# Introduction to the CGC effective theory

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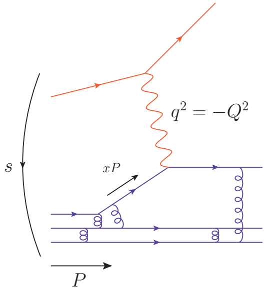
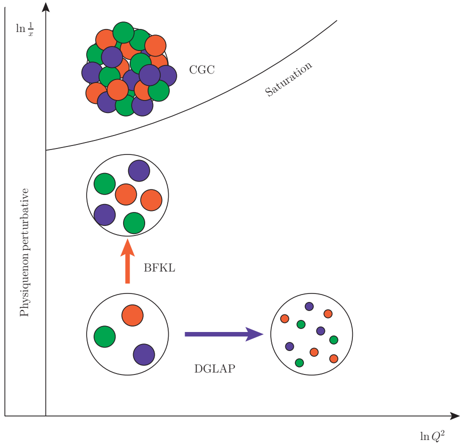
# Overview

1. Probing a single hadron
2. The Color Glass Condensate
3. Hadron-hadron collisions within the CGC framework
4. Summary

# Accessing the partonic content of hadrons with an electromagnetic probe

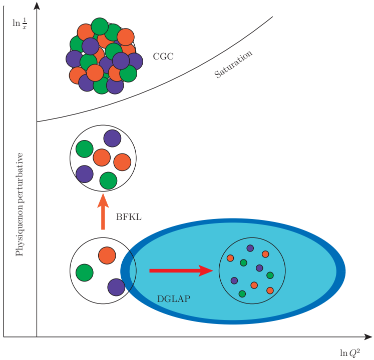



Electron-proton collision (parton model)



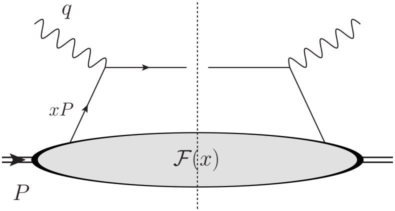
# QCD at moderate $x_B = Q^2/s$

$Q^2 \sim s$



## QCD factorization

processes with a hard scale $Q \gg \Lambda_{QCD}$




$$\sigma = \mathcal{F}(x,\mu) \otimes \mathcal{H}(x,\mu)$$

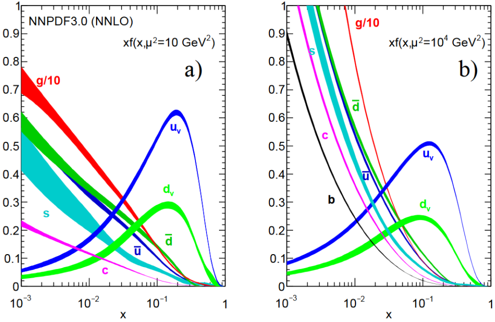
At a scale $\mu$, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(x,\mu)$
- A Parton Distribution Function (PDF) $\mathcal{F}(x,\mu)$

$\mu$ independence: DGLAP renormalization equation for $\mathcal{F}$

## Parton Distribution Functions

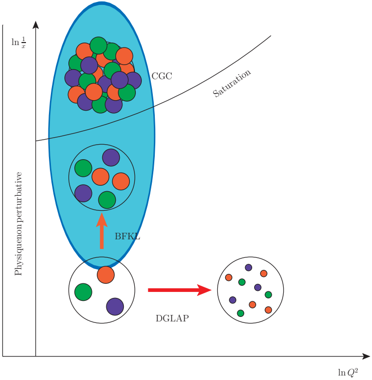
# Gluon exchanges dominate at small $x$




[NNLO NNPDF3.0 global analysis, taken from PDG2018]

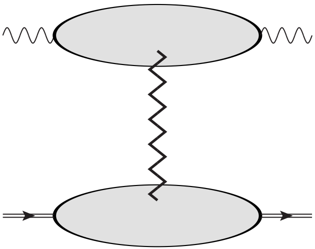
# QCD at small $x_B = Q^2/s$

$$Q^2 \ll s$$



# The Pomeron

Regge theory: for asymptotic values of $s$, an effective particle with the quantum numbers of the vacuum is exchanged

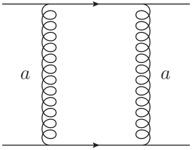


Positive $C$-parity: Pomeron exchange, negative $C$-parity: Odderon exchange

- How can we understand the Pomeron and the Odderon in perturbative QCD?
- How does it couple to hadrons?

# Naive perturbative Pomeron and Odderon

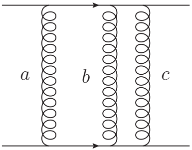
Naive perturbative description of the target hadron




Two gluons on a color singlet state

$\mathrm{tr}(t^a t^a)$

Leading Pomeron




Three gluons on a color singlet state

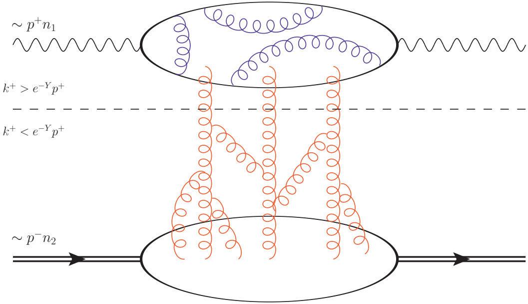
$\mathrm{tr}(t^a t^b t^c) = \frac{1}{4}(d^{abc} + i f^{abc})$

$f^{abc}$: subleading Pomeron
$d^{abc}$: leading Odderon

More involved but still for perturbative targets: BFKL, BKP, BLV...

Most general framework: small-$x$ semiclassical effective theory

# Effective semiclassical description of small $x$ QCD

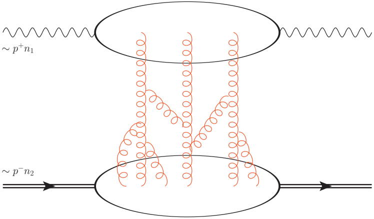


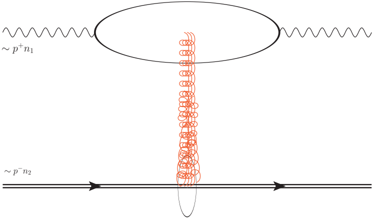

Let us split the gluonic field between "fast" and "slow" gluons

$$A^{\mu a}(k^+, k^-, \vec{k}) \rightarrow \mathcal{A}^{\mu a}_{Y_c}(|k^+| > e^{-Y_c} p^+, k^-, \vec{k}) + A^{\mu a}_{Y_c}(|k^+| < e^{-Y_c} p^+, k^-, \vec{k})$$

$e^{-Y_c} \ll 1$

# Large longitudinal boost to the projectile frame




$$A^+(x^+, x^-, \vec{x}) \longrightarrow \frac{1}{\Lambda} A^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$A^-(x^+, x^-, \vec{x}) \longrightarrow \Lambda A^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

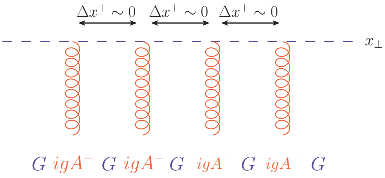
$$A^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad A^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

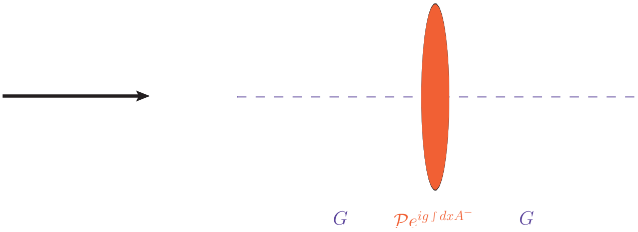
$$A^\mu(x) \to A^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

*Shockwave* approximation

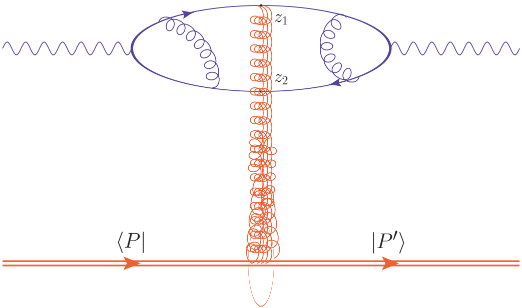
# Effective Feynman rules in the slow background field

The interactions with the background field can be exponentiated



# Factorized picture




Factorized amplitude

$$\mathcal{A}^{Y_c} = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U_{\vec{z}_1}^{Y_c} U_{\vec{z}_2}^{Y_c\dagger}) - N_c]|P\rangle$$

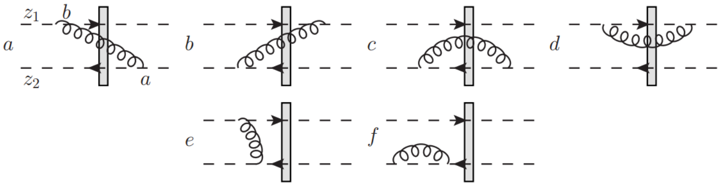
Dipole operator $\mathcal{U}_{ij}^{Y_c} = \frac{1}{N_c}\mathrm{Tr}(U_{\vec{z}_i}^{Y_c} U_{\vec{z}_j}^{Y_c\dagger}) - 1$

Written similarly for any number of Wilson lines in any color representation!

$Y_c$ independence: B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

# Evolution for the dipole operator

$$\mathcal{U}_{12}^{Y_c+\delta Y} - \mathcal{U}_{12}^{Y_c}$$




B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

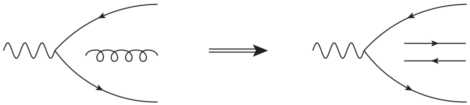
$$\frac{\partial \mathcal{U}_{12}^{Y_c}}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[ \mathcal{U}_{13}^{Y_c} + \mathcal{U}_{32}^{Y_c} - \mathcal{U}_{12}^{Y_c} + \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{Y_c}\mathcal{U}_{32}^{Y_c}}{\partial Y_c} = \ldots$$

Evolves a dipole into a double dipole

# The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky] [Kovchegov]

$$\frac{\partial \left\langle \mathcal{U}_{12}^{Y_c} \right\rangle}{\partial Y_c} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[ \left\langle \mathcal{U}_{13}^{Y_c} \right\rangle + \left\langle \mathcal{U}_{32}^{Y_c} \right\rangle - \left\langle \mathcal{U}_{12}^{Y_c} \right\rangle + \left\langle \mathcal{U}_{13}^{Y_c} \right\rangle \left\langle \mathcal{U}_{32}^{Y_c} \right\rangle \right]$$

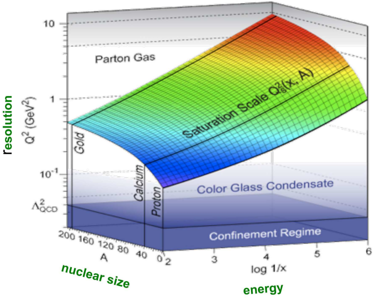
BFKL/BKP part      Triple pomeron vertex

Non-linear term : one type of saturation

Non-perturbative elements are compatible with CGC-type models

# Saturation scale: a quick estimate

## The saturation scale $Q_s$




Gluons per unit area

$$\rho \propto \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

Recombination cross section

$$\sigma_{gg\to g} \propto \frac{\alpha_s}{Q^2}$$

Saturation starts when $\rho\sigma \simeq 1$, which means $Q_s^2$ solves

$$Q_s^2 \propto \alpha_s \frac{xG_A(x, Q_s^2)}{\pi R_A^2}.$$

$$Q_s^2 \propto A^{1/3} x^{-0.3}$$

# Loop corrections: probing a single hadron

One-loop corrections with saturation effects: state of the art

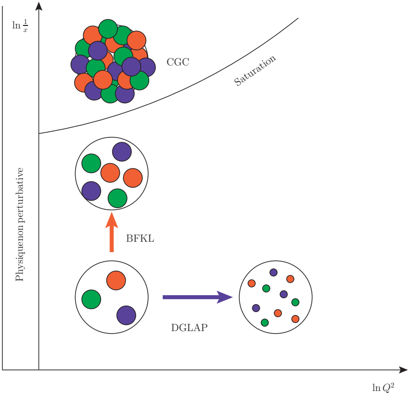
## Evolution

- Monopole and dipole evolution [Balitsky, Chirilli]
- 3-point operator evolution [Balitsky, Gerasimov, Grabovsky]
- 4-point operator evolution [Grabovsky]
- Full JIMWLK Hamiltonian [Kovner, Lublinsky, Mulian]

## Observables

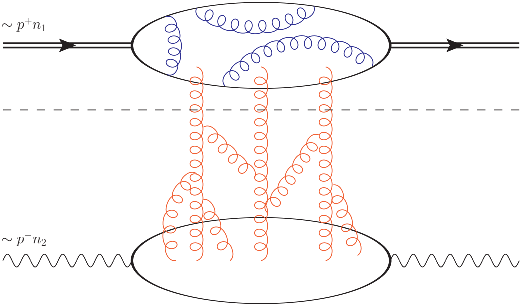
- Fully inclusive Deep Inelastic Scattering [Balitsky, Chirilli], [Beuf], [Hänninen, Lappi, Paatelainen]
- (Semi-inclusive) Photon+dijet in for *ep* and *eA* [Roy, Venugopalan]
- Exclusive dijet in *ep*, *eA*, $\gamma p$ or $\gamma A$ [RB, Grabovsky, Szymanowski, Wallon]
- Exclusive light vector meson in *ep* and *eA* [RB, Ivanov, Grabovsky, Szymanowski, Wallon]

# Summary: probing a single hadron



# Overview

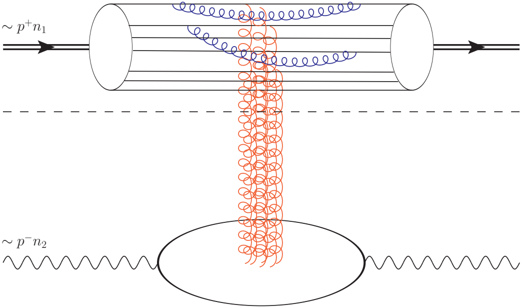
# The Color Glass Condensate




fast partons $\leftrightarrow$ valence partons

slow gluons $\leftrightarrow$ wee gluons

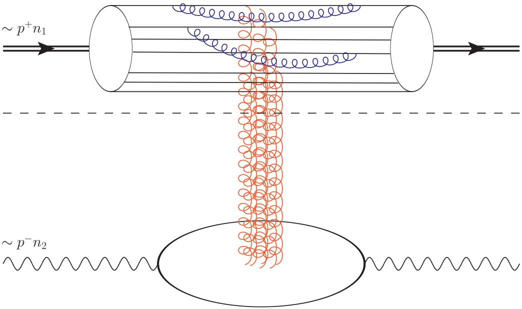
# The Color Glass Condensate




Hadron wave function = collection of static color sources

Color sources $\rho$ are classical random variables, treated with a weight function $W_Y[\rho]$

# The Color Glass Condensate



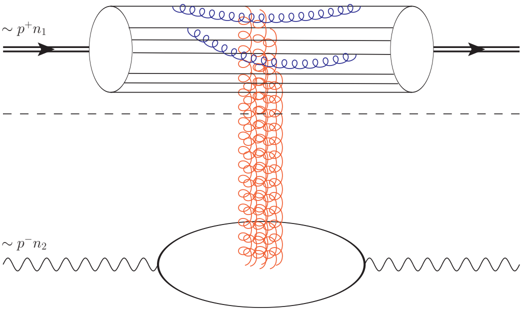

Static source = static current of color charge

$$J_a^\mu = \delta^{\mu +} \rho_a(x)$$

Wee gluons: solutions to the classical Yang-Mills equation with the source

$$[D_\nu, F^{\mu\nu}] = \delta^{\mu +} \rho_a(x) T^a$$

# The Color Glass Condensate




Target matrix elements → averages over configurations of sources and dynamical fields $A^\mu$

$$\frac{\langle P|\mathcal{O}|P\rangle}{\langle P|P\rangle} \to \langle \mathcal{O}\rangle = \int \mathcal{D}\rho\, \mathcal{D}A^\mu\, W[\rho]\, e^{iS[\rho,A]}\, \mathcal{O}[\rho, A]$$

## The MV model

### McLerran-Venugopalan model

- Sources $\simeq$ valence quarks $\Rightarrow$ number of sources $\sim N_c A$
- Transverse radius $R_A \propto A^{1/3}\Lambda_{\mathrm{QCD}}^{-1}$
- Transverse resolution of the probe $1/Q^2$
- Number of sources seens by the probe $\Delta N = \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\frac{N_c A^{1/3}}{\pi}$

If $Q^2 \ll \Lambda_{\mathrm{QCD}}^2 A^{1/3}$, a large number of sources is probed

Random distribution of sources, total color charge probed is 0:

$$\langle \mathcal{Q} \rangle = \int_{1/Q^2} d^2\vec{x} \int dx^- \rho(x^-, \vec{x}) = 0$$

## The MV model

# McLerran-Venugopalan model

- Assume that $\langle \rho_a(x^-, \vec{x}) \rangle = 0$
- Write that
$\langle \rho_a(x^-, \vec{x}) \rho_b(y^-, \vec{y}) \rangle = g_s^2 \delta_{ab} \delta(x^- - y^-) \delta(\vec{x} - \vec{y}) \lambda(x^-)$
- Assume that higher-point functions vanish

Correlators are generated from a Gaussian weight function

$$\Phi[\rho] \propto \exp\left( -\frac{1}{2} \int d^2\vec{x} \, \frac{\rho_a \, \rho_a}{\mu^2} \right), \quad \mu \propto \int dx^- \lambda(x^-)$$

Target matrix elements:

$$\frac{\langle P|\mathcal{O}|P\rangle}{\langle P|P\rangle} \to \frac{\int \mathcal{D}\rho \, \Phi[\rho] \, \mathcal{O}}{\int \mathcal{D}\rho \, \Phi[\rho]}$$

# Beyond the McLerran-Venugopalan model

Possible extensions

- Add a transverse dependence
  $\langle \rho_a(x^-, \vec{x}) \rho_b(y^-, \vec{y}) \rangle = g_s^2 \delta_{ab} \delta(x^- - y^-) \delta(\vec{x} - \vec{y}) \lambda(x^-, \vec{x})$
- Include higher-point functions, or use non-Gaussian weight functions

$$\Phi[\rho] \propto \exp\left(-\frac{1}{2}\int d^2\vec{x}\left[\frac{\rho_a\,\rho_a}{\mu^2} - \frac{d^{abc}\rho_a\rho_b\rho_c}{\kappa}\right]\right)$$

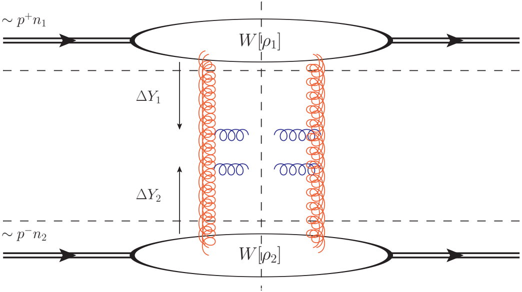
[Jeon, Venugopalan]

$\rho_a\rho_a$: Pomeron term, $d^{abc}\rho_a\rho_b\rho_c$: Odderon term

# Overview

# Hadron-hadron collisions in the CGC

## Collisions of two distributions of color sources



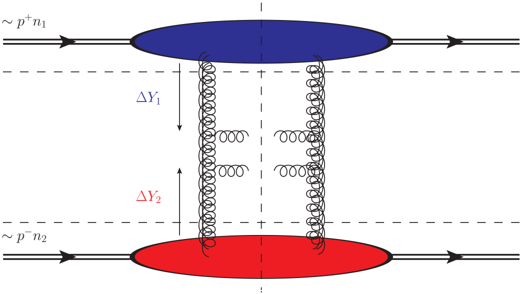

Expectation value of an operator

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\rho_1 \, \mathcal{D}\rho_2 \, W_{Y_1}[\rho_1] \, W_{Y_2}[\rho_2] \, \mathcal{O}[\rho_1, \rho_2]$$

Source terms in both light cone directions $J_1^\mu = \delta^{\mu +} \rho_1$ and $J_2^\nu = \delta^{\nu -} \rho_2 \ldots$

# Hadron-hadron collisions in the CGC

## Two different saturation scales

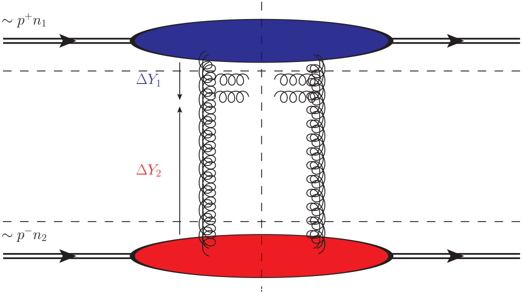



Projectile saturation scale $Q_{s1}^2 \propto (A_1/x_1)^{1/3}$

Target saturation scale $Q_{s2}^2 \propto (A_2/x_2)^{1/3}$

# Hadron-hadron collisions in the CGC

Hybrid factorization ansatz [Dumitru, Hayashigaki, Jalilian-Marian]




At forward rapidities, we can use the CGC to describe the target, while using colinear factorization to describe the projectile.

Allows to study the target with well-understood descriptions of the projectile.

# Loop corrections with the hybrid factorization ansatz

One-loop corrections with saturation effects: state of the art

Evolution

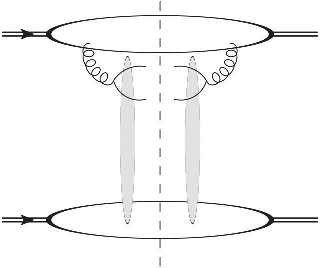
- Dipole evolution [Balitsky, Chirilli]
- 3-point operator evolution [Balitsky, Gerasimov, Grabovsky]
- 4-point operator evolution [Grabovsky]
- Full JIMWLK Hamiltonian [Kovner, Lublinsky, Mulian]

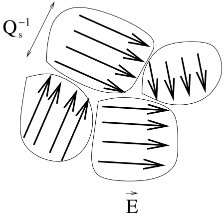
Observable

- Semi-inclusive hadron production in hybrid factorization [Chirilli, Xiao, Yuan], [Altinoluk, Armesto, Beuf, Kovner, Lublinsky]

# Hybrid processes

## Example: domain structure and the correlation limit






Picture from [Kovner, Lublisnky]

A pair of partons from the splitting of a colinear gluon from the projectile probes the target as a dipole of size $r_\perp$.

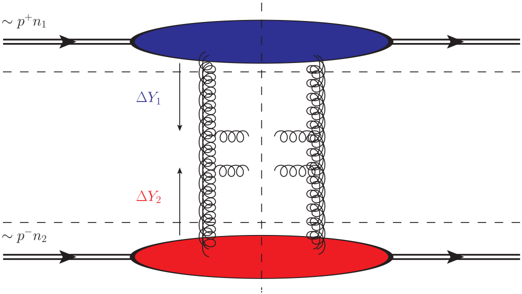
Domain structure: the target contains domains of oriented chromo-electric fields of size $1/Q_s$.

Small dipoles $|r_\perp| \ll 1/Q_s$ will probe a single domain. In momentum space, small dipole = back-to-back dijet.

Thus local correlations in the target lead to momentum correlations in the outgoing state

# Hadron-hadron collisions in the CGC

## Two different saturation scales

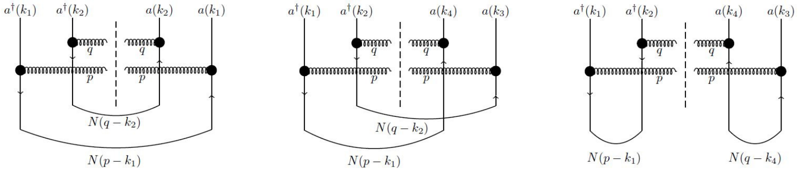



Projectile saturation scale $Q_{s1}^2 \propto (A_1/x_1)^{1/3}$

Target saturation scale $Q_{s2}^2 \propto (A_2/x_2)^{1/3}$

# Glasma graphs

An example of glasma graphs: double inclusive gluon production




Picture from [Altinoluk, Armesto]

- 2 gluons from the projectile: need to compute $\langle AAAA \rangle_{A_1}$. Assumption: each gluon comes from a different color charge density
- Scattering with the dense target via Wilson line operators: adjoint dipoles $N(p-k_1)$ and $N(q-k_2)$.
- Eikonal coupling between the $t$-channel gluons and the measured gluons via Lipatov vertices

## Glasma graphs

### Beyond glasma graphs

- Assumption: each gluon comes from a different color charge density:
  - A single charge could emit a gluon which splits into a gluon pairs [Kovner, Lublinsky, Skokov], [Kovchegov, Skokov]
  - Other assumption to relax: mean field $\langle AAAA\rangle_{A_1} \to \langle AA\rangle_{A_1}\langle AA\rangle_{A_1}$
- adjoint dipoles $N(p-k_1)$ and $N(q-k_2)$.
  - Also a possibility to relax the mean field approximation: $\langle N(p-k_1)N(q-k_2)\rangle_{A_2} \to \langle N(p-k_1)\rangle_{A_2}\langle N(q-k_2)\rangle_{A_2}$
- Eikonal coupling between the $t$-channel gluons and the measured gluons via Lipatov vertices
  - Posibility to include sub-eikonal corrections [Agostini, Altinoluk, Armesto] [Altinoluk, Beuf, Czajka, Tymowska]

# Overview

## Summary

A few typical CGC topics:

- One-loop corrections and precision phenomenology
- Target models beyond MV
- Correlations from the domain structure, from glasma graphs and beyond
- Spin effects in the CGC?
- Odd harmonics in the CGC?