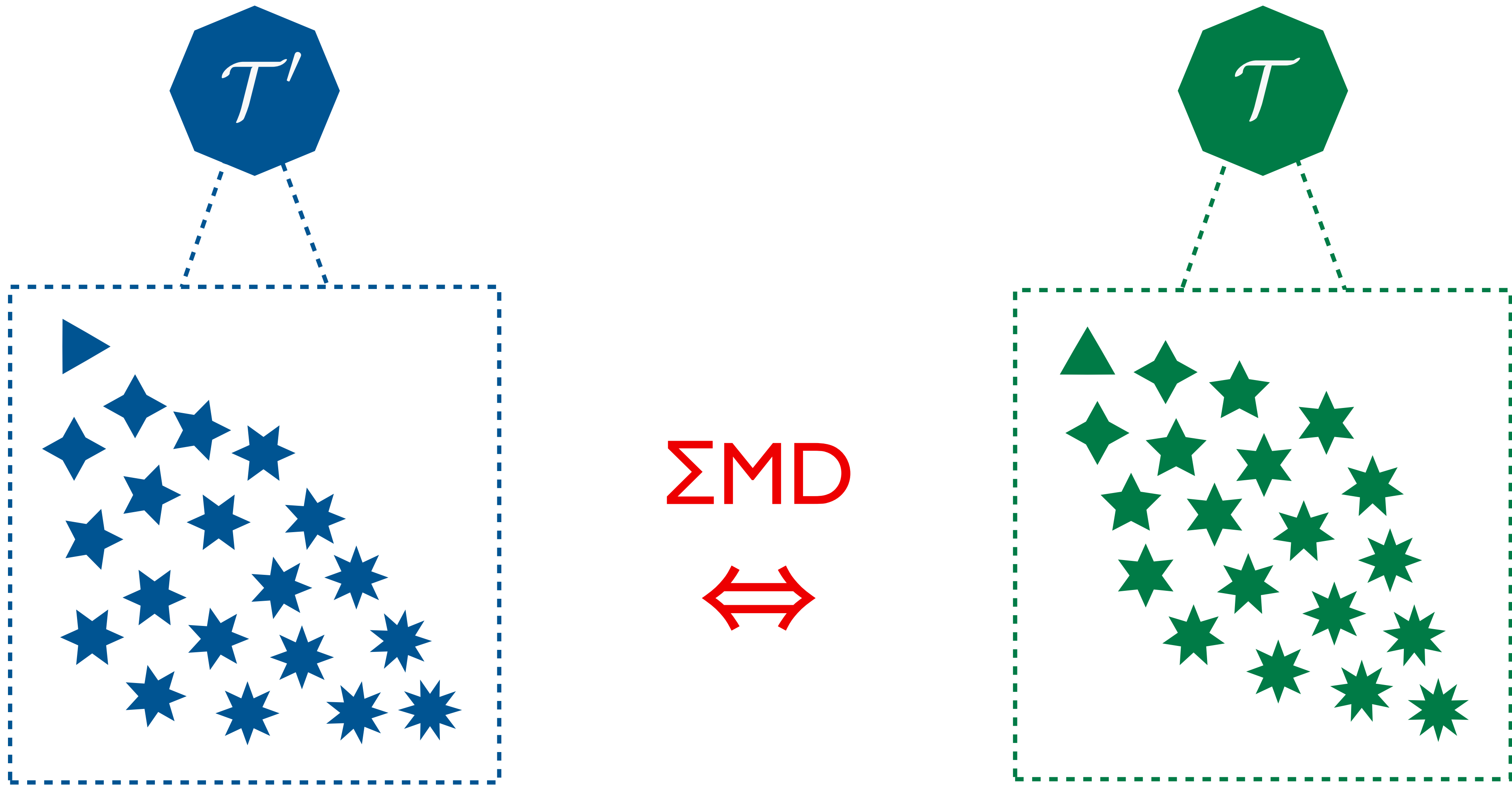
[Komiske, Metodiev, JDT, JHEP 2020]



jet
CMS Experiment at LHC, CERN
Data recorded: Wed Nov 25 12:21:51 2015 CET
Run/Event: 262548 / 14582169
Lumi section: 309
Pb
QGP
Pb
quenched jet

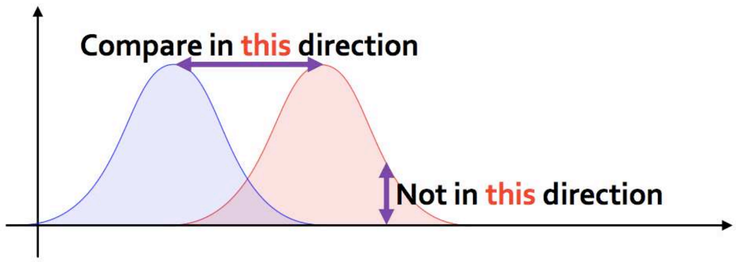
Theory Prime:
In-Medium QCD

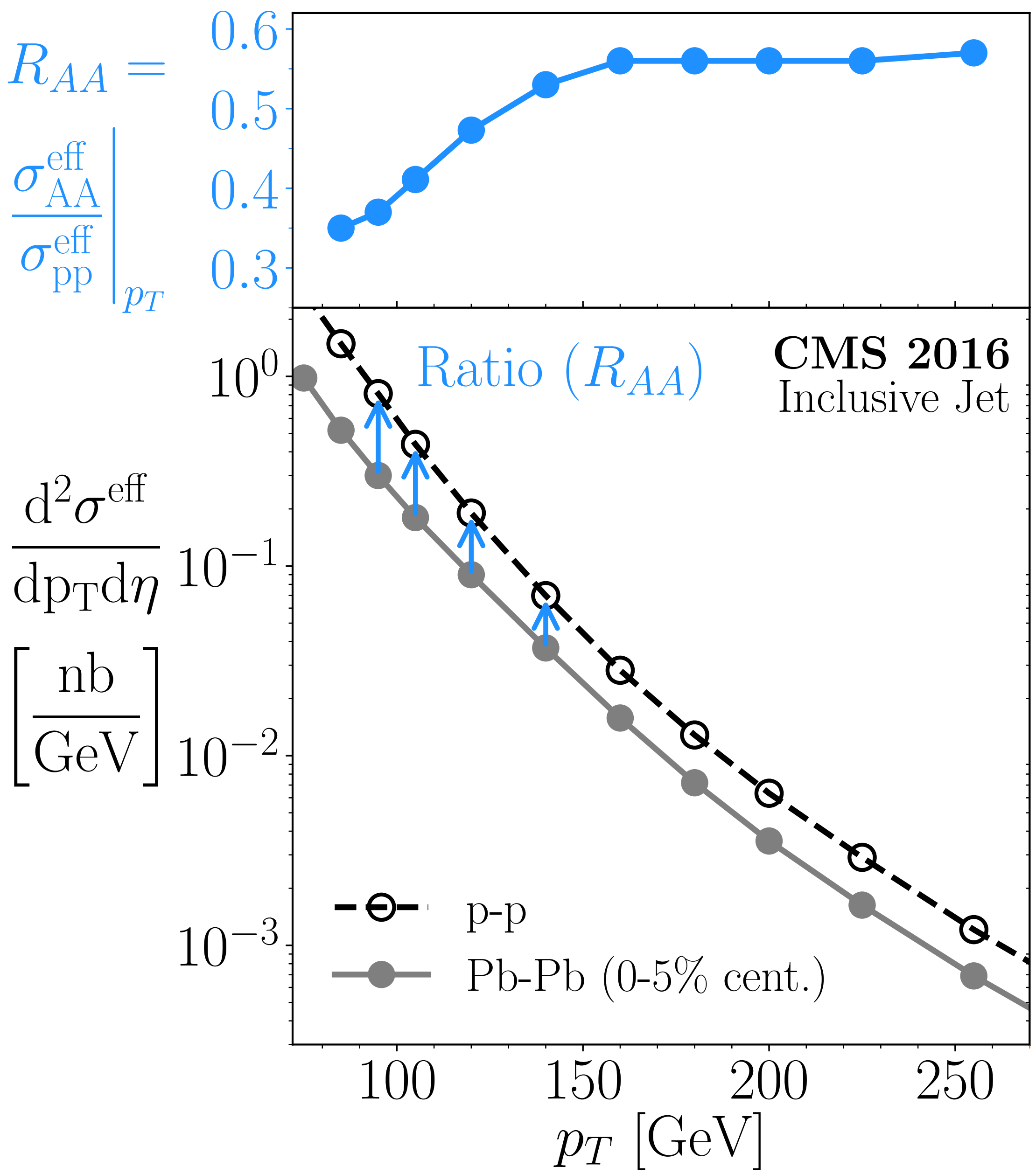
Theory:
Vacuum QCD



Optimal transportation plan defines mapping between in-medium jets and vacuum jets!

Jet Quenching via R_{AA}?

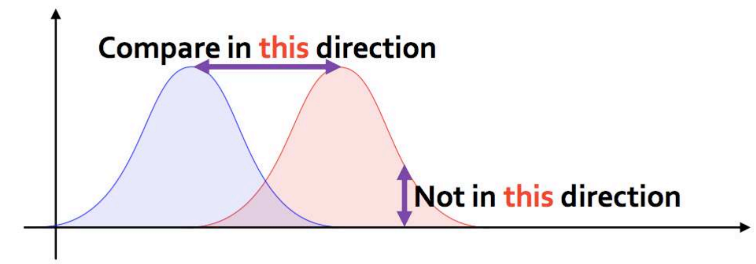




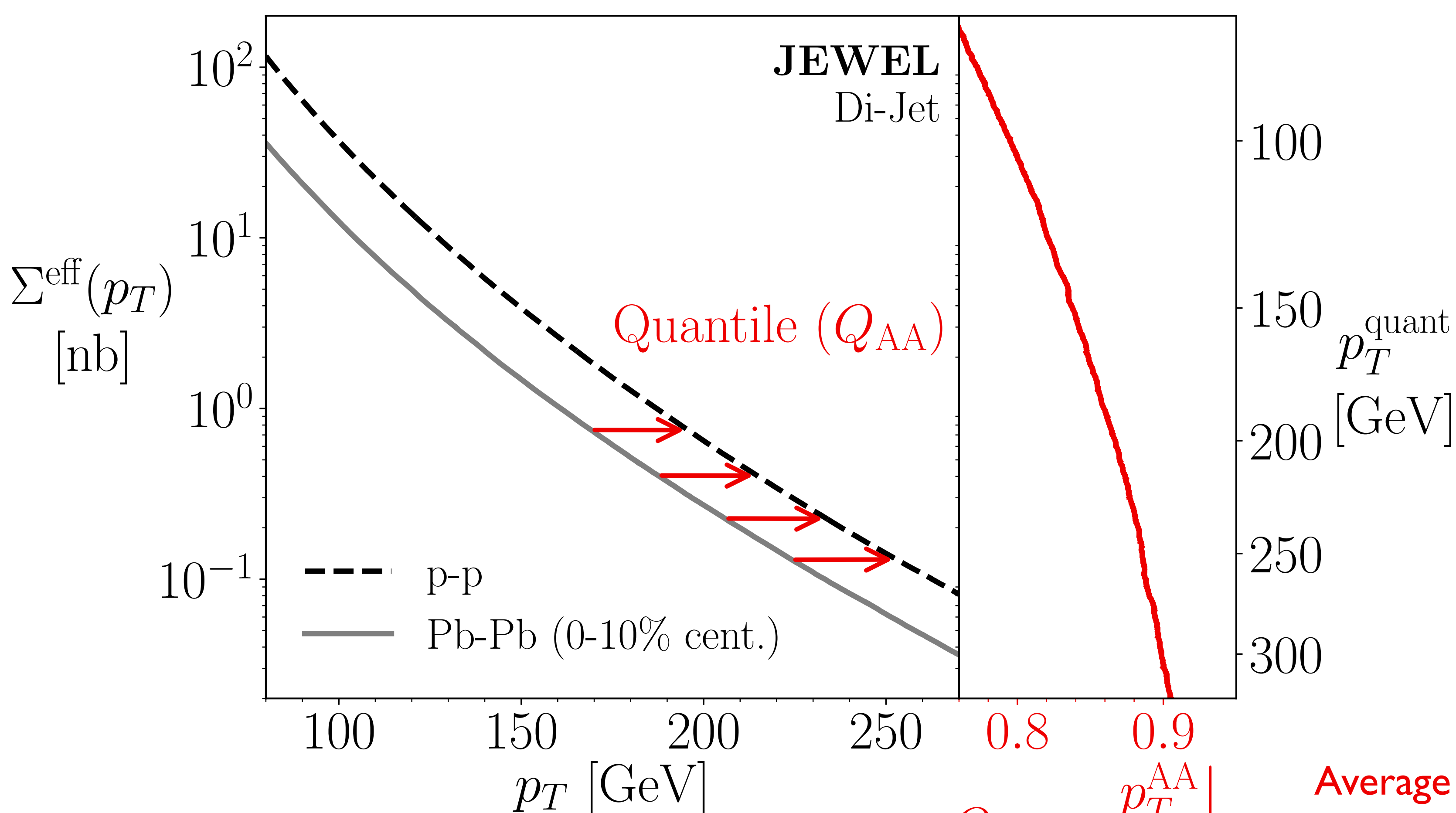
Average σ loss per p_T

Jet Quenching via QAA!

Quantile matching as optimal transport



(For the record, we had no idea about OT when we suggested this method)



$$Q_{\text{AA}} = \left.\frac{p_T^{\text{AA}}}{p_T^{\text{pp}}}\right|_{\Sigma^{\text{eff}}}$$

Average p_T loss per jet

[Brewer, Milhano, JDT, PRL 2019]