Chiral hydrodynamics of plasma in strong magnetic fields & quantum criticality

HIPSTARS 2020, ZOOM

December 3rd, 2020

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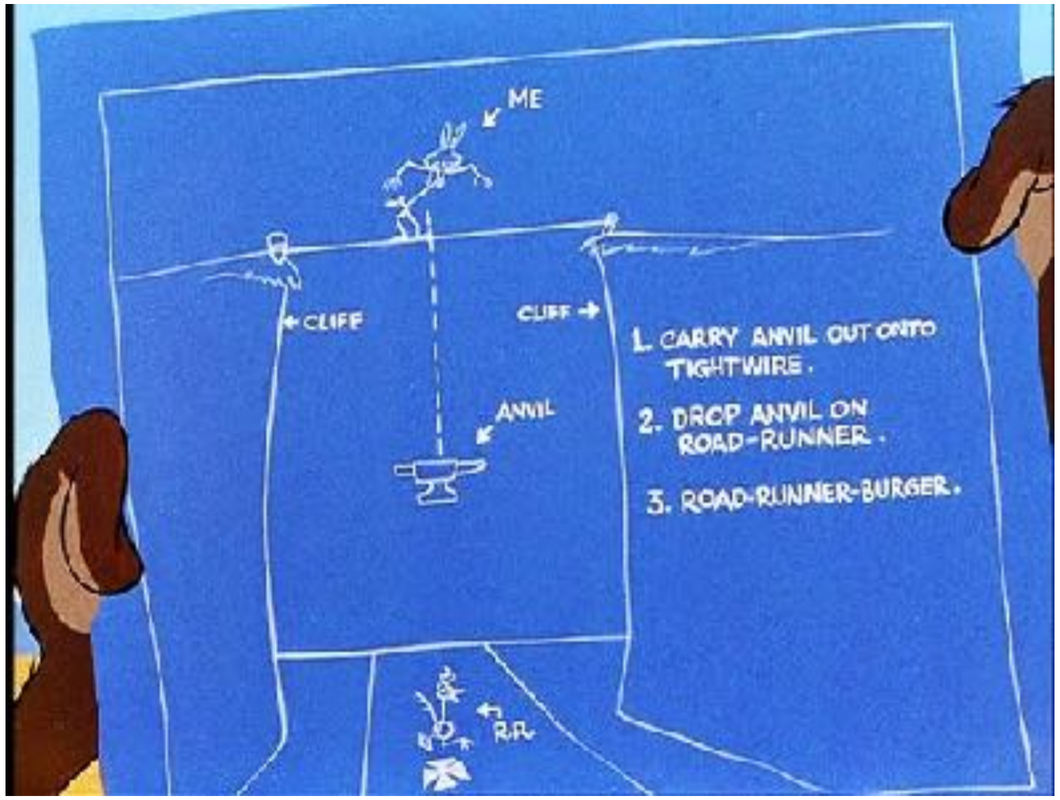
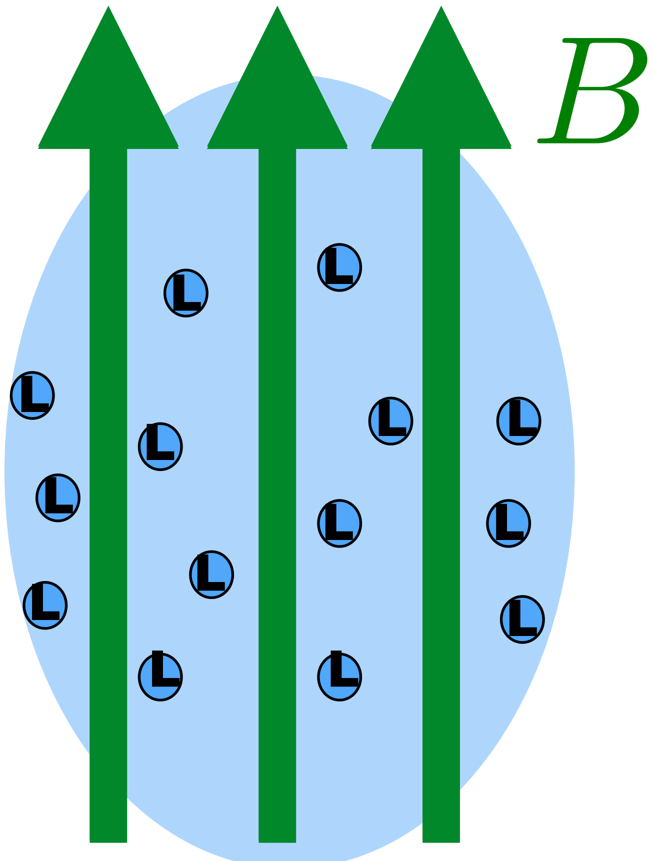
University of Alabama

Outline



1. Hydrodynamics - Methods
2. Hydrodynamic results
   - Kubo formulae
   - novel transport effects
3. Towards a quantum critical point
4. Discussion

Why should I not go and get a coffee?



charged
(3+1)-dimensional
relativistic fluid of
chiral fermions in
magnetic field

Sneak preview of results

5 novel transport effects :

- ✦ 1 perpendicular magnetic vorticity susceptibility
- ✦ 1 shear-induced conductivity
- ✦ 2 expansion-induced conductivities
- ✦ **1 *non-dissipative:***
  shear-induced Hall conductivity

$$j_x \sim c_{10}(\partial_y v_z + \partial_z v_y)$$

2 Hall viscosities & modified Hall physics

Kubo formulae for >20 transport coefficients

Reminder:
shear viscosity

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, dx\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

Novel:
expansion-induced conductivity

$$c_4 \sim \langle [j_z, T_{xx}] \rangle$$

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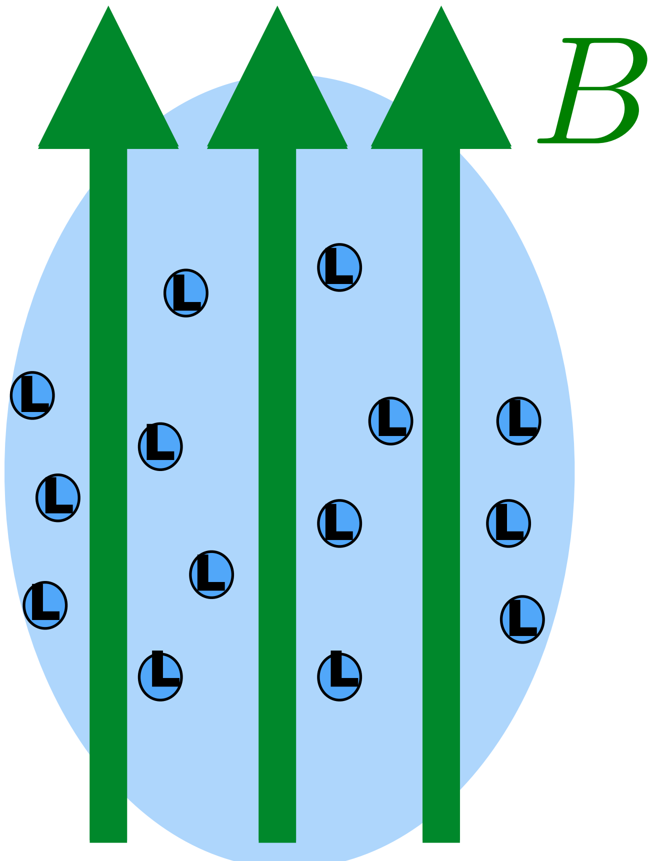


charged (3+1)-dimensional relativistic fluid of chiral fermions in magnetic field

➡Also: I need your expertise.

Sneak preview of results

5 novel transport effects :

✦ 1 perpendicular magnetic vorticity susceptibility
✦ 1 shear-induced conductivity
✦ 2 expansion-induced conductivities
✦ ***1 non-dissipative: shear-induced Hall conductivity***

$$j_x \sim c_{10}(\partial_y v_z + \partial_z v_y)$$

2 Hall viscosities & modified Hall physics

Kubo formulae for >20 transport coefficients

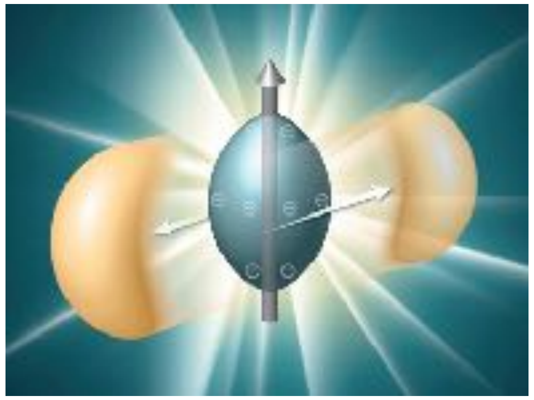
Reminder: shear viscosity $\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, dx\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$

Novel: expansion-induced conductivity $c_4 \sim \langle [j_z, T_{xx}] \rangle$

1. Hydrodynamics - Examples

Quark Gluon Plasma

- strong magnetic field B
- chiral anomaly
- chiral transport effects, e.g. ***chiral magnetic effect / wave***



[Fukushima,Kharzeev,Warringa; PRD (2008)]

[Kharzeev,McLerran,Warringa; Nucl.Phys.A (2008)]

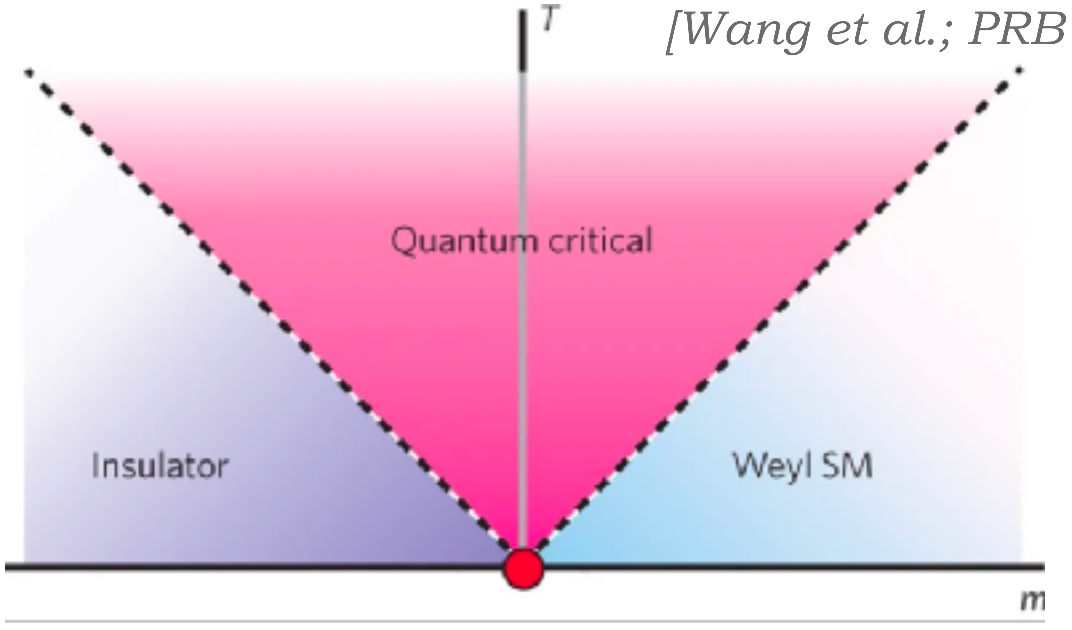
[Kharzeev,Yee; PRD (2011)]

Weyl-semimetals

- relativistic Weyl fermions + B
- chiral anomaly, ***CME / wave***
- quantum critical point in Weyl semimetals & in QGP?

[Yang et al.; Nature (2014)]

[Wang et al.; PRB (2019)]



Which are the relevant quantities in systems near equilibrium, and how can we predict their behavior?

Hydrodynamics

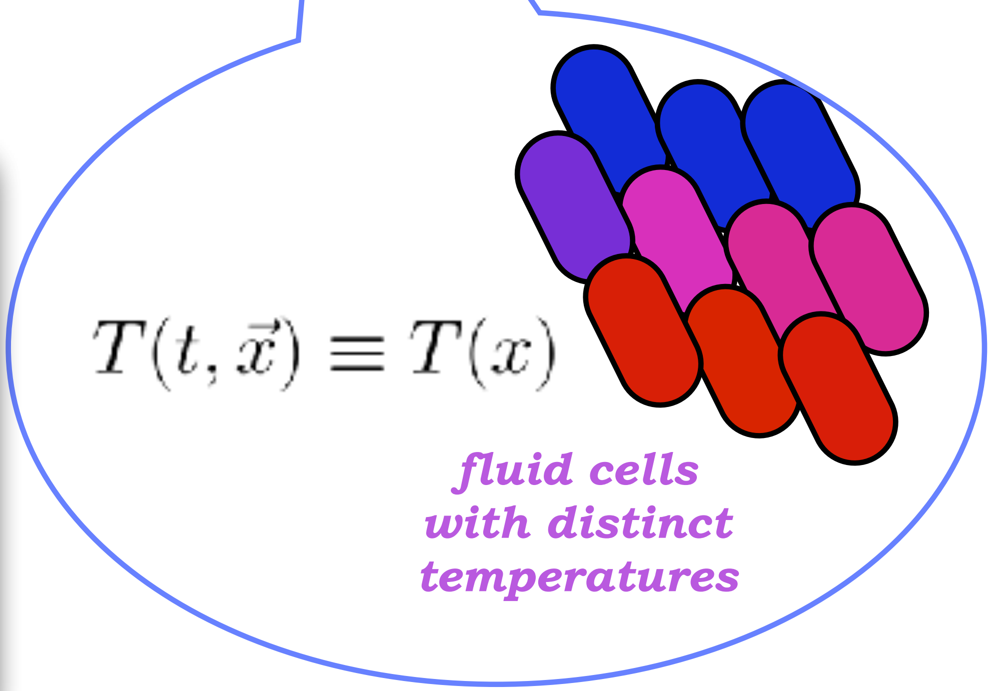
- effective description of systems at late times and large distances
- conserved quantities survive
- small gradients
- large temperature

$$\partial_t e^{-i\omega t} = -i\omega\, e^{-i\omega t}$$

$$\frac{\omega}{T} \ll 1, \qquad \frac{|\vec{k}|}{T} \ll 1$$

In this presentation also :

$$B \sim \mathcal{O}(1) \qquad B \ll T^2$$



1. Hydrodynamics - Formalism

Universal **effective field theory (EFT)**

[Baier, Romatschke, Romatschke, Son, Starinets, Stephan; JHEP (2008)]

- expansion in gradients of fields
- systematic construction
- generating functional
  [Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]
  [JHEP (2011)]
  [Banerjee et al. JHEP (2012)]

[Previously: *[Landau, Lifshitz]* phenomenological]

Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields
- constitutive equations
- conservation equations
- sources
  [Luttinger]

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Hydrodynamic limit $\frac{\omega}{T} \ll 1, \quad \frac{|\vec{k}|}{T} \ll 1$

- fields $T(x)$, $n(x)$, $u^\alpha(x)$

  temperature, *charge density*, *fluid velocity*

- constitutive equations

  $$\langle j^\alpha \rangle = n\, u^\alpha + \nu^\alpha$$

  ideal hydro, *derivative corrections*

- conservation equations

  $\nabla_\alpha \langle j^\alpha \rangle = 0$ *e.g. continuity:* $\partial_t n + \vec{\nabla} \cdot \vec{j} = 0$

- sources

  [Luttinger] $g_{\alpha\beta}(x)$, $A_\alpha(x)$

  metric, *gauge field*

[Ammon, Kaminski et al.; JHEP (2017); to be published (2020)]

[Hernandez, Kovtun; JHEP (2017)]

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} \Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$\begin{aligned}
\mathcal{E}_{\text{eq.}} = & -p + T p_{,T} + \mu p_{,\mu} + \left(T M_{5,T} + \mu M_{5,\mu} - 2 M_5\right) B{\cdot}\Omega \\
& + \left(T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1\right) s_1 \\
& + \left(T M_{2,T} + \mu M_{2,\mu} - M_2\right) s_2 \\
& + \frac{4B^2}{T^4} \left(M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2}\right) s_3 \\
& + \left(T M_{4,T} + \mu M_{4,\mu} + \frac{4B^2}{T^4} M_{1,\mu} + M_{3,\mu}\right) s_4 \,, \qquad (2.10\text{a})
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\text{eq.}} = & \; p - \tfrac{4}{3} p_{,B^2} B^2 - \tfrac{1}{3} (M_5 + 4 M_{5,B^2} B^2) B{\cdot}\Omega - \tfrac{2}{3} \left(M_2 + 2B^2 M_{2,B^2}\right) s_2 \\
& + \frac{4B^2}{3T^4} \left(M_1 - T M_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2}\right) s_3 \\
& + \frac{4B^2}{3T^4} \left(M_{1,\mu} - T^4 M_{4,B^2}\right) s_4 \,, \qquad (2.10\text{b})
\end{aligned}$$

$$\begin{aligned}
\mathcal{Q}^\mu_{\text{eq.}} = & -M_5 \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\sigma B_\rho + (2M_5 - T M_{5,T} - \mu M_{5,\mu}) \epsilon^{\mu\nu\rho\sigma} u_\nu B_\rho \partial_\sigma T / T \\
& - M_{5,B^2} \epsilon^{\mu\nu\rho\sigma} u_\nu B_\rho \partial_\sigma B^2 + (M_{5,\mu} - 2 p_{,B^2}) \epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma \qquad (2.10\text{c})
\end{aligned}$$

$$\begin{aligned}
\mathcal{T}^{\mu\nu}_{\text{eq.}} = & \; 2 p_{,B^2} \left(B^\mu B^\nu - \tfrac{1}{3} \Delta^{\mu\nu} B^2\right) + B^{\langle\mu} B^{\nu\rangle} \left(M_{5,B^2} B{\cdot}\Omega + M_{2,B^2} s_2 + (M_{4,B^2} - \tfrac{1}{T^4} M_{1,\mu}) s_4\right) \\
& + B^{\langle\mu} B^{\nu\rangle} \frac{1}{T^4} \left(T M_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} - M_1 + T^4 M_{3,B^2}\right) s_3 + M_5 B^{\langle\mu} \Omega^{\nu\rangle} \\
& + 2 M_2 B^{\langle\mu} \epsilon^{\nu\rangle\rho\sigma\alpha} u_\rho \partial_\sigma B_\alpha + \left(T M_{2,T} + \mu M_{2,\mu} - M_2\right) B^{\langle\mu} \epsilon^{\nu\rangle\alpha\rho\sigma} u_\alpha B_\rho \partial_\sigma T / T \\
& + M_{2,B^2} B^{\langle\mu} \epsilon^{\nu\rangle\alpha\rho\sigma} u_\alpha B_\rho \partial_\sigma B^2 - M_{2,\mu} B^{\langle\mu} \epsilon^{\nu\rangle\rho\sigma\alpha} u_\rho E_\sigma B_\alpha \,, \qquad (2.10\text{d})
\end{aligned}$$

1. Hydrodynamics - Constitutive relations

[Ammon, Kaminski et al.; JHEP (2017); to be published (2020)]

[Hernandez, Kovtun; JHEP (2017)]

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} \Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

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➡ **Complicated because of broken symmetries:**

- **~~Chiral symmetry~~ — microscopic chiral anomaly**
- **~~Parity~~ — axial chemical potential**
- **~~Time reversal~~ + ~~Spatial rotation symmetry~~ — strong magnetic field** $B \sim \mathcal{O}(1)$

➡ **Many novel transport effects**

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➡ Hydrodynamic 2-point functions

[Ammon,Kaminski et al.; JHEP (2017); to be published (2020)]

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➡ Many novel transport effects

2. Hydrodynamics - Kubo formulae

Two types :

1.) Thermodynamic transport coefficients

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{xz}T^{yz}}(\omega=0, k_z\hat{\mathbf{z}}) = -2\, B_0^2 \boxed{M_2}$$

novel: perpendicular magnetic vorticity susceptibility

+ *3 novel thermodynamic transport coefficients* M_1, M_3, M_4

+ *3 chiral conductivities (CME / CVE / CTE) in equilibrium,*
$\propto$ *chiral anomaly,*
measured negative magnetoresistance in 3D Weyl semimetals

+ ...

e.g. [Huang et al; PRX (2015)]

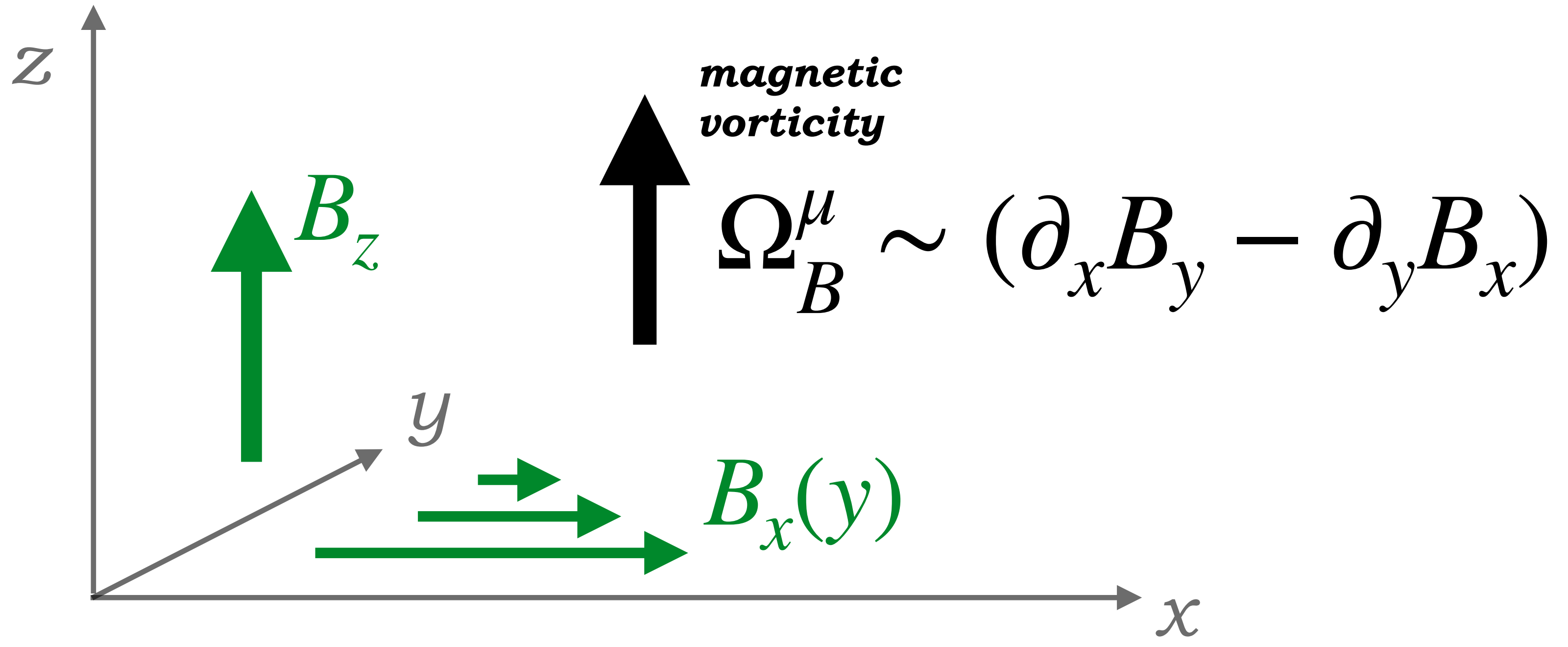
2.) Hydrodynamic transport coefficients

— boring example : $\frac{1}{\omega}\mathrm{Im}\, G_{J^zJ^z}(\omega, \mathbf{k}{=}0) = \sigma_{||} + \cdots$

parallel charge conductivity

+ *3 novel hydrodynamic transport coefficients*

Perpendicular magnetic vorticity susceptibility M_2



$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_2 B\cdot\Omega_B$$
$$\sim -M_2 B_z \partial_y B_x(y)$$

magnetic vorticity :

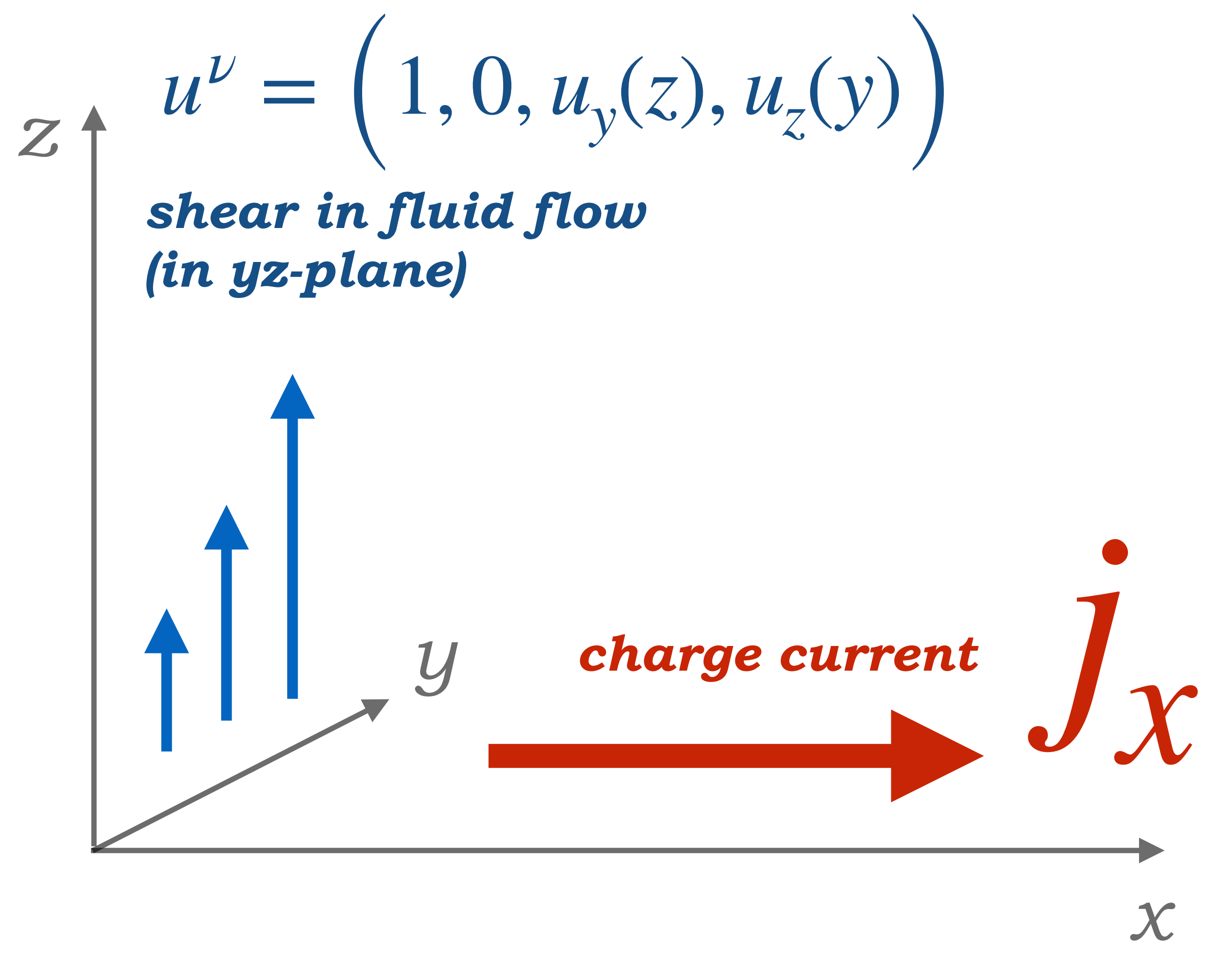
$$\Omega_B^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho B_\sigma$$

$$u^\nu = (1,0,0,0) + \mathcal{O}(\partial)$$

Recall tensor structure in W_s :

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

Shear-induced Hall conductivity c_{10}



$$j_x \sim c_{10}(\partial_y u_z + \partial_z u_y)$$

$$c_{10} \sim \frac{1}{\omega} \mathrm{Im}\, G_{T^{tx}\,T^{yz}}$$

- ➡ **novel Hall response**
- ➡ **non-dissipative**
- ➡ **interplay: shear-charge**

2. Hydrodynamics - holographic model

➡ **use as holographic dual to charged state in strong *B***

➡ **values for transport coefficients in *N*=4 Super-Yang-Mills**

[Ammon, Kaminski et al.; JHEP (2017); to be published (2020)]

Einstein-Maxwell-Chern-Simons action

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Chern-Simons term encodes chiral anomaly

Charged magnetic black branes

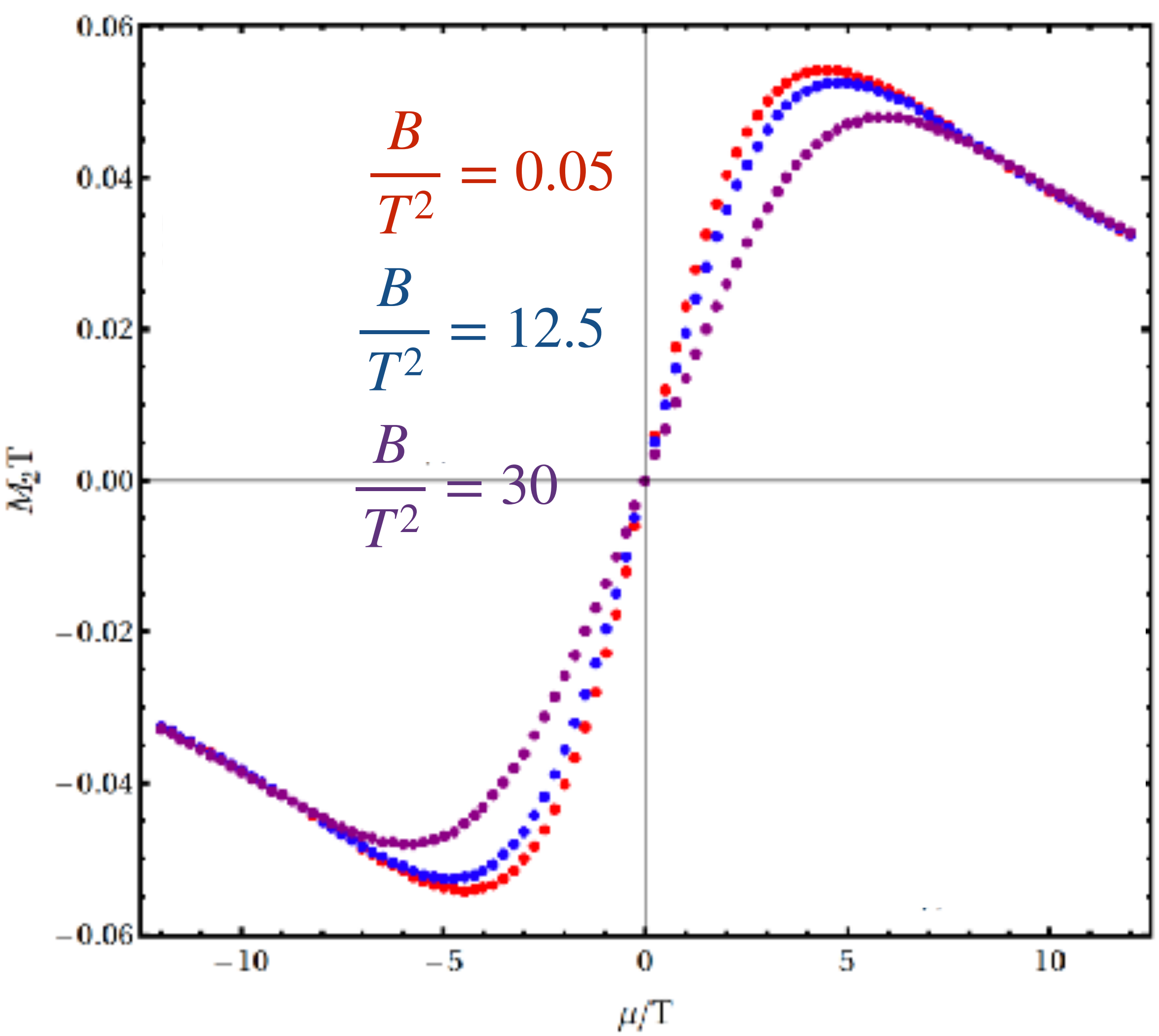
[D'Hoker, Kraus; JHEP (2010)]

- charged magnetic analog of Reissner-Nordstrom black brane
- asymptotically AdS_5

2. Hydrodynamics - holographic model

[Ammon, Kaminski et al.; to be published (2020)]

Perpendicular magnetic vorticity susceptibility M_2



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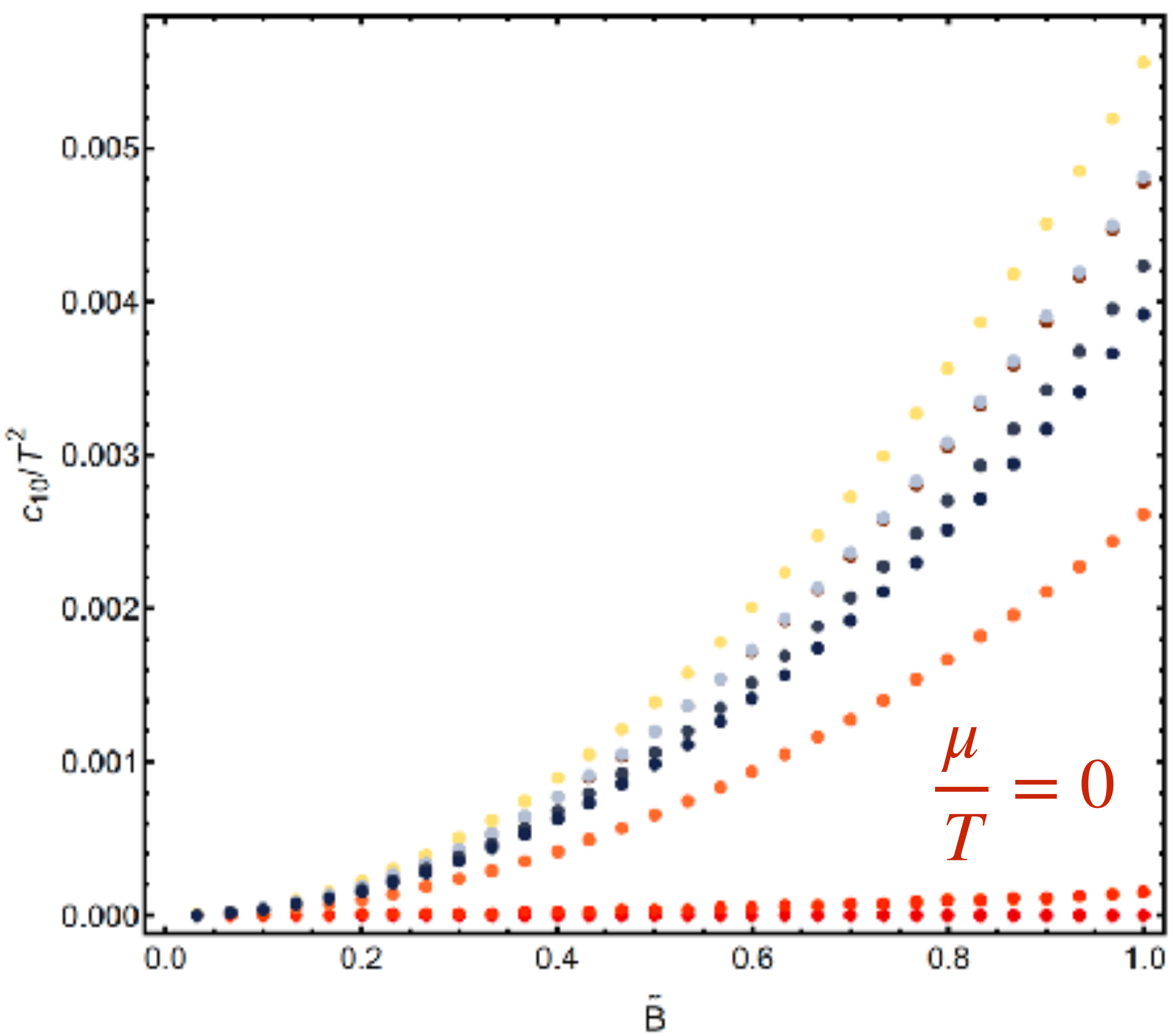

➡ **not zero, finite, Onsager satisfied**
➡ **Kubo formulae reasonable**

RECALL: 1. Hydrodynamics - Examples

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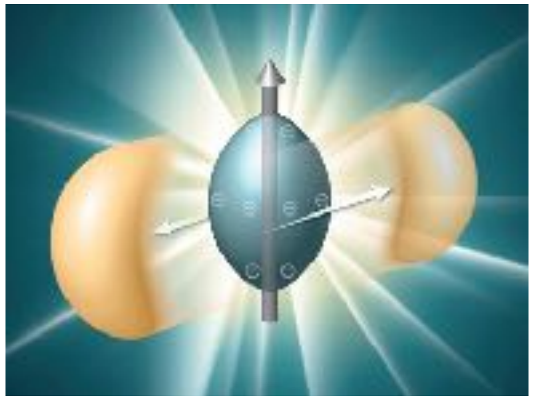

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- quantum critical point in Weyl semimetals & in QGP?

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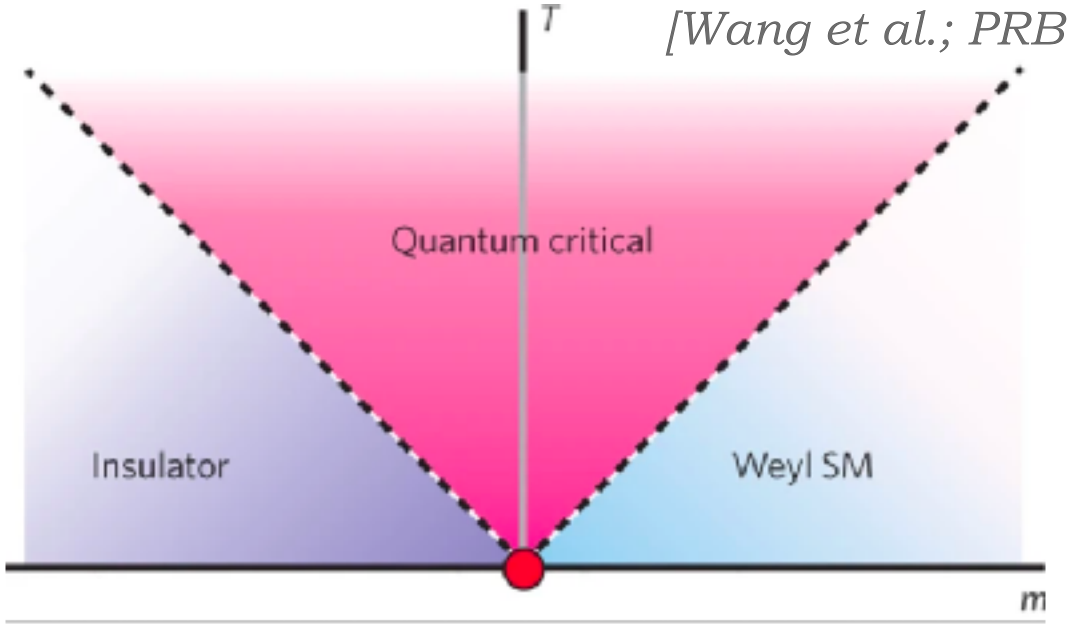


quantum critical point

magnetization

3. Towards a QCP - holo discovery tool

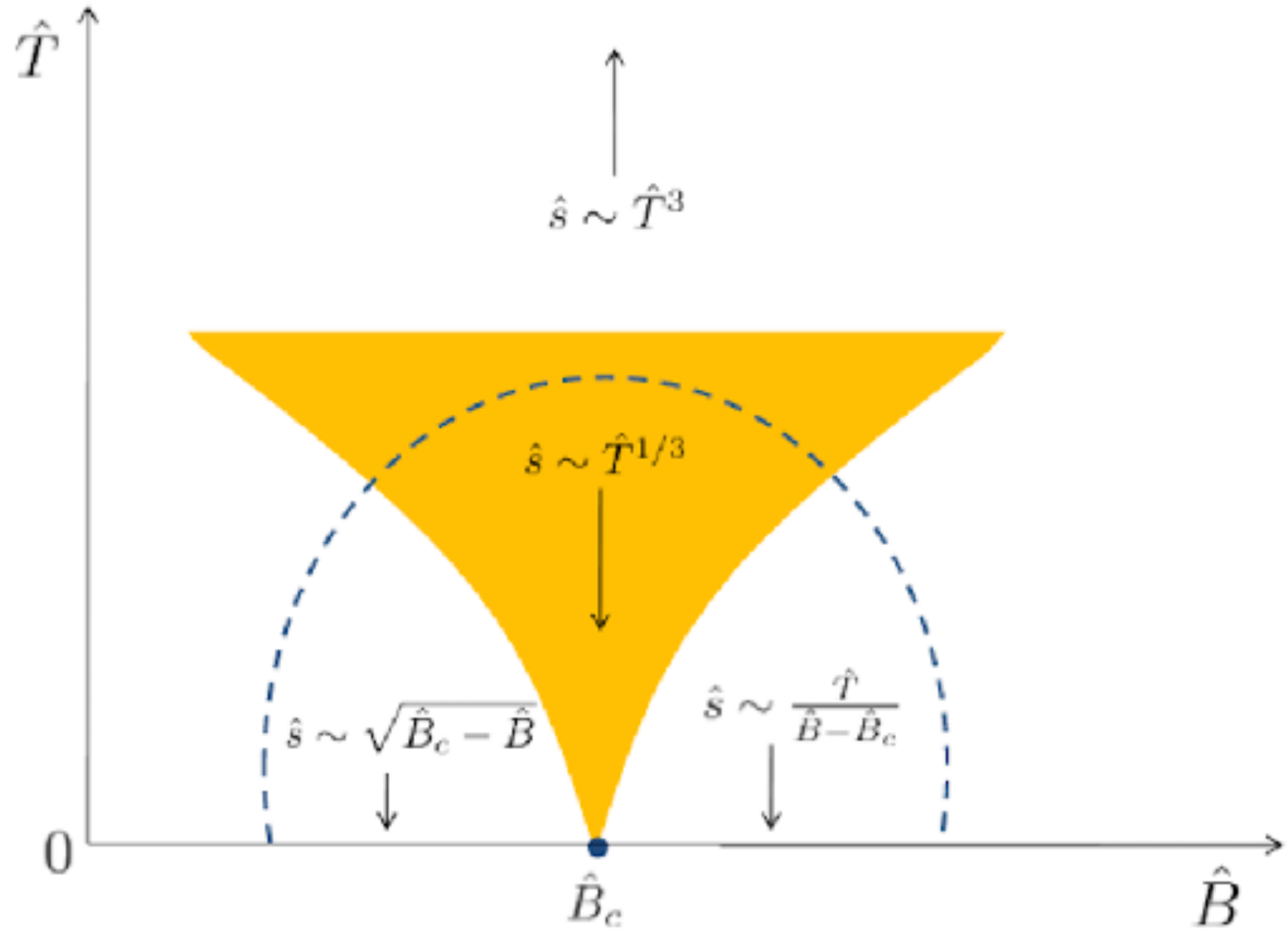
➡ **our holographic model has quantum critical point**

➡ **quantum critical effects: entanglement entropy $S_\perp$**

[Cartwright, Ingram, Kaminski; to be published (2020)]

Einstein-Maxwell-Chern-Simons charged magnetic black branes

[D'Hoker, Kraus; JHEP (2010)]



3. Discussion - Summary

Hydrodynamics

- (3+1)D hydrodynamics: charged chiral fluids in strong *B*
- 5 novel hydro transport coefficients (+3 thermo) at leading and sub-leading order in the hydrodynamic expansion
- as important as shear viscosity and charge conductivity
- Kubo formulae for 25 transport coefficients

Quantum criticality

entanglement entropy across quantum phase transition

Hydrodynamics

- far from equilibrium

[Romatschke; PRL (2018)]
[Cartwright, Kaminski; JHEP (2019)]
[Wondrak, Kaminski, Bleicher; PRB (2020)]

- quantum chaos

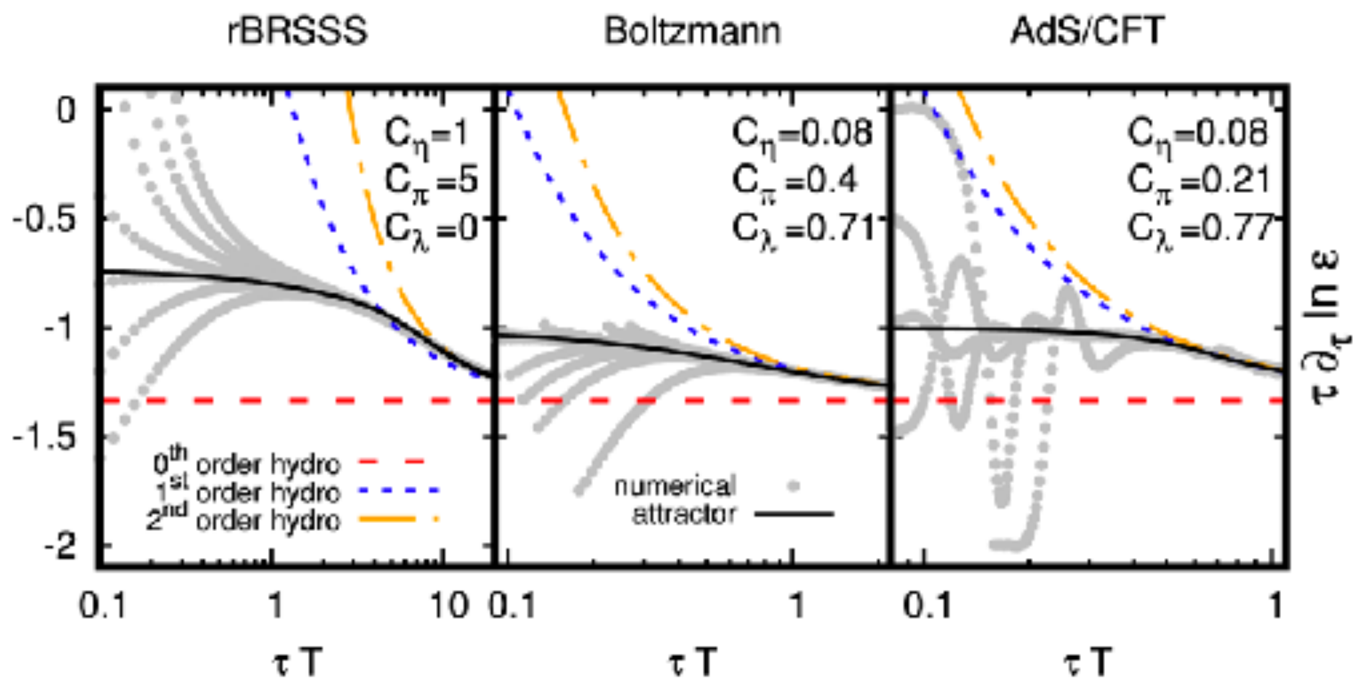
[Blake, Lee, Liu; JHEP (2018)]
[Grozdanov et al. (2019)]

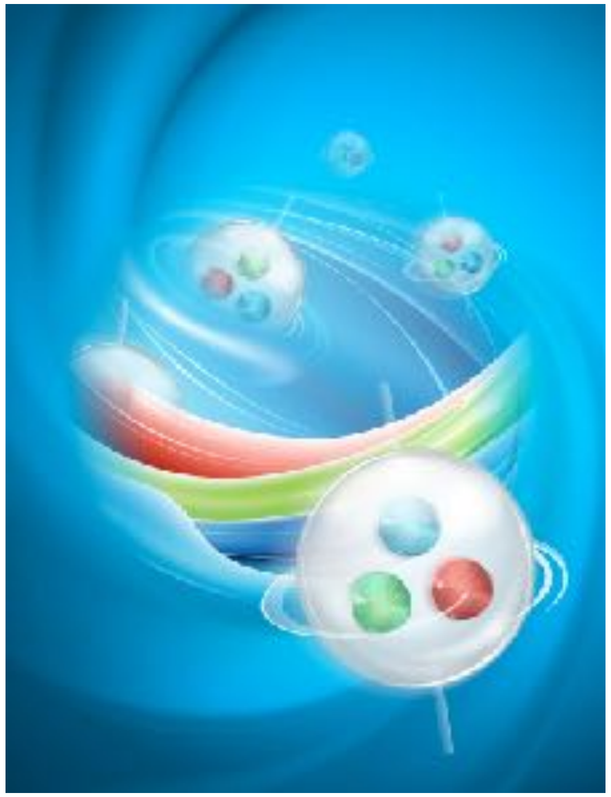
- convergence & stability

[Kovtun; JHEP (2019)]
[Grozdanov, Kovtun, Starinets, Tadic; PRL (2019)]
[Withers; JHEP (2018)]
[Heller, Janik, Witaszczyk; PRL (2013)]
[Heller, Spalinski; PRL (2018)]

- most vortical fluid

[Garbiso, Kaminski; JHEP (2019)]





[STAR; Nature (2017)]

Collaborators on these projects

Perimeter, Canada
Juan Hernandez

Thank you for listening!

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Dr. Roshan Koirala


Markus Garbiso


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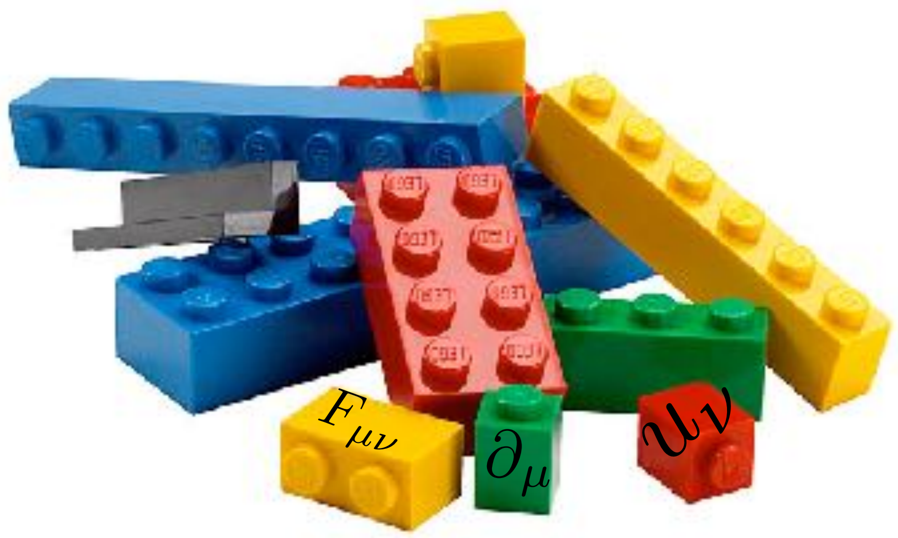
Jana Ingram


Casey Cartwright

APPENDIX

1. Hydrodynamics - Construction

1. Constitutive equations: all (pseudo)vectors and (pseudo)tensors under Lorentz group



Examples: $\nabla_\nu u^\nu$

$$\underset{\textit{vorticity}}{\omega^\mu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\nabla_\lambda u_\rho$$

2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu(n u^\mu) = 0$

3. **"Old school":** Further restricted by positivity of local entropy production: $\nabla_\mu J^\mu_s \geq 0$ *[Landau, Lifshitz]*

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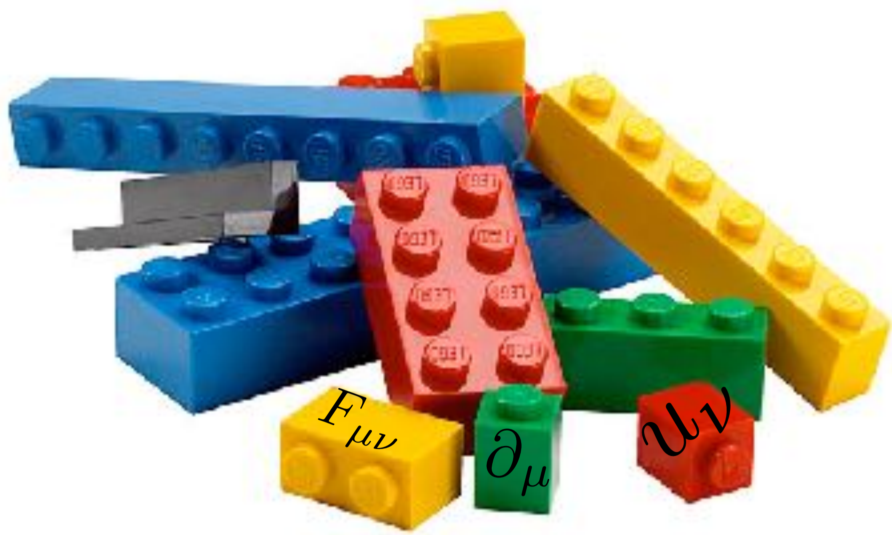

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4. **Modern:** Construct generating functional, use field theory restrictions (Onsager relations, analyticity, Ward identities) *[Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)]* *[JHEP (2011)]* *[Banerjee et al. JHEP (2012)]*

➡ **Most general hydrodynamic 1-point functions**

[Ammon, Kaminski et al.; JHEP (2017); to be published (2020)]

➡ Hydrodynamic 2-point functions

[Ammon,Kaminski et al.; JHEP (2017); to be published (2020)]

Variation of the …

… constitutive relations (1-point functions)

$$\boxed{G^R_{T^{\mu\nu},J^\alpha}} = \frac{\delta}{\delta A_\alpha} T^{\mu\nu}_{\text{on-shell}}[A,g]$$

Recall Onsager relations : $G^R_{TJ} = \eta_T \eta_J G^R_{JT}$

… equilibrium generating functional

$$\delta W_s[A,g] = \int d^4x \sqrt{-g} \left(\tfrac{1}{2} \boxed{T^{\mu\nu}_{\text{eq.}}} \underline{\delta g_{\mu\nu}} + \boxed{J^\mu_{\text{eq}}} \underline{\delta A_\mu} \right)$$

sources

$$W_s = \int d^4x \sqrt{-g} \left(p(T,\mu,B^2) + \sum_{n=1}^{5} \underline{M_n}(T,\mu,B^2) s_n + O(\partial^2) \right)$$

5 thermodynamic transport coefficients

$$s_2 = \epsilon^{\mu\nu\rho\sigma} u_\mu B_\nu \nabla_\rho B_\sigma$$

Shear-induced conductivity c_8

$$j_x \sim c_8(\partial_x u_z + \partial_z u_x)$$

Expansion-induced conductivities c_4 and c_5

$$j^\mu \sim \hat{b}^\mu(c_4 \nabla \cdot u + c_5 \hat{b}^\alpha \hat{b}^\beta \partial_\alpha u_\beta)$$

➡Fluid flow gradients create charge currents

More thermodynamic transport coefficients

Magneto-thermal susceptibility M_1 **:**

$$\mathcal{E}_{\rm eq} \sim M_1 B^\mu \partial_\mu \left(\frac{B^2}{T^4}\right)$$

Magneto-acceleration susceptibility M_3 **:**

$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_{3,B^2}\, B\cdot a$$

Magneto-electric susceptibility M_4 **:**

$$\mathcal{E}_{\rm eq} \sim M_{4,T}\, B\cdot E\,, \qquad \mathcal{P}_{\rm eq} \sim M_{4,B^2}\, B\cdot E$$

Magneto-vortical susceptibility M_5 **:**

$$\mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_5 B\cdot\Omega$$