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Generation of matter antimatter asymmetries and hypermagnetic fields by the chiral vortical effect of transient fluctuations

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Content

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- Abelian anomaly and matter asymmetries
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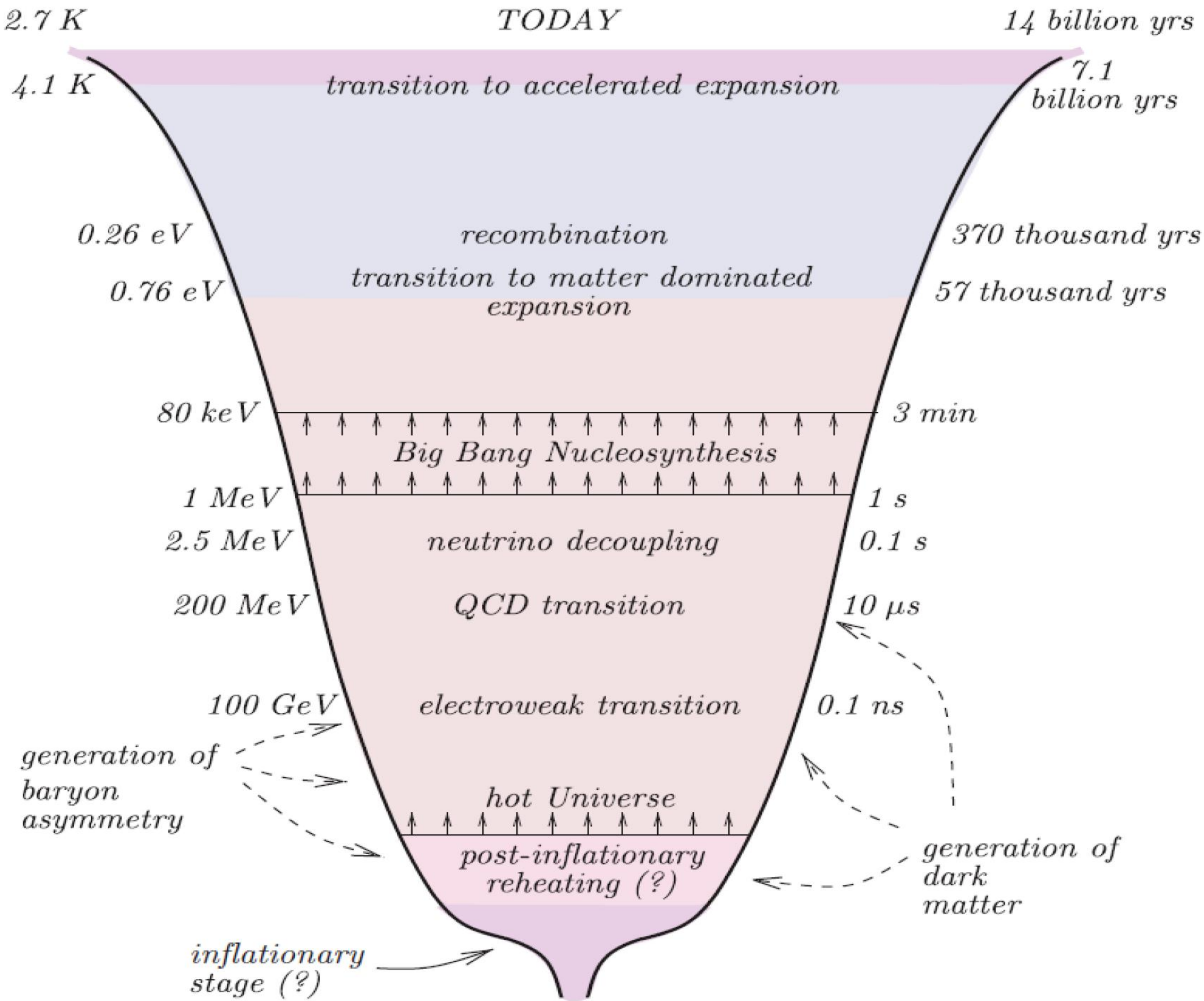
- Right-handed electrons
- Scenario



Stages of the evolution of the Universe

Observational data

- Baryon asymmetry of the Universe

  BBN and CMB: $\eta_B \sim 10^{-10}$

- Large scale magnetic fields

  Gamma rays from blazars:

$$B_0 \simeq 10^{-17} - 10^{-15}\mathrm{G} \qquad \lambda_0 \gtrsim 1\mathrm{Mpc}$$

Sakharov Conditions for Baryogenesis:

- B violation
- C and CP violation
- Departure from thermal equilibrium

The Chiral Coupling of $\mathrm{U_Y(1)}$ to Fermions

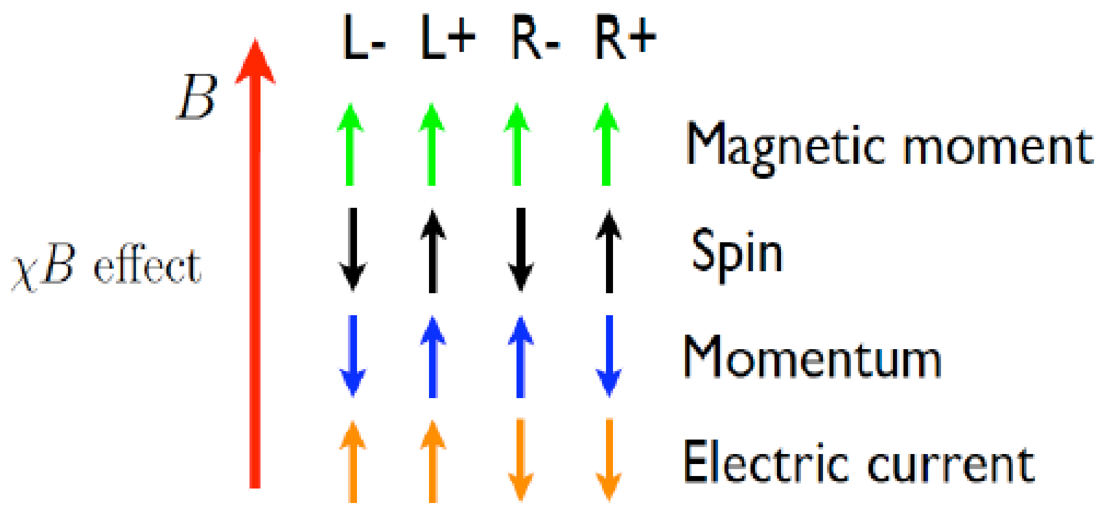
- Anomaly leading to fermion number violation

$$\partial_\mu\, j^\mu_{\rm bar} = \partial_\mu\, j^\mu_{\rm lep} = \frac{N_g}{2}\left(\frac{g^2}{16\pi^2} W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} - \frac{g'^{\,2}}{16\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right)$$

- The chiral magnetic and vortical effects

Chiral magnetic effect (CME)

The CME is the generation of the electric current parallel to the magnetic field.

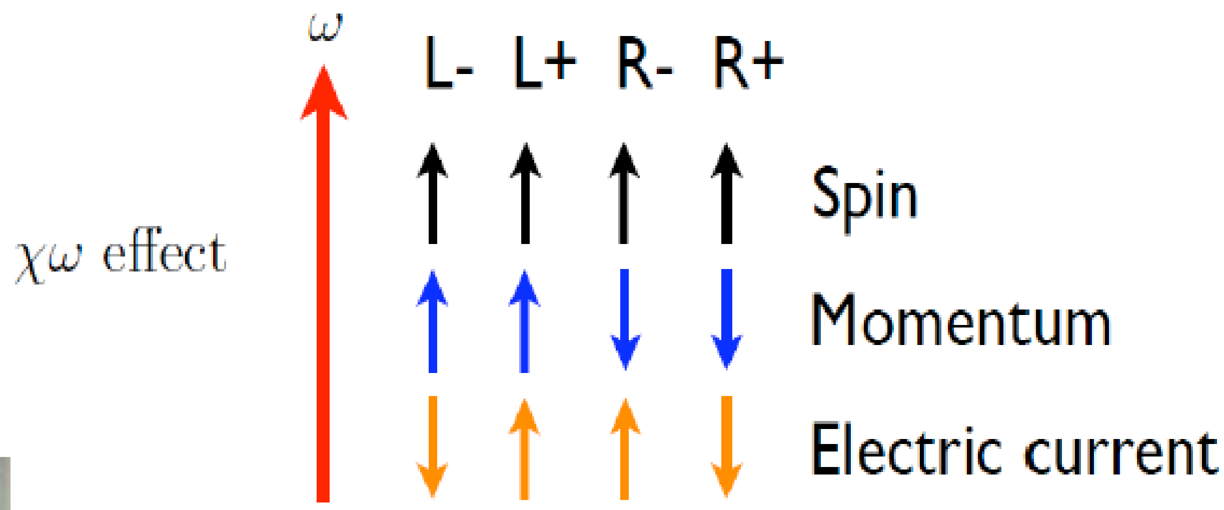


$$J_{\chi B} \propto [n(e_L^-) - n(e_R^+)] - [n(e_R^-) - n(e_L^+)]$$

$$\boldsymbol{J}_{\chi B} = \frac{e^2}{2\pi^2}\Delta\mu\, \boldsymbol{B}$$

Chiral vortical effect (CVE)

The CVE is the generation of the electric current along the vorticity field.



$$J_{\chi\omega} \propto [n(e_L^-) + n(e_R^+)] - [n(e_R^-) + n(e_L^+)]$$

$$\boldsymbol{J}_{\chi\omega} = \frac{e}{4\pi^2}\Delta\mu^2 \; \boldsymbol{\omega}$$

AMHD equations

$$\frac{1}{R}\vec{\nabla}\cdot\vec{E}_Y = 0, \quad \frac{1}{R}\vec{\nabla}\cdot\vec{B}_Y = 0,$$

$$\frac{1}{R}\vec{\nabla}\times\vec{E}_Y + \left(\frac{\partial \vec{B}_Y}{\partial t} + 2H\vec{B}_Y\right) = 0,$$

$$\frac{1}{R}\vec{\nabla}\times\vec{B}_Y - \left(\frac{\partial \vec{E}_Y}{\partial t} + 2H\vec{E}_Y\right) = \vec{J}$$

$$= \vec{J}_{\rm Ohm} + \vec{J}_{\rm cv} + \vec{J}_{\rm cm},$$

$$\vec{J}_{\rm Ohm} = \sigma\left(\vec{E}_Y + \vec{v}\times\vec{B}_Y\right),$$

$$\vec{J}_{\rm cv} = c_{\rm v}\vec{\omega},$$

$$\vec{J}_{\rm cm} = c_{\rm B}\vec{B}_Y,$$

Chiral vortical and magnetic coefficients

$$c_V(t) = \sum_{i=1}^{n_G} \Big[\frac{g'}{24} \Big(T_{R_i}^2 - T_{L_i}^2 + T_{d_{R_i}}^2 - 2T_{u_{R_i}}^2 + T_{Q_i}^2 \Big) + \frac{g'}{8\pi^2} \Big(\mu_{R_i}^2 - \mu_{L_i}^2 + \mu_{d_{R_i}}^2 - 2\mu_{u_{R_i}}^2 + \mu_{Q_i}^2 \Big) \Big]$$

$$c_B(t) = \frac{-g'^2}{8\pi^2} \sum_{i=1}^{n_G} \Big[-2\mu_{R_i} + \mu_{L_i} - \frac{2}{3}\mu_{d_{R_i}} - \frac{8}{3}\mu_{u_{R_i}} + \frac{1}{3}\mu_{Q_i} \Big]$$

The Navier-Stokes equations

$$\frac{\partial \rho}{\partial t} + \frac{1}{R}\vec{\nabla} \cdot \left[(\rho + p)\,\vec{v}\right] + 3H\,(\rho + p) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{1}{R}\left(\vec{v} \cdot \vec{\nabla}\right) + H\right]\vec{v} + \frac{\vec{v}}{\rho + p}\frac{\partial p}{\partial t}$$

$$= -\frac{1}{R}\frac{\vec{\nabla} p}{\rho + p} + \frac{\vec{J} \times \vec{B}_Y}{\rho + p} + \frac{\nu}{R^2}\left[\nabla^2 \vec{v} + \frac{1}{3}\vec{\nabla}\left(\vec{\nabla} \cdot \vec{v}\right)\right]$$

Chern-Simons configuration

$$\vec{B}_Y = (1/R)\vec{\nabla} \times \vec{A}_Y$$

$$\vec{A}_Y = \gamma(t)\,(\cos kz, \sin kz, 0)$$

$$\vec{v} = (1/R)\vec{\nabla} \times \vec{S}$$

$$\vec{S} = r(t)\,(\cos kz, \sin kz, 0)$$

The evolution equations of hypermagnetic and velocity fields

$$\vec{E}_Y = -\frac{k'}{\sigma}\vec{B}_Y + \frac{c_{\rm v}}{\sigma}k'\vec{v} - \frac{c_{\rm B}}{\sigma}\vec{B}_Y$$

$$\frac{dB_Y(t)}{dt} = \left[-\frac{1}{t} - \frac{k'^2}{\sigma} - \frac{c_{\rm B}k'}{\sigma}\right]B_Y(t) + \frac{c_{\rm v}}{\sigma}k'^2\, v(t)$$

$$\frac{\partial \vec{v}}{\partial t} = -\nu k'^2\vec{v}$$

$$\nu \simeq 1/(5\alpha_Y^2 T) \qquad k' = k/R = kT \qquad \sigma = 100T$$

Abelian anomaly equations

$$\nabla_\mu j^\mu_{e_R} = -\frac{1}{4}(Y_R^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu} = \frac{g'^2}{4\pi^2}\vec{E}_Y \cdot \vec{B}_Y,$$

$$\nabla_\mu j^\mu_{e_L} = \frac{1}{4}(Y_L^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu} = -\frac{g'^2}{16\pi^2}\vec{E}_Y \cdot \vec{B}_Y$$

FRW metric $\quad ds^2 = dt^2 - R^2(t)\delta_{ij}dx^i dx^j$

Right-handed Electrons

Chirality flip processes:

$$e_L \bar{e}_R \leftrightarrow \phi^{(0)} \qquad \nu_e^L \bar{e}_R \leftrightarrow \phi^{(+)}$$

are out of thermal equilibrium For $T > T_{RL} \simeq 10 \text{ TeV}$.

Evolution equations of the asymmetries

$$\frac{d\eta_{e_R}}{dt} = \frac{g'^2}{4\pi^2 s}\langle \vec{E}_Y \cdot \vec{B}_Y \rangle + \left(\frac{\Gamma_0}{t_{EW}}\right)\left(\frac{1-x}{\sqrt{x}}\right)(\eta_{e_L} - \eta_{e_R})$$

$$\frac{d\eta_{\nu_e^L}}{dt} = \frac{d\eta_{e_L}}{dt} = -\frac{g'^2}{16\pi^2 s}\langle \vec{E}_Y \cdot \vec{B}_Y \rangle + \left(\frac{\Gamma_0}{2t_{EW}}\right)\left(\frac{1-x}{\sqrt{x}}\right)(\eta_{e_R} - \eta_{e_L})$$

$$\frac{d\eta_{\mathrm{B}}}{dt} = \frac{3g'^2}{8\pi^2 s}\langle \vec{E}_Y \cdot \vec{B}_Y \rangle$$

$$\langle \vec{E}_Y \cdot \vec{B}_Y \rangle = \frac{B_Y^2(t)}{100}\left[-\frac{k'}{T} - \frac{6sg'^2}{4\pi^2 T^3}\left(\eta_{e_R} - \frac{\eta_{e_L}}{2} + \frac{3}{8}\eta_{\mathrm{B}}\right)\right] + \left[\frac{g'}{24}\beta[x(T)] + \frac{36s^2 g'}{8\pi^2 T^6}\left(\eta_{e_R}^2 - \eta_{e_L}^2\right)\right]\frac{k'T}{100}\langle \vec{v}(t) \cdot \vec{B}_Y(t) \rangle$$

Parameters appearing in the equations

$$\eta_f = (n_f/s) \text{ with } f = e_R, e_L, \nu_e^L \qquad s = 2\pi^2 g^* T^3/45$$

$$g^* = 106.75 \qquad x = (t/t_{\mathrm{EW}}) = (T_{\mathrm{EW}}/T)^2$$

$$\Gamma_0 = 121,\ t_{\mathrm{EW}} = \left(M_0/2T_{\mathrm{EW}}^2\right) \qquad M_0 = \left(M_{\mathrm{Pl}}/1.66\sqrt{g^*}\right)$$

Gaussian fluctuations

$$\beta[x(T)] = \Delta T^2 / T^2$$

$$\beta(x) = \frac{\beta_0}{b\sqrt{2\pi}} \exp\left[-\frac{(x - x_0)^2}{2b^2}\right]$$

$$\omega(x) = k' v(x) = \frac{k' v_0}{b\sqrt{2\pi}} \exp\left[-\frac{(x - x_0)^2}{2b^2}\right]$$

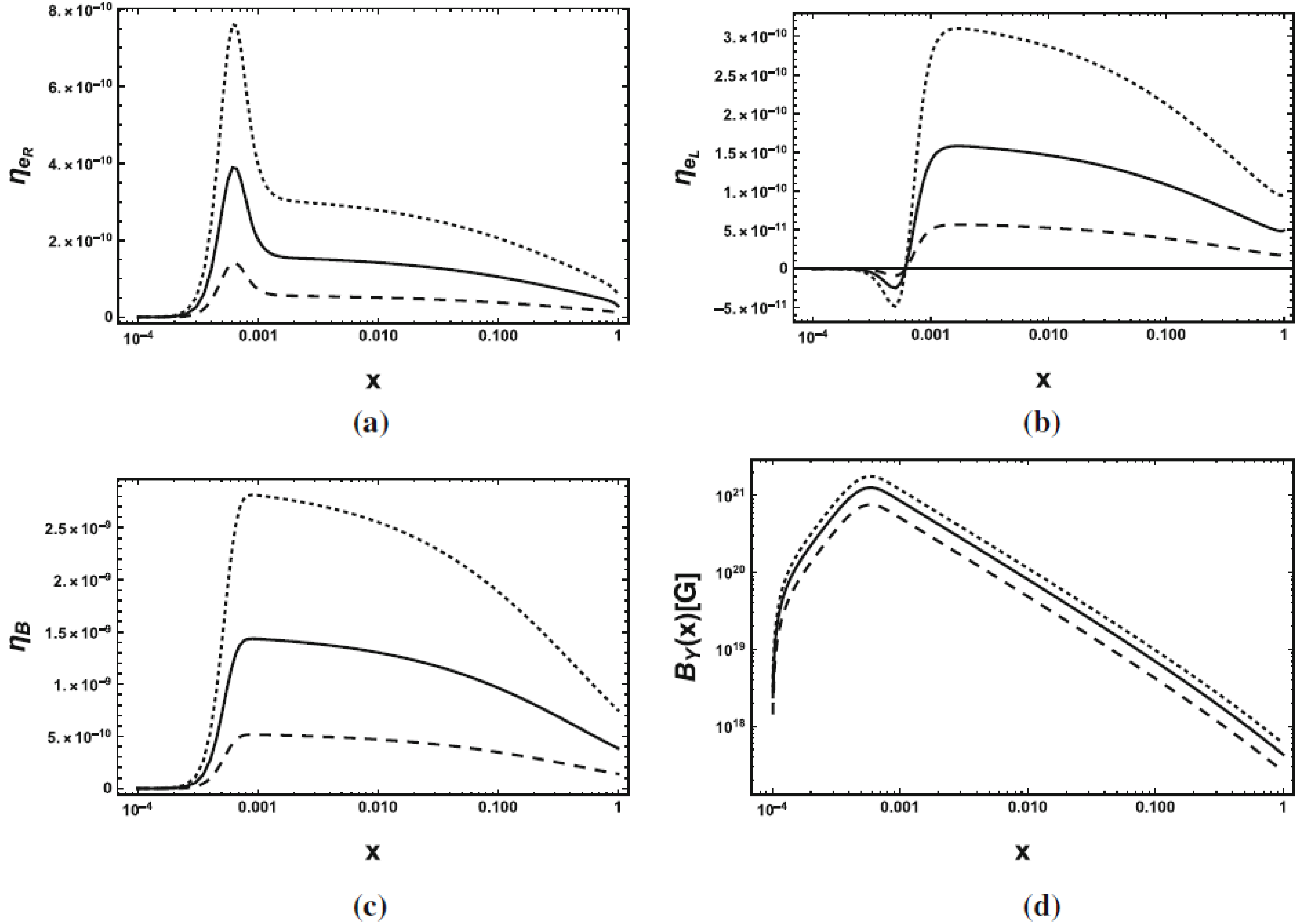


Fig. 1 Time plots of: **a** the right-handed electron asymmetry η_{e_R}, **b** the left-handed electron asymmetry η_{e_L}, **c** the baryon asymmetry η_B, and **d** the hypermagnetic field amplitude B_Y, for various values of the amplitude of temperature fluctuation of e_R. The initial conditions are: $k = 10^{-7}, B_Y^{(0)} = 0, \eta_{e_R}^{(0)} = \eta_{e_L}^{(0)} = \eta_B^{(0)} = 0, v_0 = 10^{-5}, b = 2\times 10^{-4}$, and $x_0 = 45 \times 10^{-5}$. The dashed line is for $\beta_0 = 3 \times 10^{-4}$, the solid line is for $\beta_0 = 5 \times 10^{-4}$, and the dotted line is for $\beta_0 = 7 \times 10^{-4}$

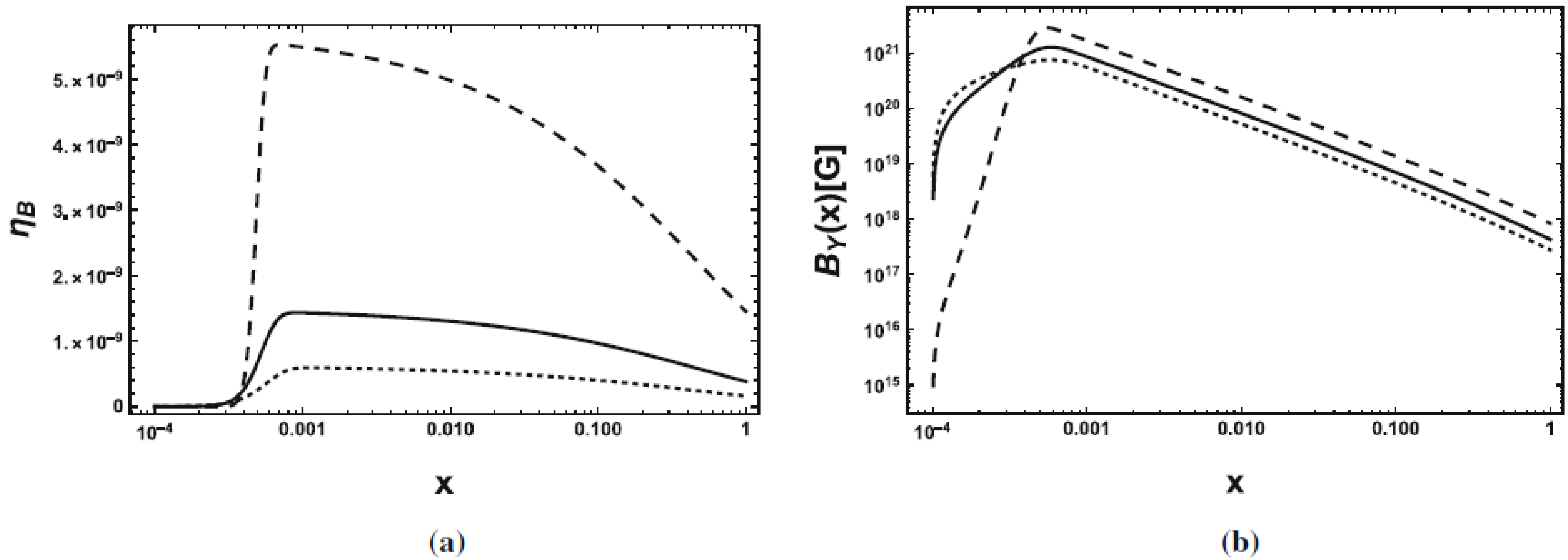


Fig. 2 Time plots of: **a** the baryon asymmetry η_B, and **b** the hypermagnetic field amplitude B_Y, for various values of the width of fluctuations. The initial conditions are: $k = 10^{-7}$, $B_Y^{(0)} = 0$, $\eta_{e_R}^{(0)} = \eta_{e_L}^{(0)} = \eta_B^{(0)} = 0$, $v_0 = 10^{-5}$, $\beta_0 = 5 \times 10^{-4}$, and $x_0 = 45 \times 10^{-5}$. The dotted line is obtained for $b = 3 \times 10^{-4}$, the solid line for $b = 2 \times 10^{-4}$, the dashed line for $b = 10^{-4}$

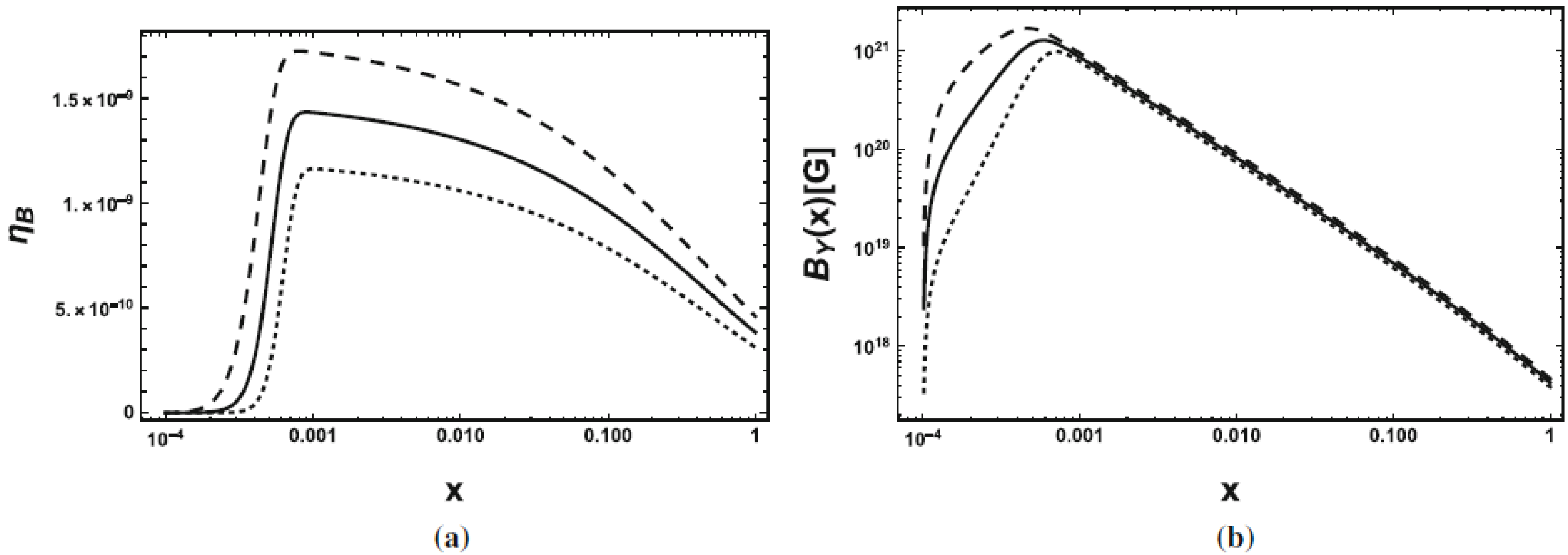


Fig. 3 Time plots of: **a** the baryon asymmetry η_B, and **b** the hypermagnetic field amplitude B_Y, for various values of the time of fluctuations. The initial conditions are: $k = 10^{-7}$, $B_Y^{(0)} = 0$, $\eta_{e_R}^{(0)} = \eta_{e_L}^{(0)} = \eta_B^{(0)} = 0$, $v_0 = 10^{-5}$, $\beta_0 = 5 \times 10^{-4}$, $b = 2 \times 10^{-4}$. The dotted line is obtained for $x_0 = 55 \times 10^{-5}$, the solid line for $x_0 = 45 \times 10^{-5}$, and the dashed line for $x_0 = 35 \times 10^{-5}$

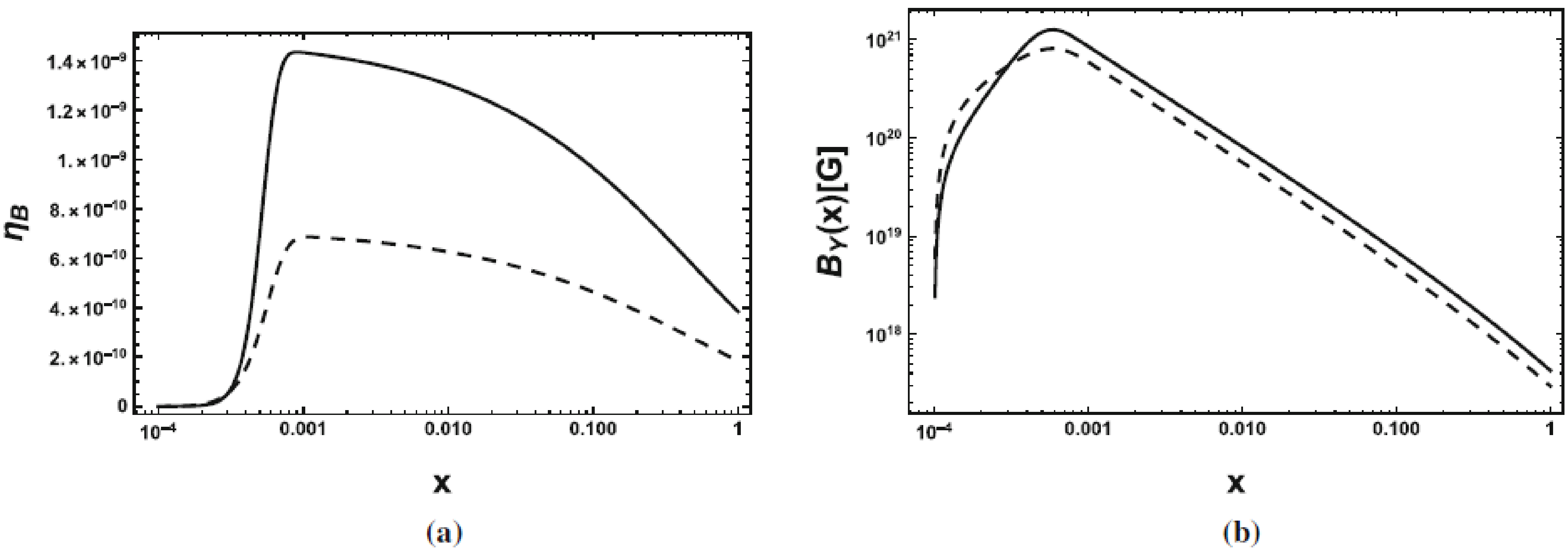


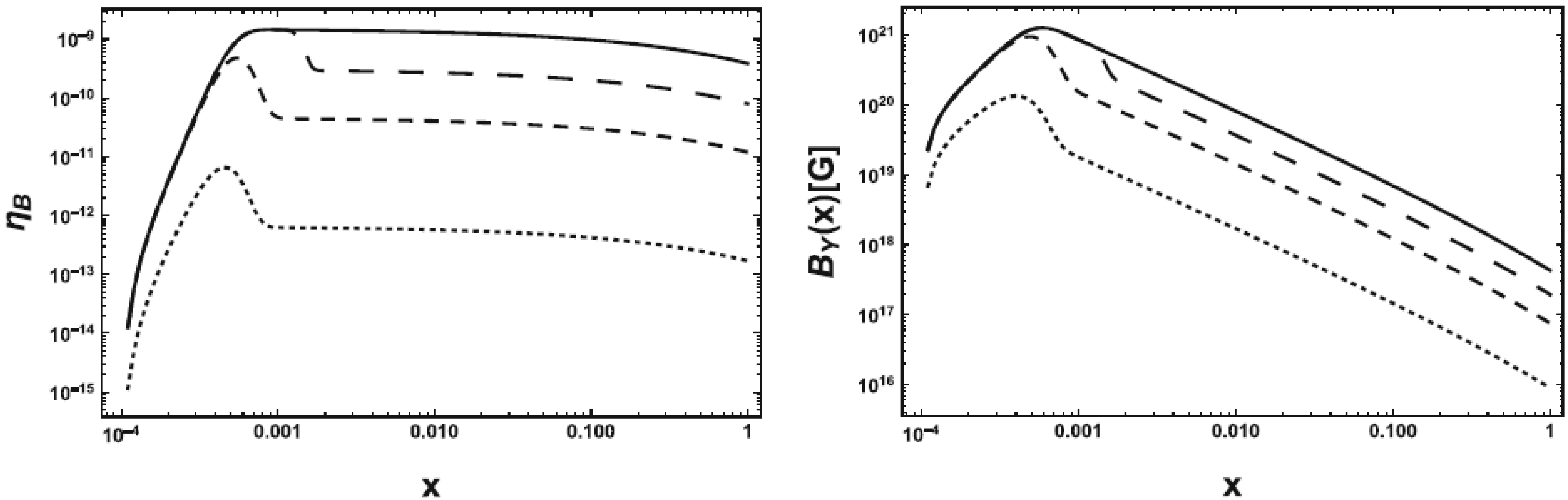
Fig. 4 Time plots of: **a** the baryon asymmetry η_B, and **b** the hypermagnetic field amplitude B_Y, for two different vorticity configurations. The initial conditions are: $k = 10^{-7}$, $B_Y^{(0)} = 0$, $\eta_{e_R}^{(0)} = \eta_{e_L}^{(0)} = \eta_B^{(0)} = 0$, $\beta_0 = 5 \times 10^{-4}$, $b = 2 \times 10^{-4}$, and $x_0 = 45 \times 10^{-5}$. The solid line is for vorticity fluctuation with $v_0 = 10^{-5}$, and the dashed line is for constant vorticity with $v_0 = 10^{-2}$

Two sets of Gaussian fluctuations

$$\beta(x) = \beta_+(x) + \beta_-(x) \qquad \beta_\pm(x) = \frac{\pm\beta_0}{b\sqrt{2\pi}} \exp\left[-\frac{(x - x_{0,\pm})^2}{2b^2}\right]$$

$$v(x) = v_+(x) + v_-(x) \qquad v_\pm(x) = \frac{v_0}{b\sqrt{2\pi}} \exp\left[-\frac{(x - x_{0,\pm})^2}{2b^2}\right]$$

$$\Delta x_0 = x_{0,+} - x_{0,-}$$



(a) **(b)**

Fig. 5 Time plots of: **a** the baryon asymmetry η_B, **b** the hypermagnetic field amplitude B_Y for two sets of successive and opposing fluctuations. The initial conditions are: $k = 10^{-7}$, $B_Y^{(0)} = 0$, $\eta_{e_R}^{(0)} = \eta_{e_L}^{(0)} = \eta_B^{(0)} = 0$, $\upsilon_{0,+} = \upsilon_{0,-} = 10^{-5}$, $b = 2 \times 10^{-4}$, $\beta_{0,+} = -\beta_{0,-} = 5 \times 10^{-4}$, and $x_{0,+} = 4.5 \times 10^{-4}$. The large dashed line is for $x_{0,-} = 1.45 \times 10^{-3} = 5b + x_{0,+}$, the medium dashed line is for $x_{0,-} = 6.5 \times 10^{-4} = b + x_{0,+}$, the dotted line is for $x_{0,-} = 4.7 \times 10^{-4} = 0.1b + x_{0,+}$, and the solid line is obtained in the absence of the second set of fluctuations

Thank You

$$\frac{d\eta_{e_R}}{dx} = [-C_1 - C_2\eta_T(x)] \left(\frac{B_Y(x)}{10^{20}G}\right)^2 x^{3/2}$$

$$+ \left[C_3\beta(x) + C_4\Delta\eta^2(x)\right] v(x) \left(\frac{B_Y(x)}{10^{20}G}\right) \sqrt{x}\Bigg] - \Gamma_0 \frac{1-x}{\sqrt{x}} \left[\eta_{e_R}(x) - \eta_{e_L}(x)\right],$$

$$\frac{d\eta_{e_L}}{dx} = -\frac{1}{4}[-C_1 - C_2\eta_T(x)] \left(\frac{B_Y(x)}{10^{20}G}\right)^2 x^{3/2}$$

$$-\frac{1}{4}\left[C_3\beta(x) + C_4\Delta\eta^2(x)\right] v(x) \left(\frac{B_Y(x)}{10^{20}G}\right) \sqrt{x}\Bigg] + \Gamma_0 \frac{1-x}{2\sqrt{x}} \left[\eta_{e_R}(x) - \eta_{e_L}(x)\right],$$

$$\frac{dB_Y}{dx} = \frac{1}{\sqrt{x}}[-C_5 - C_6\eta_T(x)] B_Y(x) - \frac{1}{x}B_Y(x) + \left[C_7\beta(x) + C_8\Delta\eta^2(x)\right] \frac{v(x)}{x^{3/2}},$$

$$\Delta\eta^2(x) = \eta^2_{e_R}(x) - \eta^2_{e_L}(x),$$

$$\eta_T(x) = \eta_{e_R}(x) - \frac{\eta_{e_L}(x)}{2} + \frac{3}{8}\eta_{\mathrm{B}(x)}$$

$$\beta(x) = \frac{\beta_0}{b\sqrt{2\pi}} \exp\left[-\frac{(x-x_0)^2}{2b^2}\right],$$

$$\upsilon(x) = \frac{\upsilon_0}{b\sqrt{2\pi}} \exp\left[-\frac{(x-x_0)^2}{2b^2}\right]$$

$$M = 2\pi^2 g^*/45$$

$$\alpha_Y = g'^2/4\pi \simeq 0.01$$

$$C_1 = 0.00096\left(\frac{k}{10^{-7}}\right)\alpha_Y,$$

$$C_2 = 865688\alpha_Y^2,$$

$$C_3 = 0.71488\left(\frac{k}{10^{-7}}\right)\alpha_Y^{3/2},$$

$$C_4 = 17152.7\left(\frac{k}{10^{-7}}\right)\alpha_Y^{3/2},$$

$$C_5 = 0.356\left(\frac{k}{10^{-7}}\right)^2,$$

$$C_6 = 3.18373\times 10^8\alpha_Y\left(\frac{k}{10^{-7}}\right),$$

$$C_7 = 262.9\times 10^{20}\sqrt{\alpha_Y}\left(\frac{k}{10^{-7}}\right)^2,$$

$$C_8 = 63\times 10^{25}\sqrt{\alpha_Y}\left(\frac{k}{10^{-7}}\right)^2,$$

Anomaly equations in the symmetric phase of the MSM

$$
\begin{aligned}
\partial_\mu j^\mu_{Q^i} &= +\tfrac{1}{2}(N_w)\frac{g_s^2}{16\pi^2}G^A_{\mu\nu}\tilde{G}^{A\,\mu\nu} + \tfrac{1}{2}(N_c)\frac{g^2}{16\pi^2}W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} + \tfrac{1}{4}(N_cN_w y_Q^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}\\
\partial_\mu j^\mu_{u^i_R} &= -\tfrac{1}{2}\frac{g_s^2}{16\pi^2}G^A_{\mu\nu}\tilde{G}^{A\,\mu\nu} - \tfrac{1}{4}(N_c y_{u_R}^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}\\
\partial_\mu j^\mu_{d^i_R} &= -\tfrac{1}{2}\frac{g_s^2}{16\pi^2}G^A_{\mu\nu}\tilde{G}^{A\,\mu\nu} - \tfrac{1}{4}(N_c y_{d_R}^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}\\
\partial_\mu j^\mu_{L^i} &= \tfrac{1}{2}\frac{g^2}{16\pi^2}W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} + \tfrac{1}{4}(N_w y_L^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}\\
\partial_\mu j^\mu_{e^i_R} &= -\tfrac{1}{4}(y_{e_R}^2)\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}
\end{aligned}
$$

$$
y_Q = \frac{1}{3}\,, \quad y_{u_R} = \frac{4}{3}\,, \quad y_{d_R} = -\frac{2}{3}\,, \quad y_L = -1\,, \quad y_{e_R} = -2
$$

$$
N_c = 3 \text{ and } N_w = 2
$$

Name	Particle Reaction	Rate	Chemical Equilibrium
Up-Type Yukawa	$d_L^i + \Phi^+ \leftrightarrow u_R^i$ $u_L^i + \Phi^0 \leftrightarrow u_R^i$	$\frac{h_{u^i}^2}{8\pi}T$	$\mu_{Q^i} + \mu_\Phi - \mu_{u_R^i} = 0$
Down-Type Yukawa	$u_L^i \leftrightarrow \Phi^+ + d_R^i$ $d_L^i \leftrightarrow \Phi^0 + d_R^i$	$\frac{h_{d^i}^2}{8\pi}T$	$\mu_{Q^i} - \mu_\Phi - \mu_{d_R^i} = 0$
Electron-Type Yukawa	$\nu_L^i \leftrightarrow \Phi^+ + e_R^i$ $e_L^i \leftrightarrow \Phi^0 + e_R^i$	$\frac{h_{e^i}^2}{8\pi}T$	$\mu_{L^i} - \mu_\Phi - \mu_{e_R^i} = 0$
Strong Sphaleron	$\sum_i (u_L^i + d_L^i) \leftrightarrow \sum_i (u_R^i + d_R^i)$	$100\alpha_{\mathrm{s}}^5 T$	$\sum_i \left(2\mu_{Q^i} - \mu_{u_R^i} - \mu_{d_R^i}\right) = 0$
Weak Sphaleron	$\sum_i (u_L^i + d_L^i + d_L^i + \nu_L^i) \leftrightarrow 0$ $\sum_i (u_L^i + u_L^i + d_L^i + e_L^i) \leftrightarrow 0$	$25\alpha_{\mathrm{w}}^5 T$	$\sum_i (3\mu_{Q^i} + \mu_{L^i}) = 0$